Heterotic Twistor-String Theory

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based on work w/ Lionel Mason

+ Witten 0504078,
  Nekrasov 0511008,
  Katz + Sharpe 0406226,
  Adams, Distler + Ernebjerg 0506263

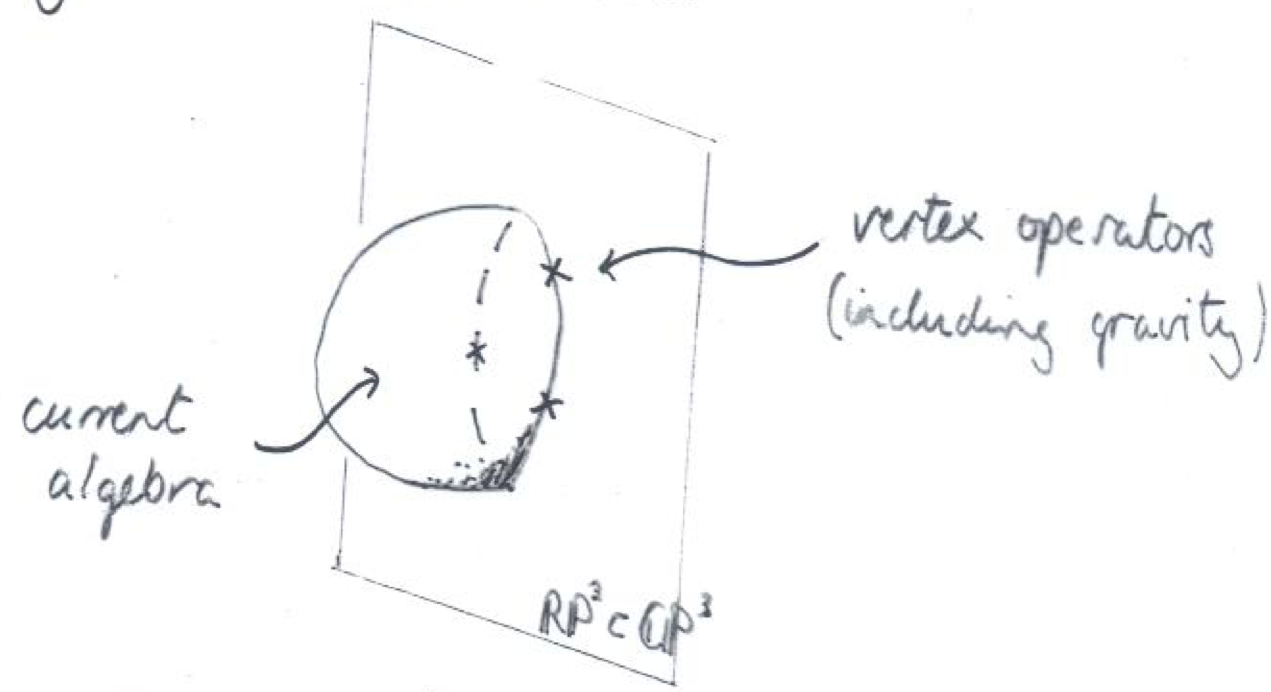
+ standard twistor-string papers.

Why heterotic?

1. GRAVITY

- Perturbatively, the open B-model does not contain gravity. We would appear to need to understand string field theory at a non-perturbati[ve] level.

- Gravity and YM arise perturbatively, <u>but unusually</u> in Berkovits' model



2. "UNIFIED VIEW" OF TWISTOR STRINGS

Heterotic

Witten	Berkovits
Dolbeault	Čech

5. OTHER PROBLEMS

- Topological B-model on supermanifold
  - is $\mathbb{P}^{3|4}$ a CY threefold?
- Role of D1-D1 strings / other D-branes?
- Measure on curves $g \geq 1$
- Spacetime signature

⋮

∴ SLOGAN

"Just as low-energy limit of physical heterotic string is sugra + SYM, for topologically twisted heterotic string these states are all that survive"

Twisted (0,2) Basics, I

$$\phi : \Sigma \to X$$

$\phi \in \mathrm{Maps}(\Sigma, X)$

Twisted (0,2) susy requires that we also introduce

$$\rho^i_{\bar z} \in \Gamma(\Sigma, \bar K_\Sigma \otimes \phi^* T_X) \qquad \rho^{\bar j} \in \Gamma(\Sigma, \phi^* \overline{T_X})$$

related to ϕ by the (0,2) transformations

$$\{\bar Q, \phi^i\} = 0 \qquad \boxed{\{\bar Q, \phi^{\bar j}\} = \rho^{\bar j}}$$

$$\boxed{\{\bar Q, \rho^i_{\bar z}\} = \partial_{\bar z}\phi^i} \qquad \{\bar Q, \rho^{\bar j}\} = 0$$

$$\left(\begin{array}{ll} \{\bar Q^\dagger_{\bar z}, \phi^i\} = \rho^i_{\bar z} & \{\bar Q^\dagger_{\bar z}, \phi^{\bar j}\} = 0 \\ \{\bar Q^\dagger_{\bar z}, \rho^i_{\bar z}\} = 0 & \{\bar Q^\dagger_{\bar z}, \rho^{\bar j}\} = \partial_{\bar z}\phi^{\bar j} \end{array}\right)$$

where $\{\bar Q, \bar Q^\dagger_{\bar z}\} = \partial_{\bar z}$, $\bar Q^2 = 0 = \bar Q^{\dagger\,2}_{\bar z}$ and $\bar Q$ is a scalar operator $\Rightarrow$ BRST operator.

$$\boxed{\text{$\bar Q$ is the $\bar\partial$ operator on the space of maps $\Sigma \to X$ and $\bar Q$-invariant configurations are holomorphic maps}}$$

Just as for the physical heterotic string, we can couple our sigma-model to a bundle $V \to X$ by introducing left-moving fermions

$$\Psi^a \in \Gamma(\Sigma, \phi^* V) \qquad \bar{\Psi}_{az} \in \Gamma(\Sigma, K_\Sigma \otimes \phi^* V^\vee)$$

with (0,2) transformations

$$\{\bar{Q}, \Psi^a\} = 0 \qquad \{\bar{Q}, \bar{\Psi}_{az}\} = \bar{r}_{az}$$

$$\{\bar{Q}, r^a_{\bar z}\} = D_{\bar z}\Psi^a + F_{i\bar{j}}{}^a{}_b \Psi^b \rho^i_{\bar z} \rho^{\bar j} \qquad \{\bar{Q}, \bar{r}_{az}\} = 0$$

(0,2) SUSY $\Rightarrow$ V a <u>holomorphic</u> bundle

$$\boxed{\begin{aligned} S &= \int_\Sigma \{\bar{Q}, g_{i\bar j}\rho^i_{\bar z}\partial_z \phi^{\bar j} + \bar{\Psi}_{az} r^a_{\bar z}\}\, d^2z \\ &= \int_\Sigma g_{i\bar j}\left(\partial_{\bar z}\phi^i \partial_z \phi^{\bar j} + \rho^i_{\bar z} \nabla_z \rho^{\bar j}\right) \\ &\qquad + \bar{\Psi}_{az} D_{\bar z}\Psi^a + \bar{\Psi}_{az} F_{i\bar j}{}^a{}_b \Psi^b \rho^i_{\bar z}\rho^{\bar j} + \bar{r}_{az} r^a_{\bar z} \end{aligned}}$$

- Can also add $i\int_\Sigma \phi^*(\omega - iB)$ if $\bar\partial B = 0$
- Reduces to the A-model action if $V = T_X$ only using $\bar{Q}$ as the BRST operator
- (4) In general ($\bar\partial B \neq 0$), target must be "Kähler + torsion"

Twistor Target

Could try $X = \mathbb{P}^{3|4}$ with no bundle, but

- can't set $\bar{\Psi} = 0$ using D-branes
- have to understand $\bar{r} (= \delta \bar{\Psi})$
  - dynamical fields

Another approach : choose $\underline{X = \mathbb{P}^3}$, $\underline{V = \mathcal{O}(1)^{\oplus 4}}$

Basic idea : $\Psi^a \in \Gamma(\Sigma, \phi^* \mathcal{O}(1)^{\oplus 4})$ are exactly same type of field as Witten / Berkovits use to describe odd directions of $\mathbb{P}^{3|4}$

$$S^{\phi}_{kin} = \int_{\Sigma} |d^2 z| \; g_{i\bar{j}} \, \partial_{\bar{z}} \phi^i \partial_z \phi^{\bar{j}}$$

$$S^{\Psi}_{kin} = \int_{\Sigma} |d^2 z| \; \bar{\Psi}_{az} \, D_{\bar{z}} \Psi^a$$

'hybrid' σ-model / $\beta\gamma$-system

⌐

Anomalies

$$S \supset \int_\Sigma |d^2z|\; g_{i\bar{j}}\, \rho^i_{\bar{z}} \nabla_z \rho^{\bar{j}} + \bar{\Psi}_{az} D_{\bar{z}} \Psi^a$$

chiral fermions $\to$ possible σ-model anomaly

$$\det{}'(\partial_{\phi^* T_X})\ \det{}'(\bar{\partial}_{\phi^* V}) \in \Gamma(\mathcal{L})$$

$$F^{(\mathcal{L})} = \int \underbrace{\phi^*(ch_2(V) - ch_2(T_X))}_{\substack{\text{usual Green-Schwarz}\\ \text{condition}}} + \underbrace{\tfrac{1}{2} c_1(T_\Sigma)\phi^*(c_1(V) - c_1(T_X))}_{\substack{\text{arises because of}\\ \text{twisting}}}$$

When $X = \mathbb{P}^3$ and $V = \mathcal{O}(1)^{\oplus 4}$ we have

$$c(T_{\mathbb{P}^3}) = c(\mathcal{O}(1)^{\oplus 4}) \quad \text{for } \underline{\text{all}} \text{ Chern classes}$$

(c.f. $ch_0(T_{\mathbb{P}^{3|4}}) = -1$, $ch_i(T_{\mathbb{P}^{3|4}}) = 0 \quad i \geq 1$)

so no σ-model anomaly.

Aside: for A-model, $V = T_X$ so always anomaly-free,
for B-model, $V = T_X^*$ so anom. free iff $c_1(T_X) = 0$

Global symmetrics

	ϕ	ρ	$\bar\rho$	ψ	$\bar\psi$	r	$\bar r$
$U(1)_R$	·	-1	$+1$	·	·	-1	$+1$
$U(1)_F$	·	·	·	$+1$	-1	$+1$	-1

$$U(1)_R : \quad \text{ind}\ \bar\partial_{\phi^* T_{\mathbb{P}^3}} = 4d + 3(1-g)$$

$$U(1)_F : \quad \text{ind}\ \bar\partial_{\phi^* \mathcal{O}(1)^{\oplus 4}} = 4(d+1-g)$$

Generically, have $4(d+1-g)$ zero modes of ψ which must be absorbed by inserted vertex operators.

Vertex operator for SYM state of helicity h has $2h+2$ associated ψs (see later...).

Amplitudes with n_h external SYM states of helicity h generically supported on curves of degree

$$\boxed{d = g - 1 + \sum^{1} \frac{h+1}{2} n_h}$$

If $\phi : \Sigma \to \mathbb{P}^3$ is holomorphic, nearby map $\phi + \delta\phi$ is too iff $\delta\phi \in H^0(\Sigma, \phi^* T_{\mathbb{P}^3})$.
But $\bar{\rho}$ zero-mode is an anticommuting element of $H^0(\Sigma, \phi^* T_{\mathbb{P}^3})$

$\bar{\rho}$ zero modes $\Longleftrightarrow$ $(0,1)$ forms on $\mathcal{M}$ (instanton moduli space)

At genus zero, can show $\overline{\mathcal{M}} = \mathbb{P}^{3+4d}$.

How do we interpret ψ zero-modes, which lie in $H^0(\Sigma, \phi^* \mathcal{V})$? Construct a sheaf W on $\mathcal{M}$ as follows:

$$\begin{tikzcd} \mathcal{M} \times \Sigma \arrow[r, "\Phi"] \arrow[d, "\pi"'] & \mathbb{P}^3 \\ \mathcal{M} & \end{tikzcd}$$

$$W(U) = \pi_* \Phi^* V(U) = \Phi^* V(\pi^{-1} U) = H^0(U \times \Sigma, \Phi^* V)$$

Then (generically) $W|_\phi = H^0(\Sigma, \phi^* V)$

At genus zero, can show $W = \mathcal{O}_{\mathcal{M}}(1)^{\oplus 4d+4}$,
and in particular

$$\left(\Lambda^{\text{top}}\, \mathcal{O}_{\mathcal{M}}(1)^{\oplus 4d+4}\right)^{\vee} \simeq K_{\mathcal{M}}$$

... analogue of $\mathbb{P}^{3+4d|4+4d}$ instanton moduli space being super-CY.

More generally $(g > 0)$ it is difficult to describe either $\mathcal{M}$ or W explicitly, but can still show $(\Lambda^{\text{top}} W)^{\vee} \simeq K_{\mathcal{M}}$

$\Rightarrow$ instanton moduli spaces at $g \geq 1$ are still "super-CY", for <u>fixed</u> worldsheet complex structure.

Vertex Operators in $\bar{Q}$ cohomology

$$T_{\bar{z}\bar{z}} = g_{i\bar{j}}\left(\partial_{\bar{z}}\phi^i \partial_{\bar{z}}\phi^{\bar{j}} + \rho^i_{\bar{z}} \nabla_{\bar{z}} \rho^{\bar{j}}\right) = \{\bar{Q}, g_{i\bar{j}}\, \rho^i_{\bar{z}}\, \partial_{\bar{z}}\phi^{\bar{j}}\}$$

so all the antiholomorphic Virasoro generators $\bar{L}_n$ are $\underline{\bar{Q}\text{-exact}}$.

$$[\bar{L}_0, \mathcal{O}] = \bar{h}\mathcal{O}\,, \quad \text{but if} \quad \bar{L}_0 = \{\bar{Q}, \bar{\varsigma}_0\} \quad \text{then}$$

$$\bar{h}\mathcal{O} = [\{\bar{Q}, \bar{\varsigma}_0\}, \mathcal{O}] = \underbrace{\{\bar{Q}, [\bar{\varsigma}_0, \mathcal{O}]\}}_{\bar{Q}\text{-exact}} + \underbrace{\{[\bar{Q}, \mathcal{O}], \bar{\varsigma}_0\}}_{=0}$$

Conclude that $\bar{Q}$ cohomology is trivial except on states with $\bar{h} = 0$.

> In the A- or B-model, similarly we'd find $h = 0$, but here there is no holomorphic susy algebra, so all $h \geq 0$ are allowed

We've found an infinite tower of vertex operators - far too many for twistor-string theory - reflecting the fact that twisted $(0,2)$ theories are CFTs (albeit with special properties) and not TFTs.

There's an interesting subset where $(h,\bar{h}) = (1,0)$ and ghost number $gh = 1$

$$B_{i\bar{j}}(\phi,\bar{\phi},\psi)\,\rho^{\bar{j}}\partial_z\phi^i \qquad \beta_{a\bar{j}}(\phi,\bar{\phi},\psi)\,\rho^{\bar{j}}\partial_z\psi$$

$$g_{i\bar{k}}\,J^i{}_{\bar{j}}(\phi,\bar{\phi},\psi)\,\rho^{\bar{j}}\partial_z\phi^{\bar{k}} \qquad M^a{}_{\bar{j}}(\phi,\bar{\phi},\psi)\,\rho^{\bar{j}}\,\bar{\psi}_{az}$$

which are non-trivial iff

$$[J] \neq 0 \in \bigoplus_{q=0}^{4} \overbrace{H^{0,1}(\mathbb{PT}', T_{\mathbb{PT}'} \otimes \Lambda^q V^*)}^{\text{linearized graviton, helicity } -}$$

$\uparrow$ $N=4$ extension

$$[M] \neq 0 \in \bigoplus_{q=0}^{4} \underbrace{H^{0,1}(\mathbb{PT}', V \otimes \Lambda^q V^*)}_{\text{four } N=4 \text{ gravitino multiplets}}$$

$$[H] \neq 0 \in \bigoplus_{q=0}^{4} \underbrace{H^{0,1}(\mathbb{PT}', \Omega^2_{\mathbb{PT}'} \otimes \Lambda^q V^*)}_{N=4 \text{ linearized gravitons, helicity } +2}$$

$H = dB$ locally (NS fieldstrength)

$$[\mathcal{H}] \neq 0 \in \bigoplus_{q=0}^{4} \underbrace{H^{0,1}(\mathbb{PT}', \Omega^1_{\mathbb{PT}'} \otimes V^* \otimes \Lambda^q V^*)}_{\text{four (CPT conjugate) gravitino multiplets}}$$

Together, these comprise the states of $N=4$ CSUGRA at the linearized level by the Penrose transform.

Coupling to Yang-Mills

Introduce a new bundle $E \to \mathbb{P}^3$ and

$$\lambda^\alpha \in \Gamma(\Sigma, K_\Sigma^{1/2} \otimes \phi^* E)$$
$$\bar{\lambda}_\alpha \in \Gamma(\Sigma, K_\Sigma^{1/2} \otimes \phi^* E^\vee)$$

w/s spinors

$$S = \int_\Sigma |d^2z| \; \bar{\lambda}_\alpha D_{\bar{z}} \lambda^\alpha + \bar{\lambda}_\alpha F_{i\bar{j}}{}^\alpha{}_\beta \lambda^\beta \rho^i \bar{\rho}^{\bar{j}} \; (+\text{ auxiliary})$$

(0,2) susy requires $E \to \mathbb{P}^3$ holomorphic.

Conf invariance " $g^{i\bar{j}} F_{i\bar{j}} = 0 \Rightarrow \boxed{c_1(E) = 0}$

No σ-model anomalies if $c_2(E) = 0$ also, implying the YM bundle on spacetime has no instantons.

In fact, we <u>can</u> couple heterotic twistor strings to instanton bundles.

· Heterotic strings contain NS branes which couple magnetically to B-field.

Physical Heterotic Theory $\Rightarrow$ NS 5-branes

Twisted $(0,2)$ on threefold $\Rightarrow$ NS 1-brane

If background contains an NS brane wrapping a 2-cycle $C \subset X$, Green-Schwarz condition is modified

$$dH = \overbrace{\mathrm{tr}\, R^2 - \mathrm{tr}\, F_V^2}^{\text{cancel}} - \mathrm{tr}\, F_E^2 + \tilde{C}$$

where $[\tilde{C}] \in H^4(X, \mathbb{Z})$ is Poincaré dual to class of C.

$$\boxed{c_2(E) = [\tilde{C}]}$$

In fact, connection between twistor curves and YM instantons known to Atiyah + Ward, Hartshorne in '70s.

eg $A = dx^\mu \sigma_{\mu\nu} \partial^\nu \log \Phi \quad ; \quad \Phi = \sum_{i=0}^{K} \dfrac{\lambda_i}{(x - x_i)^2}$

$SU(2)$ 't Hooft K-instanton

13 $\Rightarrow$ wrap NS branes on twistor $\mathbb{P}$'s corresponding to each x_i

• The $\bar{\lambda}\lambda$ system is precisely the current algebra on Witten's D-instantons or on Berkovits' worldsheet. It's incorporated very naturally in the heterotic framework.

• Get another vertex operator at $(h, \bar{h}) = (1,0)$ and $U(1)_R$-charge $+1$:

$$\bar{\lambda}_\alpha A^\alpha_{\bar{j}\,\beta}(\phi, \bar{\phi}, \psi)\, \lambda^\beta \rho^{\bar{j}}$$

non-trivial in $\bar{Q}$-cohomology iff

$$[A] \neq 0 \in H^{0,1}\left(\mathbb{PT}', \mathrm{End}\, E \otimes \Lambda^{\cdot} V^\vee\right)$$

giving an $\mathcal{N}=4$ SYM multiplet via Penrose transform.

• Henceforth choose E trivial, rank r, vanishing background connection (free current algebra)

Coupling to Worldsheet Gravity

(0,2) models inherently depend on a choice of complex structure on Σ.

$$T_{\bar{z}\bar{z}} = \{\bar{Q}, G_{\bar{z}\bar{z}}\} \qquad T_{zz} \neq \{\bar{Q}, \cdot\}$$

To integrate over moduli space $\overline{\mathcal{M}_{g,n}}$ we can use $G_{\bar{z}\bar{z}}$ as "$\bar{b}$-antighosts" to provide a top antiholomorphic form on $\overline{\mathcal{M}_{g,n}}$, but need to introduce holomorphic reparametrization ghosts

$c \in \Gamma(\Sigma, T_\Sigma) \qquad b \in \Gamma(\Sigma, K_\Sigma \otimes K_\Sigma)$

(Related fact: (0,2) models are only "half topological")

$$\boxed{Q_{BRST} = Q_{phys} + \bar{Q}}$$

$Q^2_{BRST} = 0$ requires $rk(E) = 28$ (as for Berkovits). Exactly analogous to $c_{YM} = 16$ in physical heterotic string. Troubling result!!!

	Physical Heterotic	Twistor String
c_{YM}	16	28
spacetime field theory	$SO(32)$, $E_8 \times E_8$, $E_8 \times U(1)^{248}$, $U(1)^{496}$	$SU(2) \times U(1)$ $U(1)^4$
modular invariance	$SO(32)$, $E_8 \times E_8$	?

- Change level of current algebra ?
- Include additional fields to make up c ?
- Avoid introducing bc system so requirement becomes $c_{YM} = 2$? (... sits well with $U(1)^4$)

Clear that modular invariance is key test - at present, $g > 0$ twistor strings seem <u>not</u> to be SYM + Conf. SUGRA, but just plain inconsistent!

 ... perhaps that's not such a bad thing!

Amplitudes

(Fixed) vertex operators in the string theory must be diff × Weyl invariant (so must have $(h,\bar{h}) = (0,0)$) and linear in the c-ghost.

$$V_{\text{string}} = c^z U_z$$

σ-model vertex operator of $(h,\bar{h}) = (1,0)$

eg $$V_{YM} = c\,\bar{\lambda}_\alpha A_{\bar{j}}(\phi,\bar{\phi},\psi)^\alpha{}_\beta \lambda^\beta \rho^{\bar{j}}$$

Out of the entire sheaf of chiral algebras, only these moduli survive as vertex operators in the string theory.

There's a standard 'descent' procedure to produce an integrated vertex operator, involving both the $\bar{Q}^+_{\bar{z}}$ supercharge and b antighost zero-mode.

$\boxed{17}$

Wish to compute

$$\left\langle V^1_{YM} \cdots V^n_{YM} \right\rangle$$

$$\overset{?}{=} \int [d\Phi]\, e^{-S} \prod_{i=1}^{3g-3+n} (\mu^i, b)(\bar{\mu}^i, \bar{G})$$

$$\times V^1_{YM} \cdots V^n_{YM}$$

Anomaly in bc-system ghost number:

$$\text{ind}\ \bar{\partial}_{T_\Sigma} = 3 - 3g$$

completely absorbed by $3g-3+n$ factors of (μ^i, b) together with n vertex operators.

Anomaly in $U(1)_R$ (or $(0,2)$ ghost number):

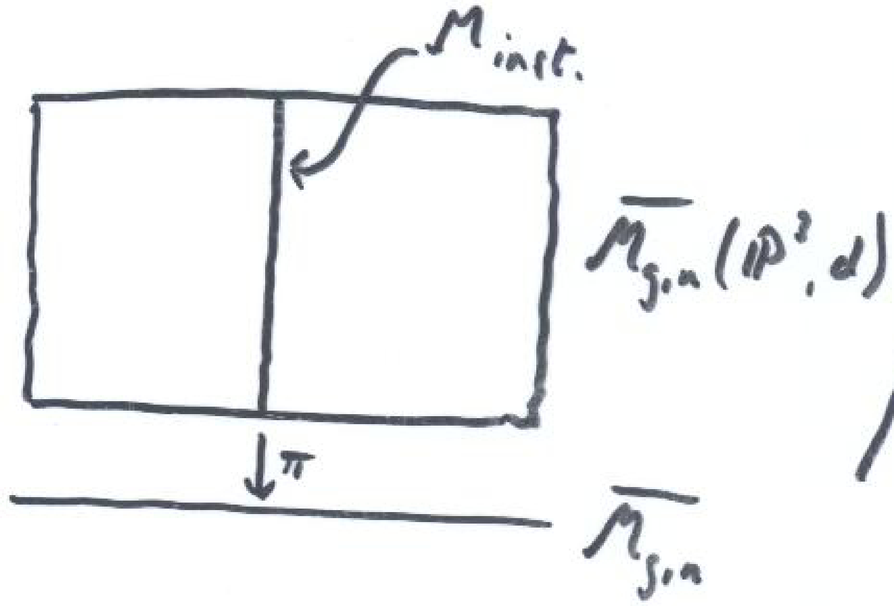
$$\text{ind}\ \bar{\partial}_{\phi^* T_{\mathbb{P}^3}} = 3 - 3g + 4d$$

but now anomaly of $4d$ still remains.

$\overline{19})$

Easy to interpret : path integral of zero modes is over "moduli space of stable maps" $\overline{\mathcal{M}}_{g,n}(\mathbb{P}^3, d)$

(roughly,



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Easy to compute $\dim_{\mathbb{C}} \overline{\mathcal{M}}_{g,0}(\mathbb{P}^3, d) = 4d$.

- Path integral provides a $(4d, 0)$-form on this moduli space.

- Should be integrating over a half-dimensional real slice - i.e. a contour.

How do we implement this in the path integral?

- Pick a contour of appropriate dimension and find Poincaré dual.

- This $4d$-form will have a $(0,4d)$ component $\Gamma_{\bar{A}_1 \cdots \bar{A}_{4d}} d\bar{t}^{\bar{A}_1} \wedge \cdots \wedge d\bar{t}^{\bar{A}_{4d}}$

- Pick a basis of $\bar{\rho}$ zero-modes $\{\bar{\rho}^{\bar{A}}\}$ with $\bar{\rho}^{\bar{J}} = \bar{\rho}^{\bar{A}} \dfrac{\partial \phi^{\bar{J}}}{\partial \bar{t}^{\bar{A}}}$; ϕ holomorphic

- Insert

$$\Gamma := \Gamma_{\bar{A}_1 \cdots \bar{A}_{4d}} \bar{\rho}^{\bar{A}_1} \cdots \bar{\rho}^{\bar{A}_{4d}}$$

into path integral

The remaining anomaly is now saturated, and the path integral reduces to an integral over the contour. From here on, can follow [illegible] calculation word-for-word

Is there a more systematic way to include the contour?

Contour is <u>real slice</u> of $\overline{M}_{g,0}(\mathbb{P}^3, d)$, and at $d = 1$ is just a real slice of complexified spacetime.

Given real structures on Σ and on $\mathbb{P}^3$ we can induce a prefered real structure on the (moduli) space of maps by considering only equivariant maps

$$\begin{tikzcd}
\Sigma \arrow[r, "\text{c.c.}"] \arrow[d, "\phi"'] & \Sigma \arrow[d, "\phi"] \\
\mathbb{P}^3 \arrow[r, "\text{c.c.}"'] & \mathbb{P}^3
\end{tikzcd}$$

Simplest example $Z^\alpha \mapsto \overline{Z^\alpha}$, $z \mapsto 1/\bar{z}$

- Fixes $\mathbb{RP}^3 \subset \mathbb{CP}^3$

  $S^1 \subset \mathbb{P}^1 \quad (g=0)$

- Equivariant maps must have

  $\phi(\partial\Sigma) \longrightarrow \mathbb{RP}^3$

  with vertex operators inserted only on $\partial\Sigma$

- Amounts to <u>orientifold</u> <u>projection</u> of the string theory, with $\mathbb{RP}^3$ an orientifold fixed plane (not a D-brane!)

Also possible to construct a projection giving Euclidean signature (though less successful)
Not possible to get Lorentz signature this way

Relation to Berkovits' model

$$\int_{D\subset\mathbb{C}} dz\wedge d\bar{z}\ \delta^2(z,\bar{z})\, f(z) = \frac{1}{2\pi i}\oint_{\Gamma} \frac{f(z)}{z}\, dz$$

$$= \frac{i}{2\pi i}\int_{|z|=r} f(re^{i\theta})\, d\theta$$

We'd like a path integral analogue of the second integral above so as to apply Cauchy's th$^{\underline{m}}$

Standard relation (Witten, Nekrasov)

$$(0,2) \text{ models} \iff \beta\gamma \text{ systems}$$

Consider sigma model $\phi : \Sigma \to U \subset \mathbb{P}^3$ for U a contractible open patch. Within U can choose flat metric $g_{i\bar{j}} = \delta_{i\bar{j}}$ and

$$H^{0,p}(U,\cdot) = 0 \quad \text{for } p>0$$

$\Rightarrow$ non-trivial vertex operators are independent of $\bar{\rho}$ and may equally well be constructed from the action

$$S' = \int_\Sigma |d^2z| \; \beta_{iz} \partial_{\bar{z}} \gamma^i + \bar{\Psi}_{az} \partial_{\bar{z}} \Psi^a$$

where $\gamma^i = \phi^i$, $\beta_{iz} = \delta_{i\bar{j}} \partial_z \phi^{\bar{j}}$.

Note that target space susy has been restored.

Obtain full theory by piecing together such $\beta\gamma$ systems over the whole target, subject to consistency conditions on the overlaps.

Higher vertex operators represented by hol functions on overlaps that cannot be split into difference of two holomorphic f^{n}s on open sets.

A QFT version of Čech - Dolbeault isomorphism

Towards the Googly Problem?

Stringy vertex operators $\Longleftrightarrow$ Cohomology grps on $\mathbb{PT}$ $\Leftrightarrow$ Sol^{n}s of LINEARIZED field equations

However, descendant vertex operators give infinitesimal deformations of the worldsheet action

eg. $c\delta A_{\bar{J}}{}^{\alpha}{}_{\beta}\,\bar{\lambda}_{\alpha}\lambda^{\beta}\rho^{\bar{J}}$ pointlike vertex operator for δA

$\Updownarrow$

$$\delta S = \int_{\Sigma} \bar{\lambda}_{\alpha}\,\phi^{*}(\delta A)^{\alpha}{}_{\beta}\lambda^{\beta} + \bar{\lambda}_{\alpha}(D_{i}\,\delta A_{\bar{J}})^{\alpha}{}_{\beta}\,\lambda^{\beta}\rho^{i}\rho^{\bar{J}}$$

integrated (1,1)-form descendant

so, just as in the A- and B-models, we have

$$\boxed{\text{Stringy vertex operators} \Leftrightarrow \text{Tangent space to ``moduli space of (0,2) model''}}$$

What is this moduli space? With a non-trivial B-field, target space of (0,2) model must be "Kähler + torsion", or a twisted generalized complex manifold. (Hitchin Gualtieri)

Can get some insight as follows:

When integrating out non-zero modes from (0,2) path integral (in physical gauge) one meets the expression

$$\exp\left(i\int_C B\right) \frac{\det(\bar{\partial}_E)}{\mathrm{Ber}'(\bar{\partial}_{N_{C|\mathbb{P}^{3|4}}})} .$$

On shell with fixed background, this ratio is just a constant (provided $c_{YM} = 28$) but in SFT natural to consider how this object varies as $f^{=}$ of background J, H, $\bar{\partial}+A$.

This is exactly the way instanton corrections to spacetime ($d=4$) superpotential are computed in the physical heterotic string.

Precisely this type of structure is also found in twistor space "lift" of csugra action.