R-R Cohomology and O(D,D) T-duality

Kanghoon Lee

(CQUeST, Sogang University, Seoul)

Strings 2012

Talk Based on works
(collaboration with Imtak Jeon, Jeong-Hyuck Park)

- **Differential geometry with a projection: Application to double field theory**
  JHEP 1104:014 (2011), arXiv:1011.1324
- **Double field formulation of Yang-Mills theory**
  Phys. Lett. B 701:260 (2011), arXiv:1102.0419
- **Stringy differential geometry, beyond Riemann**
  Phys. Rev. D 84:044022 (2011), arXiv:1105.6294
- **Incorporation of fermions into double field theory**
  JHEP 1111:025 (2011), arXiv:1109.2035
- **Supersymmetric Double Field Theory : Stringy Reformulation of Supergravity**
  Phys. Rev. D Rapid Comm. (2012), arXiv:1112.0069
- **Ramond-Ramond Cohomology and O(D,D) T-duality**
  arXiv : 1206.3478
- **Type II Democratic Double Field Theory**
  In preparation

Fundamental variables in SDFT

– **Dilaton** : e^{-2d} ,   **Double-vielbein** : $V_{Ap}\ \ \bar{V}_{A\bar{p}}$,   **R-R potential** : $\mathcal{C}^{\alpha}{}_{\bar{\alpha}}$,

– **Dilatinos** : $\rho^{\alpha}\,,\quad \rho'^{\bar{\alpha}}\,,$ **Gravitinos** : $\psi^{\alpha}_{\bar{p}}\,,\quad \psi'^{\bar{\alpha}}_{p}$.

✓ **Symmetries of DFT and their indices** :

- $\mathbf{O}(D,D)$ global T-duality transformation.
- Double gauge transformation (generalized Lie derivative)
- Double local Lorentz transformation : $\mathbf{Pin}(1,D{-}1)_L \times \mathbf{Pin}(D{-}1,1)_R$

$$A\,,B\,,\cdots:\mathbf{O}(D,D)\ \ \&\ \ \text{Double gauge vector}$$

$$p\,,q\,,\cdots:\mathbf{Pin}(1,D{-}1)_L\ \ \text{vector}$$

$$\bar{p}\,,\bar{q}\,,\cdots:\mathbf{Pin}(D{-}1,1)_R\ \ \text{vector}$$

$$\alpha\,,\beta\,,\cdots:\mathbf{Pin}(1,D{-}1)_L\ \ \text{spinor}$$

$$\bar{\alpha}\,,\bar{\beta}\,,\cdots:\mathbf{Pin}(D{-}1,1)_R\ \ \text{spinor}$$

✓ **Double-vielbein** generates a pair of 'projections' :

$$V_{Ap}V_B{}^p = P_{AB} \qquad \bar{V}_{A\bar{p}}\bar{V}_B{}^{\bar{p}} = \bar{P}_{AB}$$

where $P_A{}^C P_C{}^B = P_A{}^B$, $\bar{P}_A{}^C \bar{P}_C{}^B = \bar{P}_A{}^B$, and $P_{AB} - \bar{P}_{AB} = \mathcal{H}_{AB}$.

✓ **R-R potential**, $\mathcal{C}^{\alpha}{}_{\bar{\alpha}}$, is in the **bi-spinorial representation** of the double local Lorentz group, $\mathbf{Pin}(1, D-1)_L \times \mathbf{Pin}(D-1, 1)_R$.

✓ Chirality condition : $\mathcal{C} = \pm\gamma^{(D+1)}\mathcal{C}\bar{\gamma}^{(D+1)}$

Covariant Derivatives

- For each of the DFT gauge symmetry, we introduce a corresponding **connection** and set **master semi-covariant derivative :**

$$\mathcal{D}_A = \partial_A + \Gamma_A + \Phi_A + \bar{\Phi}_A$$

which satisfies following compatibility conditions,

$$\mathcal{D}_A d = 0\,, \qquad \mathcal{D}_A V_{Bp} = 0\,, \qquad \mathcal{D}_A \bar{V}_{B\bar{p}} = 0\,,$$

- Connection Γ can be uniquely determined in terms of fundamental variables :

$$\begin{aligned}\Gamma = \;& 2\left(P\partial_C P\bar{P}\right)_{[AB]} + 2\left(\bar{P}_{[A}{}^{D}\bar{P}_{B]}{}^{E} - P_{[A}{}^{D}P_{B]}{}^{E}\right)\partial_D P_{EC} \\ & -\tfrac{4}{D-1}\left(\bar{P}_{C[A}\bar{P}_{B]}{}^{D} + P_{C[A}P_{B]}{}^{D}\right)\left(\partial_D d + (P\partial^E P\bar{P})_{[ED]}\right)\end{aligned}$$

R-R field Cohomology

- We define R-R field strength $\mathcal{F}^{\alpha}{}_{\bar{\beta}}$ as

$$\mathcal{F} := \mathcal{D}_{+}\mathcal{C} = \gamma^{A}\mathcal{D}_{A}\mathcal{C} + \gamma^{(D+1)}\mathcal{D}_{A}\mathcal{C}\bar{\gamma}^{A}$$

where $\mathcal{D}_{+}$ is a **covariant nilpotent operator,** $\mathcal{D}_{+}^{2} = 0$.

- R-R gauge transformation :

$$\delta\mathcal{C} = \mathcal{D}_{+}\Delta \longrightarrow \delta\mathcal{F} = \mathcal{D}_{+}^{2}\Delta = 0$$

- We impose **self-duality relation** by hand :

$$\mathcal{F} = \gamma^{(D+1)}\mathcal{F}$$

- The R-R sector Lagrangian :

$$\mathcal{L}_{RR} = -\tfrac{1}{2}e^{-2d}\mathcal{F}^{\alpha\bar{\alpha}}\mathcal{F}_{\alpha\bar{\alpha}}$$

Parametrization and gauge fixing

- Double-vielbein takes the most general **parametrization**,

$$V_{Ap} = \frac{1}{\sqrt{2}} \begin{pmatrix} (e^{-1})_p{}^{\mu} \\ (B+e)_{\nu p} \end{pmatrix} \qquad \bar{V}_{A\bar{p}} = \frac{1}{\sqrt{2}} \begin{pmatrix} (\bar{e}^{-1})_{\bar{p}}{}^{\mu} \\ (B+e)_{\nu\bar{p}} \end{pmatrix}$$

where $e_{\mu}{}^{p}$, $\bar{e}_{\mu}{}^{\bar{p}}$ are two copies of the D-dimensional vielbein,

$$e_{\mu}{}^{p} e_{\nu}{}^{q} \eta_{pq} = -\bar{e}_{\mu}{}^{\bar{p}} \bar{e}_{\nu}{}^{q} \bar{\eta}_{\bar{p}\bar{q}} = g_{\mu\nu}$$

- We may choose an alternative parametrization, (c.f. Andriot, Hohm, Larfors, Lust, Patalong)

$$V_A{}^{p} = \frac{1}{\sqrt{2}} \begin{pmatrix} (\beta+\tilde{e})^{\mu p} \\ (\tilde{e}^{-1})^{p}{}_{\nu} \end{pmatrix} \qquad \bar{V}_A{}^{\bar{p}} = \frac{1}{\sqrt{2}} \begin{pmatrix} (\beta+\bar{\tilde{e}})^{\mu\bar{p}} \\ (\bar{\tilde{e}}^{-1})^{\bar{p}}{}_{\nu} \end{pmatrix}$$

- **Diagonal gauge fixing** of double local Lorentz transformation

$$e_{\mu}{}^{p} \equiv \bar{e}_{\mu}{}^{\bar{p}}$$

This gauge fixing breaks $\mathbf{Pin}(1,D-1)_L \times \mathbf{Pin}(D-1,1)_R$ into diagonal subgroup $\mathbf{Pin}(1,D-1)_D$.

- To preserve the diagonal gauge fixing, $O(D,D)$ transformation rule must be modified to ***accompany a compensating*** $\mathbf{Pin}(D-1,1)_R$ ***Lorentz transformations***.

$$\mathcal{C} \longrightarrow \mathcal{C}S_{\bar{L}}^{-1} \qquad \rho \longrightarrow \rho \qquad \psi_{\bar{p}} \longrightarrow (\bar{L}^{-1})_{\bar{p}}{}^{\bar{q}}\psi_{\bar{q}}$$

$$\mathcal{F} \longrightarrow \mathcal{F}S_{\bar{L}}^{-1} \qquad \rho' \longrightarrow S_{\bar{L}}\rho' \qquad \psi'_{p} \longrightarrow S_{\bar{L}}\psi'_{p}$$

- Especially, if $\det(\bar{L}) = -1$, the modified $O(D,D)$ rotation **flips the chirality** of the theory. (mechanism for exchanging type IIA and IIB supergravity)

$$\bar{\gamma}^{(D+1)}S_{\bar{L}} = \det(\bar{L})\, S_{\bar{L}}\bar{\gamma}^{(D+1)}$$

- Diagonal gauge fixing reduces the R-R field to the usual **democratic formulation of supergravity**,

$$\mathcal{C} \equiv \left(\tfrac{1}{2}\right)^{\frac{D+2}{4}} \sum_p{}' \tfrac{1}{p!}\, \mathcal{C}_{a_1 a_2 \cdots a_p} \gamma^{a_1 a_2 \cdots a_p}$$

$$\mathcal{F} = \mathcal{D}_+ \mathcal{C} \equiv \left(\tfrac{1}{2}\right)^{\frac{D}{4}} \sum_p{}' \tfrac{1}{(p+1)!}\, \mathcal{F}_{a_1 a_2 \cdots a_{p+1}} \gamma^{a_1 a_2 \cdots a_{p+1}}$$

where $\mathcal{F}_{a_1 a_2 \cdots a_p} \simeq p \left(D_{[a_1} \mathcal{C}_{a_2 \cdots a_p]} - \partial_{[a_1} \phi\, \mathcal{C}_{a_2 \cdots a_p]} \right) + \frac{p!}{3!(p-3)!} H_{[a_1 a_2 a_3} \mathcal{C}_{a_4 \cdots a_p]}$

- The nilpotent operator $\mathcal{D}_+$ reduces to an **exterior derivative :**

$$\mathcal{D}_+ \longrightarrow \mathrm{d} + (\mathcal{H} - \mathrm{d}\phi)\wedge$$

- $\mathbf{O}(D,D)$ spinorial representation of R-R field (Fukuma, Oota, Tanaka; Hohm, Kwak, Zwiebach) can be mapped to our bi-spinorial representation.

Type II SDFT (fermion leading order)

$$\mathcal{L} = e^{-2d}\left[\tfrac{1}{8}(P^{AB}P^{CD} - \bar{P}^{AB}\bar{P}^{CD})S_{ACBD} - \tfrac{1}{2}\mathrm{Tr}[\mathcal{F}\bar{\mathcal{F}}] - i\left(\bar{\rho}\mathcal{F}\rho' - \bar{\psi}_{\bar{p}}\gamma^{q}\mathcal{F}\bar{\gamma}^{\bar{p}}\psi'_{q}\right)\right.$$
$$\left. + i\tfrac{1}{2}\bar{\rho}\gamma^{p}\mathcal{D}_{p}\rho - i\bar{\psi}^{\bar{p}}\mathcal{D}_{\bar{p}}\rho - i\tfrac{1}{2}\bar{\psi}^{\bar{p}}\gamma^{p}\mathcal{D}_{p}\psi_{\bar{p}} + i\tfrac{1}{2}\bar{\rho}'\bar{\gamma}^{\bar{p}}\mathcal{D}_{\bar{p}}\rho' - i\bar{\psi}'^{p}\mathcal{D}_{p}\rho' - i\tfrac{1}{2}\bar{\psi}'^{p}\bar{\gamma}^{\bar{p}}\mathcal{D}_{\bar{p}}\psi'_{p}\right]$$

$$\delta d = -\tfrac{i}{2}\bar{\varepsilon}\rho + \tfrac{i}{2}\bar{\varepsilon}'\rho'\,,$$

$$\delta V_{Ap} = -i\bar{V}_{A}{}^{\bar{q}}\bar{\varepsilon}\gamma_{p}\psi_{\bar{q}} - i\bar{V}_{A}{}^{\bar{q}}\bar{\varepsilon}'\bar{\gamma}_{\bar{q}}\psi'_{p}\,,$$

$$\delta\bar{V}_{A\bar{p}} = iV_{A}{}^{q}\bar{\varepsilon}\gamma_{q}\psi_{\bar{p}} + iV_{A}{}^{q}\bar{\varepsilon}'\bar{\gamma}_{\bar{p}}\psi'_{q}\,,$$

$$\delta\mathcal{C} = -i\tfrac{1}{2}\left(\gamma^{p}\varepsilon\bar{\psi}'_{p} + \rho\bar{\varepsilon}' - \varepsilon\bar{\rho}' - \psi_{\bar{p}}\bar{\varepsilon}'\bar{\gamma}^{\bar{p}}\right) + \delta_{\varepsilon}d\,\mathcal{C} - \tfrac{1}{2}\bar{V}_{\bar{q}}^{A}\delta_{\varepsilon}V_{Ap}\gamma^{(d+1)}\gamma^{p}\mathcal{C}\bar{\gamma}^{\bar{q}}\,,$$

$$\delta\rho = -\gamma^{A}\mathcal{D}_{A}\varepsilon\,, \qquad \delta\psi_{\bar{p}} = \bar{V}^{A}{}_{\bar{p}}\mathcal{D}_{A}\varepsilon + \mathcal{F}\bar{\gamma}_{\bar{p}}\varepsilon'\,,$$

$$\delta\rho' = -\bar{\gamma}^{A}\mathcal{D}_{A}\varepsilon'\,, \qquad \delta\psi'_{p} = V^{A}{}_{p}\mathcal{D}_{A}\varepsilon' - \bar{\mathcal{F}}\gamma_{p}\varepsilon\,.$$

c.f. Type II theory in generalized geometry (Coimbra, Strickland-Constable, Waldram)