New Aspects of Heterotic Geometry and Phenomenology

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Work done in collaboration with:

J. Gray, A. Lukas, and E. Palti: arXiv: 1106.4804, 1202.1757

J. Gray, A. Lukas and B. Ovrut: arXiv: 1010.0255, 1102.0011, 1107.5076, 1208.????

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Motivation

- String theory is a powerful extension of quantum field theory, but extracting low-energy physics from string geometry is mathematically challenging...

Higher dimensional geometry → String Comp. → $4d$ physics

- Need a good toolkit in any corner of string theory to extract the full low energy physics: (missing structure in the $N = 1$ lagrangian, coupings, moduli stabilization, etc.)
- Rules for "top down" model building? Patterns/Constraints/Predictions?

An algorithmic approach:

- Rather than attempting to engineer/tune a single model, can we develop general techniques? Produce a large number (i.e. billions) of consistent, global models and then scan for the desired properties? Identify Patterns?

A smooth $E_8 \times E_8$ heterotic compactification:

- The geometric ingredients include:
  - A Calabi-Yau 3-fold, X
  - Two holomorphic vector bundles, $V_1 \subset$ "Visible E_8", $V_2 \subset$ "Hidden E_8" on X (with structure group $G_i \subset E_8$).
- Compactifying on X leads to $\mathcal{N}=1$ SUSY in $4D$, while V breaks $E_8 \to G \times H$, where H is the Low Energy GUT group
  - $G = SU(n)$, $n = 2, 3, 4, 5$ leads to $H = E_7, E_6, SO(10), SU(5)$
- Matter and Moduli:

$$\mathbf{248}_{E_8} \to [(\mathbf{1},\mathbf{24}) \oplus (\mathbf{5},\bar{\mathbf{10}}) \oplus (\bar{\mathbf{5}},\mathbf{10}) \oplus (\mathbf{10},\mathbf{5}) \oplus (\bar{\mathbf{10}},\bar{\mathbf{5}}) \oplus (\mathbf{24},\mathbf{1})]_{\mathbf{SU(5)\times SU(5)}}$$

SU(5)-Charged	$n_{10} = h^1(V)$, $n_{\overline{10}} = h^1(V^*)$, $n_5 = h^1(\wedge^2 V^*)$, $n_{\bar{5}} = h^1(\wedge^2 V)$
Moduli (n_1)	$X \Rightarrow h^{1,1}(X)$ Kähler, $h^{2,1}(X)$ Complex Structure $V \Rightarrow h^1(X, V \otimes V^*)$ Bundle moduli, (+ Dilaton, $M5$...)

Two new ideas, both based on Vector bundles, V_i, and conditions for $N = 1$ SUSY: (Hermitian-Yang-Mills) $\delta\chi = 0 \Rightarrow \begin{cases} F_{ab} = F_{\bar{a}\bar{b}} = 0 \\ g^{a\bar{b}} F_{a\bar{b}} = 0 \end{cases}$

Model Building

- Idea: Make V_1 simpler. Easier to solve $g^{a\bar{b}} F_{a\bar{b}} = 0$.
- A simpler construction for the visible sector vector bundle allows us to find many models with Standard Model spectra. Provides a probe into general moduli space.

Moduli Stabilization

- Idea: Make V_2 more complicated. $F_{\bar{a}\bar{b}} = 0$ constrains moduli.
- Must determine what the moduli of the theory are and how to fix them. Values appear in Yukawa couplings, gauge couplings, etc.
- It is possible to choose vector bundles, V, which are only holomorphic at special points in complex structure moduli space of their base X.

Observation: At special loci in moduli space, bundle structure groups can (and often do) "split", causing the low energy gauge group to enhance.

Bundle	4d Symmetry
$SU(5) \to S[U(4) \times U(1)]$	$SU(5) \to SU(5) \times U(1)$
$V \to U_4 \oplus L \quad (c_1(U_4) + c_1(L) = 0)$	

Can consider "maximal" splitting

$$SU(5) \to S[U(1)^5] \qquad\qquad SU(5) \to SU(5) \times U(1)^4$$

$$V \to \bigoplus_a^5 L_a \quad (\sum_a c_1(L_a) = 0)$$

- $V_{split} = \bigoplus_a L_a \Rightarrow$ much easier to handle technically (HYM easy!)
- V_{split} resides in a larger, non-Abelian bundle moduli space and still carries many of the properties of generic, non-Abelian bundles
- The enhanced $U(1)$ symmetries are Green-Schwarz anomalous (massive) and most matter becomes charged under them.

- Scanned over $\sim 10^{12}$ bundles V_{split} over 65 Calabi-Yau 3-folds with $h^{1,1} \leq 5$.
- Found 2122 models with exact MSSM spectrum and no exotics.
- Can group these into 407 families with distinct symmetries.
- Discrete remnants of anomalous $U(1)$ symmetries constrains the superpotential, Kahler potential, etc...
- Further (hopefully exhaustive) scans ongoing. Still a long way to go...
  - Only 198 (of 407) free from dim $4, 5$ proton decay
  - 45 with $\text{rk}(Y^{(u)}) > 0$, etc.

Now on to moduli stabilization and the hidden sector...

- Holomorphy: $F_{\bar{a}\bar{b}} = 0$.
- In 10d:

$$S_{partial} \sim \int_{M_{10}} \sqrt{-g}\{({F^{(1)}}_{ab}{F^{(1)}}_{\bar{a}\bar{b}}g^{a\bar{a}}g^{a\bar{a}}) + ({F^{(2)}}_{ab}{F^{(2)}}_{\bar{a}\bar{b}}g^{a\bar{a}}g^{a\bar{a}})\ldots\}$$

- For the $4d$ Theory: Holomorphy obstructions should appear through $4d$ F-terms. Superpotential $W = \int_X \Omega \wedge H$ where $H = dB - \frac{3\alpha'}{\sqrt{2}}\left({\omega_3}^{\mathrm{YM}} - {\omega_3}^{\mathrm{L}}\right)$ and ${\omega_3}^{\mathrm{YM}} = A \wedge dA - \frac{1}{3}A \wedge A \wedge A$.
  But how to explicitly compute?
- Start with $F_{\bar{a}\bar{b}} = 0$, w.r.t a fixed complex structure. What happens as we vary the complex structure? Must a bundle stay holomorphic for any variation $\delta\mathfrak{z}^I v_I \in h^{2,1}(X)$? $\Rightarrow$ No.
- Infinitesimally, we must solve:

$$\delta\mathfrak{z}^I v^c_{I[\bar{a}}F^{(0)}_{|c|\bar{b}]} + 2D^{(0)}_{[\bar{a}}\delta A_{\bar{b}]} = 0$$

We must take the holomorphy condition

$$\delta \mathfrak{z}^I v^c_{I[\bar{a}} F^{(0)}_{|c|\bar{b}]} + 2D^{(0)}_{[\bar{a}} \delta A_{\bar{b}]} = 0$$

into account to determine the real moduli of a heterotic compactification.

Naively: $h^{1,1}(X) + h^{2,1}(X) + h^1(End_0(V_1)) + h^1(End_0(V_2))$. Not that simple...

Physics Idea Use bundle holomorphy to constrain C.S. Moduli of X. New moduli stabilization tool.

Math Question Given a CY, X, how to find/engineer bundles that are holomorphic only at special loci (ideally isolated pts) in CS moduli space?

Deformation Space – $Def(X, V)$: Simultaneous holomorphic deformations of V and X. The tangent space is $H^1(X, \mathcal{Q})$, defined via Atiyah Sequence

$$0 \to V \otimes V^\vee \to \mathcal{Q} \xrightarrow{\pi} TX \to 0$$

$\mathcal{Q}$ is the projectivized total space of the bundle $\mathbb{P}(V) \to X$.

- The long exact sequence in cohomology gives us

$$0 \to H^1(V \otimes V^\vee) \to H^1(\mathcal{Q}) \xrightarrow{d\pi} H^1(TX) \xrightarrow{\alpha} H^2(V \otimes V^\vee) \to \dots$$

- We must determine: $H^1(X, \mathcal{Q}) = H^1(X, V \otimes V^*) \oplus Ker(\alpha)$ where

$$\alpha = [F^{1,1}] \in H^1(V \otimes V^\vee \otimes TX^\vee)$$

  is the Atiyah Class

- C.S. moduli allowed $\alpha(\delta \mathfrak{z} v) = 0 \quad (0 \in H^2(V \times V^\vee))$. I.e. in $Ker(\alpha)$,

$$\delta \mathfrak{z}^I v^c_{I[\bar{a}} F^{(0)}_{|c|\bar{b}]} = -2D^{(0)}_{[\bar{a}} \delta A_{\bar{b}]}$$

$H^1(X, \mathcal{Q})$ are really the moduli

A simple example

- Consider an $SU(2)$ bundle defined by a extension in $Ext^1(\mathcal{L}^\vee, \mathcal{L})$:

$$0 \to \mathcal{L} \to V \to \mathcal{L}^\vee \to 0$$

In principle, such a bundle can stabilize arbitrarily many moduli.

- For example, consider $\mathcal{L} = \mathcal{O}(-2, -2, 1, 1)$ on the CY, $X = \left[\begin{array}{c|c} \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \end{array}\right]^{4,68}$
- Why this one? Here $Ext^1(\mathcal{L}^\vee, \mathcal{L}) = H^1(X, \mathcal{O}(-4, -4, 2, 2)) = 0$ generically. Hence cannot define the bundle for general complex structure!
- Happily, cohomology can "jump" at higher co-dimensional loci in Complex Structure moduli space.
- This is a easy example of "structural" C.S. dependence in V. Many others... (Monads/Spectral covers/Serre Construction, etc.)

- Jumping cohomology: Suppose $H^1(X, \mathcal{L}^{\otimes 2}) \neq 0$ for starting pt., p_0, in C.S.

  Question: How can we vary $p = p_0 + \delta p$ so that $H^1(X, \mathcal{L}^{\otimes 2}) \neq 0$?

  $\Rightarrow$ Can prove that this is equivalent to Atiyah computation for this case

In field theory:

- E_7 Singlets: $C_+ \in H^1(\mathcal{L}^{\otimes 2})$, $C_- \in H^1(\mathcal{L}^{\vee \otimes 2})$. ("Jump" together)
- Superpotential: $W = \lambda_{ia}(\mathfrak{z})\, C_+^i C_-^a$
  - Choose Vacuum: $< C_+ > \neq 0$ and $< C_- >= 0$ (Corresponds to non-trivial $0 \to \mathcal{L} \to V \to \mathcal{L}^{\vee} \to 0$)
  - Non-trivial F-term: $\frac{\partial W}{\partial C_-^a} = \lambda_{ia}(\mathfrak{z}) < C_+^i >= 0$
  - In fluctuation $\delta\left(\frac{\partial W}{\partial C_-^b}\right) = \frac{\partial \lambda_{ib}}{\partial \mathfrak{z}_\perp^I} < C_+^i > \delta \mathfrak{z}_\perp^I$

$\frac{\partial \lambda_{ia}}{\partial \mathfrak{z}^I}$ vanishes along locus with $\lambda = 0$. $\perp$ to locus, $\delta \mathfrak{z}_\perp^I$ gets a mass. All agree with Atiyah Computation.

Global questions...

- Everything we have discussed so far involves fluctuation around a point in C.S. moduli space. Big limitiation: You have to know where to start.
- Hard to find isolated solutions (wanted for moduli stabilization) this way.
- New Idea: Represent CS Loci (vacuum solutions to the F-terms) as an algebraic variety: $\frac{\partial W}{\partial C_{-}^{a}} = \lambda_{ia}(\mathfrak{z}) < C_{+}^{i} >= 0$
- Tools exist in computational algebraic geometry to analyze the properties of the solution space...
- Now we can scan over all possible starting points!

Disconnected Loci

- Let's perform the analysis for the example... $\mathcal{L} = \mathcal{O}(-4, -4, 2, 2)$ on $X^{4,68}/\mathbb{Z}_2 \times \mathbb{Z}_4$.
- For this bundle, 27 distinct loci in Complex Structure Moduli Space
- Branches to the solution space of $\lambda_{ia}(\mathfrak{z}) < C_+^i >= 0$ range in dimension from 7 to zero in $\mathfrak{z}$.
- Having found isolated point-like solutions, we might think we can declare victory...
- But for the given values of C.S., we still have to check transversality of the CY: $p = 0 = dp$
- By Bertini's Theorem, a generic CICY is smooth, but once we are fixed to very special points in CS, singularities are a real concern...

Singularities in the CY

Dimension in CS	7	5	4	3	2	1	0
Dimension of Singularities in X	0	0	smooth	0	0	0	2

- For this example, of the 27 branches to the solution space, all but one force the CY to be singular somewhere.
- Can we do anything with the singular solutions?
- Locally, for dim(Sing)≤ 1 we can imagine resolving (i.e. blowing-up) the singularities.
- Unfortunately, for the case at hand, all isolated point-like solutions are too badly singular. However, we can consider one of the larger loci...

Resolution

- Consider the 5-dimensional locus with pt-like singularities in CY
- Locally, we can resolve these singular pts. But have to worry about global issues: CY condition? What happens to the bundle? Symmetries?
- Happily, there are some resolutions of singular CYs that we have good control over: Conifold Transitions.
- CY defining poly takes the form $p = f_1 f_3 - f_2 f_4 = 0$. Topologically a cone over $S^3 \times S^2$. Can be resolved by introducing new $\mathbb{P}^1$ direction

$$\begin{pmatrix} f_1 & f_2 \\ f_3 & f_4 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \end{pmatrix} = 0$$

- Can explicitly track divisors, extension bundles, and symmetries through this geometric transition.

- Consider the following resolution of the Tetraquadric:

$$X = \left[\begin{array}{c|c} \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \end{array} \right] \Rightarrow \left[p_{(2,2,2,2)} \to f^1{}_{(2,0,2,0)} f^3{}_{(0,2,0,2)} - f^2{}_{(2,0,2,0)} f^4{}_{(0,2,0,2)} = 0 \right] \Rightarrow X_{res} = \left[\begin{array}{c|cc} \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 2 & 0 \\ \mathbb{P}^1 & 0 & 2 \\ \mathbb{P}^1 & 2 & 0 \\ \mathbb{P}^1 & 0 & 2 \end{array} \right]$$

- Quotienting both sides by $\mathbb{Z}_2 \times \mathbb{Z}_4$, this "factored" locus intersects the dim 5 CS locus above, leads to 8 singular pts on the CY. Resolving such pts, $X \to X_{res}$.
- To check, can repeat the analysis independently with $\mathcal{L} = \mathcal{O}(0, -2, -2, 1, 1)$ on X_{res}.
- This time, 14 branches ranging from dimension 3 to 0. The resolution X gives a 2 dimensional locus in the CS space of a smooth X_{res}.
- All CICYs connected by such transitions. Reid's Fantasy?
- Not yet dynamical transitions, but provides interesting web of "stabilizing" bundles on CYs...

Conclusions

- It is possible to build phenomenologically relevant heterotic models using the simplest possible gauge configurations – sums of line bundles. $\Rightarrow$ A computationally accessible arena for probing more general geometries.
- The presence of a holomorphic vector bundle constrains C.S. moduli $\Rightarrow$ Can be used as a hidden sector mechanism for moduli stabilization.
  - Allows us to keep Kähler geometric model-building toolkit
  - Stabilized values fully determined for use in physical couplings
- Both new tools give interesting hints into connections/patterns in web of CY 3-folds linked via geometric transitions
  - Complex structure fixing bundles can carry through resolutions
  - Patterns in SM data set linking bundles over conifold pair CYs

The End