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(In)Stabilities and Complementarity in AdS/CFT

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Based on works with J.L.F Barbon

Based on work with R. Auzzi, S. Elitzur and S.B. Gudnason

- J. L. F. Barbon and E. Rabinovici, “AdS Crunches, CFT Falls And Cosmological Complementarity,” JHEP **1104**, 044 (2011) [arXiv:1102.3015 [hep-th]].
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- S. Elitzur, A. Giveon, M. Porrati , E. Rabinovici
- S. de Haro, I. Papadimitriou , A. C. Petkou
- J. Orgera, J. Polchinski,; D. Harlow
- J. Maldacena

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- Complementarity
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INTRODUCTION

Dualities

- Geometry
- Topology
- Number of dimensions, small and large
- (non-)Commutativity
- Singularity structure
- Associativity

=

- Singularities express a breakdown of our knowledge/approximations
- In general-covariantly invariant theories, singularities can hide behind horizons
- Finite black hole entropy can be reconstructed from the outside
- Can infinite entropy of a crunch be reconstructed as well?

ADS SET UP


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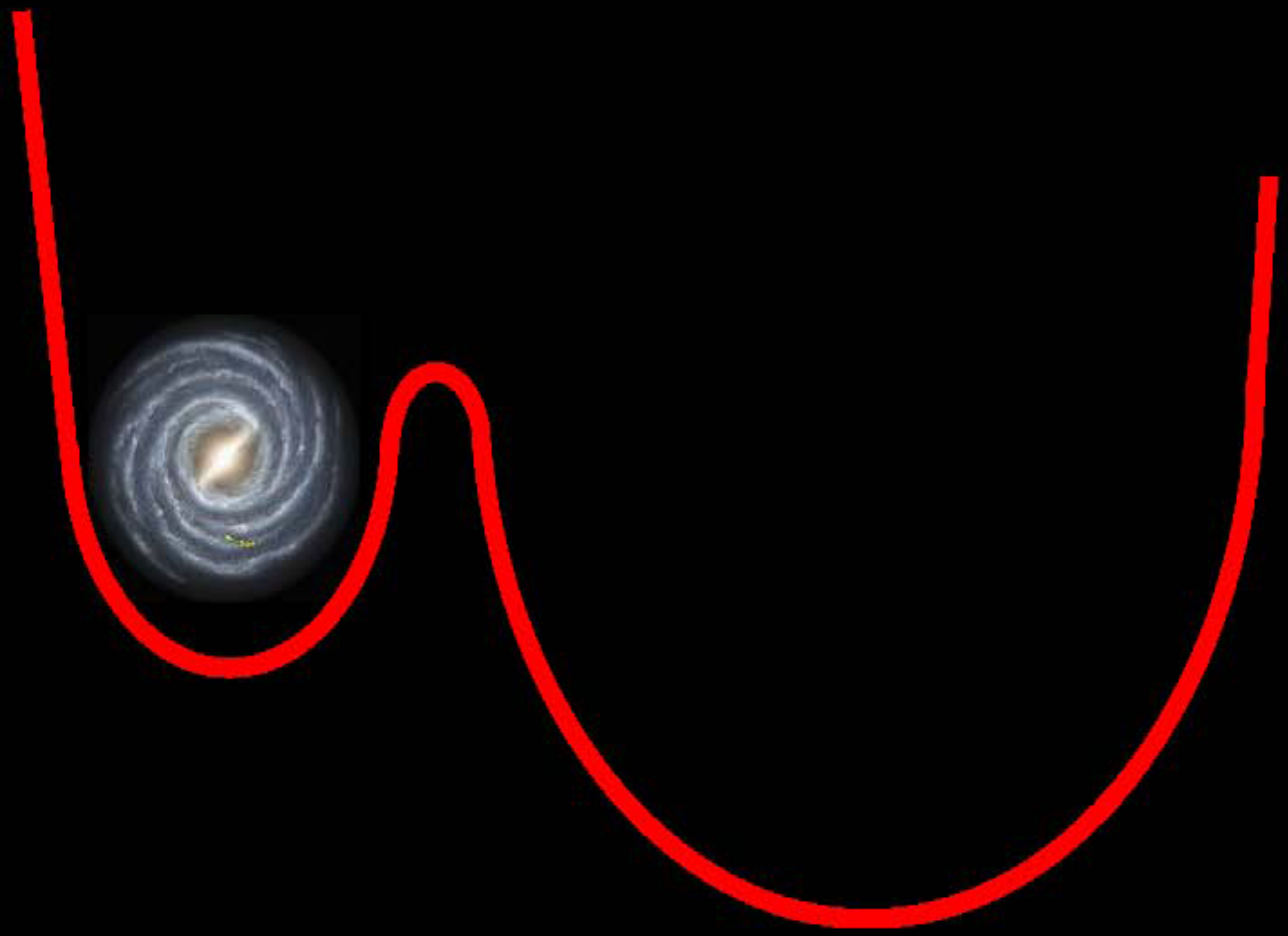
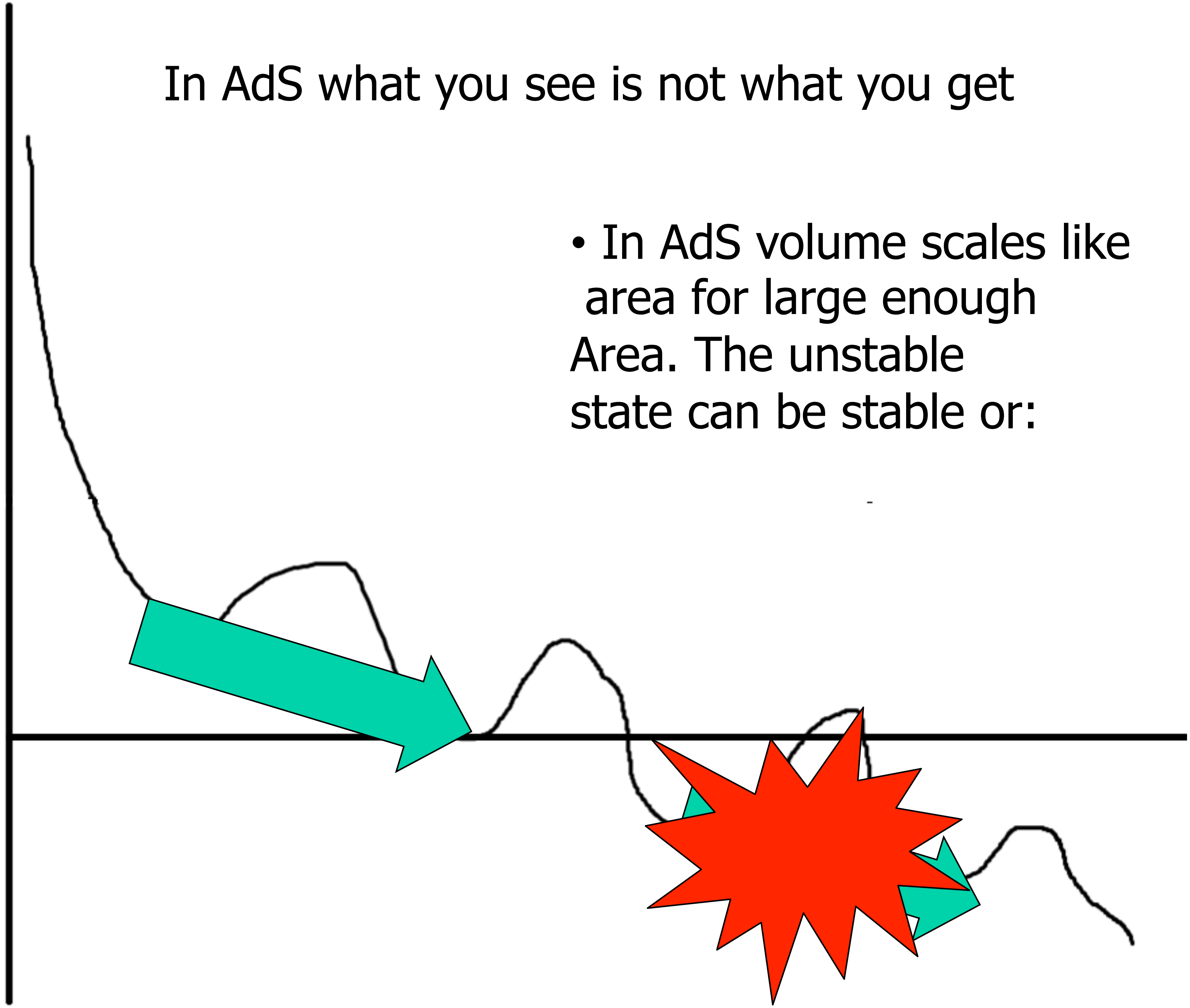
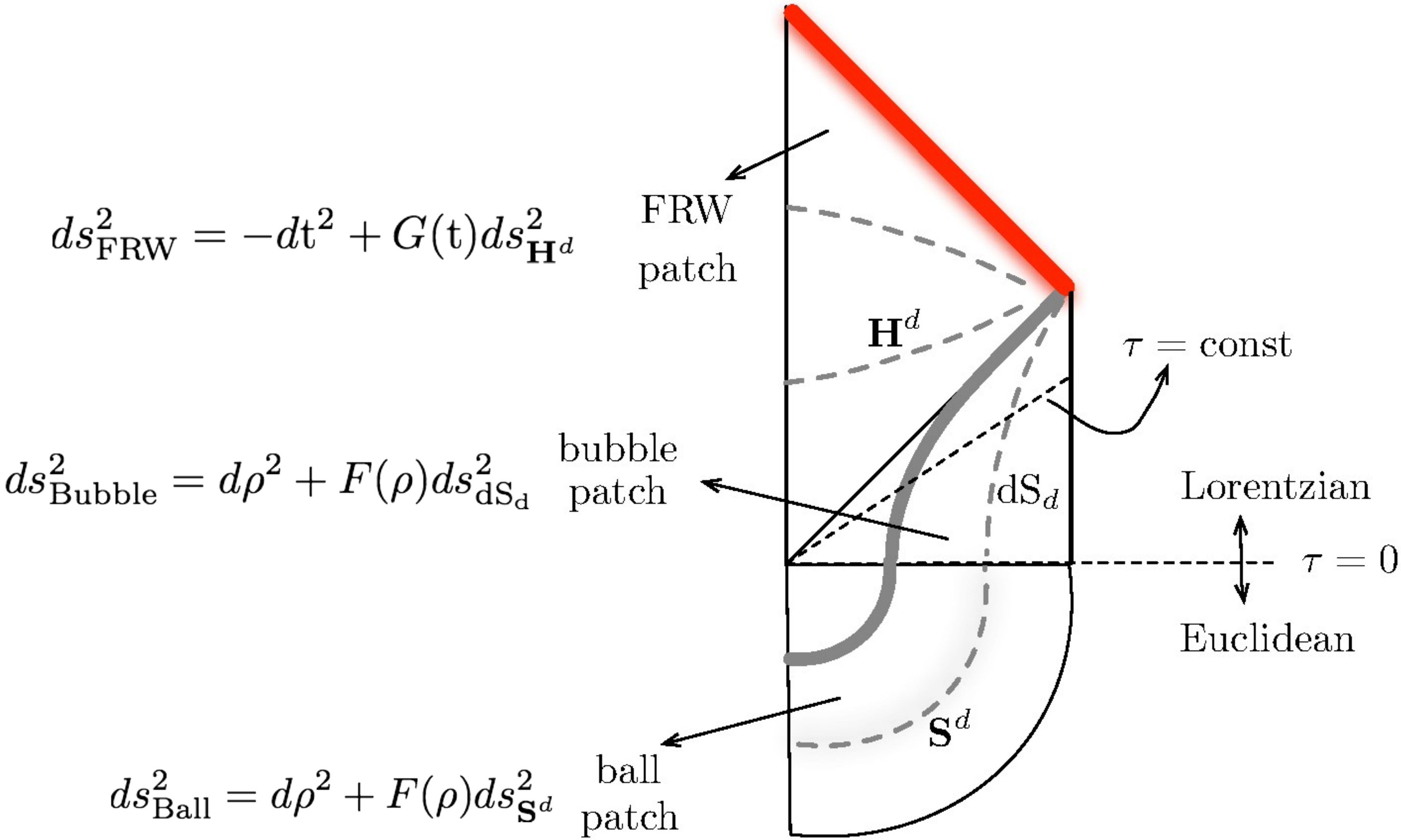


In AdS what you see is not what you get

• In AdS volume scales like
area for large enough
Area. The unstable
state can be stable or:





$ds^2_{\mathrm{FRW}} = -d\mathrm{t}^2 + G(\mathrm{t})ds^2_{\mathbf{H}^d}$
FRW
patch
$\mathbf{H}^d$
$\tau = \mathrm{const}$
$ds^2_{\mathrm{Bubble}} = d\rho^2 + F(\rho)ds^2_{\mathrm{dS_d}}$
bubble
patch
dS_d
Lorentzian
$\tau = 0$
Euclidean
$\mathbf{S}^d$
$ds^2_{\mathrm{Ball}} = d\rho^2 + F(\rho)ds^2_{\mathbf{S}^d}$
ball
patch

We shall refer to 'AdS-crunches' as a particular class of FRW cosmologies with $O(d,1)$-invariant spatial sections (i.e. d-hyperboloids $\mathbf{H}^d$),

$$ds^2_{\rm FRW} = -d{\rm t}^2 + G({\rm t})\, ds^2_{\mathbf{H}^d}\ .$$

backgrounds in terms of the Euclidean versions with $O(d+1)$ isometries. Let us consider the metric

$$ds^2_{\rm Ball} = d\rho^2 + F(\rho)\, d\Omega_d^2\ ,$$

satisfying the field Equations with an $O(d+1)$-symmetric matter distribution $\varphi(\rho)$. We term it 'the ball' on account of its $O(d+1)$ symmetry, even if it may be non-compact in general. Smoothness at the center of the ball requires $F(\rho) \approx \rho^2$ and $\varphi(\rho) \approx \varphi_0 + \frac{1}{2}\varphi_0''\rho^2$ as $\rho \to 0$.

Writing $d\Omega_d^2 = d\theta^2 + \cos^2(\theta)\, d\Omega_{d-1}^2$, we generate a Lorentz-signature metric with $O(d,1)$ symmetry by the analytic continuation $\theta = i\tau$. We call this metric 'the bubble':

$$ds^2_{\rm Bubble} = d\rho^2 + F(\rho)\, \left(-d\tau^2 + \cosh^2(\tau)\, d\Omega_{d-1}^2\right) = d\rho^2 + F(\rho)\, ds^2_{{\rm dS}_d}\ ,$$

where the group $O(d,1)$ acts on global de Sitter sections dS_d. By construction, the matter fields are de Sitter-invariant functions $\varphi(\rho)$, so all features of the metric and matter fields expand like a de Sitter space-time, i.e. we have a generalized notion of an 'expanding bubble'. This bubble background is time-symmetric around $\tau = 0$, where it can be formally matched to the Euclidean $O(d+1)$-invariant 'ball'. Therefore, we may interpret this construction as a time-symmetric cosmology with bang and crunch, or as a crunching cosmology that evolves from a particular initial condition obtained from some quantum-cosmological tunneling event, *a la* Hartle–Hawking [10].

At ρ=0 the dS_d sections become null and they may be further extended as the nearly null $\mathbf{H}^d$ sections of the FRW patch. By mimicking the pure AdS case, we can achieve this matching by the coordinate redefinition $\rho = i\mathrm{t}$ and $y = \tau + i\pi/2$:

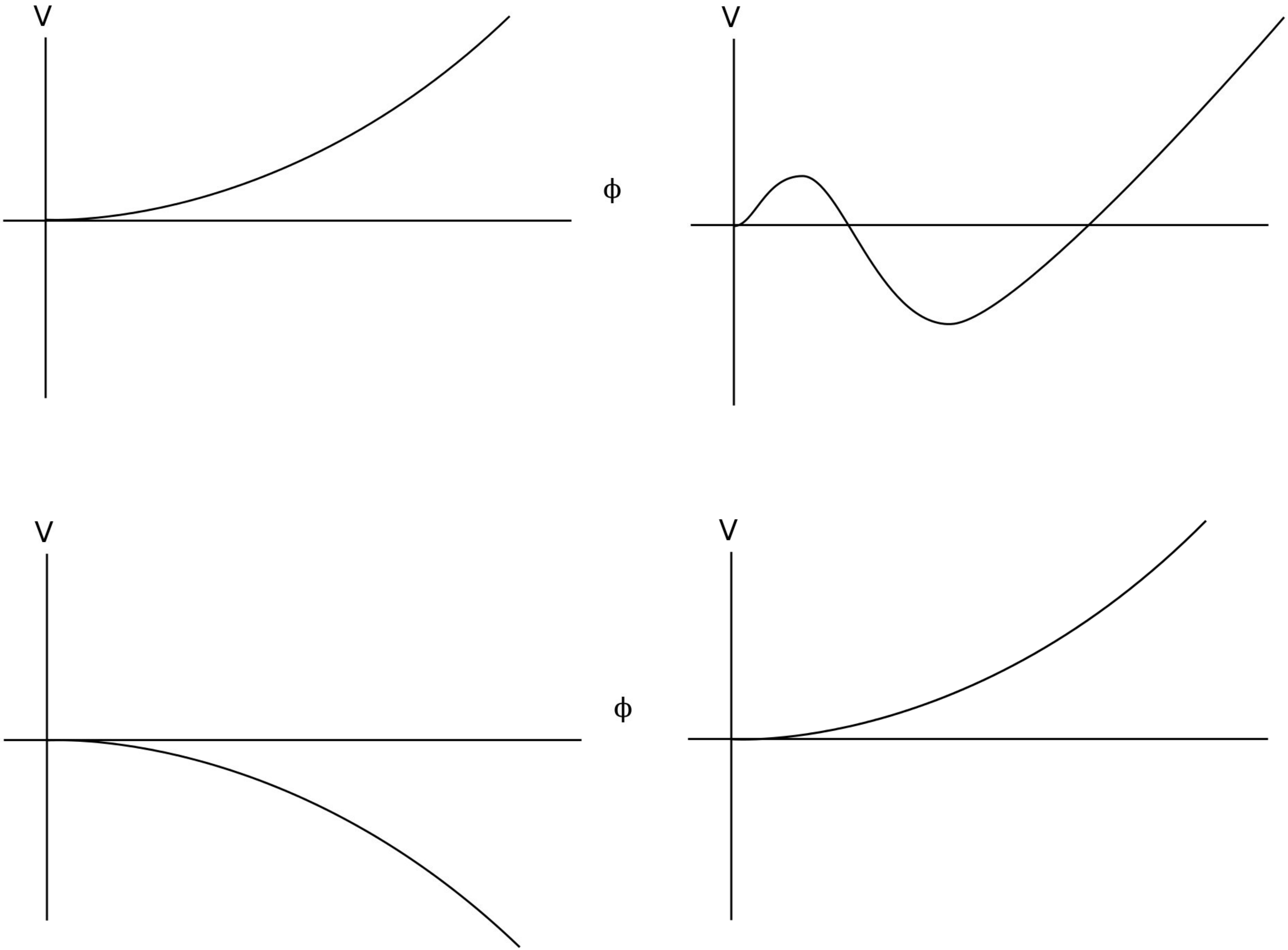
$$ds^2_{\mathrm{FRW}} = -d\mathrm{t}^{\,2} + G(\mathrm{t})\,\left(dy^2 + \sinh^2(y)\,d\Omega^2_{d-1}\right) = -d\mathrm{t}^{\,2} + G(\mathrm{t})\,ds^2_{\mathbf{H}^d}\;,$$

where the smooth matching requires $G(\mathrm{t}) \approx \mathrm{t}^2 \approx -F(i\mathrm{t})$ near $\mathrm{t} = 0$. For the rest of the fields, $\varphi(\rho)$ continues to a $O(1,d\,)$-invariant function $\varphi(\mathrm{t})$ with small t behavior $\varphi(\mathrm{t}) \approx \varphi_0 - \frac{1}{2}\varphi_0''\,\mathrm{t}^2$. Hence, the result is a FRW model with negative spatial curvature (2.1), which eventually crunches barring fine-tuning.

Non perturbative definition of the theory.

There are several possible QFT duals on the bondary

BOUNDARY

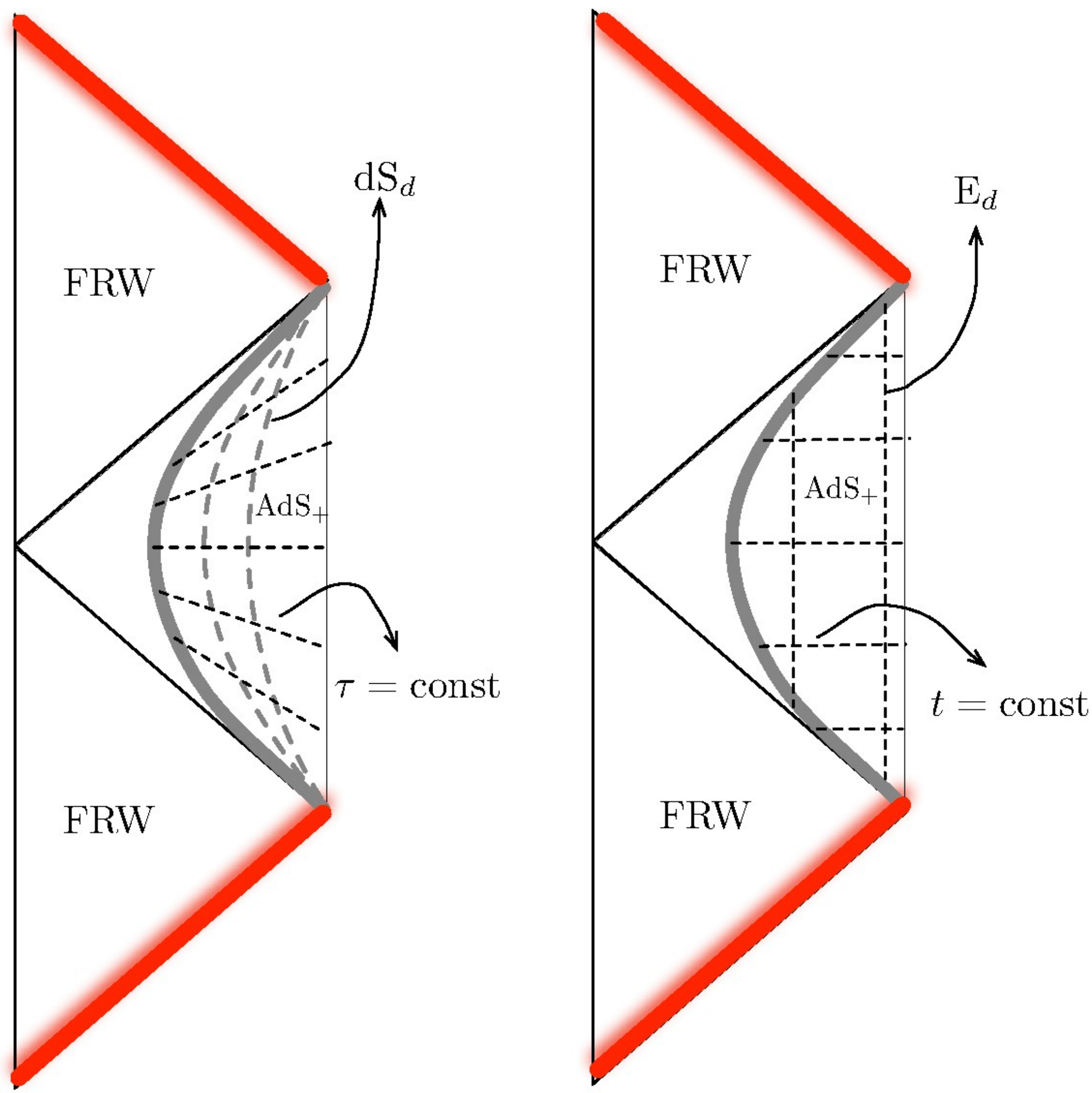

V
ϕ
V
ϕ
V
ϕ
V
ϕ

- If the boundary theory is well defined so is the crunch in the bulk.
- For the bulk crunch example above the boundary theory is well defined. Possible to describe a crunch.
- It is well defined on a world volume which is dS but there is no gravitational coupling.
- To see the crunch change coordinates on the boundary.

$$ds^2_{\mathrm{E}} = -dt^2 + d\Omega^2_{d-1},$$

$$ds^2_{\mathrm{dS}} = \Omega^2(t)\, ds^2_{\mathrm{E}}\,, \qquad \Omega(t) = \cosh(\tau) = \frac{1}{\cos(t)}\,,$$

$$t = \int \Omega^{-1}(\tau) d\tau = 2\tan^{-1}\left[\tanh(\tau/2)\right].$$



FRW
FRW
dS$_d$
AdS$_+$
τ = const
FRW
FRW
E$_d$
AdS$_+$
t = const

In the dS frame:

- The World Volume expands(consider a slow expansion relative to other scales).
- Time extends from -∞ to ∞
- The couplings in the Lagrangian are time INDEPENDENT

In the E frame:

- The world volume is static when it exists.
- Time has a finite extension.
- The relevant couplings in the Lagrangian are time dependent and explode at the end of time. The marginal operators remain time independent.

A simple classical model: O(N) on de Sitter

$$S_{\mathrm{dS}}[\vec{\phi}\,] = -\int_{\mathrm{dS}_4} \left(\frac{1}{2}\partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi} + \frac{\vec{\phi}^{\,2}}{R^2} + \lambda \left(\vec{\phi}^{\,2} \right)^2 + \varepsilon\, M^2 \vec{\phi}^{\,2} \right)$$

$MR \gg 1$ ⟶ Phases are clear-cut

$\varepsilon > 0$ $\qquad \mathrm{UV}_{O(N)} \xrightarrow{\text{RG flow}} \mathrm{IR}_{\mathrm{gap}}$

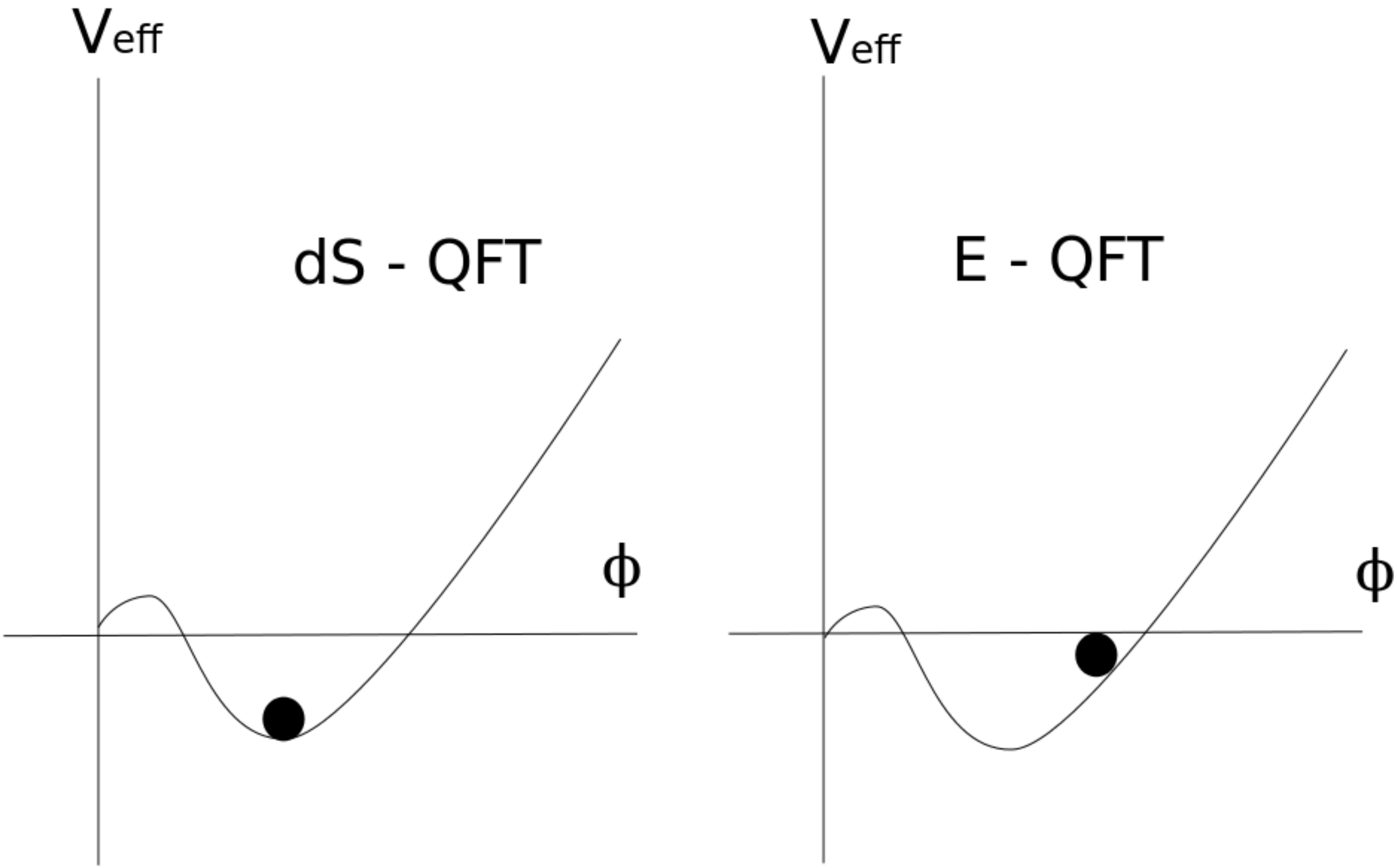
$\varepsilon < 0$ $\qquad \mathrm{UV}_{O(N)} \xrightarrow{\text{RG flow}} \mathrm{IR}_{O(N-1)}$

Over at the E-frame...

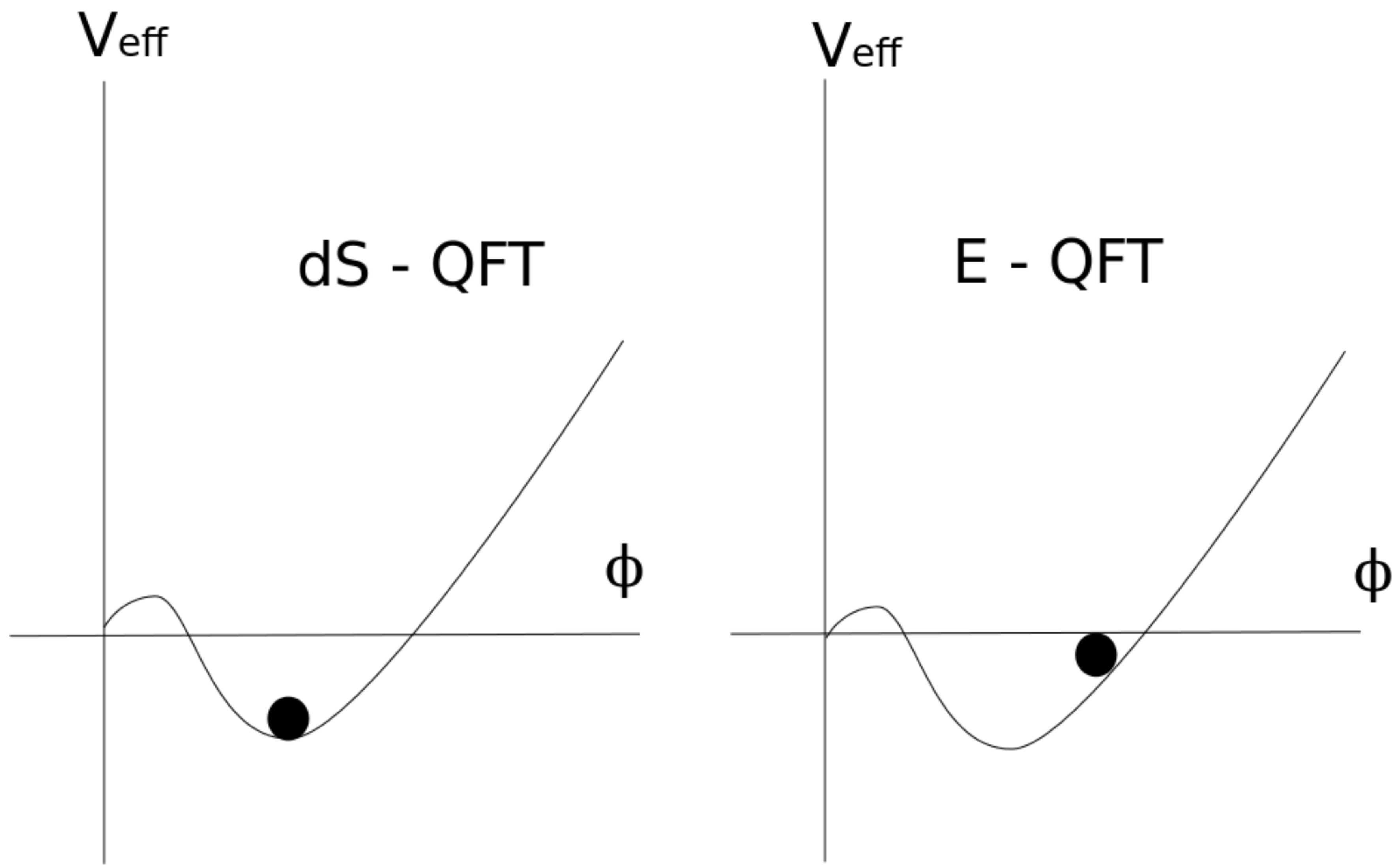
$$S_{\mathrm{E}}[\vec{\phi}\,] = -\int_{\mathrm{E}_4} \left(\frac{1}{2}\partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi} + \frac{\vec{\phi}^{\,2}}{2R^2} + \lambda \left(\vec{\phi}^{\,2} \right)^2 + \varepsilon\, \Omega(t)^2\, M^2 \vec{\phi}^{\,2} \right)$$

Mass term blows to $\varepsilon\,\infty$
in finite time

$$\int_{\mathrm{dS}_4} \mathcal{L}_{\vec{\phi}} = -\int_{\mathrm{dS}_4} \left(\frac{1}{2}\, \partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi} + \vec{\phi}^{\,2} + g_4 \left(\vec{\phi}^{\,2} \right)^2 + g_2\, \Lambda^2 \vec{\phi}^{\,2} \right) \ ,$$



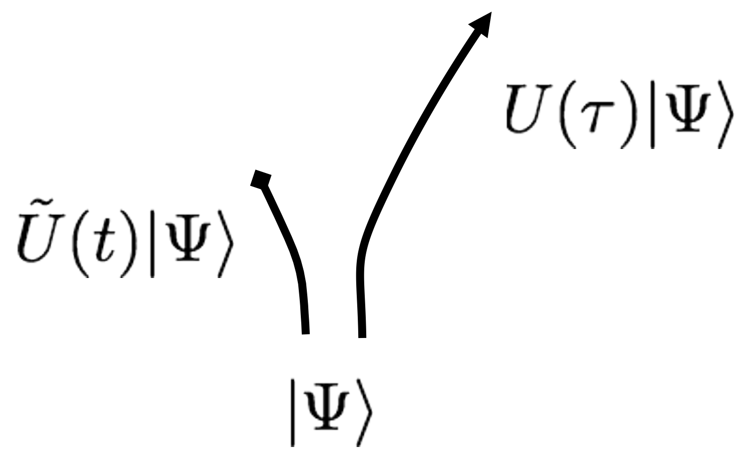
Veff
dS - QFT
ϕ
Veff
E - QFT
ϕ



Veff
dS - QFT
φ
Veff
E - QFT
φ

- A crunch can be described by a regular QFT on dS or by evolving with a state by a Hamiltonian which is well defined for a finite time range and then ceases to exist.
- The two Hamiltonians do NOT commute.

One can build quantum mechanical models with two non-commuting Hamiltonians

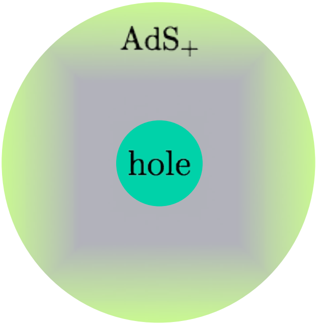


t-evolution crunches and τ -evolution is eternal

but they are complementary as both time evolution operators are related by a unitary canonical map

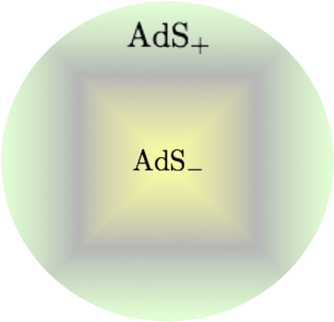
Bulk analogs

$$MR \gg 1$$
$$\varepsilon > 0$$



bubble of nothing
Continues to dS gap
and E-decoupling

$$MR \gg 1$$
$$\varepsilon < 0$$



Domain wall flow
Continues to dS condensate
and E-crunch

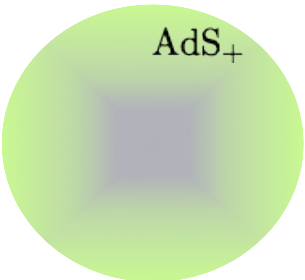
What about $MR \ll 1$?

This is the slightly massive UV CFT on a finite box

Detailed dynamics should depend on quantum effects after large-N summation

In Bulk, we get linearized scalar flows which crunch for either sign of ε

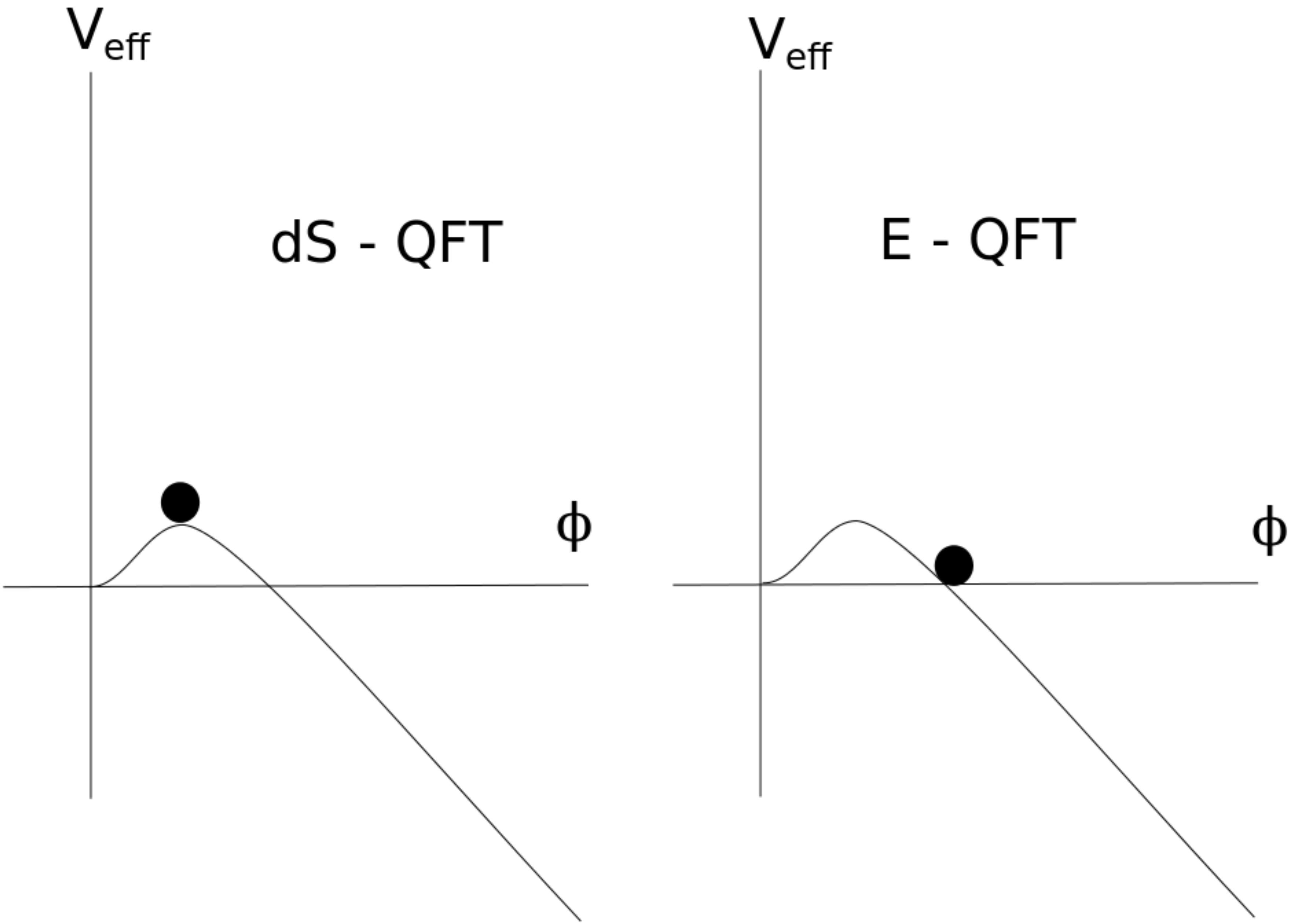
(Maldacena)



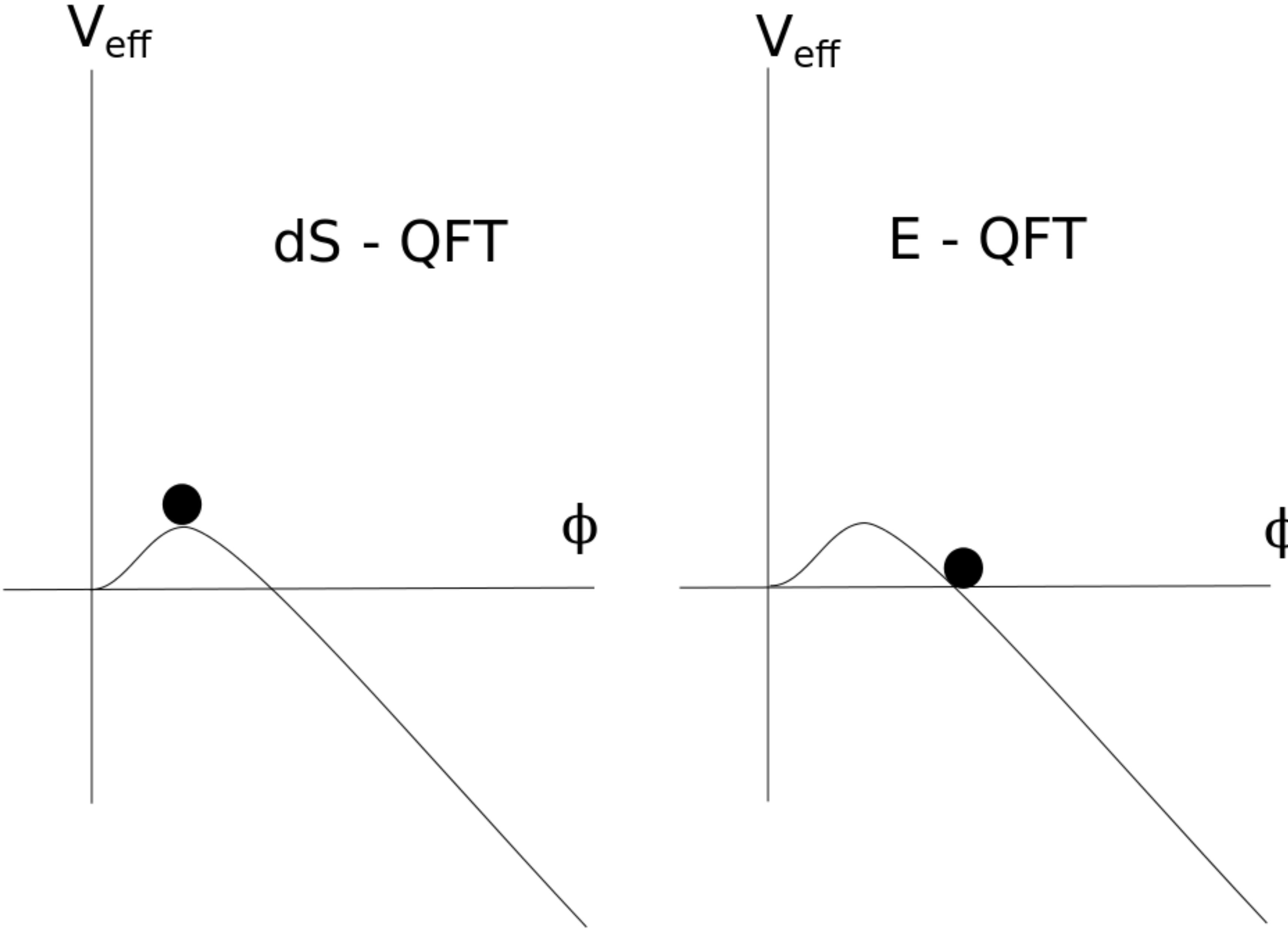
Small scalar flow
small large-N dS condensate
E-frame still crunches

- An unstable marginal operator on the boundary is related to a Coleman de Luccia bubble in the bulk.

$$\mathcal{L}_{\rm eff}[\phi] = -\frac{1}{2}\,(\partial\phi)^2 - \frac{d-2}{8(d-1)}\mathcal{R}_d\,\phi^2 - \lambda\,\phi^{\frac{2d}{d-2}} + \mathcal{O}\left(\phi^{\frac{2(d-4)}{d-2}},\partial^4\right)\,.$$



V_{eff}
dS - QFT
ϕ
V_{eff}
E - QFT
ϕ

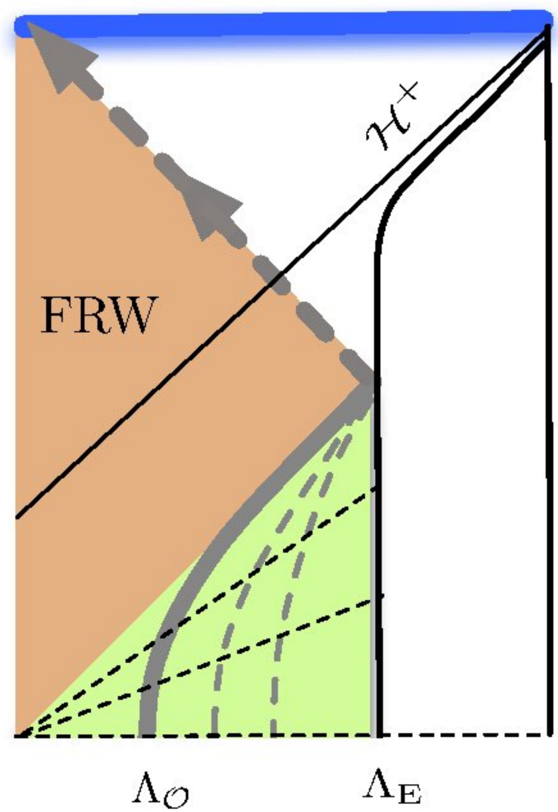


V_eff
dS - QFT
ϕ
V_eff
E - QFT
ϕ

- As seen on the boundary this crunch situation involves a flow to infinity at a finite time and need not be healed in the bulk.

BULK

COMPLEMENTARITY



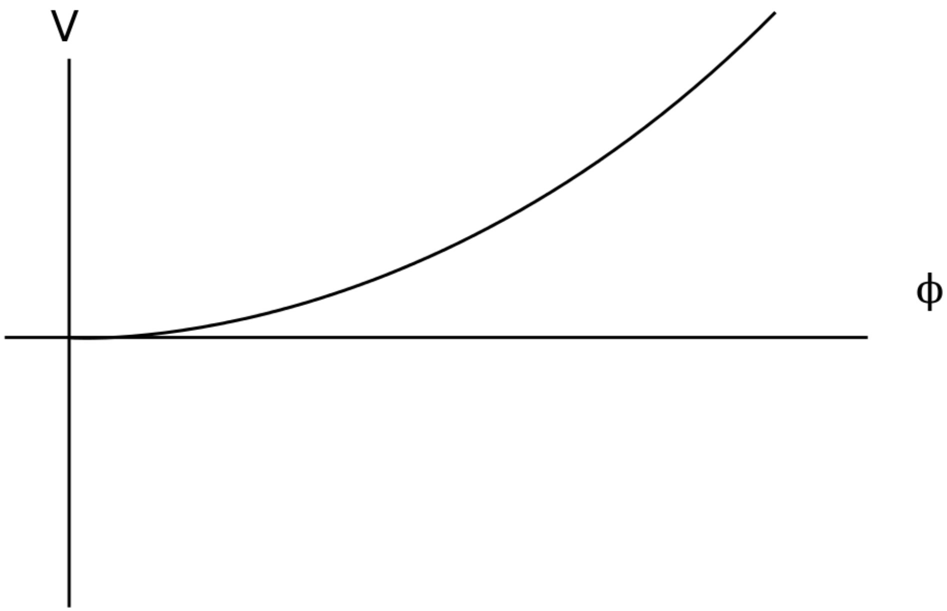
FRW
$\mathcal{H}^+$
$\Lambda_{\mathcal{O}}$
$\Lambda_{\rm E}$

BUTTERFLIES

Time dependent butterfly-like boundary potential

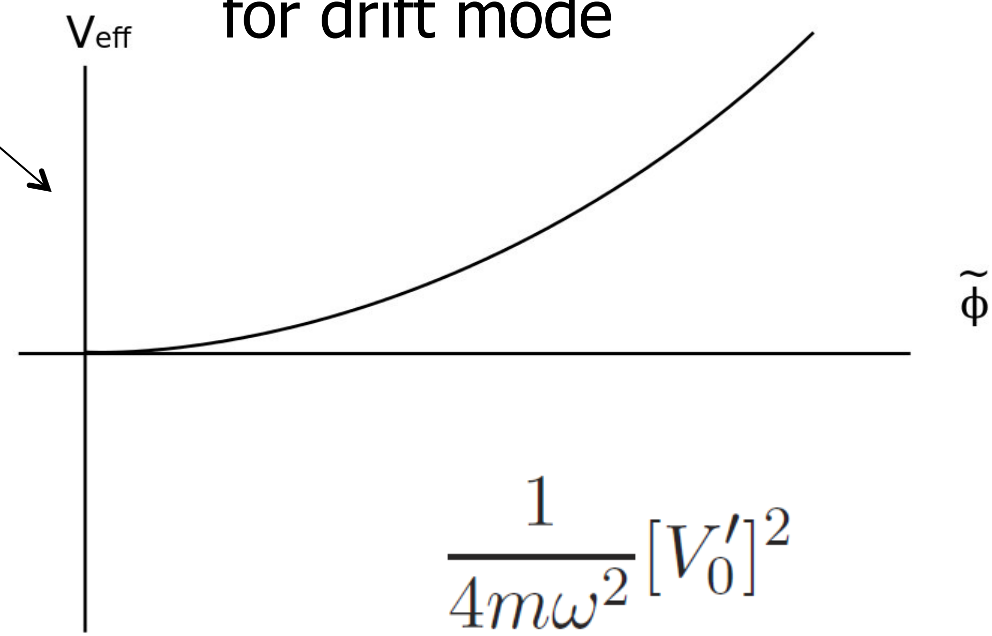
$$V(\phi,t) = \frac{\lambda}{4}\cos(\omega t)\phi^4 \, ,$$

- Stable?
- What is the dual theory in the bulk?



Positive effective potential for drift mode

High frequencies of oscillation



$$\frac{1}{4m\omega^2}[V_0']^2$$

Free Field theory

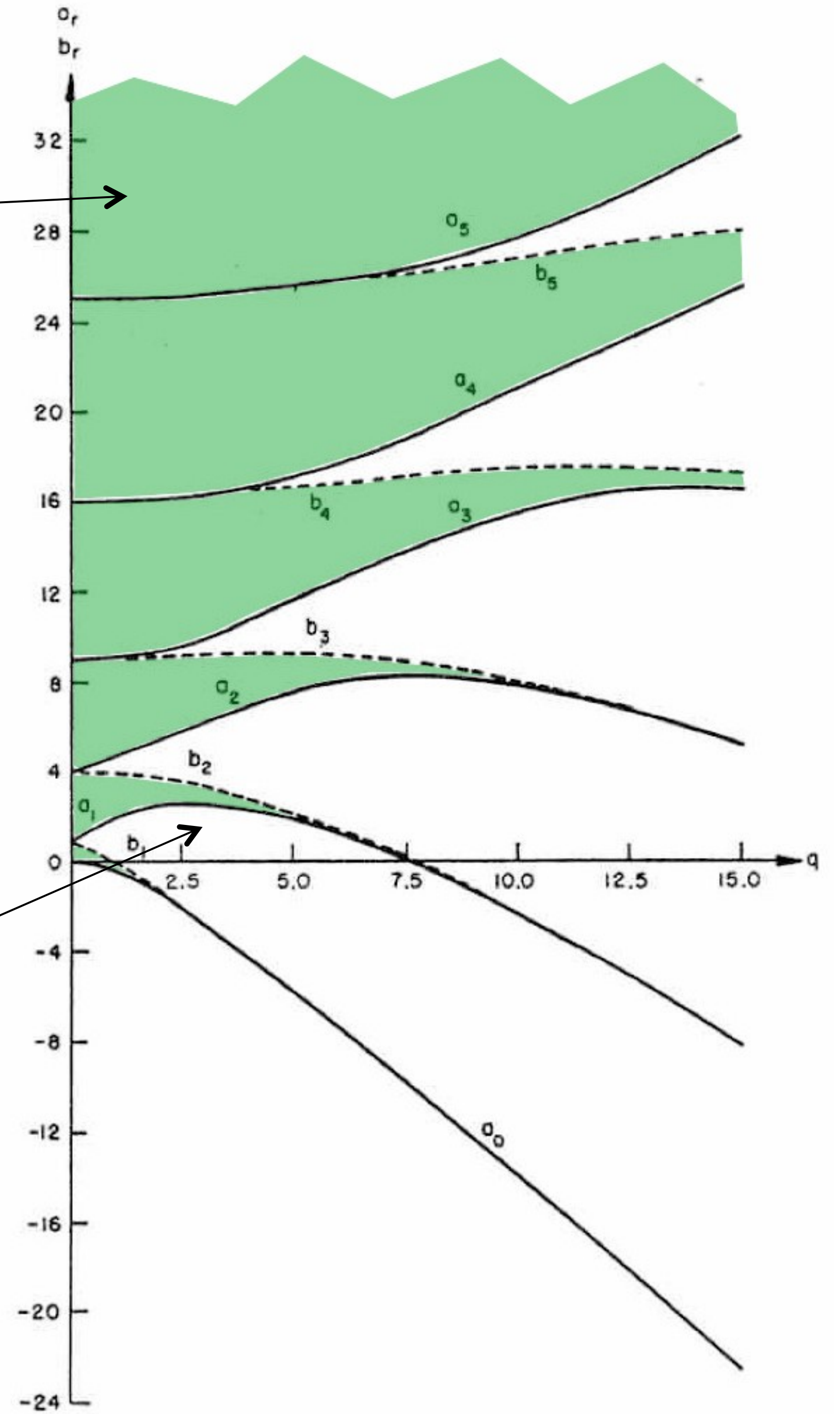
Stability zones

Mathieu equation

$$\ddot{\phi} + [a - 2q\cos(2\tau)]\,\phi = 0\,,$$

Instability zones

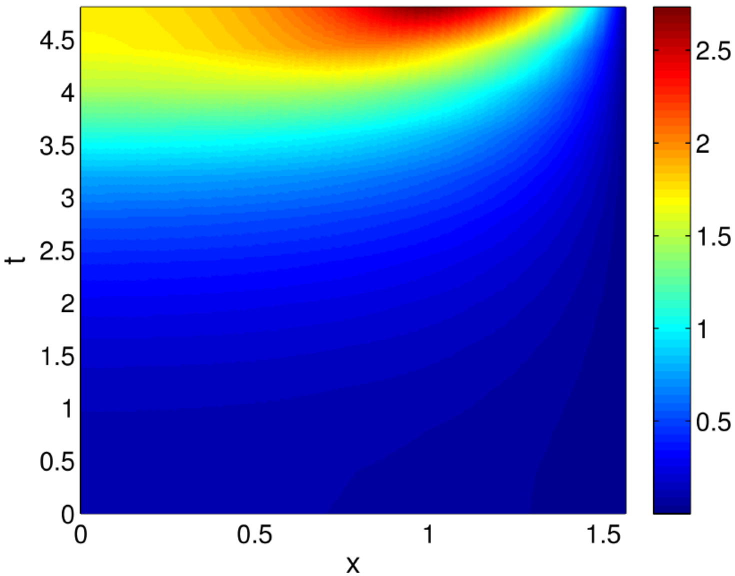
$$a \equiv \frac{4\vec{k}^2}{\omega^2}\,, \qquad q \equiv \frac{2m^2}{\omega^2}\,.$$



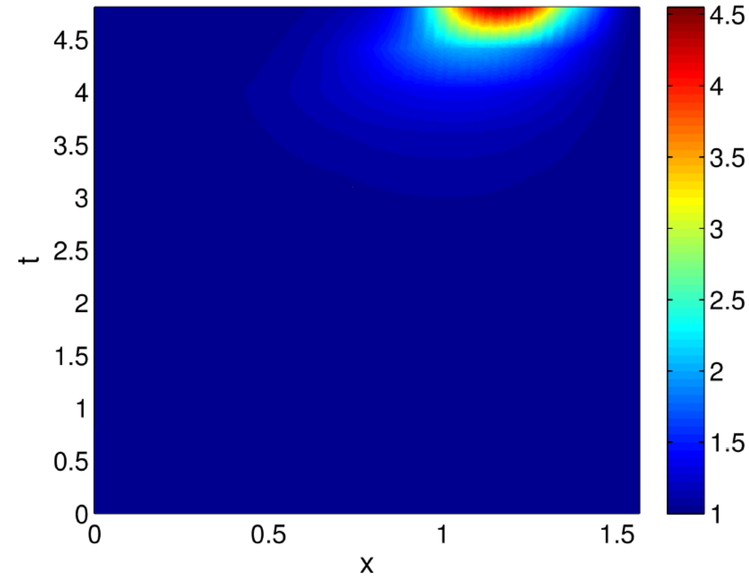
- For $a < 2q$ instabilities are cured
- For $a > 2q$ resonances may appear
- For compactified world volume resonances can be avoided (number theory results)
- What about interacting boundary field theory?
  - It should thermalize
- Go to the bulk

Expectations

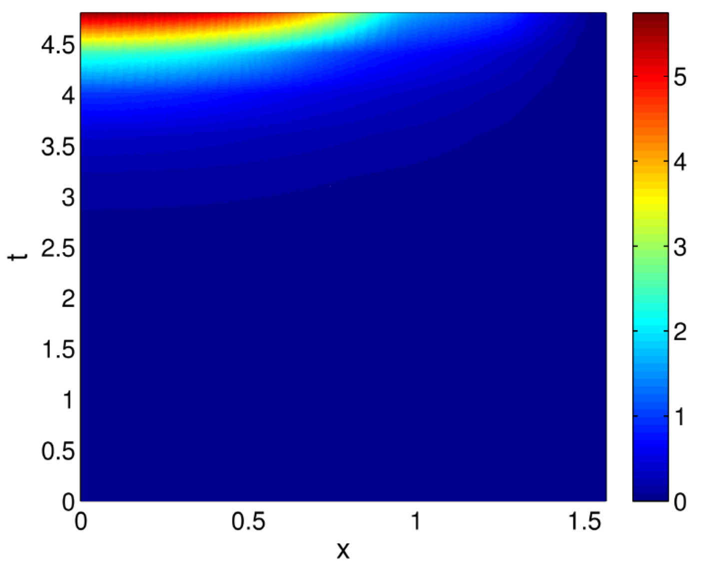
- When the boundary theory is unstable, the bulk would crunch?
- When the boundary theory is stabilized, then the bulk is healed?
- An interacting boundary theory can thermalize and produce a black hole in the bulk?
- Using AdS/CFT dictionary and numerical analysis



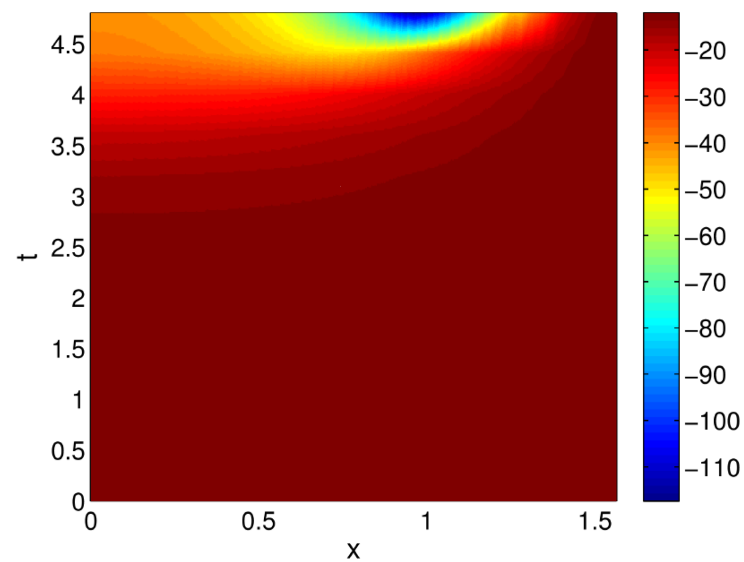
(a) $\phi(x,t)$



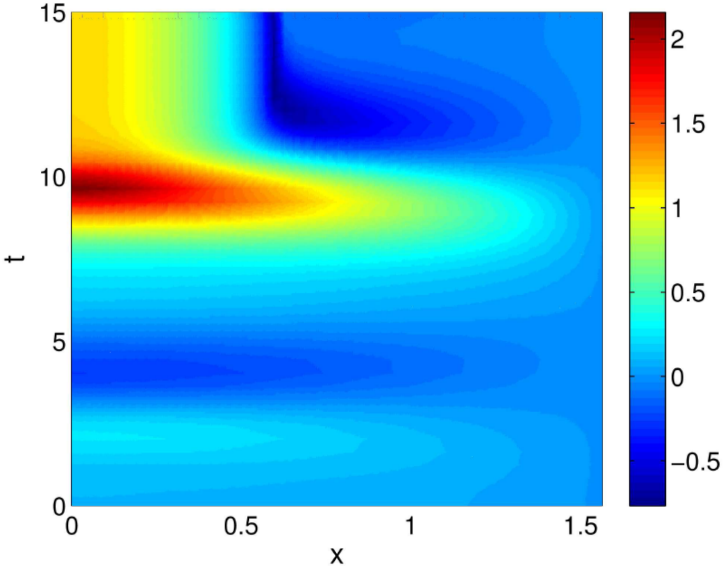
(b) $A(x,t)$



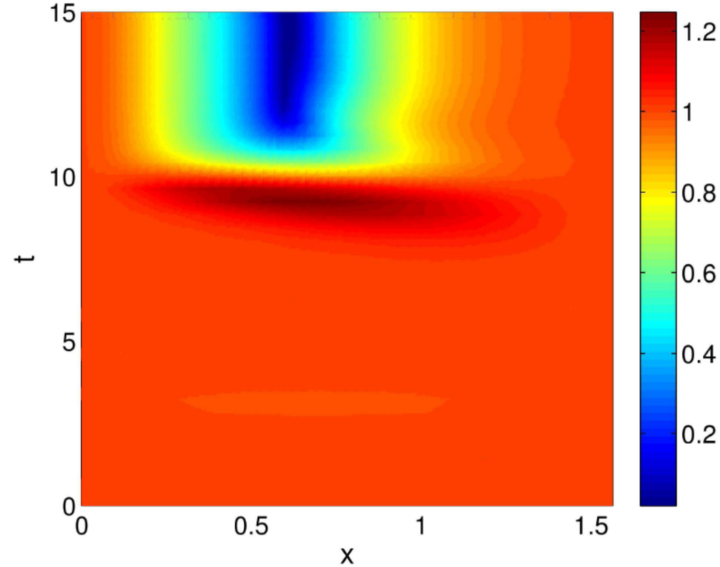
(c) $\delta(x,t)$



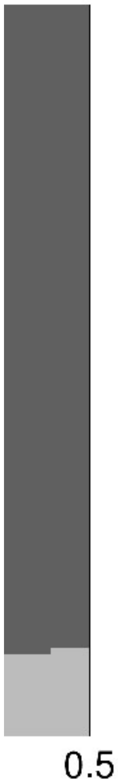
(d) scalar curvature $R(x,t)$

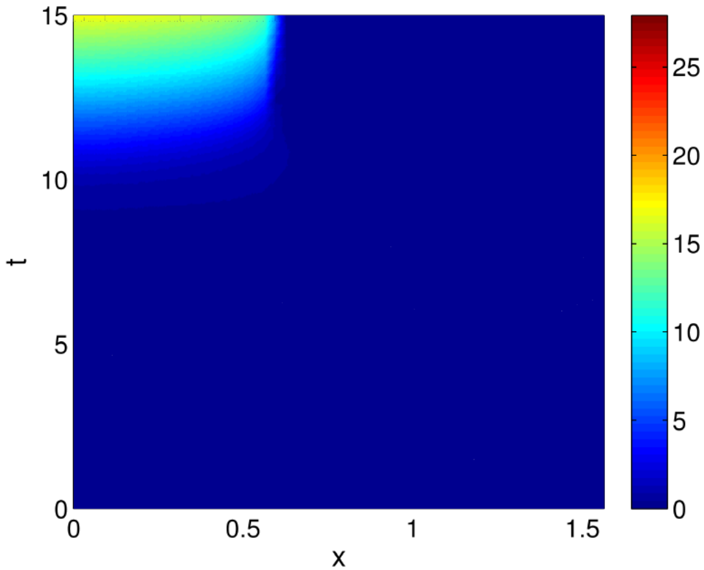


(a) $\phi(x,t)$

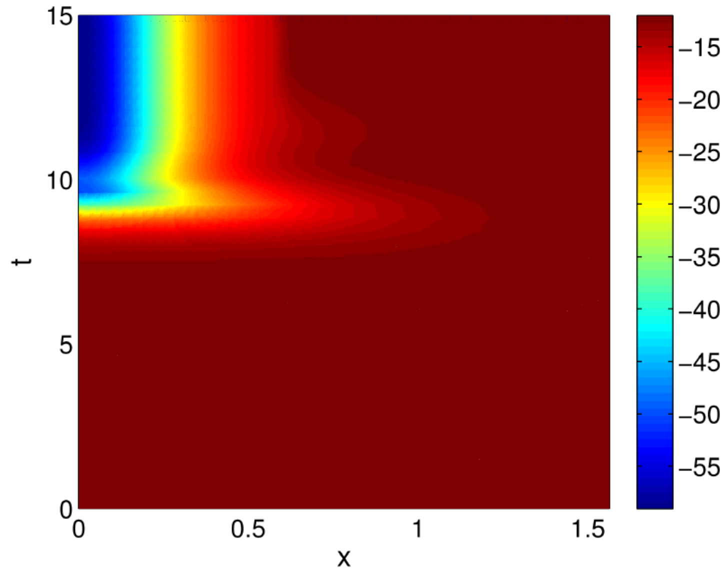


(b) $A(x,t)$

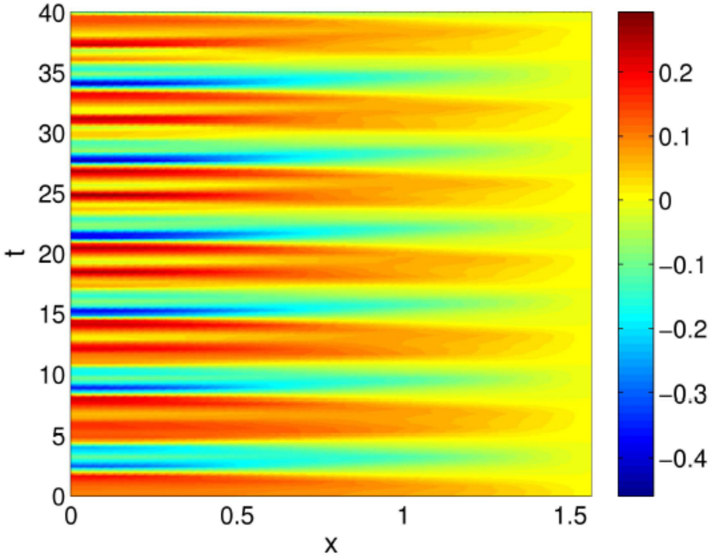




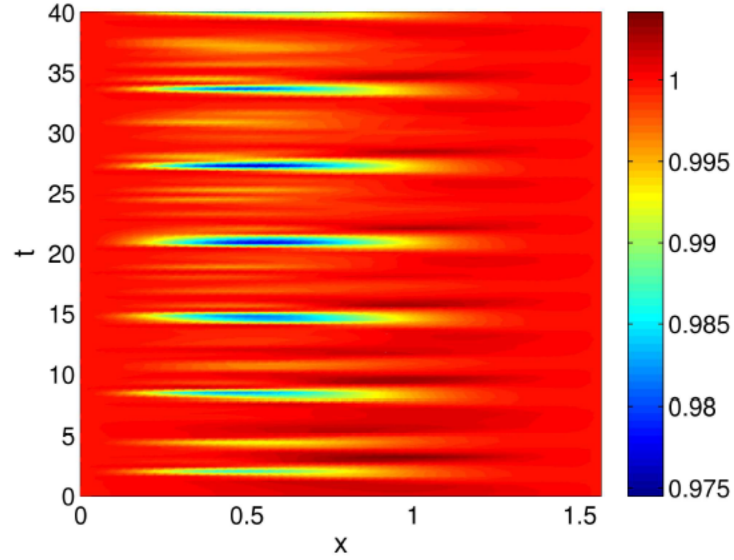
(c) $\delta(x,t)$



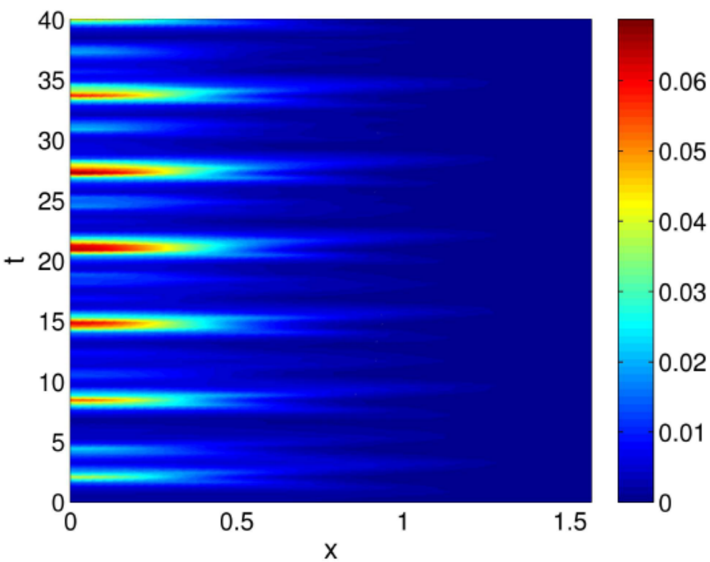
(d) scalar curvature $R(x,t)$



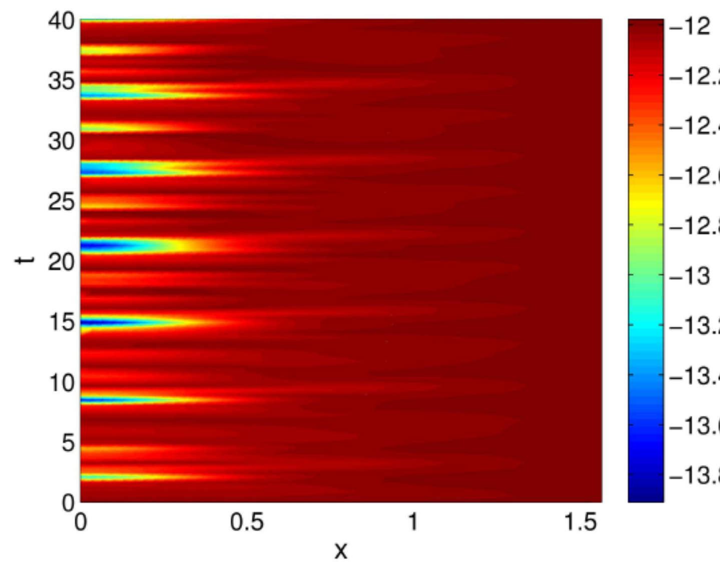
(a) $\phi(x, t)$



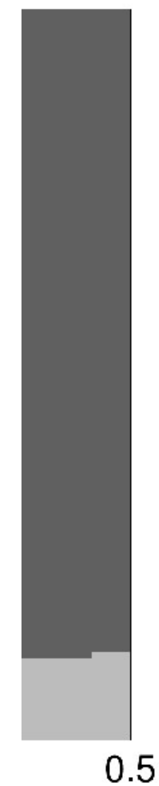
(b) $A(x, t)$



(c) $\delta(x, t)$



(d) scalar curvature $R(x, t)$



Conclusions

- Crunches can be described by complementary non commuting Hamiltonians. With or without drama.
- Time dependent boundary Hamiltonians can heal crunch singularities.