Covariant Non-Commutative Geometry From String Theory

Jonathan J. Heckman

Institute for Advanced Study

hep-th/1208.???? w/ H. Verlinde

Before I Start...

Since today is “string pheno day”
I will mention a prediction from a class of
F-theory models: (Bouchard JJH Seo Vafa ’09)

min F-GUT 2009: $\theta_{13}^{\nu} \sim \theta_C \sim \sqrt{\alpha_{GUT}} \sim 0.2$
(order of magnitude estimate)

Daya Bay Expt. 2012: $\theta_{13}^{\nu} \sim 0.15$

Before I Start...

Since today is "string pheno day"
I will mention a prediction from a class of
F-theory models: (Bouchard JJH Seo Vafa '09)

Mechanism involves fluxes / instantons
/ non-commutativity in internal directions

(c.f. JJH Vafa '08; Bouchard JJH Seo Vafa '09; Cecotti Cheng JJH Vafa '09; Marchesano Martucci '09; ...)

Covariant Non-Commutative Geometry From String Theory

Jonathan J. Heckman

Institute for Advanced Study

hep-th/1208.???? w/ H. Verlinde

4D Non-Commutativity

(Throughout work in Euclidean signature)

An old idea for regulating QFT:

$$[x_\mu, x_\nu] = i\theta_{\mu\nu}$$

Minimal Resolution: $\Delta(x_\mu)\Delta(x_\nu) \geq \ell_\theta^2$

c.f. Heisenberg, Moyal, von Neumann, Snyder, ...

String Realizations

(see also L. Landau)

E.g.: Zero Slope Limit of Strings in a B-field

(c.f. Connes, Douglas, Schwarz; Nekrasov Schwarz;
Seiberg Witten; V. Shomerus,...)

E.g.: OSFT, M(atrix) Theory, ...

But...

$[x, x] \neq 0$ comes at a (steep) price:

Lorentz invariance is lost!

Why? We introduced: $\langle \theta_{\mu\nu} \rangle \neq 0$

Covariant N.C.

Consider family of $\theta_{\mu\nu}$'s in orbit of $so(4)$:

$$[x_\mu, x_\nu]_{n_j} = i\theta_{\mu\nu}(n_j)$$

Although a given $\theta_{\mu\nu}$ breaks $so(4)$,
Note: $\langle\theta_{\mu\nu}(n_j)\rangle_{\text{orbit}} = 0$

Main Example

$[x_\mu, x_\nu] = i\ell_{NC}^2 \; n_j \cdot \eta_{\mu\nu}^j$

Where: $\sum_{j=1}^3 (n_j)^2 = 1$ (i.e. an S^2)

$\eta_{\mu\nu}^j$ = three 't Hooft matrices (note $\eta^j = *\eta^j$)

Aims / Goals

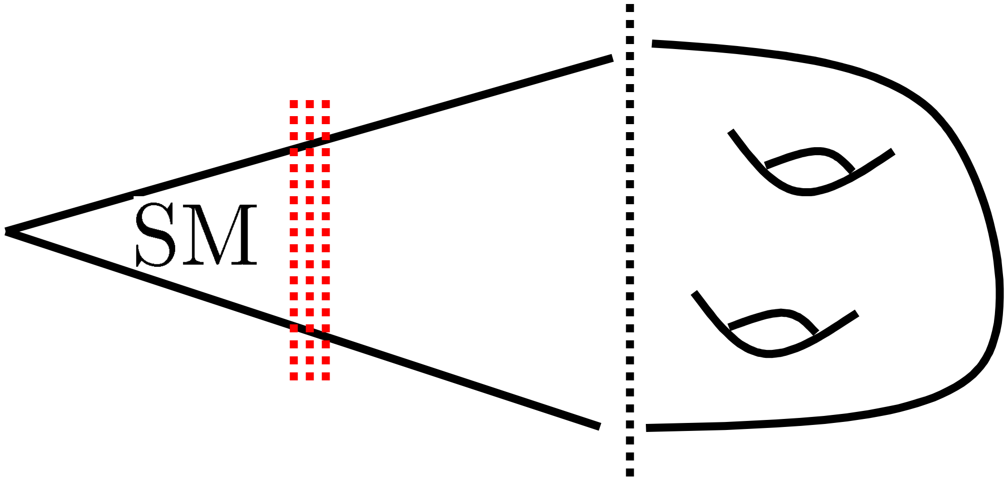
- Find a stringy realization of covariant N.-C.
- Study effects of covariant regulators for QFT
- An extra reason to study:
  appearance of 4D Gravity

A Future Goal

Use this to simplify local model building:

I) “Easy Step” Decouple gravity, build a QFT

II) “Hard Step” Recouple to 4D gravity



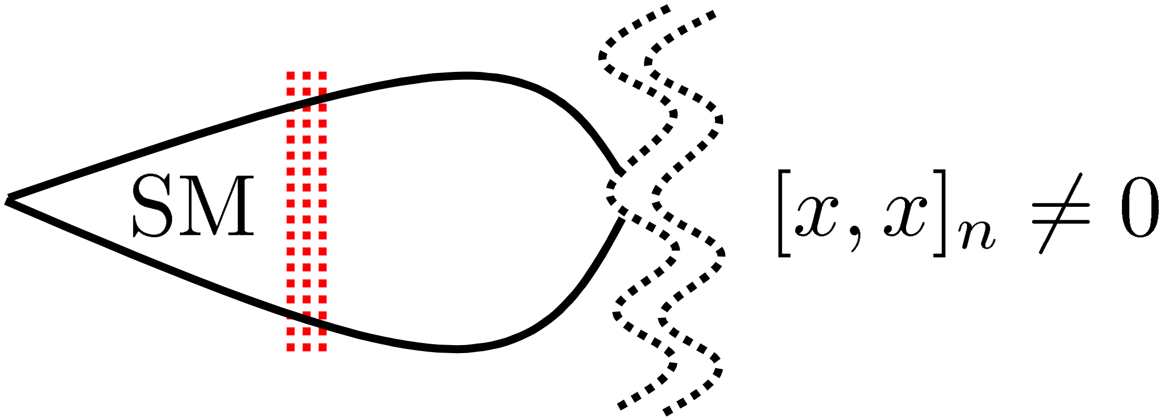
Does $\mathcal{M}_6$ need to be geometric?

A Future Goal

Use this to simplify local model building:

I) “Easy Step” Decouple gravity, build a QFT

II) “Hard Step” Recouple to 4D gravity



Outline

- Worldsheet Realization

- Holographic Dual & Dilaton Compactification

- Summary & Future Directions

Worldsheet Realization

Review: Abelian N.-C.

Throughout hold $2\pi\alpha'$ fixed

Consider N D3-Branes on $\mathbb{R}^4$ with $B_{NS} \neq 0$

Gauge Equiv: Switch on $U(1)_{D3}$ flux

$$G^{(cl)}_{\mu\nu} \to \epsilon^{1/2} \text{ and } g^{(cl)}_s \to \epsilon \text{ and } G^{(op)}_{\mu\nu} \to \epsilon^{-1/2}$$

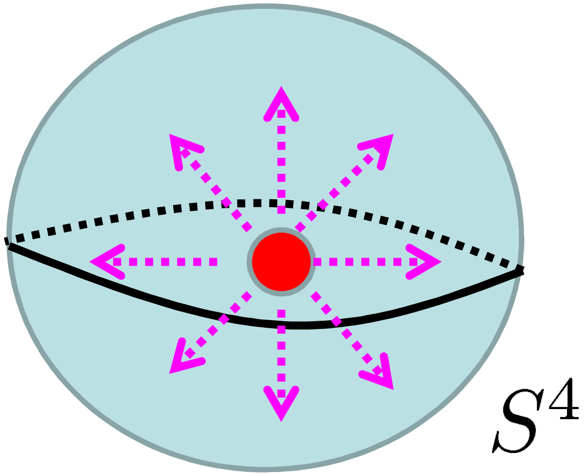
c.f. Seiberg, Witten '99

Fuzziness: $[x_\mu, x_\nu] = i \left[\frac{1}{B_{NS}}\right]_{\mu\nu}$

Non-Abelian Generalization

Open string in a non-abelian flux?

Main Focus: Flux = Yang monopole bkgnd:



On S^4 looks like a homog. instanton of $SU(2)$ YM

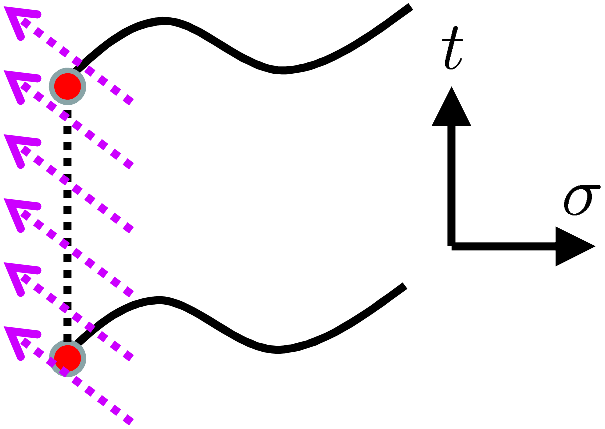
Endpoint Dynamics

Embed $SU(2)_{Yang} \rightarrow U(N)_{Chan-Paton}$ via:

$$J_z^{SU(2)} = \mathrm{diag}(+\tfrac{N-1}{2}, ..., -\tfrac{N-1}{2})$$

$\Rightarrow$ Operator Insertion:

$$\mathrm{Tr}_N P \exp i \int_{\partial\Sigma} A_{flux}$$



Or Equivalently,

Introduce a triplet of endpoint modes $n_i(t)$:

$$S = \frac{1}{4\pi\alpha'}\int_\Sigma G^{(cl)}_{\mu\nu}\partial^a X^\mu \partial_a X_\nu$$

$$-\frac{i}{2}\int_{\partial\Sigma}\left(n_i B^i_{\mu\nu} X^\mu \partial_t X^\nu + (N-1)\frac{\epsilon_{3ij} n_i \partial_t n_j}{1+n_3}\right)$$

This Term Enforces $su(2)$ algebra

Seiberg-Witten Limit?

This is the same as in the abelian flux case

But now we have endpoint modes n_i

Commutator Algebra

c.f. 4D Quant. Hall Effect Hu, Zhang '01; Bernevig et al. '02, Fabinger '02

$$[n_i, n_j] = \tfrac{2}{N-1}\epsilon_{ijk} n_k$$ (i.e. $su(2)$, rescale n's)

$$[x_\mu, n_i] = f_{ik}(n_j)\eta^k_{\mu\nu} x^\nu$$ (i.e. a Lor. transfrm)

$$[x_\mu, x_\nu] = i\ell^2_{NC}\ n_i \eta^i_{\mu\nu} + \text{subleading}$$

Geometry: $4 + 2_{top}$

As $N \to \infty$, open string sees

space of unit norm ASD 2-forms over S^4:

$$\begin{tikzcd} S^2 \arrow[r, dotted] & \mathbb{CP}^3 \arrow[d, dotted] \\ & S^4 \end{tikzcd}$$

i.e., it sees a $\mathbb{CP}^3$

$SO(4)$ "Breaking"

For each point $n_i \in S^2$, have a cplx structure on tangent space $T_x S^4 \simeq \mathbb{R}^4 \simeq \mathbb{C}^2$

$SO(4)$ "broken" to $U(2)$

coset space $= SO(4)/U(2) \simeq \mathbb{CP}^1_{top}$

What Happened to $SO(4)$?

Extend $Y_I = X_\mu \oplus n_i$ into bulk

this is accomplished via Poisson σ-model (c.f. Cattaneo Felder '99)

Parameterize as: $Z^\alpha = (iX^{\dot{a}a}\pi_a, \pi_a)$

$\mathbb{CP}^3$ coord

$\mathbb{CP}^1_{top}$ coord

BRST cohomology: $f(X) = \oint_\gamma f(Z^\alpha)\pi^a d\pi_a$

What Happened to SUSY?

This has also been preserved:

Parameterize as: $(Z^\alpha, \psi^i) = (iX^{\dot{a}a}\pi_a, \pi_a, \theta^{ia}\pi_a)$

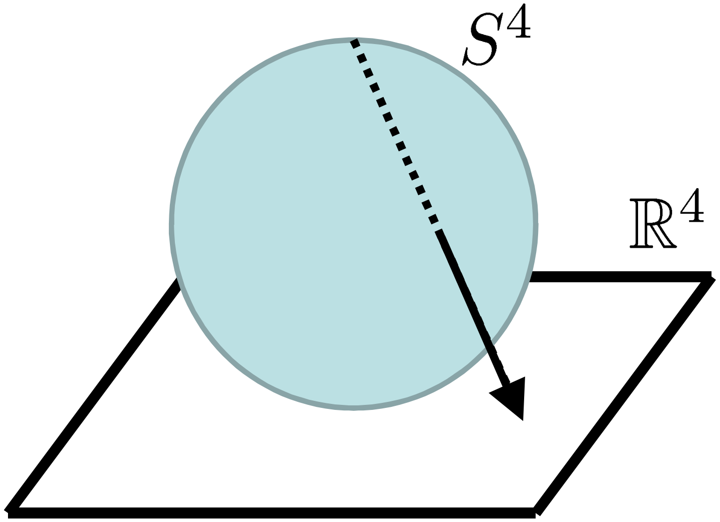
$\mathbb{CP}^{3|4}$ coord

$\mathbb{CP}^1_{top}$ coord

What Happened to P_μ?

This has also been preserved:

Contraction: $so(5) \rightarrow so(4) \rtimes$ translations



Chan-Paton Twistors

Most of these steps well-known from twistors

c.f. Penrose '67

$\mathbb{CP}^3$ = twistor space

The map $f(Z^\alpha) \to f(X)$ is the

c.f. CAT scans

"Penrose-Radon transform"

Fuzzy Gauge Theory

Chan-Paton Indices: $N \to N \otimes N_c$

Flux in $U(N)$ factor as before

Looks like a triplet of B-fields to $U(N_c)$ factor

Adiabatic Limit

Each $n_i \in \mathbb{CP}^1_{top}$ specifies a N.C. Gauge Theory:

Gauge Field: $\widehat{A}(x, n_i)$

Field Strength: $\widehat{F} = d\widehat{A} + \widehat{A} *_n \widehat{A}$

$$S[n^i] = -\frac{1}{2g^2_{YM}} \int d^4x \mathrm{Tr} \widehat{F}_{\mu\nu} *_n \widehat{F}^{\mu\nu}$$

Recap So Far

Summarizing, we have open string realization of:

$\mathcal{N} = 4$ SYM + covariant non-commutativity

Holographic Dual

&

Dilaton Compactification

’t Hooft Limit?

$U(N_c)$ $\mathcal{N} = 4$ SYM $+ \theta_{\mu\nu}(n_i)$ looks sensible

$\Rightarrow$ Should expect closed string dual description

General Strategy

$$\text{IR} \xrightarrow{\text{Energy Scale} \equiv u} \text{UV}$$

Build up dual from IR to UV, i.e. “Bottom Up”

General Strategy

$$\text{IR} \xrightarrow{\text{Energy Scale} \equiv u} \text{UV}$$

Build up dual from IR to UV, i.e. “Bottom Up”

In Deep IR, we have $\mathcal{N} = 4$ SYM
with instanton and $\overline{\text{instanton}}$ chem potls.

So, for small u, we have $AdS_5 \times S^5$

IR Region

In this regime, a family of IIB SUGRA solutions

Each one is specified by $n_i \in S^2$

i.e. a choice of self-dual (on $\mathbb{R}^4$) B_{NS} and B_{RR}

“Averaging” reflected in $\nabla^2 e^{\phi} = \rho_{inst} = -|H_{NS}|^2$

SUGRA Solutions

c.f. Hashimoto, Itzhaki '99; Maldacena, Russo '99; Das, Rama, Trivedi '99

JJH Verlinde '12

$$ds^2_{str} = e^{\phi/2} L^2_{AdS} \left[\sqrt{\tfrac{u^4}{1+a^4u^4}} ds^2_{\mathbb{R}^4} + \sqrt{\tfrac{1+a^4u^4}{u^4}} (du^2 + u^2 d\Omega^2_{(5)}) \right]$$

$$e^{\phi} = g_{IR} \left(\tfrac{1-b^4u^4}{1+a^4u^4} \right) \text{ + RR and NS Fluxes}$$

IR: $u \ll a^{-1}$, looks like AdS_5

UV: $u \simeq b^{-1} = a^{-1} \left(\frac{2\tau_+}{\tau_- - \tau_+} \right)^{1/4}$

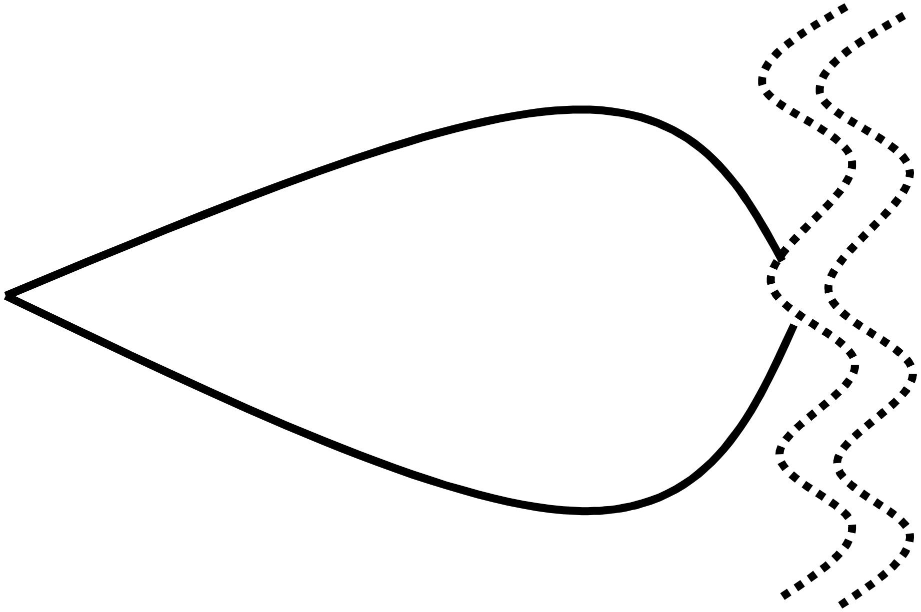
IR chemical potentials

τ_+ for instantons

τ_- for $\overline{\text{instantons}}$

Dilaton Compactification

Note: String frame metric → 0 at $u \leq \infty$:



Boundary exists provided $\tau_+ - \tau_- \leq 0$

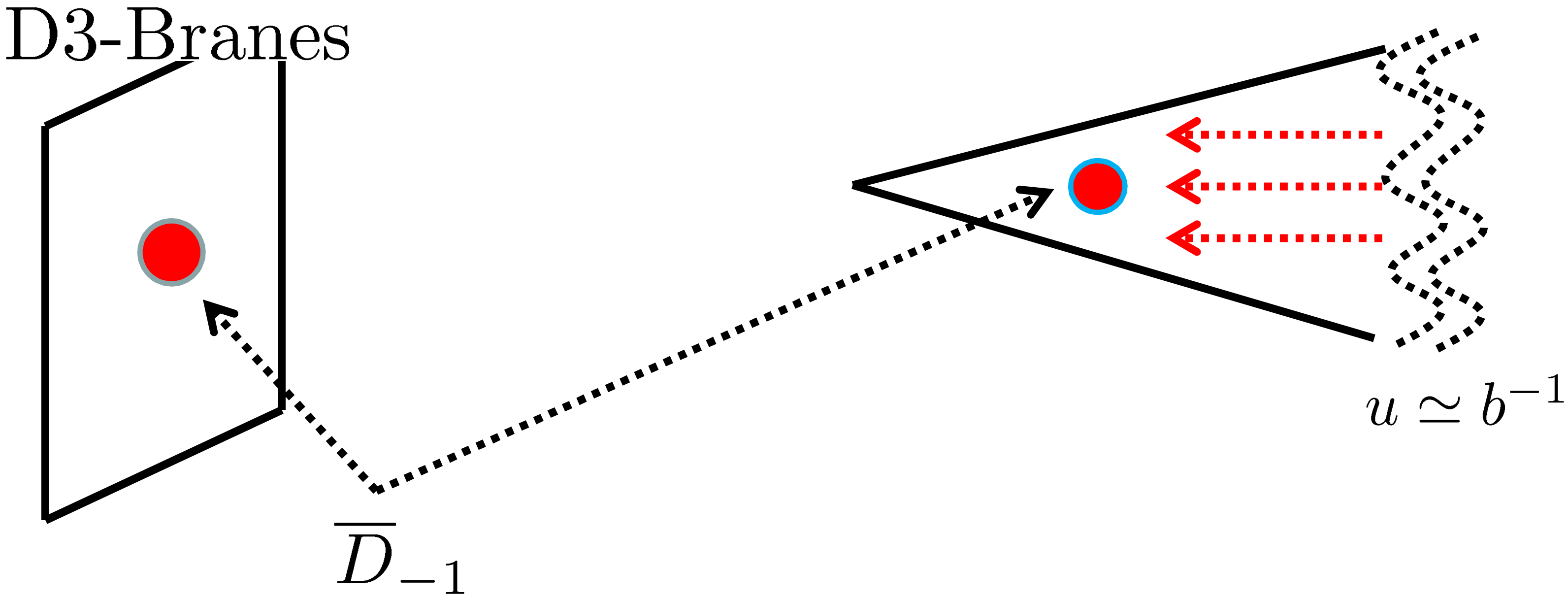
Phase Boundary

Note: $ds^2_{str} = e^{\phi/2} ds^2_{Ein} \to 0$ as $u \to b^{-1}$:

$\tau_+ - \tau_- < 0$	$\tau_+ - \tau_- = 0$	$\tau_+ - \tau_- > 0$
“compactified”	$\mathbb{R}^{10}$ “boundary”	no boundary
$0 < b < \infty$	$b = 0$	b imaginary
UV completion necessary	c.f. Hashimoto, Itzhaki ’99; Maldacena, Russo ’99; Das, Rama Trivedi ’99	×

The Wall

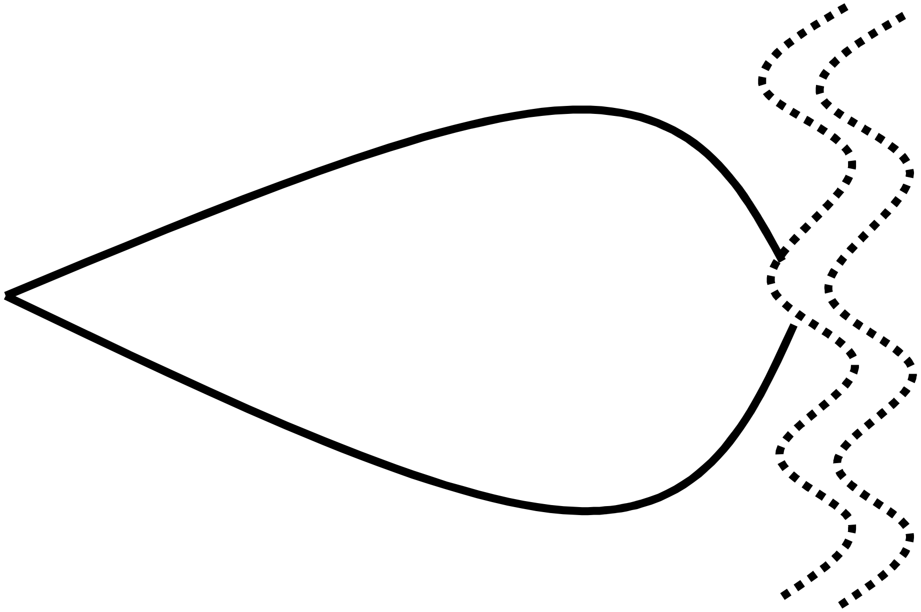
$\overline{\text{instantons}}$ repelled from UV region ($\tau_- \to \infty$):



Agrees with no small $\overline{\text{instanton}}$ in gauge thry.

Dilaton Compactification

Note: String frame metric $\to 0$ at $u \leq \infty$:

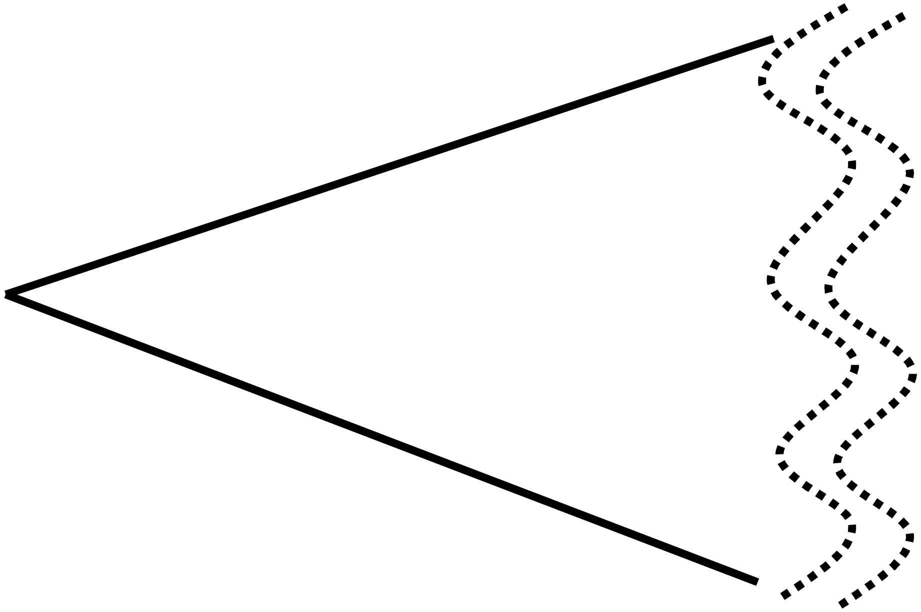


Boundary exists provided $\tau_+ - \tau_- < 0$

Einstein Frame?

But: Einstein frame metric remains non-zero:

$\Rightarrow$ working with a finite “cutoff” $\Rightarrow$ gravity on bdry



Boundary exists provided $\tau_+ - \tau_- < 0$

UV Region?

As $e^{\phi} \rightarrow 0$, F1 string tension $\rightarrow 0$

Adiabatic approximation not valid here:
all $\mathbb{CP}^3$ directions “equally physical”

$\Rightarrow$ Need Strings with $sl(4|4) \rightarrow isom(S^{4|8})$

Towards UV Completion

$\Rightarrow$ Need Strings with $sl(4|4) \to isom(S^{4|8})$

- Twistor String Theory
  c.f. Witten '03, Berkovits '04

  symmetries: $sl(4|4)$

- Twistor Matrix Model
  c.f. JJH Verlinde '11

  symmetries: $isom(S^{4|8})$

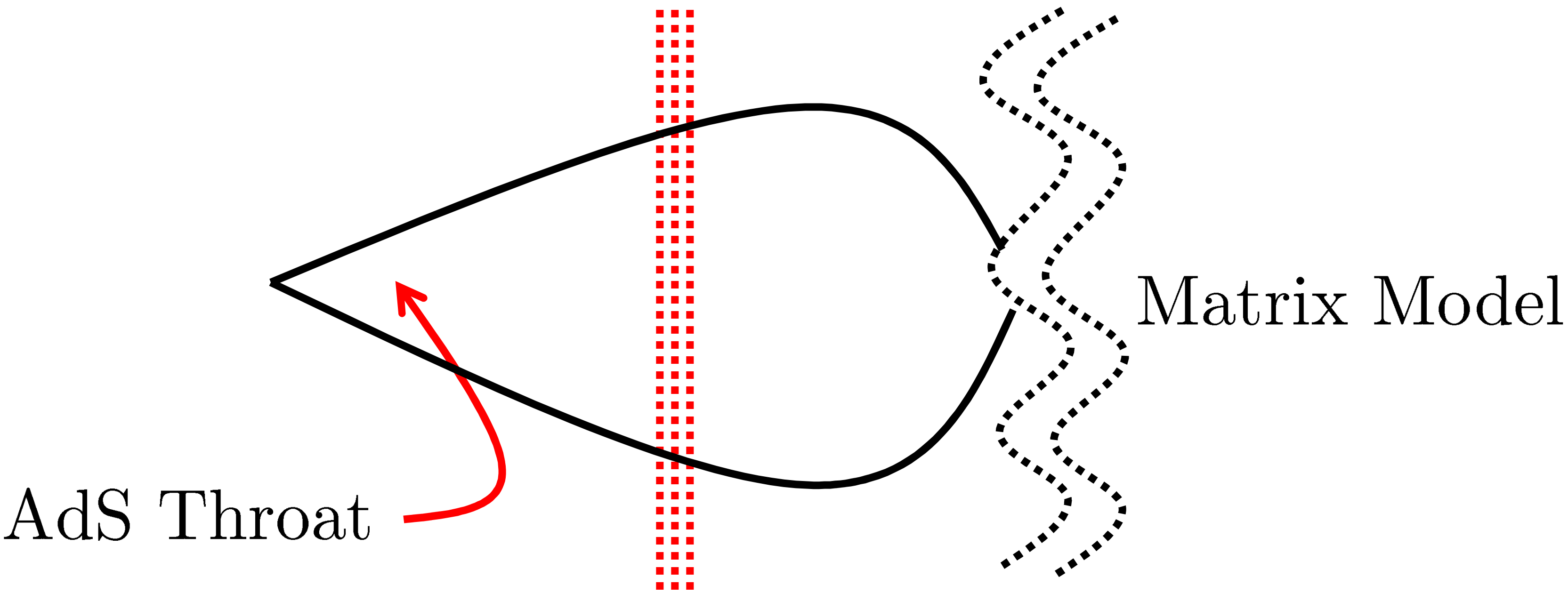
Twistor Matrix Model

Low energy limit of: [Self-Dual Yang-Mills
in Yang monopole bkgnd

- Symmetries: $isom(S^{4|8})$
- Correlators compute tree level (so far) gluon and Einstein gravity amplitudes

Recap: String Frame

Large N_c $\mathcal{N} = 4$ SYM + $\theta_{\mu\nu}(n_j)$ dual to:



Summary

&

Future Directions

Summary

- $[x_\mu, x_\nu]_n = \theta_{\mu\nu}(n^i) \rightarrow 0$ over orbit

- Lorentz Invariance & Emergent Twistors

- 4D Gravity and a covariant cutoff

Future Directions 1 / 3

Dimensions: $10 + 2_{top}$

This is reminiscient of F-theory...

But it involves a $\mathbb{CP}^1_{top}$, not a T^2...

Future Directions 2 / 3

Conjecture: (we just gave an example...)

4D QFT + Covariant Cutoff $\Rightarrow$ 4D Gravity

Fine print: $\mathbb{R}^{3,1}$ vs $\mathbb{R}^4$ vs unitarity?

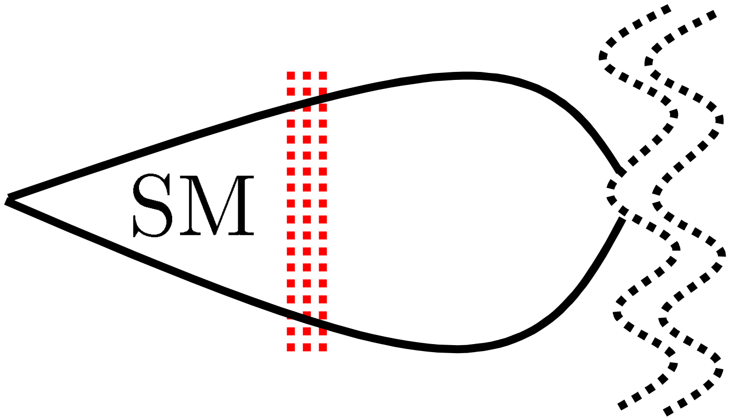
c.f. Jacobson '95, '12

Future Directions 3 / 3

Big simplification of local model building?

I) "Easy Step" Decouple gravity, build a QFT

II) "Hard Step" Recouple to 4D gravity



$$[x, x]_n \neq 0$$