Electromagnetic analysis of microwave cavities for dark-matter axion detectors

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- The *empty* rectangular cavity
- Excitation of microwave cavities: coupling
- The BI-RME method
- Examples
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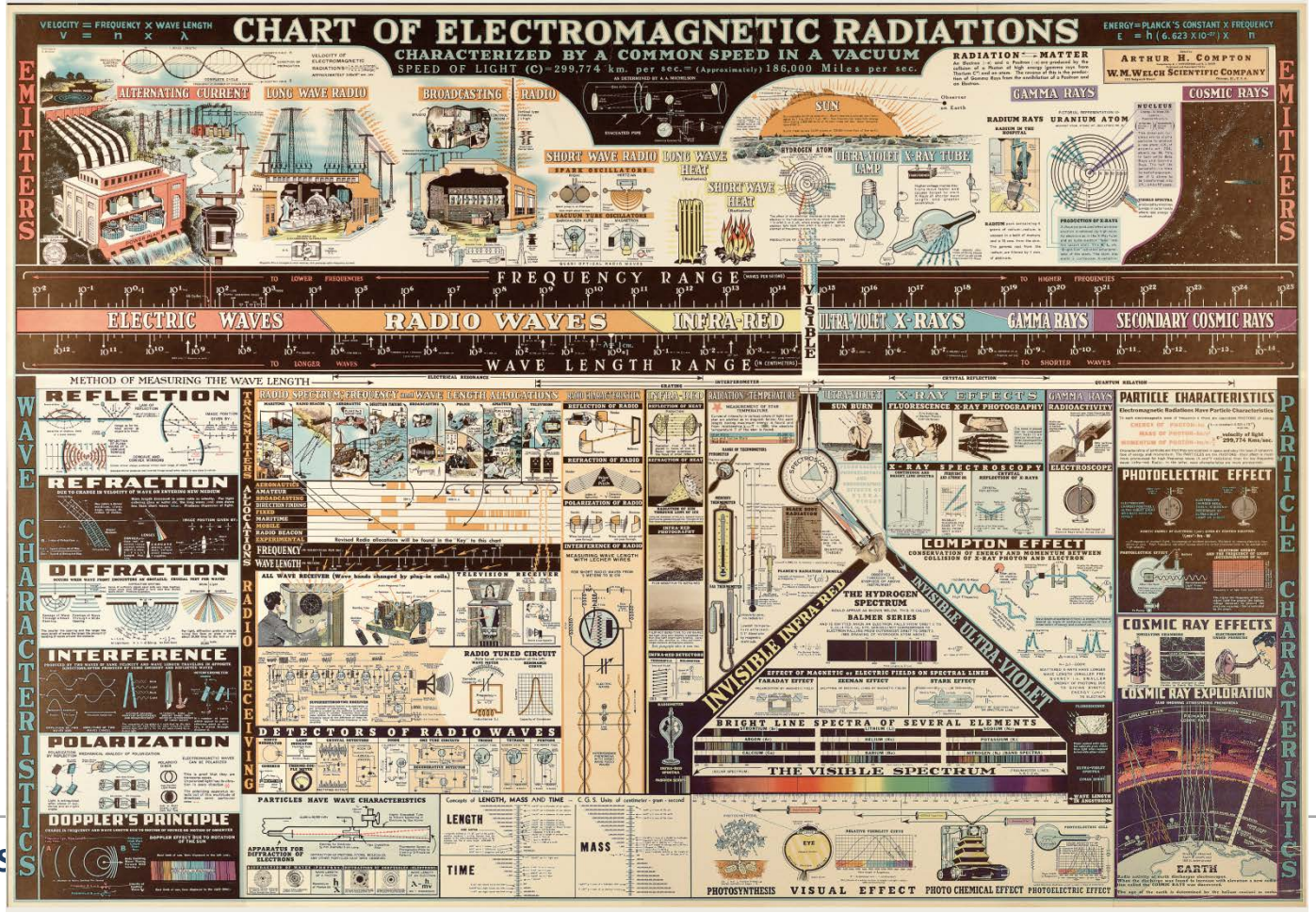
Introduction

- During these days we have been talking about dark matter axions, now the question is: *how to detect them?*
- Microwave resonators have been proposed in the literature for dark-matter axions detectors.
- The objective of this talk is to review the most relevant feautures and performances of microwave cavities from the theoretical perspective of the Classical Electrodynamics, in order to analyze their applications in the research field of axion detectors physics.



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The rectangular waveguide

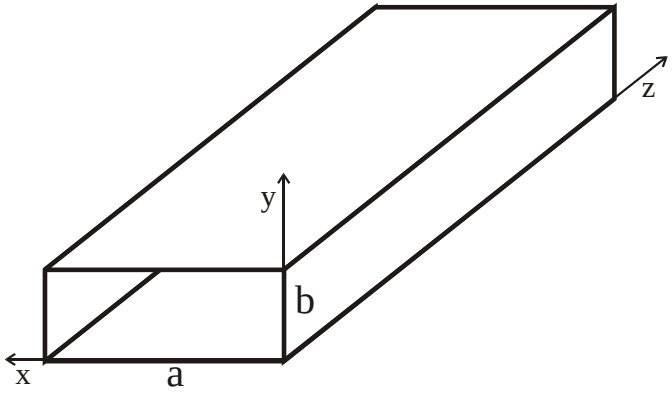
- The complete set of solenoidal modes of an *empty* rectangular cavity can be obtained short-circuiting both sides of a rectangular waveguide.
- The modes of a rectangular waveguide are wellknown:

TE^z modes:

$$H_x = B_{mn}\frac{\gamma_z}{k_{c,mn}^2}\frac{m\pi}{a}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\exp(-\gamma_z z)$$

$$H_y = B_{mn}\frac{\gamma_z}{k_{c,mn}^2}\frac{n\pi}{b}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\exp(-\gamma_z z)$$

$$H_z = B_{mn}\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\exp(-\gamma_z z)$$



$$E_x = Z_{TE}H_y = B_{mn}\frac{j\omega\mu_0}{k_{c,mn}^2}\frac{n\pi}{b}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\exp(-\gamma_z z)$$

$$E_y = -Z_{TE}H_x = -B_{mn}\frac{j\omega\mu_0}{k_{c,mn}^2}\frac{m\pi}{a}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\exp(-\gamma_z z)$$

$$E_z = 0$$

The rectangular waveguide

TM^z modes:

$$E_x = -A_{mn}\frac{\gamma_z}{k_{c,mn}^2}\frac{m\pi}{a}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\exp(-\gamma_z z)$$

$$E_y = -A_{mn}\frac{\gamma_z}{k_{c,mn}^2}\frac{n\pi}{b}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\exp(-\gamma_z z)$$

$$E_z = A_{mn}\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\exp(-\gamma_z z)$$

$$k_{c,mn}^2 = k^2 + \gamma_z^2$$

$$H_x = -\frac{E_y}{Z_{TM}} = A_{mn}\frac{j\omega\varepsilon_0\varepsilon_r(1-jtg\delta)}{k_{c,mn}^2}\frac{n\pi}{b}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\exp(-\gamma_z z)$$

$$H_y = \frac{E_x}{Z_{TM}} = -A_{mn}\frac{j\omega\varepsilon_0\varepsilon_r(1-jtg\delta)}{k_{c,mn}^2}\frac{m\pi}{a}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\exp(-\gamma_z z)$$

$$H_z = 0$$

Relevant relationships of the rectangular waveguide (uniformly dielectric loaded):

$$k_{c,mn} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\gamma_z = \sqrt{k_{c,mn}^2 - k^2} = \alpha_z + j\beta_z$$

$$k^2 = \omega^2\mu_0\varepsilon_0\varepsilon_r(1-jtg\delta)$$

$$tg\delta = \frac{\varepsilon''}{\varepsilon'}$$

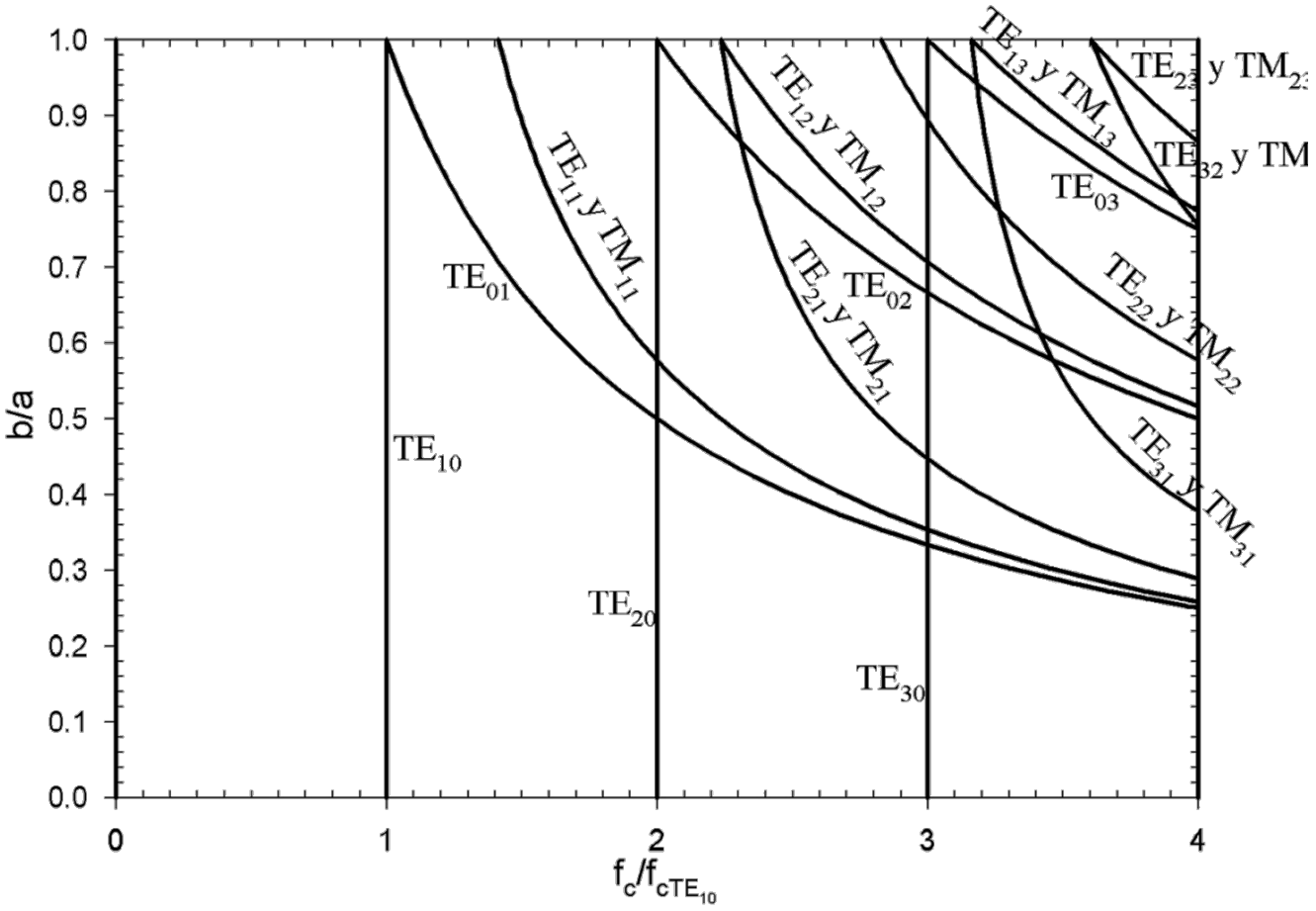
$$Z_{TE} = \frac{j\omega\mu_0}{\gamma_z} = \frac{jk}{\gamma_z}\eta$$

$$Z_{TM} = \frac{\gamma_z}{j\omega\varepsilon_0\varepsilon_r(1-jtg\delta)} = \frac{\gamma_z}{jk}\eta$$

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_r(1-jtg\delta)}}$$

The rectangular waveguide

Modal spectrum of a rectangular waveguide for different ratio b/a:



The rectangular waveguide

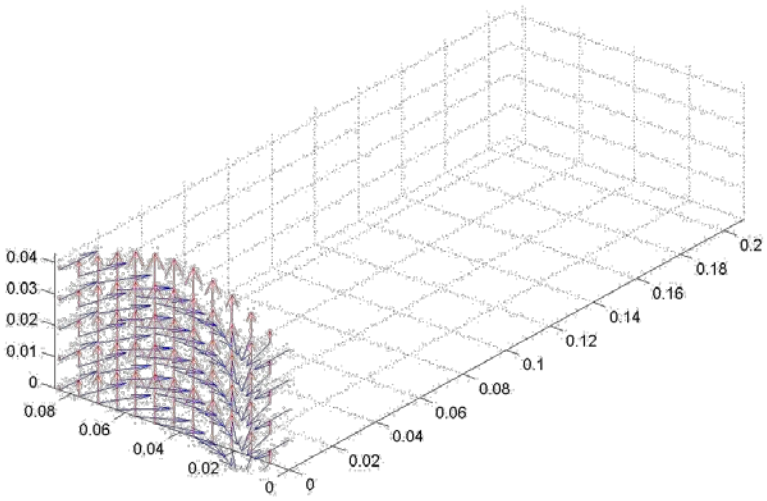
The fundamental TE^z_{10} mode:

$$H_z = B \cdot \cos\left(\frac{\pi}{a} \cdot x\right) \cdot e^{\mp\gamma \cdot z}$$

$$\vec{H}_t = \mp\frac{\gamma}{k_c^2} \cdot \nabla_t \cdot H_z = \pm B \cdot \frac{\gamma \cdot a}{\pi} \cdot sen\left(\frac{\pi}{a} \cdot x\right) \cdot \hat{x} \cdot e^{\mp\gamma \cdot z}$$

$$\vec{E}_t = Z_{TE} \cdot \left(\vec{H}_t \, x(\pm \hat{z})\right) = -j \cdot B \cdot \left(\frac{f}{f_c}\right) \cdot \eta \cdot sen\left(\frac{\pi}{a} \cdot x\right) \cdot \hat{y} \cdot e^{\mp\gamma \cdot z}$$

$$f_c = \frac{1}{2 \cdot a \cdot \sqrt{\mu \cdot \varepsilon}} \qquad \lambda_c = 2a$$

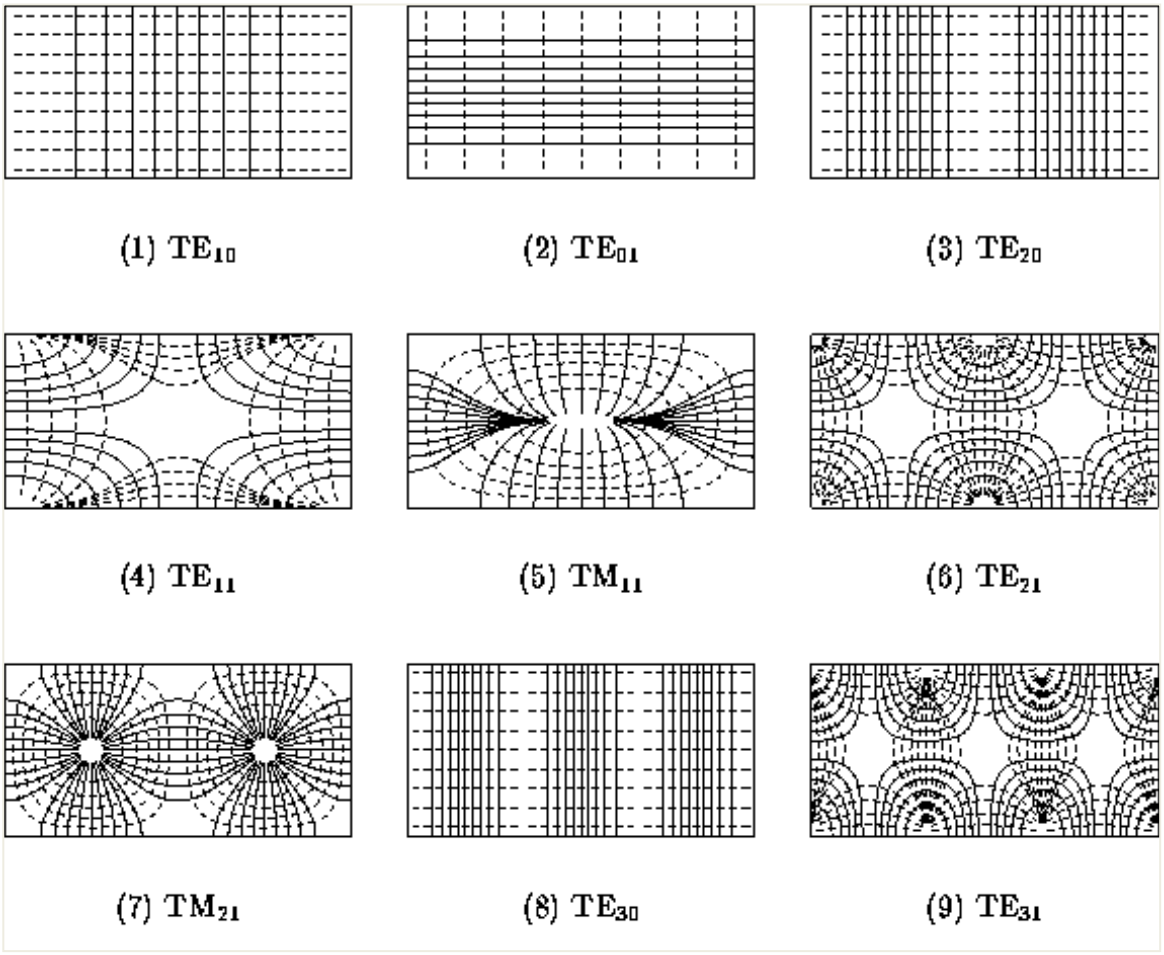


$$\gamma = \sqrt{k_c^2 - k^2} = \sqrt{\left(\frac{\pi}{a}\right)^2 - \omega^2 \cdot \mu \cdot \varepsilon}$$

$$Z_{TE} = \frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \qquad \gamma_i = \cdot\sqrt{\left(\frac{2\pi}{\lambda_c}\right)^2 - \omega^2 \mu_o \mu_{ri} \varepsilon_o \varepsilon_{ri}}$$

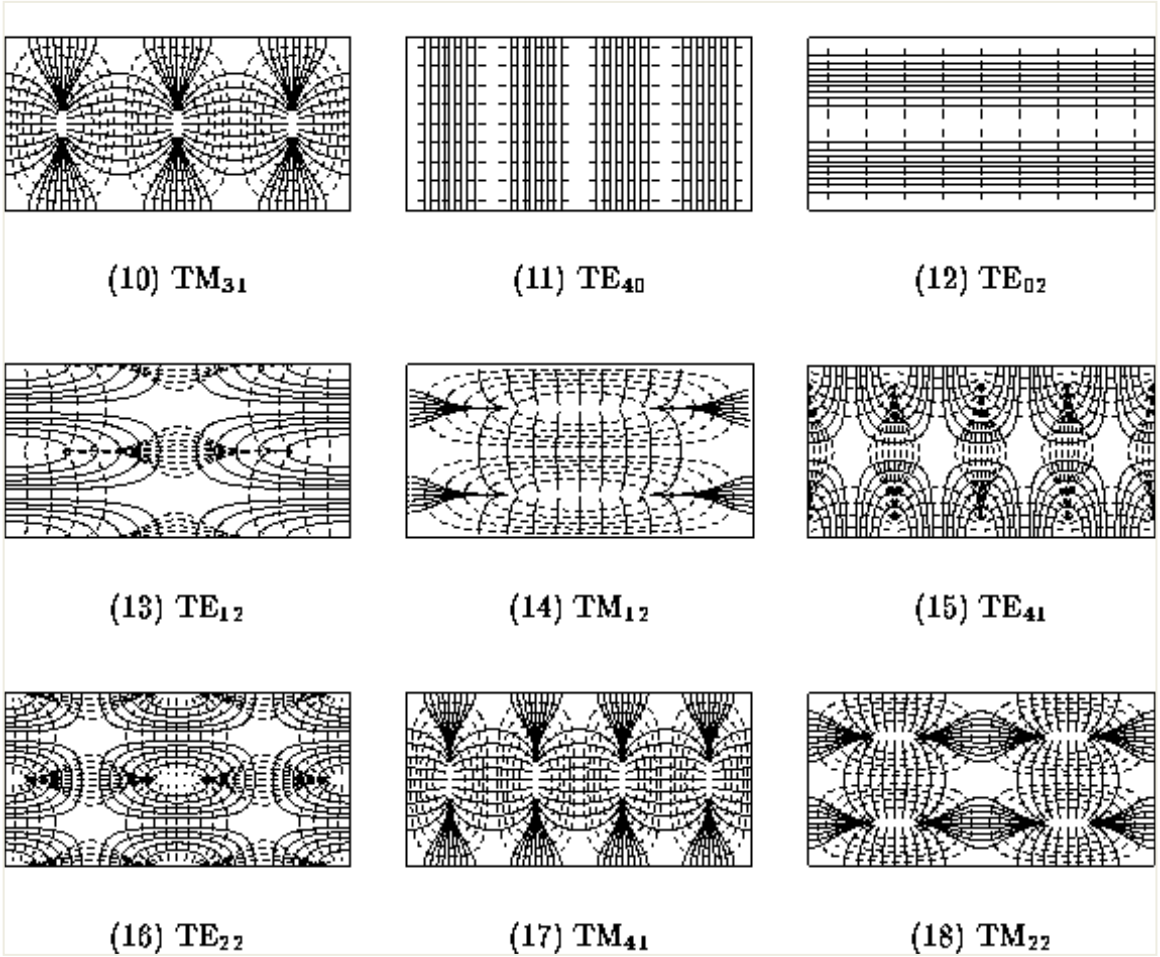
The rectangular waveguide

Electromagnetic fields distribution of a rectangular waveguide with b=a/2:



The rectangular waveguide

Electromagnetic fields distribution of a rectangular waveguide with b=a/2:



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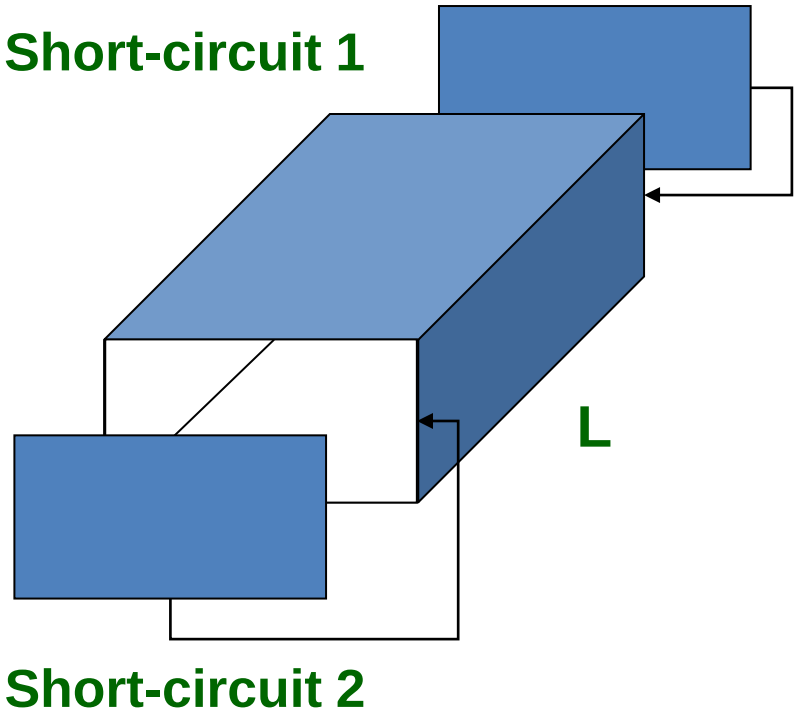
The *empty* rectangular cavity

- The complete set of solenoidal modes of an *empty* rectangular cavity can be obtained short-circuiting both sides of a rectangular waveguide:



The *empty* rectangular cavity

- The cavity modes can be easily constructed as the superposition of an incident wave and a reflected wave:

TE^z modes:

$$\vec{E} = \vec{E}_{0t}(x,y)\left(e^{-j\beta z} - \rho e^{j\beta z}\right) \qquad \text{with } \rho=1$$

$$\vec{H} = \vec{H}_{0t}(x,y)\left(e^{-j\beta z} + \rho e^{j\beta z}\right) + \vec{H}_{0z}(x,y)\left(e^{-j\beta z} - \rho e^{j\beta z}\right)$$

$$E_x = B_{mn}\frac{\omega\mu_0}{k_{c,mn}^2}\frac{n\pi}{b}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{L}z\right)$$

$$E_y = -B_{mn}\frac{\omega\mu_0}{k_{c,mn}^2}\frac{m\pi}{a}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{L}z\right)$$

$$E_z = 0$$

$$H_x = B_{mn}\frac{j\beta_z}{k_{c,mn}^2}\frac{m\pi}{a}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{L}z\right)$$

$$H_y = B_{mn}\frac{j\beta_z}{k_{c,mn}^2}\frac{n\pi}{b}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{L}z\right)$$

$$H_z = -jB_{mn}\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{L}z\right)$$

m=0,1,2… ; n=0,1,2…; p=1,2,3… (the case m=n=0 is not allowed)

The *empty* rectangular cavity

TM^z modes:

$$\vec{E} = \vec{E}_{0t}(x,y)\left(e^{-j\beta z} - \rho e^{j\beta z}\right) + \vec{E}_{0z}(x,y)\left(e^{-j\beta z} + \rho e^{j\beta z}\right) \qquad \text{with } \rho=1$$

$$\vec{H} = \vec{H}_{0t}(x,y)\left(e^{-j\beta z} + \rho e^{j\beta z}\right)$$

$$E_x = -A_{mn}\frac{\beta_z}{k_{c,mn}^2}\frac{m\pi}{a}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{L}z\right) \qquad H_x = A_{mn}\frac{j\omega\varepsilon_0\varepsilon_r}{k_{c,mn}^2}\frac{n\pi}{b}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{L}z\right)$$

$$E_y = -A_{mn}\frac{\beta_z}{k_{c,mn}^2}\frac{n\pi}{b}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{L}z\right) \qquad H_y = -A_{mn}\frac{j\omega\varepsilon_0\varepsilon_r}{k_{c,mn}^2}\frac{m\pi}{a}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{L}z\right)$$

$$E_z = A_{mn}\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{L}z\right) \qquad H_z = 0$$

m=1,2,3… ; n=1,2,3…; p=0,1,2…

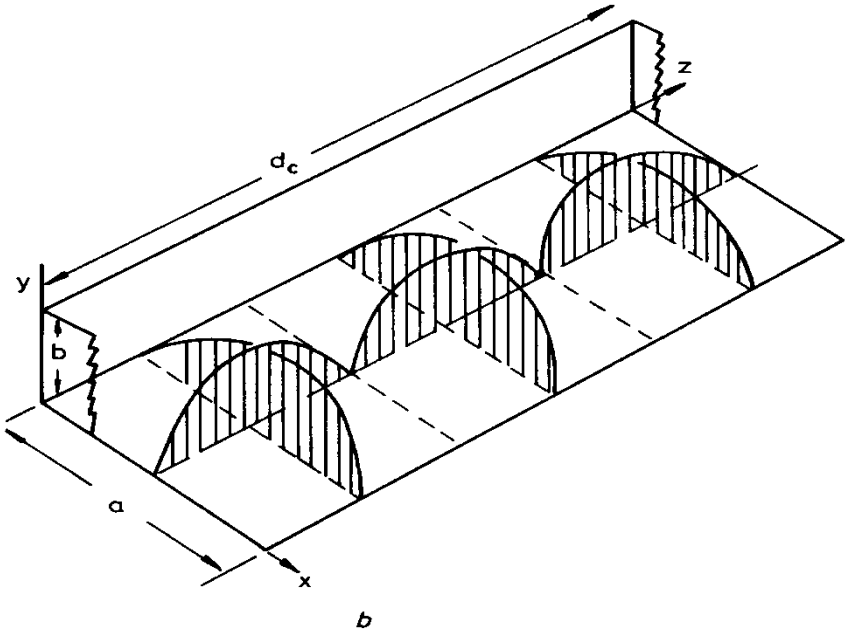
Relevant equations of the rectangular cavity (uniformly dielectric loaded):

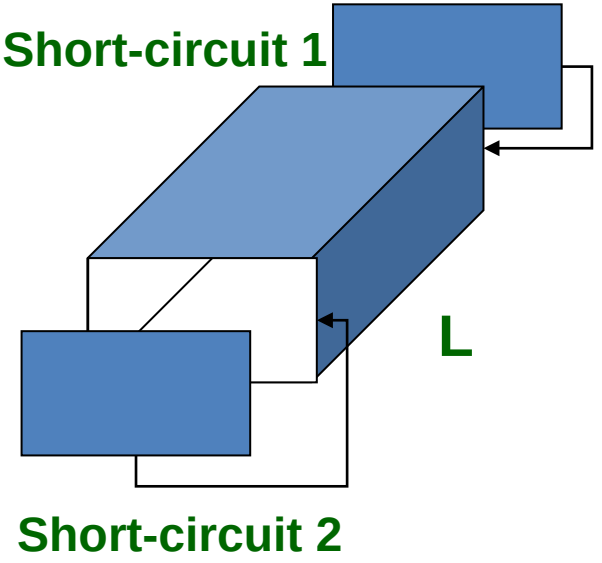
$$\omega^2\mu_0\varepsilon_0\varepsilon_r(1-jtg\delta) = k_{c,mn}^2 + \beta_z^2$$

$$\omega = \omega_u\left(1+\frac{j}{2Q_d}\right) \qquad Q_d = \frac{1}{tg\delta} \qquad \omega_u = \sqrt{\frac{k_{c,mn}^2+\beta_z^2}{\mu_0\varepsilon_0\varepsilon_r}} = \frac{1}{\sqrt{\mu_0\varepsilon_0\varepsilon_r}}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2+\left(\frac{p\pi}{L}\right)^2} = \frac{\omega_{u0}}{\sqrt{\varepsilon_r}}$$

The *empty* rectangular cavity

Example1: TE^z_{10p} modes



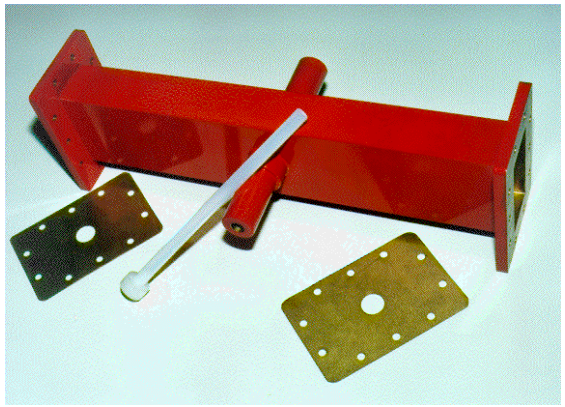


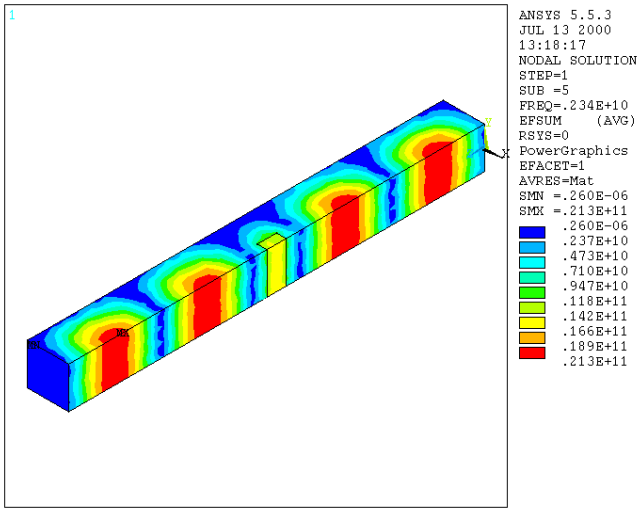
$$f_r\big|_{TE_{mnp}} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2+\left(\frac{p}{L}\right)^2}$$

The *empty* rectangular cavity

Example2: Rectangular cavity in WR-340

$$a=86.36\ mm,\ b=43.18\ mm$$





$$f_r\big|_{TE_{10p}} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{p}{L}\right)^2} = 2.45GHz$$

p	***L* (mm.)**
1	86.7
2	173.4
3	260.1
4	346.8
5	433.5
6	520.1
7	606.8
8	693.5
9	780.2
10	866.9
11	953.6

The *empty* rectangular cavity

- The time-average electric and magnetic energies stored by each mode of a microwave cavity are given by:

$$W_e = \frac{\varepsilon_0 \varepsilon_r}{4} \int_{V_c} \vec{E} \cdot \vec{E}^* dV = \frac{\varepsilon_0 \varepsilon_r}{4} \int_{V_c} \left| \vec{E} \right|^2 dV \qquad W_m = \frac{\mu_0}{4} \int_{V_c} \vec{H} \cdot \vec{H}^* dV = \frac{\mu_0}{4} \int_{V_c} \left| \vec{H} \right|^2 dV$$

which can be calculated for both family of rectangular modes:

TE^z modes:

$$W_e = \frac{\varepsilon_0 \varepsilon_r}{4} \frac{B_{mn}^2 \omega_u^2 \mu_0^2}{k_{c,mn}^2} \frac{abL}{8} \varepsilon_m \varepsilon_n \qquad W_m = \frac{\mu_0}{4} B_{mn}^2 \frac{abL}{8} \varepsilon_m \varepsilon_n \left(1 + \frac{\beta_z^2}{k_{c,mn}^2} \right) \qquad \varepsilon_p = \begin{cases} 2, & p = 0 \\ 1, & p \neq 0 \end{cases}$$

TM^z modes:

$$W_e = \frac{\varepsilon_0 \varepsilon_r}{4} A_{mn}^2 \frac{abL}{8} \varepsilon_p \left(1 + \frac{\beta_z^2}{k_{c,mn}^2} \right) \qquad W_m = \frac{\mu_0}{4} \frac{A_{mn}^2 \omega_u^2 \varepsilon_0^2 \varepsilon_r^2}{k_{c,mn}^2} \frac{abL}{8} \varepsilon_p \qquad \varepsilon_p = \begin{cases} 2, & p = 0 \\ 1, & p \neq 0 \end{cases}$$

The *empty* rectangular cavity

- The power dissipated within a microwave cavity considering losses in the metallic walls as well as in a uniform lossy dielectric filling the cavity are given by:

$$\sigma_e = \omega\varepsilon_0\varepsilon'' = \omega\varepsilon_0\varepsilon_r tg\delta \qquad P_d = \frac{\sigma_e}{2}\int_{V_c}\left|\vec{E}\right|^2 dS \qquad P_0 = \frac{R_s}{2}\int_{S_c}\left|\vec{H}_{\tan}\right|^2 dS$$

- The quality factor of a microwave cavity is defined as follows:

$$Q_d = \omega\frac{W_e + W_m}{P_d}\bigg|_{\omega=\omega_u} \qquad Q_d = \omega_u\frac{\varepsilon_0\varepsilon_r}{\sigma_e} = \frac{1}{tg\delta}$$

$$Q_u = \omega\frac{W_e + W_m}{P_d + P_0}\bigg|_{\omega=\omega_u}$$

$$\frac{1}{Q_u} = \frac{1}{Q_c} + \frac{1}{Q_d}$$

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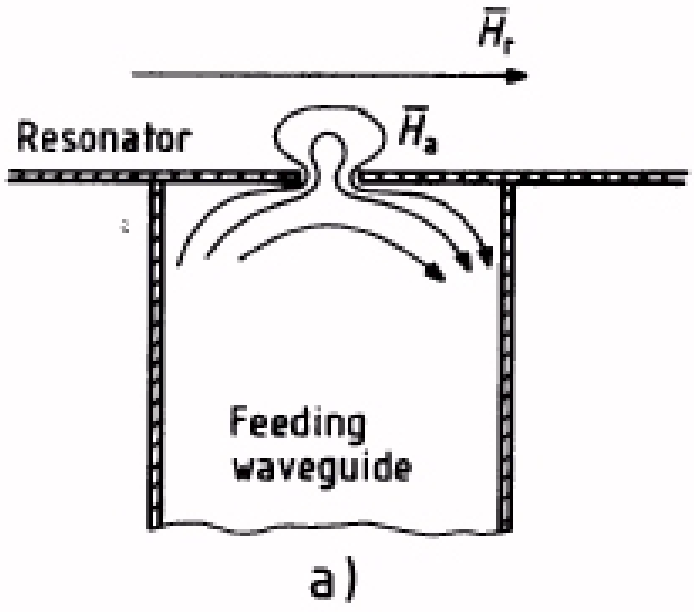
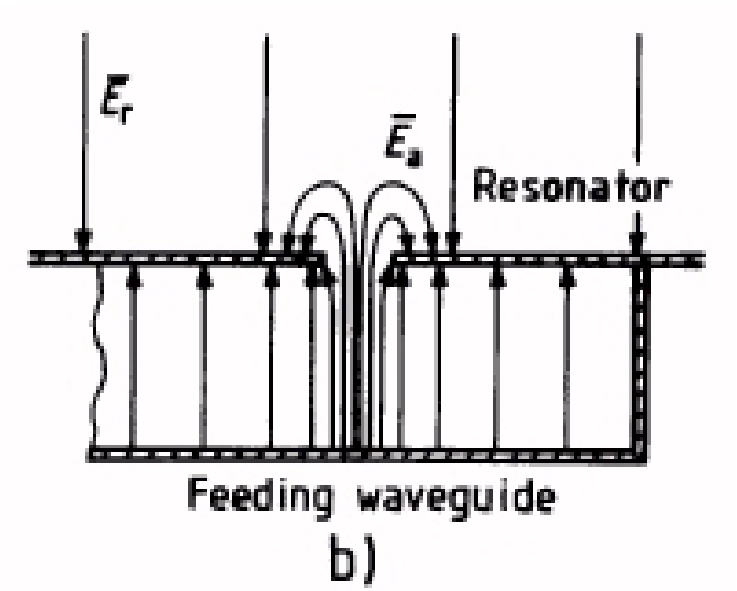
Excitation of microwave cavities: coupling

- An *empty* microwave cavity has to be feeded with an external component in order to excite an electromagnetic field within the cavity.
- There are different mechanisms to excite a microwave cavity:

- Waveguide Iris (holes performed on one cavity wall):

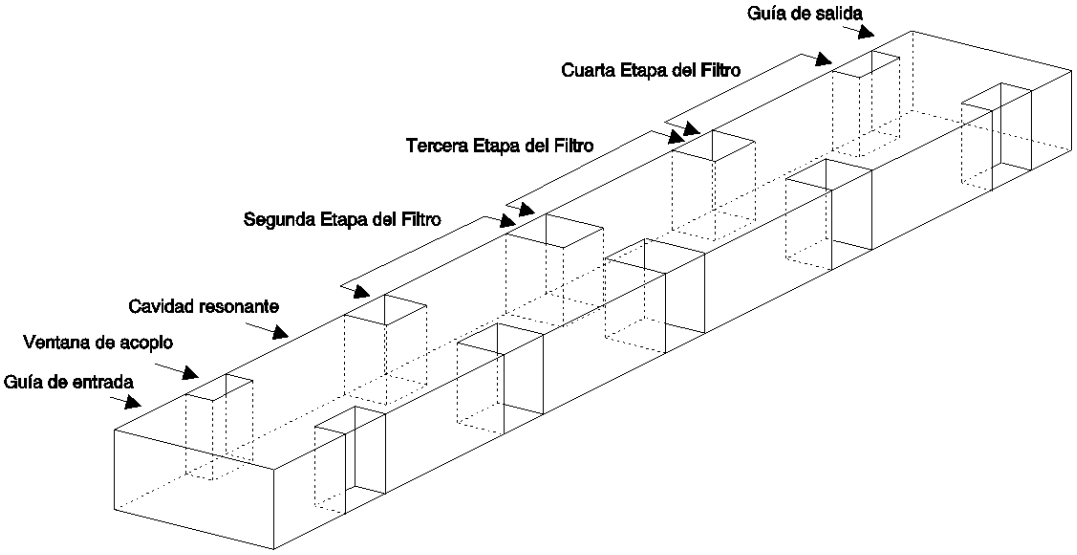
Magnetic (a) and electric (b) coupling apertures

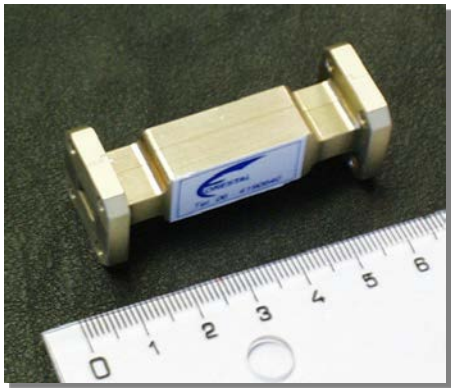


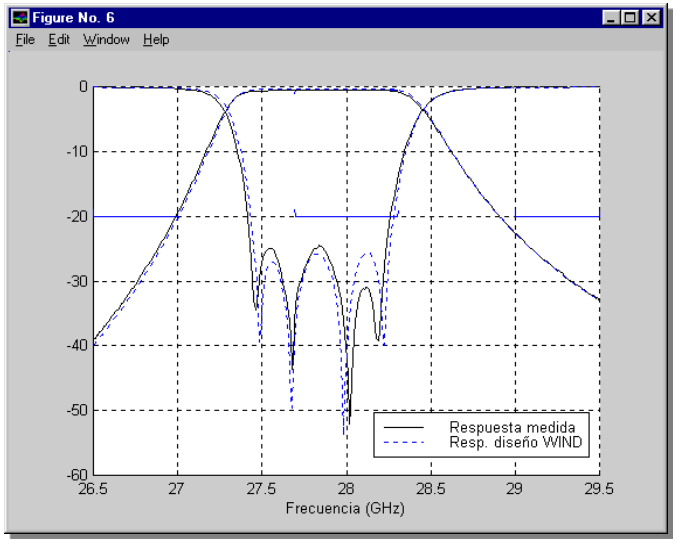


Excitation of microwave cavities: coupling

Example1: Inductive band-pass filter implemented in rectangular waveguide for LMDS technology at 28 GHz



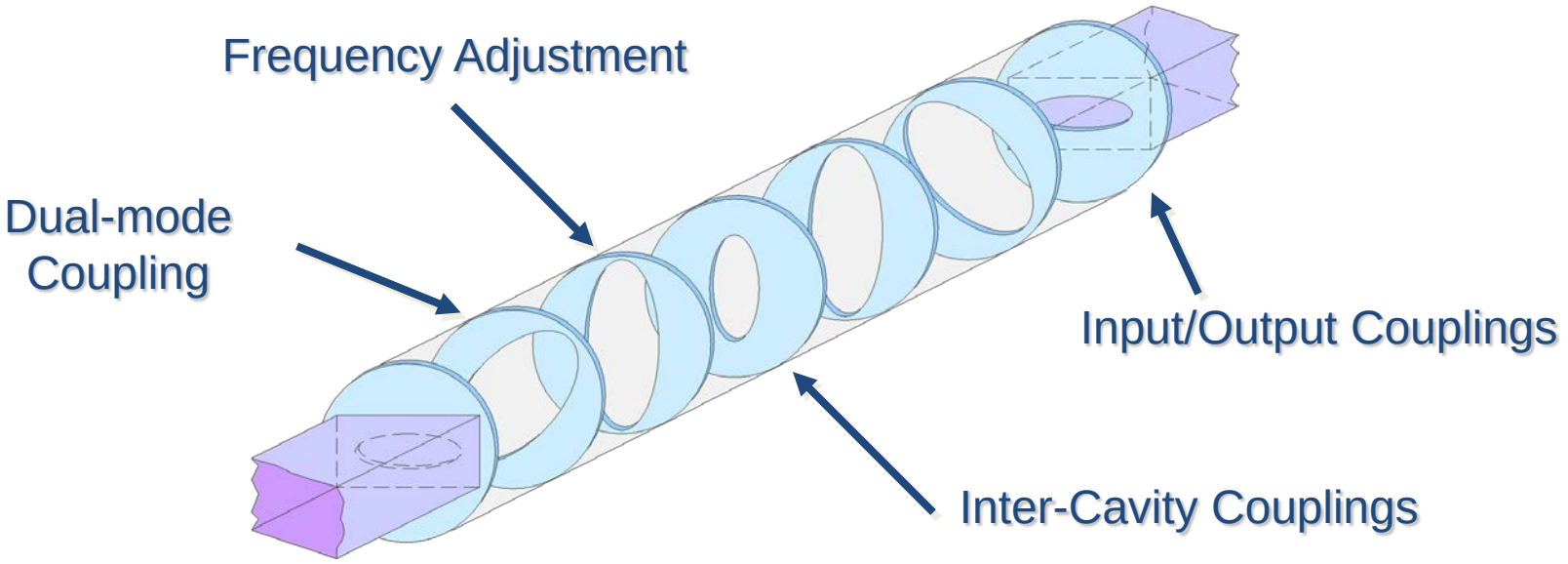




Courtesy of ESTEC-ESA

Excitation of microwave cavities: coupling

Example2: Dual-mode band-pass filter implemented in circular waveguide with elliptical irises for space telecommunications applications

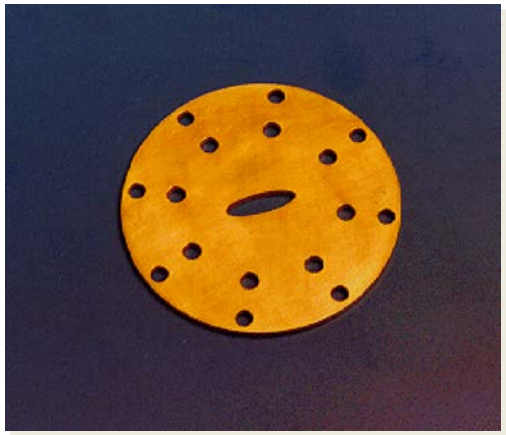
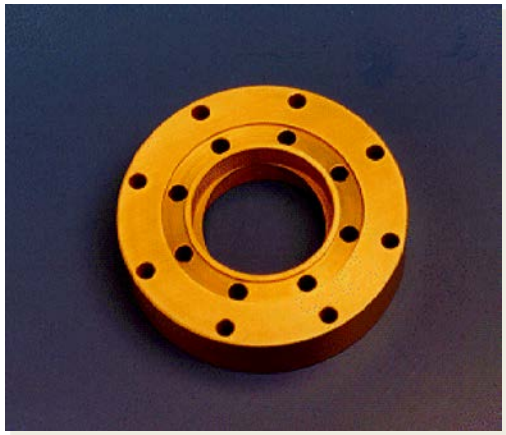
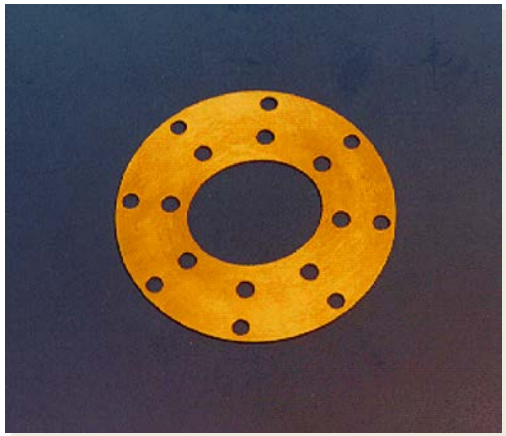
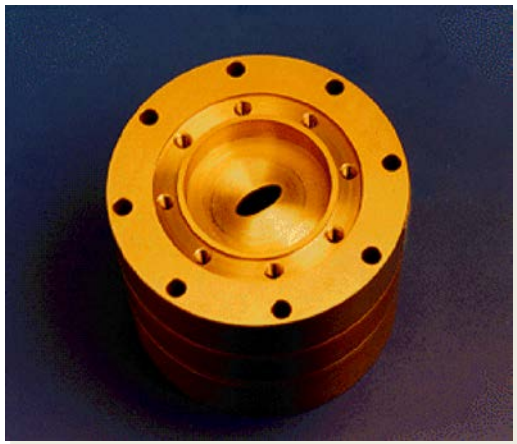


B. Gimeno, M. Guglielmi, "Full Wave Network Representation for Rectangular, Circular, and Elliptical to Elliptical Waveguide Junctions", IEEE Transactions on Microwave Theory and Techniques , vol. 45, no. 3, pp. 376-384, March 1997

Excitation of microwave cavities: coupling

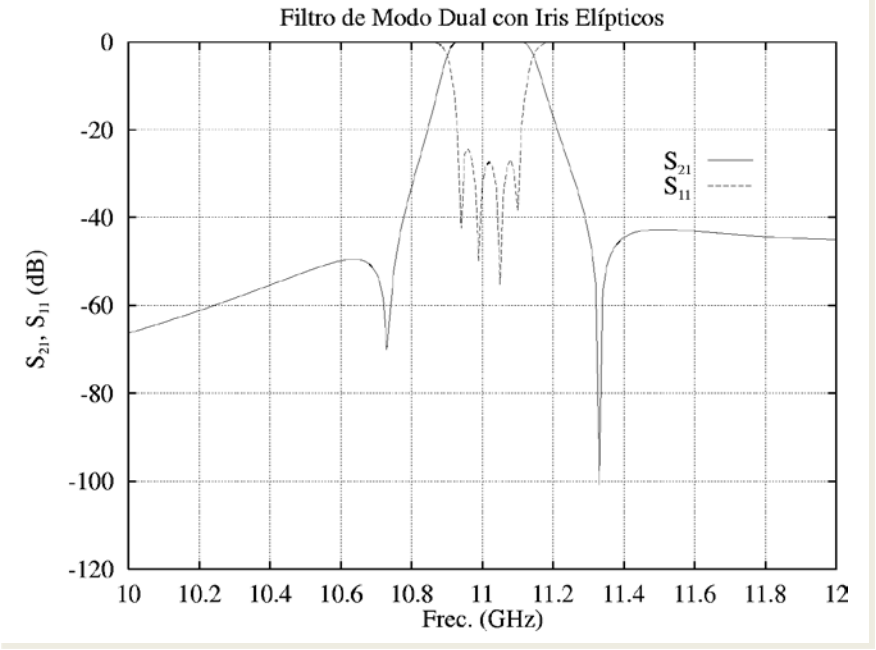
Manufactured prototype (courtesy by ESA-ESTEC):

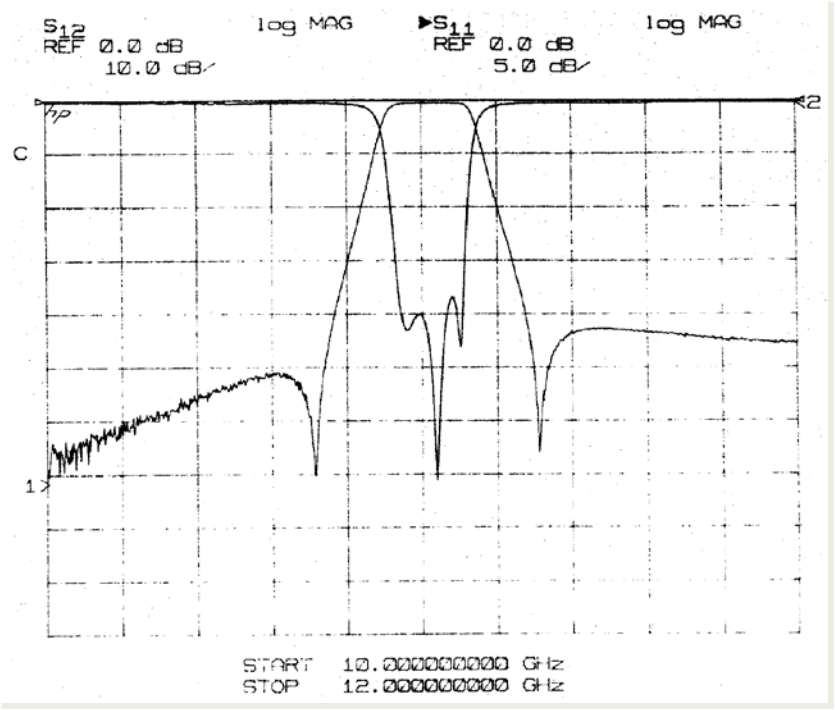
ESA PATENT: “Alternative implementation for dual-mode filters in circular waveguide”, reference number: 9606337, EU-USA-Canada


Excitation of microwave cavities: coupling

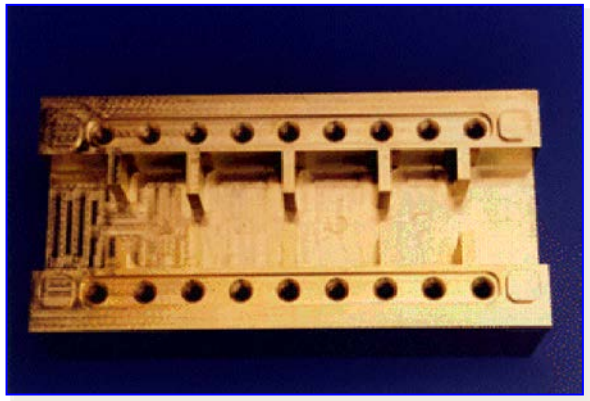
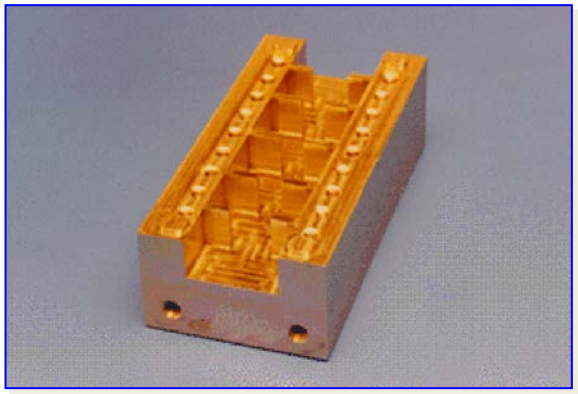
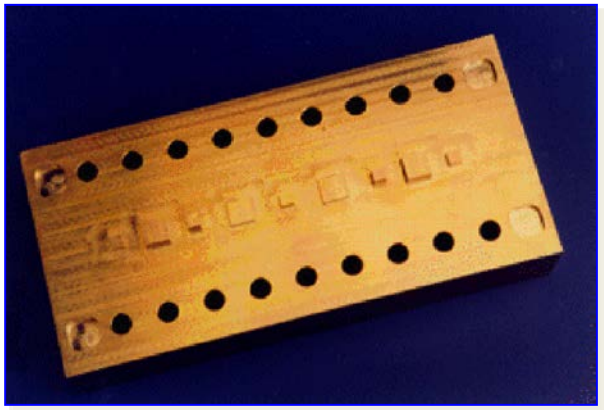
Electrical response: comparison between theory (left) and experiment (right)



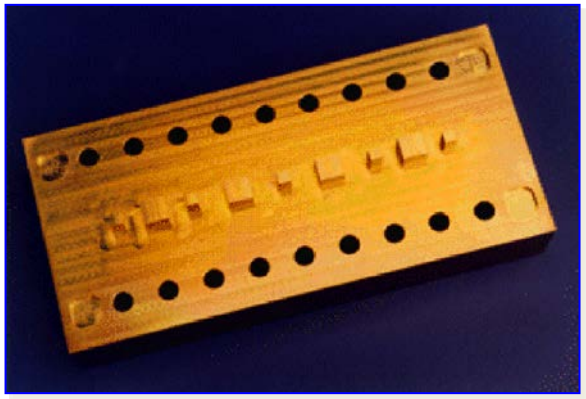
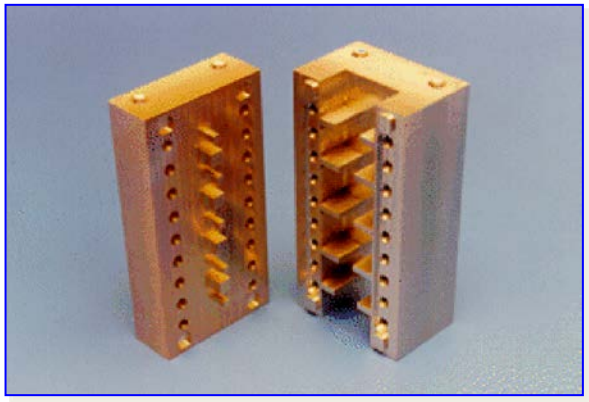


Excitation of microwave cavities: coupling

Example3: Coupled-Cavities band-pas filter with tuneable response for space telecommunications applications

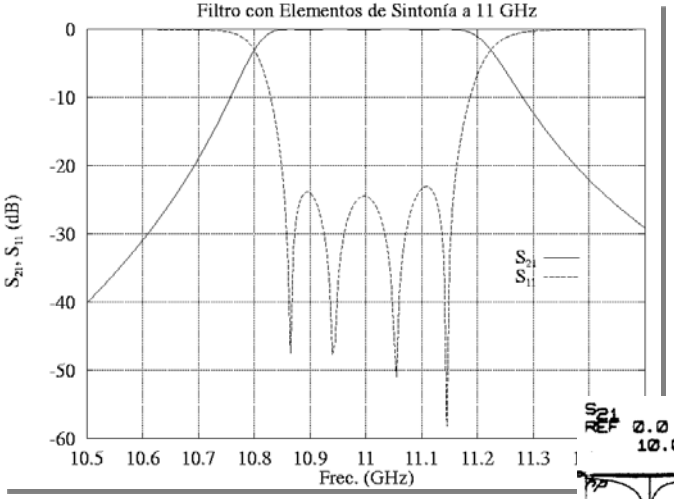
Manufactured Prototype

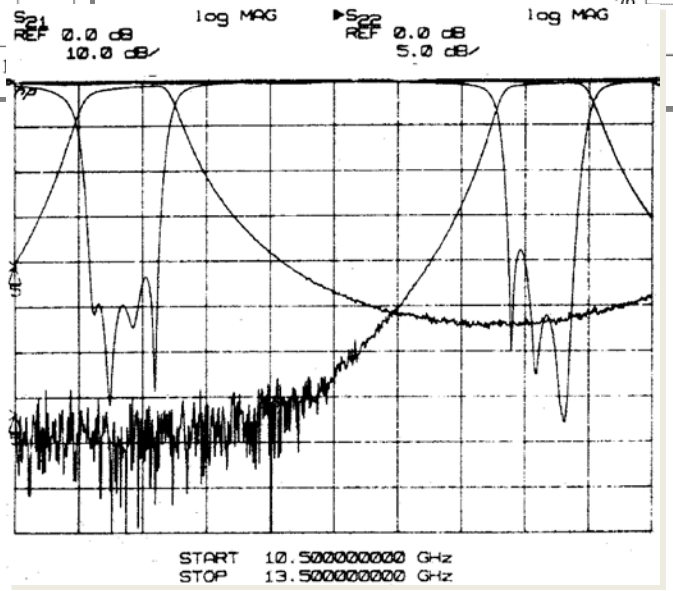



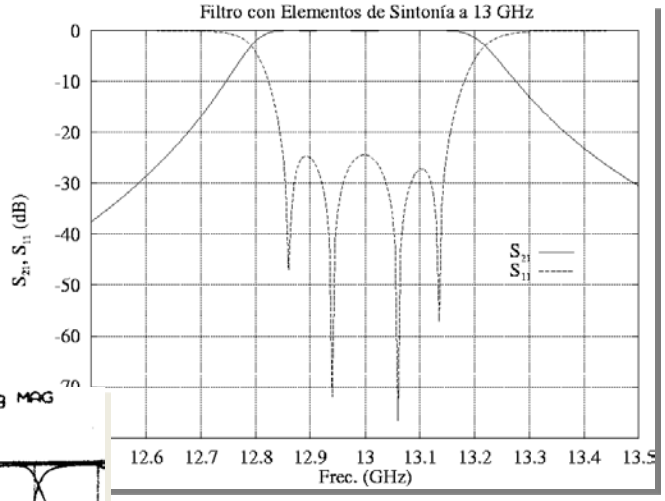
Courtesy of ESTEC-ESA

Excitation of microwave cavities: coupling

Comparison between theory (up) and experiment (down)

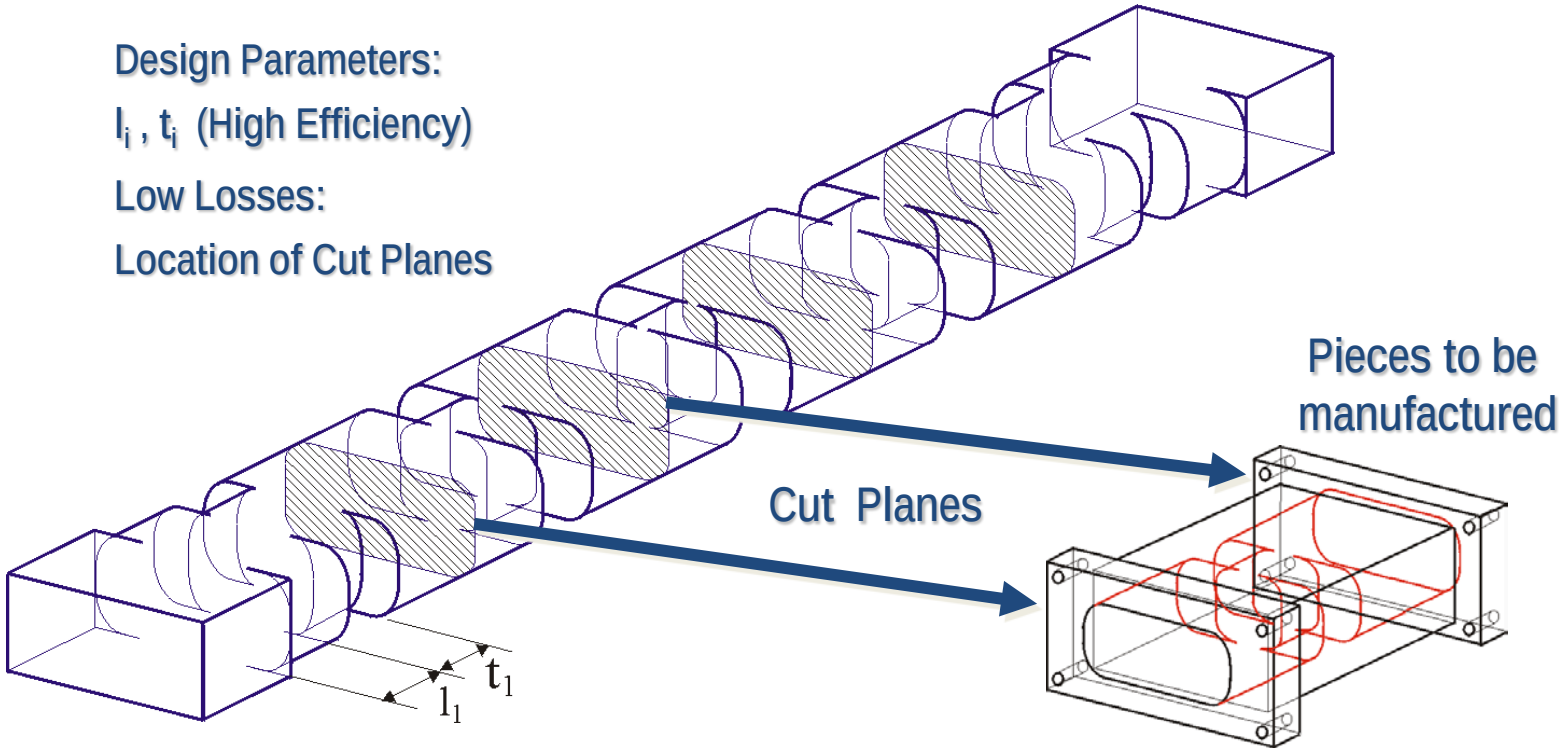






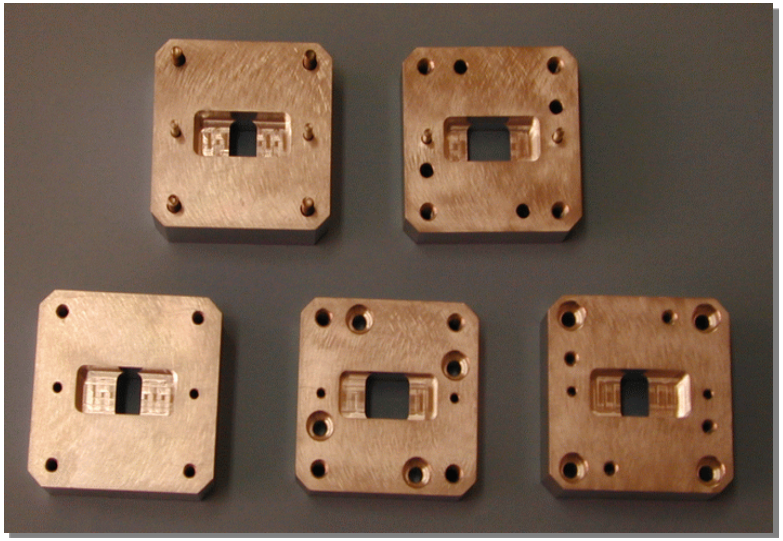
Excitation of microwave cavities: coupling

Example4: Cuasi-inductive band-pass filter with rounded corners for space telecommunications applications

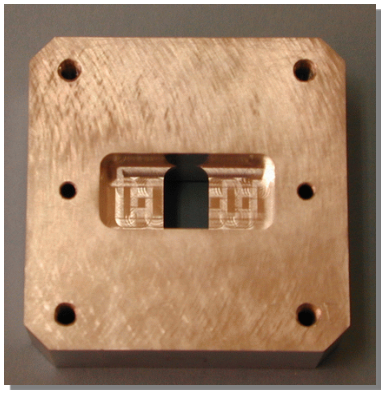


Excitation of microwave cavities: coupling

Courtesy of ESTEC-ESA

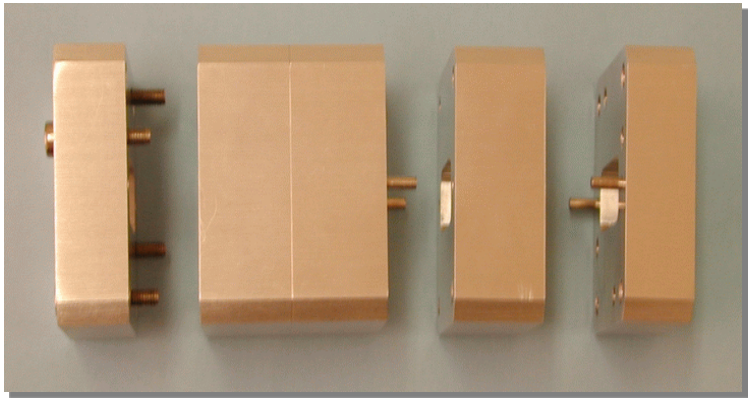


Prototype Pieces



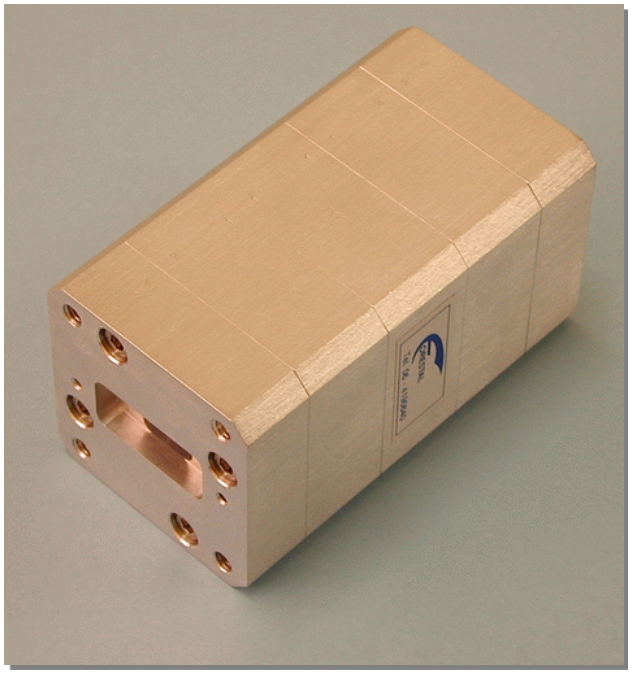
Detailed View of a Piece

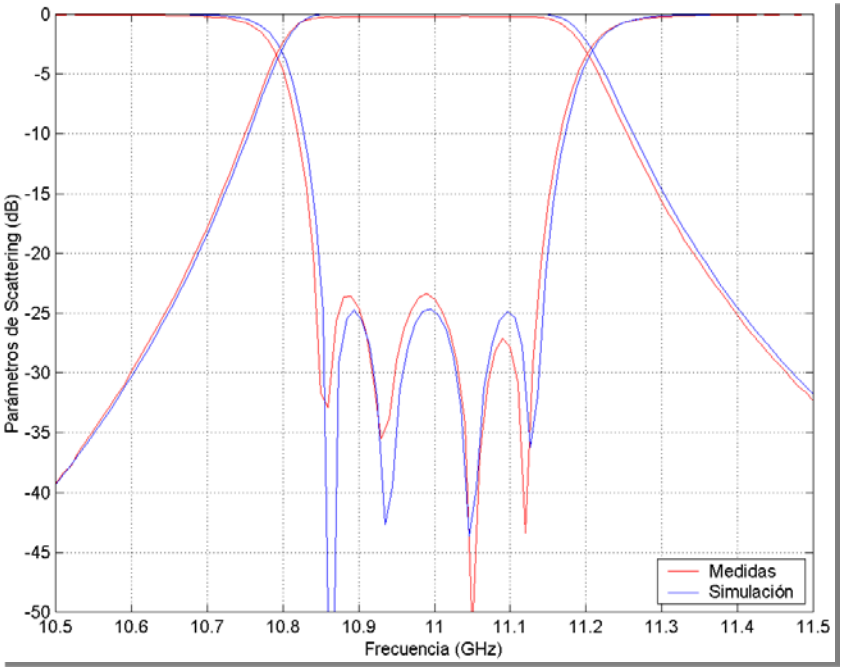
Assembling Process of the Pieces



Excitation of microwave cavities: coupling

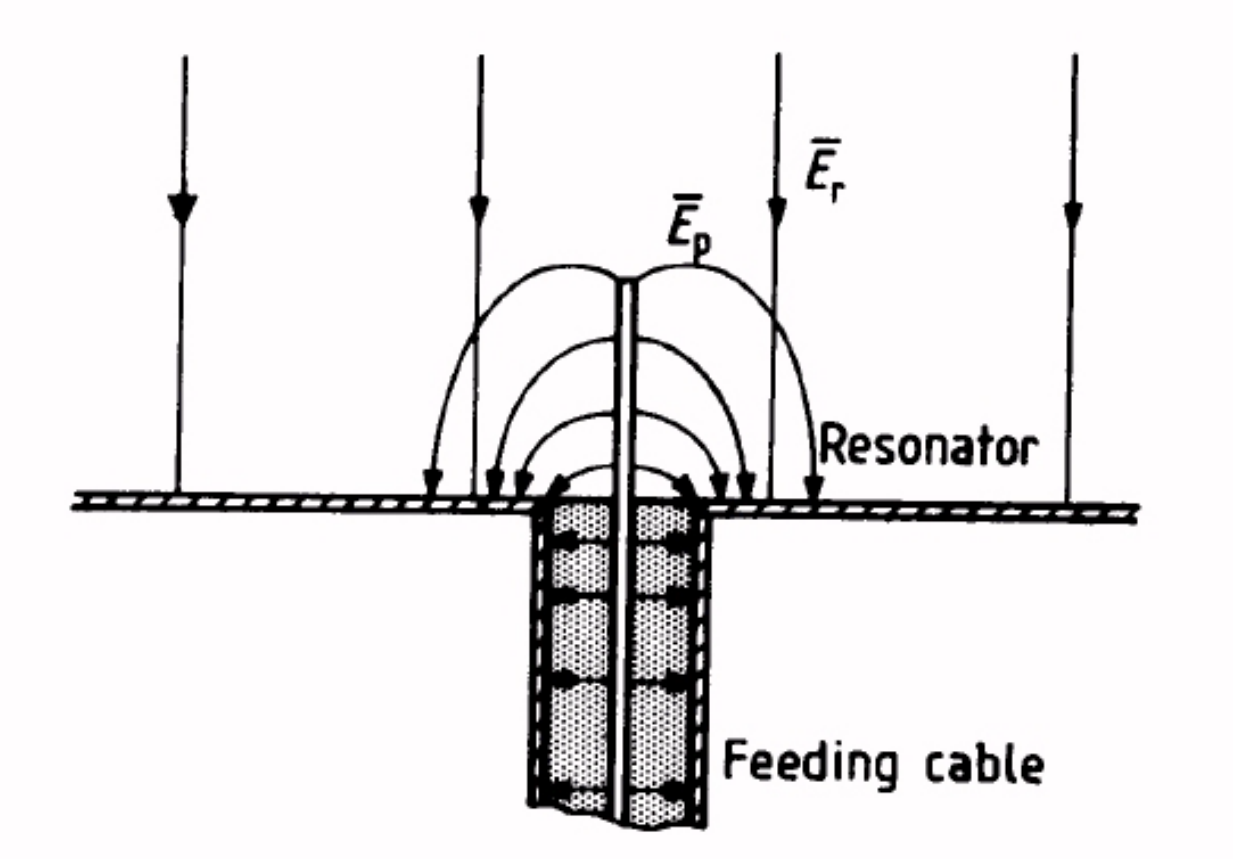
Comparison between theory and experiment





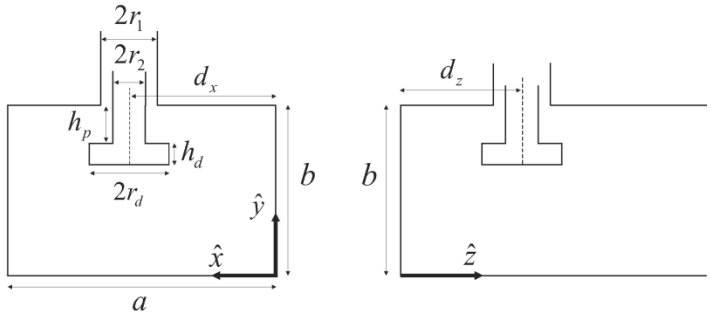
Excitation of microwave cavities: coupling

- Electric probe: Coaxial probe inserted in the cavity



Excitation of microwave cavities: coupling

Example1: Connector between a coaxial transmission line and a rectangular waveguide



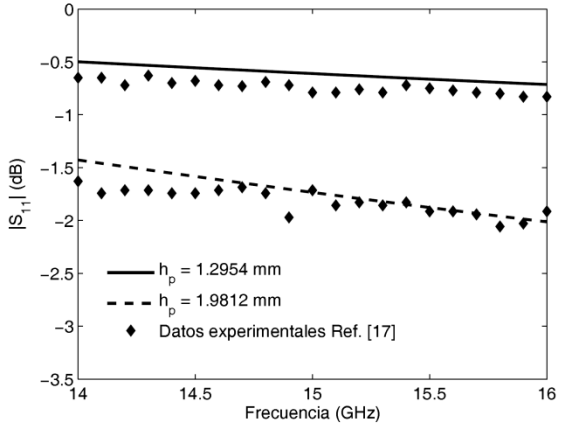


Figura 4.6: Pérdidas de retorno para una guía rectangular semi-infinita implementada en la guía estándar WR-62 excitada mediante sonda coaxial convencional ($r_1 = 1{,}6764$ mm, $r_2 = 0{,}635$ mm, $Z_0 = 50\,\Omega$, $d_z = 5{,}0$ mm, $d_x = a/2$). En la figura se estudian dos casos en función del valor de la profundidad de penetración de la sonda h_p. Los resultados de la simulación se comparan con los datos experimentales presentados en [17].

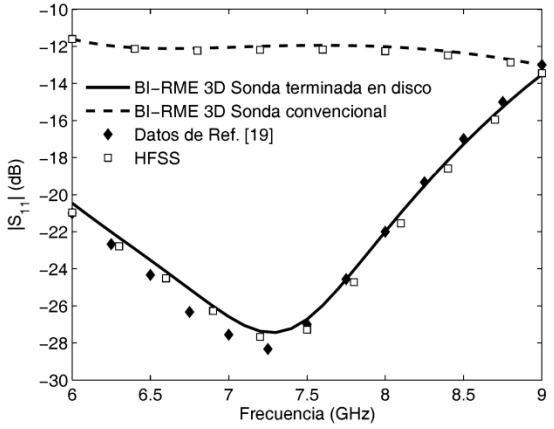


Figura 4.12: Pérdidas de retorno para una guía rectangular semi-infinita implementada en la guía estándar WR-137 excitada mediante una sonda coaxial terminada en disco ($r_1 = 4{,}8768$ mm, $r_2 = 1{,}524$ mm, $r_d = 4{,}0$ mm, $h_p = 3{,}4$ mm, $h_d = 4{,}5$ mm, $Z_0 = 50\,\Omega$, $d_z = 11{,}6$ mm, $d_x = a/2$). Los resultados de la simulación se comparan con los datos proporcionados por HFSS y con los datos extraídos de [19]. En la misma figura se presenta en trazo punteado la respuesta de una sonda coaxial convencional con $h_p' = 7{,}9$ mm.

Excitation of microwave cavities: coupling

Example2: Inductive band-pass filter excited with a coaxial connector

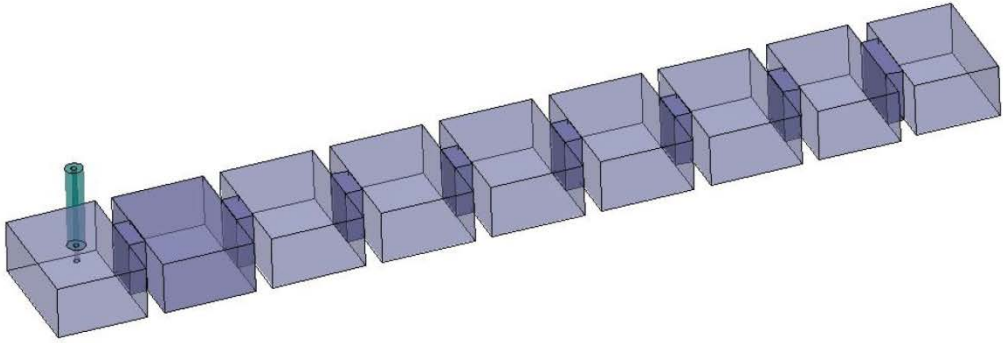


Figura 4.15: Filtro inductivo de 7 cavidades excitado mediante sonda coaxial convencional.

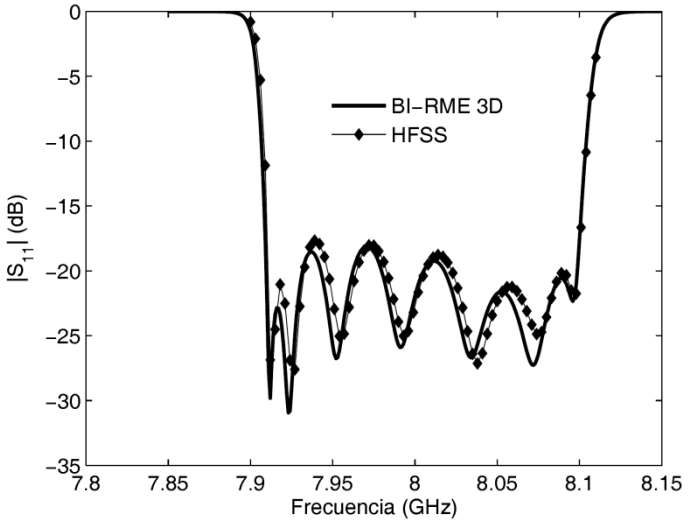


Figura 4.16: Pérdidas de retorno asociadas al filtro inductivo excitado mediante sonda coaxial representado en la figura 4.15.

Excitation of microwave cavities: coupling

Example3: Analysis of comb-line diplexers excited with coaxial connectors for space telecommunication subsystems

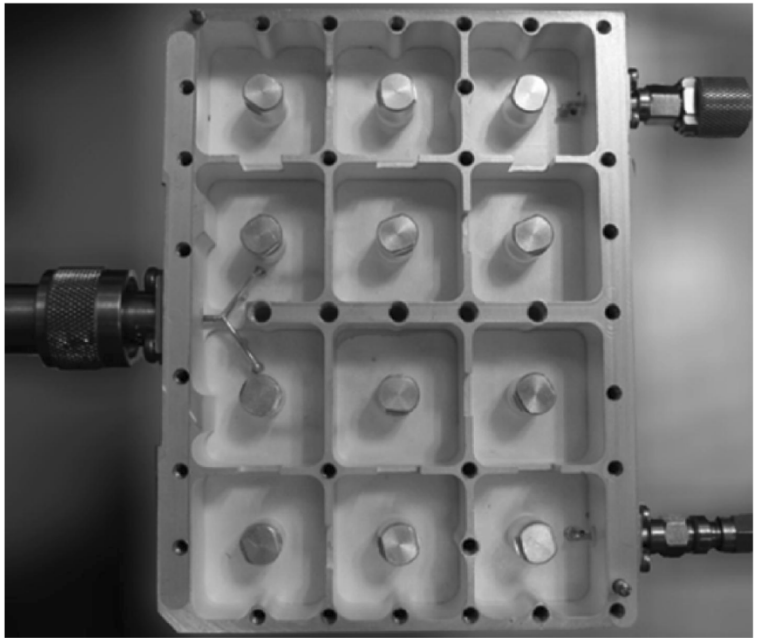


Figura 5.1: Diplexor en configuración *comb-line* implementado en tecnología guiada. El dispositivo consta de 12 resonadores rectangulares cargados con postes conductores de geometría cilíndrica (realmente, se trata de 2 filtros, cada uno con 6 resonadores rectangulares).

Excitation of microwave cavities: coupling

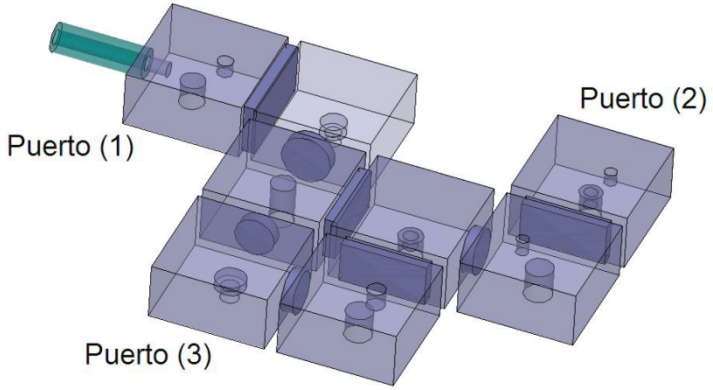


Figura 5.29: Dispositivo *comb-line* de 8 cavidades y 3 puertos de acceso, siendo uno de tipo coaxial y el resto de tipo rectangular. Esta estructura se utiliza para validar la herramienta de simulación relativa a la etapa 5.

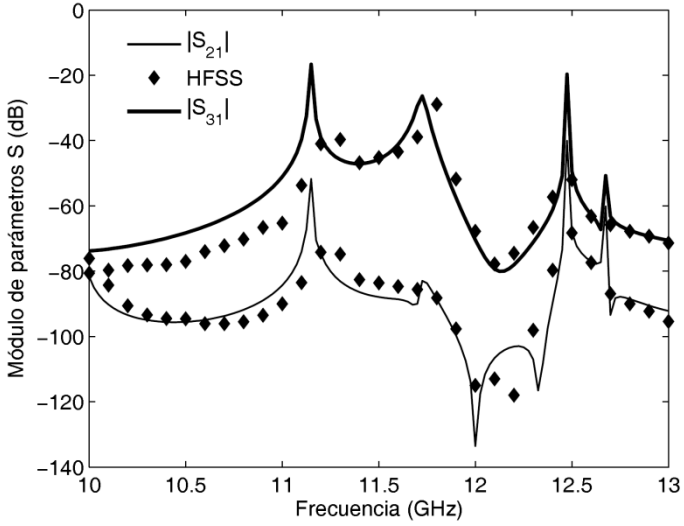
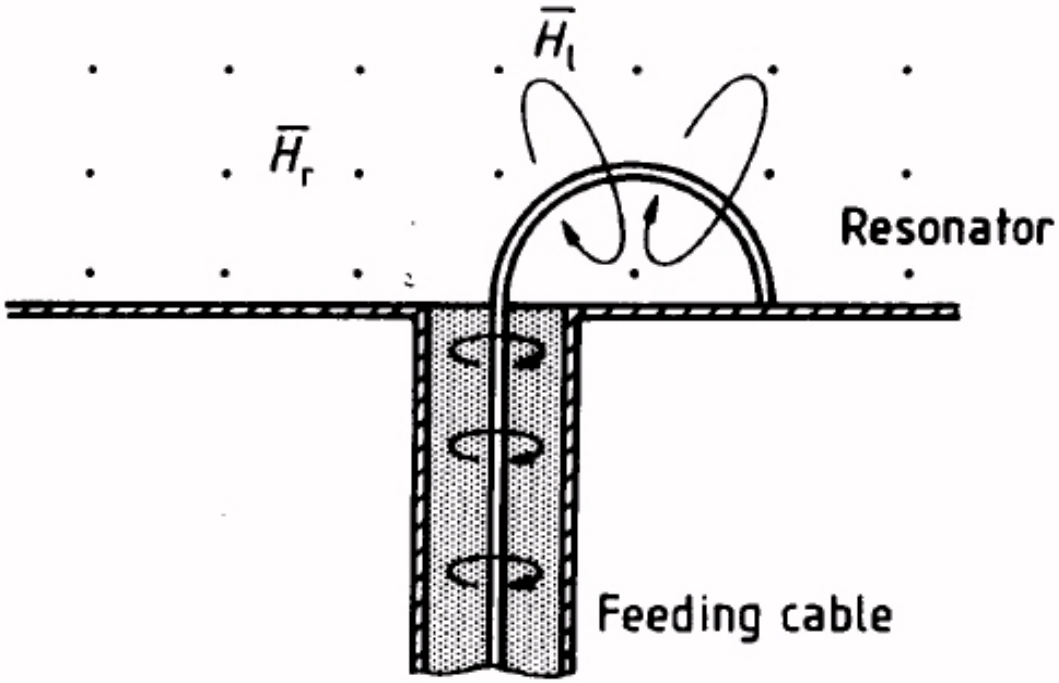


Figura 5.30: Módulo de los parámetros de dispersión del dispositivo *comb-line* de la figura 5.29. Los resultados obtenidos con la herramienta de simulación se comparan con los datos proporcionados por HFSS.

Excitation of microwave cavities: coupling

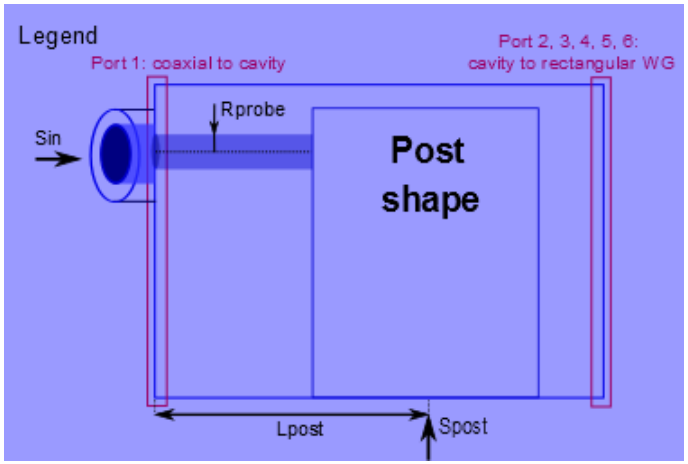
- Magnetic current loop: current loop contacting the metallic surface of the cavity

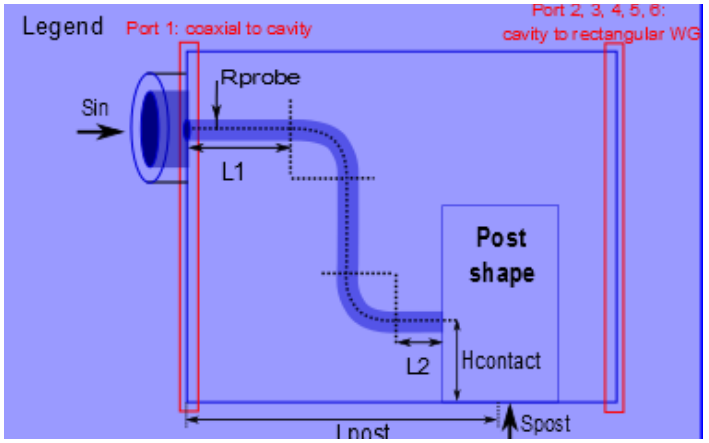


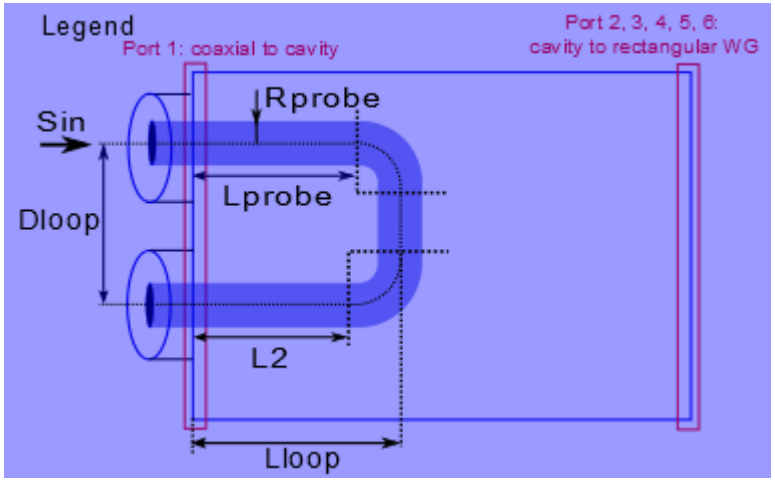
The magnetic field ($\bar{\mathbf{H}}_i$) of a coupling loop couples to the resonance mode through the magnetic field ($\bar{\mathbf{H}}_r$) perpendicular to the plane of the loop.

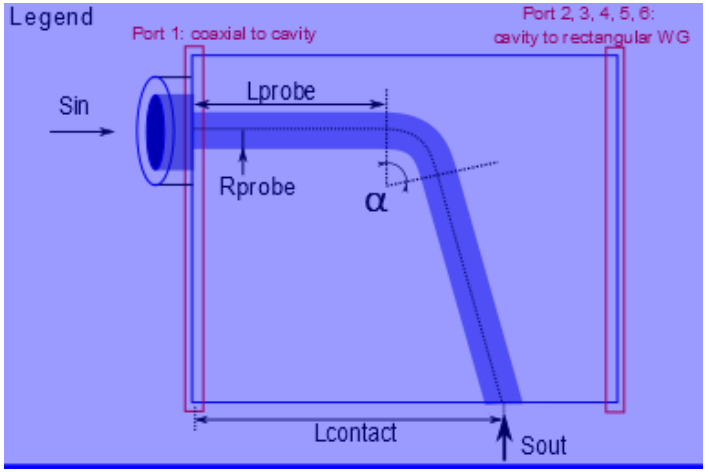
Excitation of microwave cavities: coupling

Examples: There are a lot of different kind of current loops used in microwave circuits









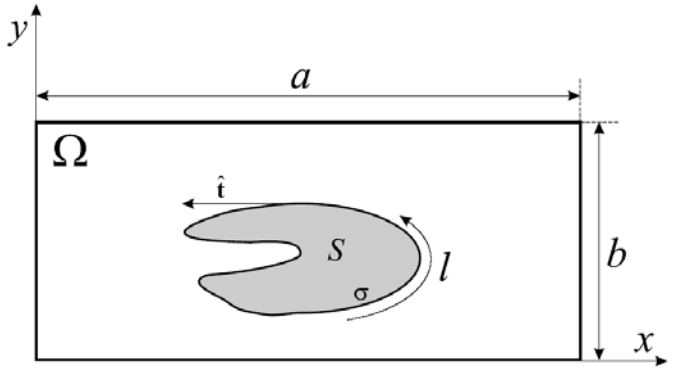
INDEX

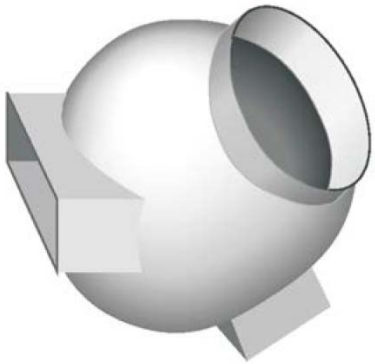
- Introduction
- The rectangular waveguide
- The *empty* rectangular cavity
- Excitation of microwave cavities: coupling
- The BI-RME method
- Examples
- Conclusions

The BI-RME method

- **BI-RME**: Boundary-Integral Resonant Mode Expansion
- Method developed at University of Pavia (Italy) by Prof. Giuseppe Conciauro, Prof. Marco Bressan, and Prof. Paolo Arcioni:

  G. Conciauro, M. Guglielmi, R. Sorrentino, *Advanced Modal Analysis*, Wiley, 2000
- During last 20 years it has been applied to the rigorous and efficient modal analysis of uniform arbitrarily-shaped waveguides (**BI-RME 2D**), and cavities including metallic and dielectric objects with arbitrary shape (**BI-RME 3D**).



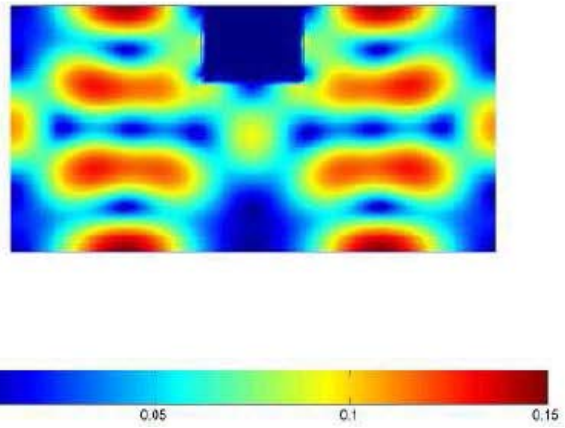


- There are more than 50 articles published in international journals and transactions directly related with BI-RME applications.

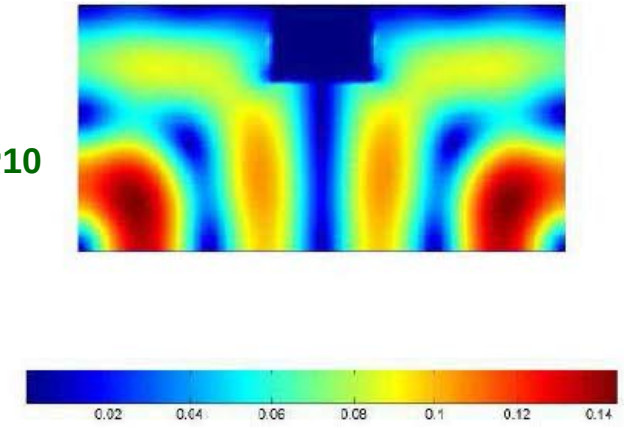
The BI-RME method

- Both versions of the BI-RME method transform an integral equation of Fredholm of first kind into an algebraic eigenvalue problem which is numerically solved with standard diagonalization subrutines.
- Example of **BI-RME 2D:** computation of the full modal spectrum of a ridge rectangular waveguide. The cutoff frequencies (eigenvalues) and the electromagnetic fields (eigenvectors) are computed for a set of modes.

TM_{11}



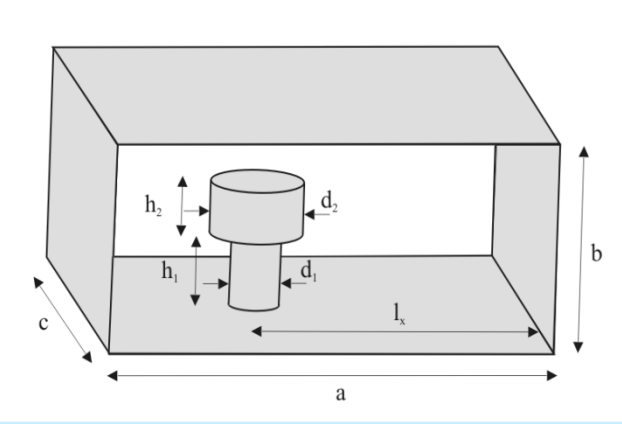
TE_{10}



S. Cogollos, S. Marini, V. E. Boria, P. Soto, A. Vidal, H. Esteban, J. V. Morro, B. Gimeno, 'Efficient Modal Analysis of Arbitrarily Shaped Waveguides Composed of Linear, Circular, and Elliptical Arcs Using the BI-RME Method', IEEE Transactions on Microwave Theory and Techniques, vol. 51, no. 12, pp. 2378-2390, Dec. 2003

The BI-RME method

- Example1 of **BI-RME 3D:** computation of the full modal spectrum of a rectangular microwave cavity loaded with a metallic cylindrical 'mushroom'. The resonant frequencies (eigenvalues) and the electromagnetic fields (eigenvectors) are computed for a set of modes.



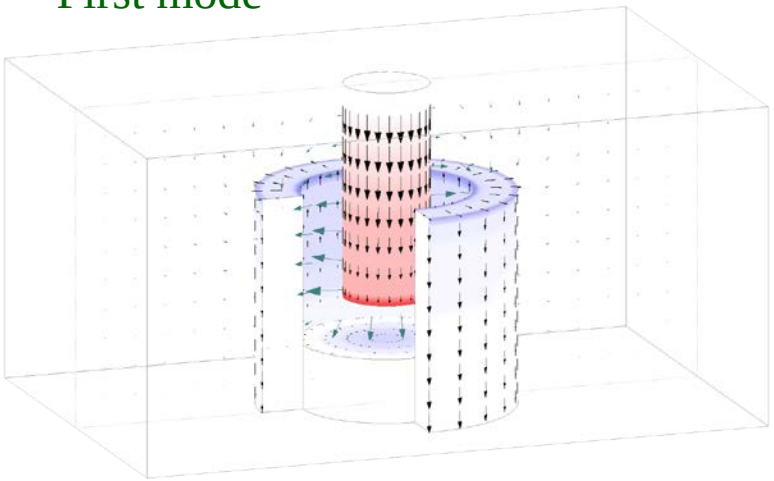
a=30.00mm b=15.00mm c=20.00mm
d_t=4.00mm d_i=8.00mm d_e=12.00mm
h_i=7.00mm h_e=10.00mm h_t=10.00mm

Resonant frequencies (GHz) and relative error (%)				
FB type II				FEM
a_{II}		b_{II}		
6.3545	0.07	6.3516	0.02	6.3501
12.9092	0.04	12.9068	0.02	12.904
15.0777	0.04	15.0723	0.01	15.071
15.3396	0.04	15.3334	0.00	15.334
16.2404	0.06	16.2369	0.04	16.230
16.8115	0.05	16.8079	0.03	16.803
17.6677	0.00	17.6672	0.00	17.667
18.4687	0.04	18.4666	0.02	18.462
19.3133	0.05	19.3089	0.03	19.304
20.1044	0.03	20.1019	0.02	20.098

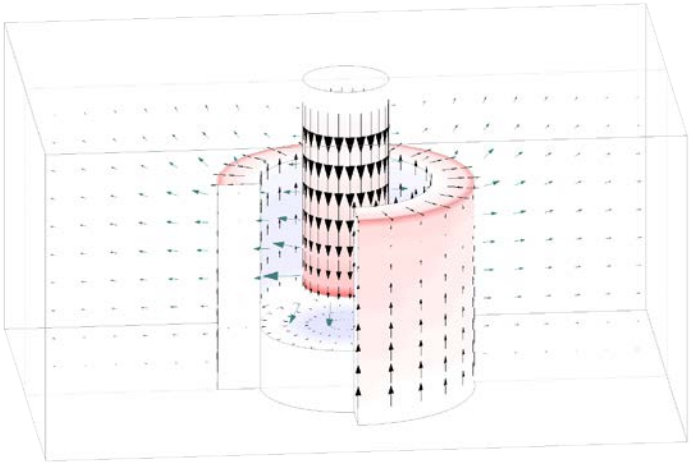
F. Mira, M. Bressan, G. Conciauro, B. Gimeno, V. E. Boria, 'Fast S-Domain Modeling of Rectangular Waveguides With Radially Symmetric Metal Insets', IEEE Transactions on Microwave Theory and Techniques, vol. 53, no. 4, pp. 1294-1303, April 2005

The BI-RME method

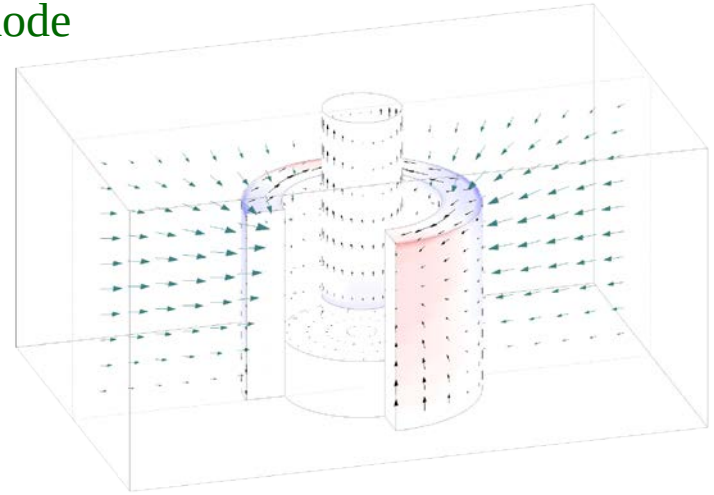
First mode



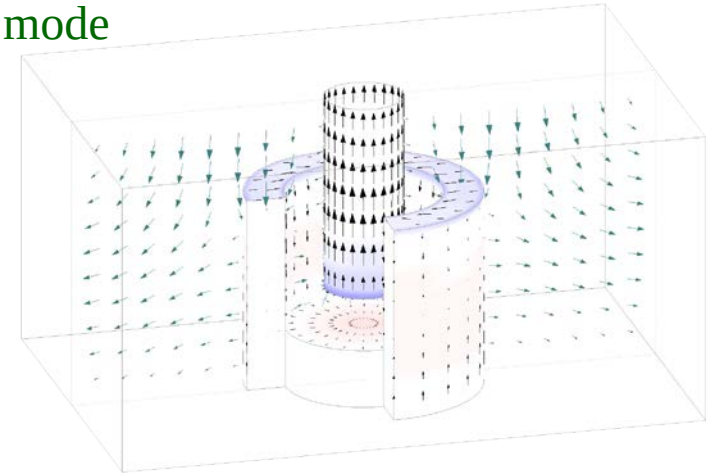
Second mode



Third mode



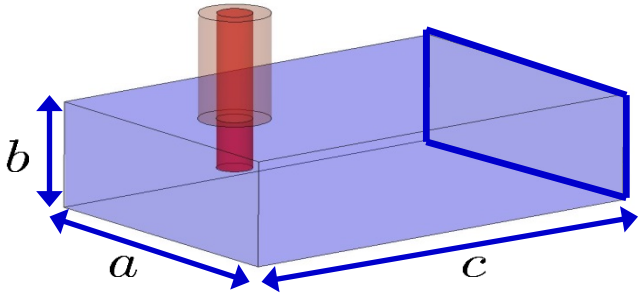
Fourth mode

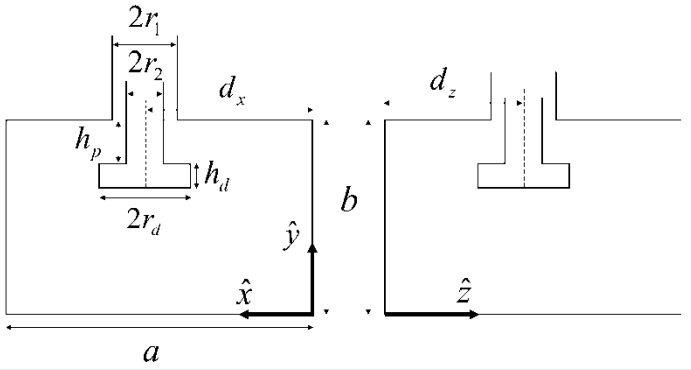


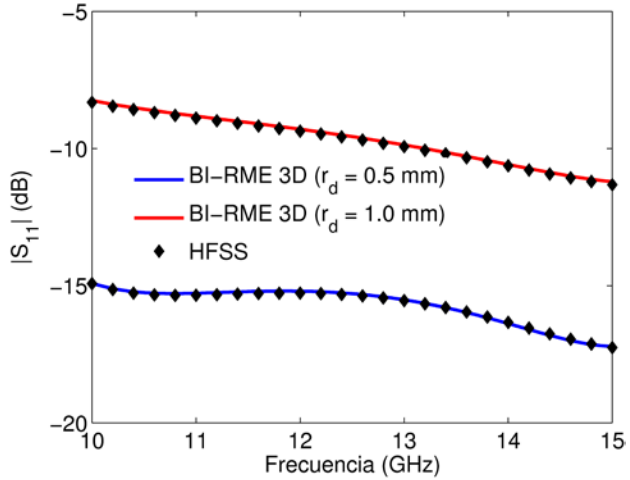
Electric field lines and surface current density lines have been plotted.

The BI-RME method

- Example2 of **BI-RME 3D:** frequency response computation of a junction between a coaxial line with a rectangular waveguide.







Dimensions (mm)

$a = 19.05,\ b = 9.525$ (WR-75)

Coaxial: $r_1 = 3.18,\ r_2 = 0.3$

$\varepsilon_r = 2.09,\ d_x = a/2,\ d_z = 6.8$

$h_p = 4.7625,\ h_d = 1.5$

CPU time: 151.6 s (500 frequency poinys) and 16 min (25 frequency points) with HFS

Á. A. San Blas, F. Mira, V. E. Boria, B. Gimeno, M. Bressan, P. Arcioni, 'On the Fast and Rigorous Analysis of Compensated Waveguide Junctions using Off-Centered Partial-Height Metallic Posts', IEEE Trans. Microwave Theory Tech., vol. 55, no. 1, pp. 168-175, Jan. 2007

The BI-RME method

- Example3 of **BI-RME 3D:** calculation of the resonant modes of the ELETTRA accelerating cavity placed in Stanford Linear Accelerator (USA).

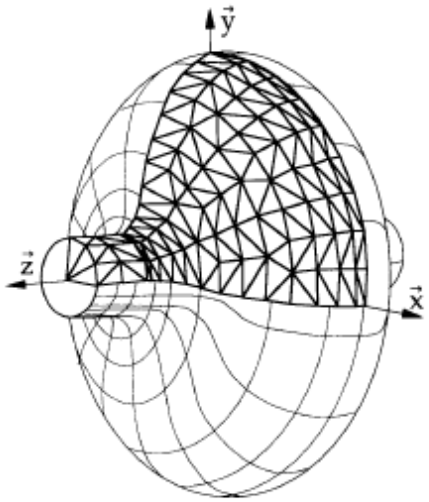


Fig. 4. The ELETTRA accelerating cavity. The mesh shown is that used to obtain the results of Table I.

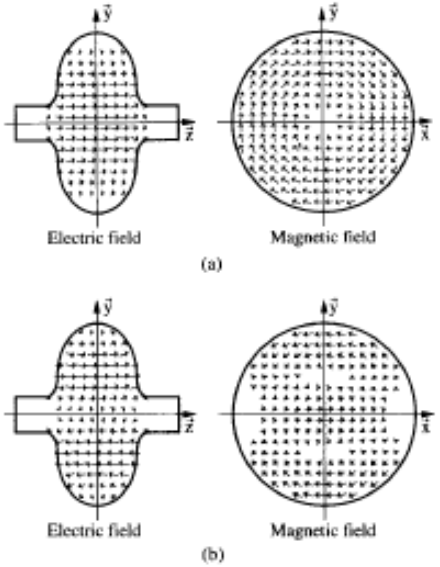


Fig. 5. Arrow plots representing two resonant fields of the ELETTRA cavity. (a) Fundamental mode; (b) first deflecting mode.

P. Arcioni, M. Bressan, L.Perregrini, 'A New Boundary Integral Approach to the Determination of the Resonant Modes of Arbitraily Shaped Cavities', IEEE Trans. Microwave Theory Tech., vol. 43, no. 8, pp. 1848-1855, Aug. 1995

The BI-RME method

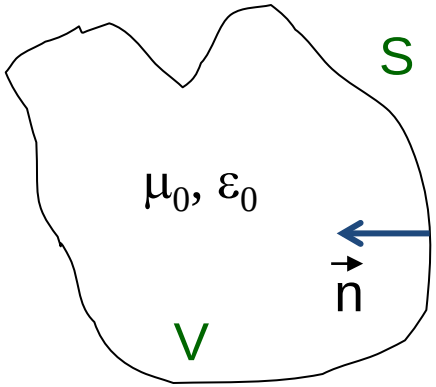
- **BI-RME 3D** technique is applied in two steps:

  (1) Calculation of the full set of resonant modes of the closed cavity

  (2) Calculation of the coupling between the closed cavity and the input/ouput waveguide ports.

- (1) Calculation of the full set of resonant modes of the closed cavity:

  - We have an empty microwave closed cavity constructed with a metallic wall:



In principle we will consider lossless conductor walls.

The BI-RME method

- Maxwell equations (expressed in frequency domain) gobern the electromagnetic fields existing within the cavity, ρ and **J** being the time-harmonic electric sources (representing a metallic inset inside the cavity):

$$\nabla \cdot \vec{E} = \rho/\varepsilon_0 \quad , \quad \nabla \times \vec{E} = -j\omega\mu_0\vec{H}$$
$$\nabla \cdot \vec{H} = 0 \quad , \quad \nabla \times \vec{H} = j\omega\varepsilon_0\vec{E} + \vec{J}$$

- The total electromagnetic field inside the cavity can be expressed as a superposition of solenoidal and irrotational modes:

$$\vec{E} = \sum_n E_n\vec{e}_n + \sum_n F_n\vec{f}_n$$
$$\vec{H} = \sum_n H_n\vec{h}_n + \sum_n G_n\vec{g}_n$$

where E_n, F_n, H_n and G_n are the expansion coefficients.

The BI-RME method

- The normalized modal vectors are clasified in 4 groups:

Solenoidal electric modes (resonant modes):

$$\nabla \times \vec{e}_n \neq \vec{0}$$
$$\nabla \cdot \vec{e}_n = 0$$

Boundary condition on the cavity surface:

$$\vec{n} \times \vec{e}_n = \vec{0}$$

$$\nabla^2 \vec{e}_n + k_n^2 \vec{e}_n = 0$$

$$k_n = \frac{\omega_n}{c} \longrightarrow \text{PHYSICAL RESONANCES}$$

Irrotational electric modes (non-resonant modes):

$$\nabla \times \vec{f}_n = \vec{0}$$
$$\nabla \cdot \vec{f}_n \neq 0$$

Boundary condition on the cavity surface:

$$\vec{n} \times \vec{f}_n = \vec{0}$$

$$\vec{f}_n = \frac{\nabla v_n}{\xi_n}$$

$$\nabla^2 v_n + \xi_n^2 v_n = 0$$

$$v_n = 0, \text{ on the cavity surface}$$

The BI-RME method

Solenoidal magnetic modes (resonant modes):

$$\nabla \times \vec{h}_n \neq \vec{0}$$
$$\nabla \cdot \vec{h}_n = 0$$

Boundary condition on the cavity surface:

$$\vec{n} \cdot \vec{h}_n = 0$$

$$\nabla^2 \vec{h}_n + k_n^2 \vec{h}_n = 0$$

Irrotational magnetic modes (non-resonant modes):

$$\nabla \times \vec{g}_n = \vec{0}$$
$$\nabla \cdot \vec{g}_n \neq 0$$

Boundary condition on the cavity surface:

$$\vec{n} \cdot \vec{g}_n = 0$$

$$\vec{g}_n = \frac{\nabla w_n}{\nu_n}$$

$$\nabla^2 w_n + \nu_n^2 w_n = 0$$

$$\frac{\partial w_n}{\partial n} = 0, \text{ on the cavity surface}$$

- Solenoidal electric and magnetic modes satisfy these reciprocal equations:

$$\nabla \times \vec{e}_n = k_n \vec{h}_n$$
$$\nabla \times \vec{h}_n = k_n \vec{e}_n$$

The BI-RME method

- The normalized modal vectors satisfy the following orthonormalization relationships:

$$\int_V \vec{e}_n \cdot \vec{e}_m \mathrm{d}V = \delta_{nm}$$

$$\int_V \vec{f}_n \cdot \vec{f}_m \mathrm{d}V = \delta_{nm}$$

$$\int_V \vec{e}_n \cdot \vec{f}_m \mathrm{d}V = 0$$

$$\int_V \vec{h}_n \cdot \vec{h}_m \mathrm{d}V = \delta_{nm}$$

$$\int_V \vec{g}_n \cdot \vec{g}_m \mathrm{d}V = \delta_{nm}$$

$$\int_V \vec{h}_n \cdot \vec{g}_m \mathrm{d}V = 0$$

As a direct consequence, the expansion coeffients can be obtained as follows,

$$E_n = \int_V \vec{E} \cdot \vec{e}_n \mathrm{d}V \qquad H_n = \int_V \vec{H} \cdot \vec{h}_n \mathrm{d}V$$

$$F_n = \int_V \vec{E} \cdot \vec{f}_n \mathrm{d}V \qquad G_n = \int_V \vec{H} \cdot \vec{g}_n \mathrm{d}V$$

The BI-RME method

- Next step is to expand the curl of the electric (magnetic) field as a magnetic-like (electric-like) field function. After inserting these results in Maxwell equations we find:

$$\int_V \nabla \times \vec{E} \cdot \vec{h}_m dV = -j\omega\mu_0 \int_V \vec{H} \cdot \vec{h}_m dV$$

$$\int_V \nabla \times \vec{H} \cdot \vec{e}_m dV = j\omega\varepsilon_0 \int_V \vec{E} \cdot \vec{e}_m dV + \int_V \vec{J} \cdot \vec{e}_m dV$$

$$\int_V \nabla \times \vec{H} \cdot \vec{f}_m dV = j\omega\varepsilon_0 \int_V \vec{E} \cdot \vec{f}_m dV + \int_V \vec{J} \cdot \vec{f}_m dV$$

which allows to obtain the expansion coefficients

$$E_m = \frac{j\omega\mu_0}{k^2 - k_m^2} \int_V \vec{J} \cdot \vec{e}_m \, dV$$

$$H_m = \frac{-k_m}{k^2 - k_m^2} \int_V \vec{J} \cdot \vec{e}_m \, dV$$

$$F_m = \frac{j}{\omega\varepsilon_0} \int_V \vec{J} \cdot \vec{f}_m \, dV$$

and $G_m = 0$.

The BI-RME method

- By combining the previous equations we find:

$$\vec{E} = \sum_m \frac{j\omega\mu_0}{k^2 - k_m^2} \int_V \vec{J}' \cdot \vec{e}'_m \, dV' \vec{e}_m + \sum_m \frac{j}{\omega\varepsilon_0} \int_V \vec{J}' \cdot \vec{f}'_m \, dV' \vec{f}_m = -\nabla V^E - j\omega \vec{A}$$

$$\vec{H} = \sum_m \frac{-k_m}{k^2 - k_m^2} \int_V \vec{J}' \cdot \vec{e}'_m \, dV' \vec{h}_m = \frac{1}{\mu_0} \nabla \times \vec{A}$$

Therefore, the expressions of both electric scalar potential and magnetic vector potential are written as

$$V^E(\vec{r}) = \frac{1}{\varepsilon_0} \sum_i \frac{1}{\xi_i^2} \int_V v_i(\vec{r'}) \rho(\vec{r'}) dV' \; v_i(\vec{r}) = \frac{1}{\varepsilon_0} \int_V g^E(\vec{r}, \vec{r'}) \rho(\vec{r'}) dV'$$

$$g^E(\vec{r}, \vec{r'}) = \sum_i \frac{1}{\xi_i^2} \, v_i(\vec{r}) \, v_i(\vec{r'}) \longrightarrow$$

ELECTRIC SCALAR GREEN'S FUNCTION IN THE COULOMB GAUGE

and

The BI-RME method

$$\vec{A}(\vec{r}) = \mu_0 \sum_i \frac{-1}{k^2 - k_i^2} \int_V \vec{J}(\vec{r'}) \cdot \vec{e}_i(\vec{r'}) dV' \; \vec{e}_i(\vec{r}) = \mu_0 \int_V \vec{\mathbf{G}}^{\mathrm{A}}(\vec{r}, \vec{r'}) \cdot \vec{J}(\vec{r'}) dV'$$

$$\vec{\mathbf{G}}^{\mathrm{A}}(\vec{r}, \vec{r'}) = \sum_i \frac{-1}{k^2 - k_i^2} \vec{e}_i(\vec{r}) \, \vec{e}_i(\vec{r'}) \longrightarrow$$

MAGNETIC DIADIC GREEN'S FUNCTION IN THE COULOMB GAUGE

A **dyadic expression** is a second-order contravariant cartesian tensor defined in $\mathcal{R}^3$ which can be formulated in terms of a matrix:

$$\vec{R}\,\vec{R} = \begin{pmatrix} R_x R_x & R_x R_y & R_x R_z \\ R_y R_x & R_y R_y & R_y R_z \\ R_z R_x & R_z R_y & R_z R_z \end{pmatrix}$$

The BI-RME method

- It can be easily demonstrated that the electric scalar potential in the Coulomb gauge satisfies the Poisson differential equation:

$$\begin{aligned} \nabla^2 V^E &= -\frac{\rho}{\varepsilon_0} \text{ en } V \\ V^E &= 0 \text{ en } S \end{aligned}$$

As a consequence, the electric scalar Green's function satisfies the following differential equation:

$$\begin{aligned} \nabla^2 g^E(\vec{r},\vec{r'}) &= -\delta(\vec{r}-\vec{r'}) \text{ en } V \\ g^E &= 0 \text{ en } S \end{aligned}$$

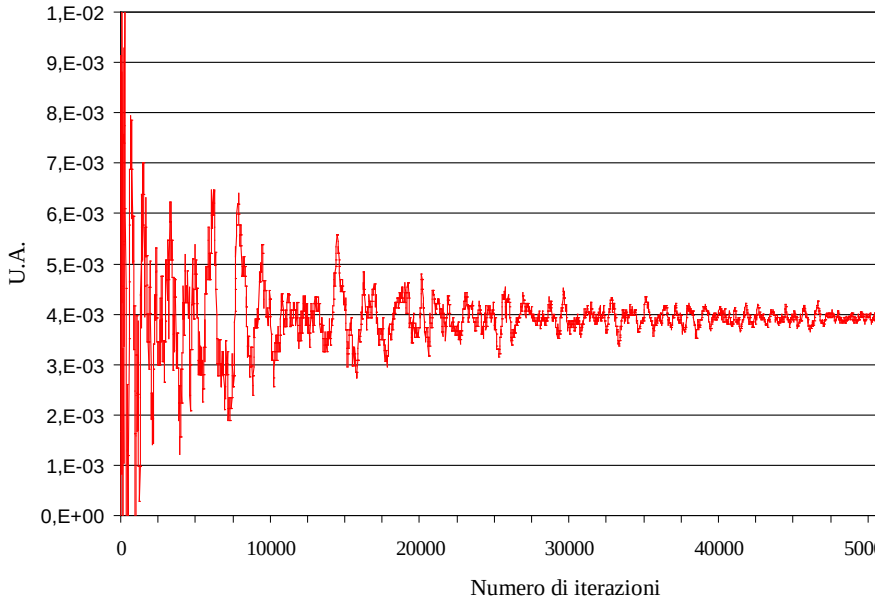
whose solution can be splitted in a singular term and a regular one:

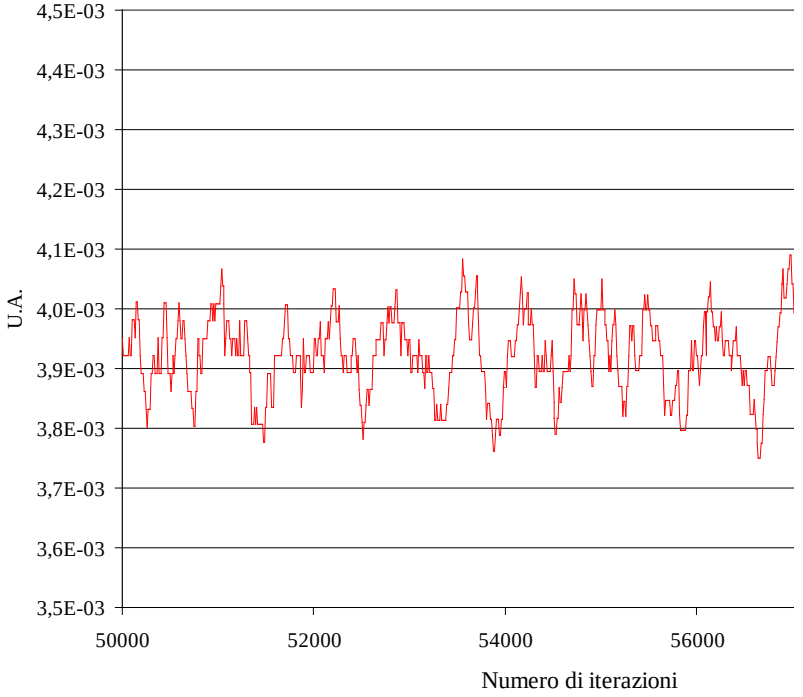
$$\begin{aligned} g^E(\vec{r},\vec{r'}) &= \sum_i \frac{1}{\xi_i^2} v_i(\vec{r}) v_i(\vec{r'}) = g_s^E(\vec{r},\vec{r'}) + g_r^E(\vec{r},\vec{r'}) \\ g_s^E(\vec{r},\vec{r'}) &= \frac{1}{4\pi R} \end{aligned}$$

The BI-RME method

From a computational point of view, this electric scalar Green function is not easy to be calculated in a rectangular cavity ($\kappa^2 = \kappa_x^2 + \kappa_y^2 + \kappa_z^2$):

$$g(\mathbf{r},\mathbf{r}') = \frac{8}{abc}\sum_{m,n,p=1}^{\infty} \frac{\sin(\kappa_x x)\sin(\kappa_y y)\sin(\kappa_z z)\sin(\kappa_x x')\sin(\kappa_y y')\sin(\kappa_z z')}{\kappa^2}$$

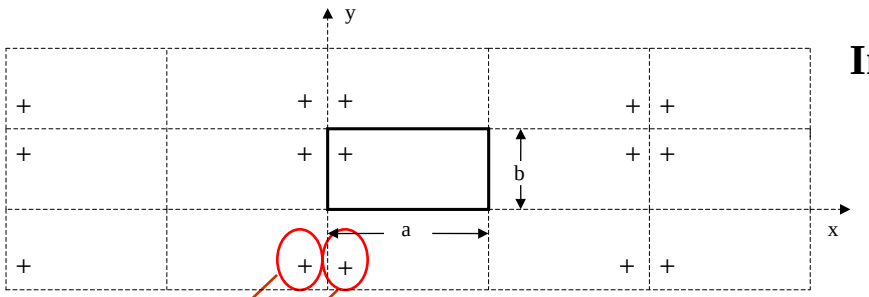




This infinite serie converges very slowly !!

The BI-RME method

Ewald technique has to be used for an efficient computation of the infinite series:



Images technique is used as an alternative representation of the series

3D Gaussians distributions are added and substracted in order to improve convergence series

$$g(\vec{r},\vec{r}') = \varphi_1(\vec{r},\vec{r}') + \varphi_2(\vec{r},\vec{r}')$$

P.P. Ewald, Ann der Physik, vol. 64, 1921

The BI-RME method

Both series have to be treated in a very different way:

$$\nabla^2\varphi_1 = -\sum_{mnp=-\infty}^{\infty} \sum_{i_x,i_y,i_z=-1,1} \left[\rho\left(\vec{r},\vec{r}'_s\right) - 1\right]$$

$$\nabla^2\varphi_2 = -\sum_{mnp=-\infty}^{\infty} \sum_{i_x,i_y,i_z=-1,1} \left[\delta(\vec{r}-\vec{r}'_s) - \rho\left(\vec{r},\vec{r}'_s\right)\right]$$

Solution in **spectral domain**

Solution in **spatial domain**

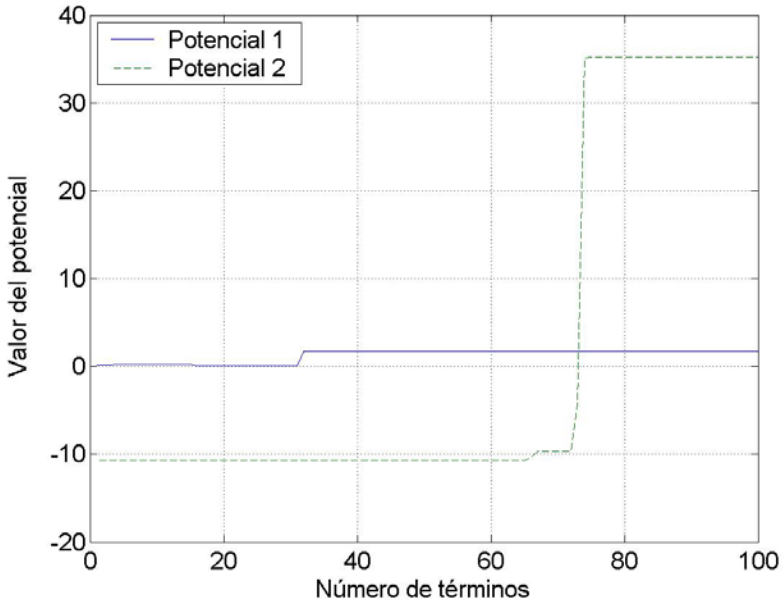
$$\varphi_1 = \frac{1}{V} \sum_{\substack{n_x,n_y,n_z=0 \\ n_x=n_y=n_z=0 \text{ excluded}}}^{\infty} \epsilon_{n_x}\epsilon_{n_y}\epsilon_{n_z} \frac{e^{-\eta^2\kappa^2/4}}{\kappa^2} C_x C_y C_z C'_x C'_y C'_z$$

$$\varphi_2 = \frac{1}{4\pi} \sum_{n_x,n_y,n_z=-\infty}^{\infty} \sum_{i_x,i_y,i_z=-1,1} \frac{\operatorname{erfc}(R/\eta)}{R} - \frac{\eta^2}{4V}$$

The argument of the serie exponentially vanishs when κ is increased

The argument of the series vanishs controled by the **erfc** function (which tends to zero when R tends to •)

The BI-RME method



Exponential convergence for both series:
40 terms for φ_1 and 80 terms for φ_2

Efficient computacional efficiency:
CPU time of 0.06 seconds for the same accuracy than in the previous case

The BI-RME method

- It can also be easily demonstrated that the magnetic vector potential in the Coulomb gauge satisfies this differential equation:

$$\begin{aligned} \nabla \times \nabla \times \vec{A} - k^2 \vec{A} &= \mu_0 \vec{J} - j\omega\mu_0\varepsilon_0 \nabla V^E \text{ en } V \\ \vec{n} \times \vec{A} &= 0 \text{ en } S \end{aligned}$$

As a consequence, the magnetic diadic Green's function satisfies the following differential equation:

$$\begin{aligned} \nabla \times \nabla \times \vec{\mathbf{G}}^{\mathrm{A}}(\vec{r},\vec{r'}) - k^2 \vec{\mathbf{G}}^{\mathrm{A}}(\vec{r},\vec{r'}) &= \vec{\mathbf{I}}\delta(\vec{r}-\vec{r'}) - \nabla\nabla' g^E(\vec{r},\vec{r'}) \text{ en } V \\ \vec{n} \times \vec{\mathbf{G}}^{\mathrm{A}} &= 0 \text{ en } S \end{aligned}$$

whose solution can be splitted in a singular term and a regular one (**this is not trivial !!**):

$$\vec{\mathbf{G}}^{\mathrm{A}}(\vec{r},\vec{r'}) = \sum_i \frac{\vec{e}_i(\vec{r})\,\vec{e}_i(\vec{r'})}{k_i^2 - k^2} = \vec{\mathbf{G}}^{\mathrm{A}}_{\mathbf{s}}(\vec{r},\vec{r'}) + \vec{\mathbf{G}}^{\mathrm{A}}_{\mathbf{r}}(\vec{r},\vec{r'})$$

$$\vec{\mathbf{G}}^{\mathrm{A}}_{\mathbf{s}}(\vec{r},\vec{r'}) = \frac{1}{8\pi R}\left(\vec{\mathbf{I}} + \frac{\vec{R}\,\vec{R}}{R^2}\right)$$

$$\vec{\mathbf{G}}^{\mathrm{A}}_{\mathbf{r}}(\vec{r},\vec{r'}) = -k^2 \sum_i \frac{\vec{e}_i(\vec{r})\,\vec{e}_i(\vec{r'})}{k_i^2(k^2 - k_i^2)}$$

The BI-RME method

- Finally, electric and magnetic fields inside the cavity can be expressed in terms of integrals containing the aforementioned Green's functions:

$$\vec{E}(\vec{r}) = \frac{\eta}{jk}\nabla\int_V g^e(\vec{r},\vec{r}')\,\nabla'\cdot\vec{J}(\vec{r}')\,dV' - jk\eta\int_V \vec{\mathbf{G}}^{\mathrm{A}}(\vec{r},\vec{r}')\cdot\vec{J}(\vec{r}')\,dV'$$

$$\vec{H}(\vec{r}) = \int_V \nabla\times\vec{\mathbf{G}}^{\mathrm{A}}(\vec{r},\vec{r}')\cdot\vec{J}(\vec{r}')\,dV'$$

where $\eta=(\mu_0/\varepsilon_0)^{1/2}$ is the free space characteristic impedance. Inserting expansions of both Green's functions we find,

$$\vec{E}(\vec{r}) = \frac{\eta}{jk}\nabla\int_V g^e(\vec{r},\vec{r}')\,\nabla'\cdot\vec{J}(\vec{r}')\,dV' - jk\eta\int_V \vec{\mathbf{G}}^{\mathrm{A}}_{\mathbf{s}}(\vec{r},\vec{r}')\cdot\vec{J}(\vec{r}')\,dV' + jk\eta\,k^2\sum_i \frac{\vec{e}_i(\vec{r})}{k_i^2(k^2-k_i^2)}\int_V \vec{e}_i(\vec{r}')\cdot\vec{J}(\vec{r}')dV'$$

$$\vec{H}(\vec{r}) = \int_V \nabla\times\vec{\mathbf{G}}^{\mathrm{A}}_{\mathbf{s}}(\vec{r},\vec{r}')\cdot\vec{J}(\vec{r}')\,dV' - k^2\sum_i \frac{\vec{h}_i(\vec{r})}{k_i^2(k^2-k_i^2)}\int_V \vec{e}_i(\vec{r}')\cdot\vec{J}(\vec{r}')dV'$$

The BI-RME method

- Calculation of the resonant modes of the cavity is performed by imposing that the tangential component of the electric field is zero on the surface of the metallic obstacles contained in the cavity:

$$\vec{E}(\vec{r})|_{tangent} = \left(\frac{\eta}{jk} \nabla \int_S g^e(\vec{r},\vec{r}\,') \, \nabla' \cdot \vec{J}_\sigma(\vec{r}\,') \, dS' - jk\eta \int_S \vec{\mathbf{G}}_{\mathbf{s}}^{\mathrm{A}}(\vec{r},\vec{r}\,') \cdot \vec{J}_\sigma(\vec{r}\,') \, dS' \right.$$
$$\left. + \; jk\eta \, k^2 \sum_i \frac{\vec{e}_i(\vec{r})}{k_i^2(k^2-k_i^2)} \int_S \vec{e}_i(\vec{r}\,') \cdot \vec{J}_\sigma(\vec{r}\,') dS' \right) \Big|_{tangent} = \vec{0}$$

This homogeneous system admits non-trivial solutions only for particular values of k, which are the resonant wavenumbers of the cavity. The surface current density **J** is the unknown function of this integral equation of Fredholm of first kind, and it has to be expanded as:

$$\mathbf{J}_\sigma = \frac{j}{\eta} \sum_{n=1}^{P} i_n \mathbf{W}_n - \frac{jk}{\eta} \sum_{n=1}^{Q} q_n \mathbf{V}_n$$

$$\nabla_{\mathrm{S}} \cdot \mathbf{W}_n = 0$$

$$\nabla_{\mathrm{S}} \cdot \mathbf{V}_n = v_n \neq 0$$

where $\mathbf{w}_n$ and $\mathbf{v}_n$ are a complete set of solenoidal and non-solenoidal vectorial basis functions defined on the surface of the metallic object, respectivelly.

The BI-RME method

By inserting the current expansion into the definition of a_m coefficients

$$a_m = \frac{jk\eta}{\kappa_m^2(\kappa_m^2 - k^2)} \int_\sigma \mathbf{e}_m \cdot \mathbf{J}_\sigma \, d\sigma \qquad \text{m = 1,2,3…M}$$

we find this algebraic linear system:

$$[K]^4[a] - k^2[K]^2[a] - k^2[R]\,[q] + k[S]\,[i] = 0 \qquad (*)$$

where [a], [q] and [i] are unknown vectors and [K]=diag{κ_1, κ_2…κ_M}; the [a] coefficients are defined as

$$R_{mn} = \int_\sigma \mathbf{e}_m \cdot \mathbf{V}_n \, d\sigma \qquad (m = 1, 2, \ldots, M; \;\; n = 1, 2, \ldots, Q)$$

and

$$S_{mn} = \int_\sigma \mathbf{e}_m \cdot \mathbf{W}_n \, d\sigma \qquad (m = 1, 2, \ldots, M; \;\; n = 1, 2, \ldots, P)$$

The BI-RME method

Using the Method of Moments with Galerkin technique (the expansion basis functions are used to dot-multiplying both sides of the integral equation) we find:

$$[C]\,[q] + k[T]\,[i] - k^2\,[V]\;[q] + k^2[R]^{\mathrm{T}}[a] = 0 \qquad (**)$$

$$-k[L]\,[i] + k^2\,[T]^{\mathrm{T}}[q] + k^2[S]^{\mathrm{T}}[a] = 0$$

where

$$C_{np} = \int_\sigma \int_\sigma v_n(\mathbf{r})\, g^{\mathrm{E}}(\mathbf{r},\mathbf{r}')\, v_p(\mathbf{r}')\, d\sigma\, d\sigma' \qquad n,p = 1,2,\ldots,Q$$

$$V_{np} = \int_\sigma \int_\sigma \mathbf{V}_n(\mathbf{r}) \cdot \overline{\mathbf{G}}^{\mathrm{A}}(\mathbf{r},\mathbf{r}',0) \cdot \mathbf{V}_p(\mathbf{r}')\, d\sigma\, d\sigma' \qquad n,p = 1,2,\ldots,Q$$

$$L_{np} = \int_\sigma \int_\sigma \mathbf{W}_n(\mathbf{r}) \cdot \overline{\mathbf{G}}^{\mathrm{A}}(\mathbf{r},\mathbf{r}',0) \cdot \mathbf{W}_p(\mathbf{r}')\, d\sigma\, d\sigma' \qquad n,p = 1,2,\ldots,P$$

$$T_{np} = \int_\sigma \int_\sigma \mathbf{V}_n(\mathbf{r}) \cdot \overline{\mathbf{G}}^{\mathrm{A}}(\mathbf{r},\mathbf{r}',0) \cdot \mathbf{W}_p(\mathbf{r}')\, d\sigma\, d\sigma' \qquad \begin{matrix} n = 1,2,\ldots,Q \\ p = 1,2,\ldots,P \end{matrix}$$

The BI-RME method

The last equation allows to obtain

$$[i] = k[L]^{-1}\left([S]^{\mathrm{T}}[a] + [T]^{\mathrm{T}}[q]\right)$$

Inserting this vector into (*) and (**) we find

$$\boxed{\left([A] - k^{-2}[B]\right)[x] = 0}$$

where

$$[A] = \left(\begin{array}{c|c} [K]^2 - [S]\,[L]^{-1}[S]^{\mathrm{T}} & [R] - [T]\,[L]^{-1}[S]^{\mathrm{T}} \\ \hline [R]^{\mathrm{T}} - [S]\,[L]^{-1}[T]^{\mathrm{T}} & [V] - [T]\,[L]^{-1}[T]^{\mathrm{T}} \end{array}\right)$$

$$[B] = \left(\begin{array}{c|c} [K]^4 & [0] \\ \hline [0] & [C] \end{array}\right) \qquad [x] = \left(\begin{array}{c} [a] \\ \hline [q] \end{array}\right)$$

The BI-RME method

It can be demostrated that both matrices [A] and [B] are symmetric and definite positive, so the solution of the previous matricial equation has positive real eigenvalues, representing the solution of the problem.
Electric and magnetic fields within the cavity can be calculated as:

$$\mathbf{E}_i = \sum_{n=1}^{Q} q_{ni} \left(\mathbf{F}_n(\mathbf{r}) - k_i^2 \mathbf{A}'_n(\mathbf{r}) \right) + k_i \sum_{n=1}^{P} i_{ni} \mathbf{A}''_n(\mathbf{r}) - k_i^2 \sum_{m=1}^{M} a_{mi}\, \mathbf{e}_m(\mathbf{r})$$

$$\begin{aligned}
\mathbf{F}_n(\mathbf{r}) &= -\nabla \int_\sigma g^{\mathrm{E}}(\mathbf{r}, \mathbf{r}')\, v_n(\mathbf{r}')\, d\sigma' \\
\mathbf{A}'_n(\mathbf{r}) &= \int_\sigma \overline{\mathbf{G}}^{\mathrm{A}}(\mathbf{r}, \mathbf{r}', 0) \cdot \mathbf{V}_n(\mathbf{r}')\, d\sigma' \\
\mathbf{A}''_n(\mathbf{r}) &= \int_\sigma \overline{\mathbf{G}}^{\mathrm{A}}(\mathbf{r}, \mathbf{r}', 0) \cdot \mathbf{W}_n(\mathbf{r}')\, d\sigma'
\end{aligned}$$

The BI-RME method

$$\mathbf{H}_i = -k_i \sum_{n=1}^{Q} q_{ni}\mathbf{B}'_n(\mathbf{r}) + \sum_{n=1}^{P} i_{ni}\mathbf{B}''_n(\mathbf{r}) - k_i \sum_{m=1}^{M} a_{mi}\,\kappa_m \mathbf{h}_m(\mathbf{r})$$

$$\mathbf{B}'_n(\mathbf{r}) = \int_\sigma \nabla\times\overline{\mathbf{G}}^{\mathrm{A}}(\mathbf{r},\mathbf{r}',0)\cdot\mathbf{V}_n(\mathbf{r}')\,d\sigma'$$

$$\mathbf{B}''_n(\mathbf{r}) = \int_\sigma \nabla\times\overline{\mathbf{G}}^{\mathrm{A}}(\mathbf{r},\mathbf{r}',0)\cdot\mathbf{W}_n(\mathbf{r}')\,d\sigma'$$

- Calculation of the Q-factor of each resonant mode can be easily computed in this scenario:

$$Q_i^{(c)} = \frac{k_i\eta}{R_s\int_{S_V}\mathbf{H}_i\cdot\mathbf{H}_i\,dS_V} \qquad R_s = \frac{1}{\sigma\,\delta} \qquad \delta = \sqrt{\frac{2}{\omega\,\mu\,\sigma}}$$

$$\int_{S_V}\mathbf{H}_i\cdot\mathbf{H}_i\,dS_V = \int_{S_V-\sigma}\mathbf{H}_i\cdot\mathbf{H}_i\,dS_V + \int_\sigma \mathbf{H}_i\cdot\mathbf{H}_i\,d\sigma$$

The BI-RME method

- For the selection of the expansion vectorial functions we have used the Rao-Wilton-Glisson (RWG) vectorial basis functions defined on triangular patches:

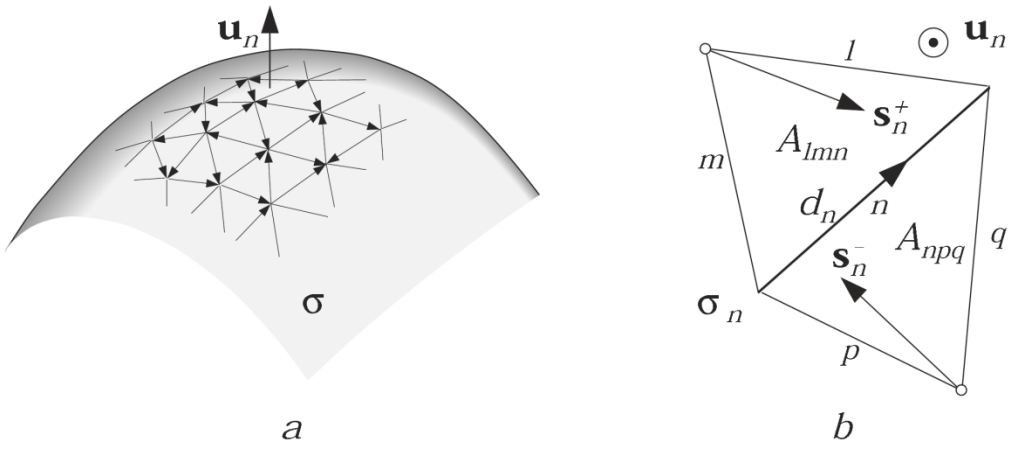


Figure 5.17: *a* - Modelling of σ by triangular patches; *b* - the support of the RWG function $\mathbf{f}_n$.

$$\mathbf{f}_n = \begin{cases} \dfrac{d_n}{2A_{\ell mn}}\,\mathbf{s}_n^+ & \text{on the left triangle} \\ -\dfrac{d_n}{2A_{npq}}\,\mathbf{s}_n^- & \text{on the right triangle} \end{cases}$$

The BI-RME method

In this case, these RWG basis functions have to be rewritten in terms of solenoidal and non-solenoidal vectorial basis functions:

$$\mathbf{W}_i = -d_\ell^{-1}\mathbf{f}_\ell - d_m^{-1}\mathbf{f}_m + d_n^{-1}\mathbf{f}_n - d_p^{-1}\mathbf{f}_p + d_q^{-1}\mathbf{f}_q + d_r^{-1}\mathbf{f}_r$$

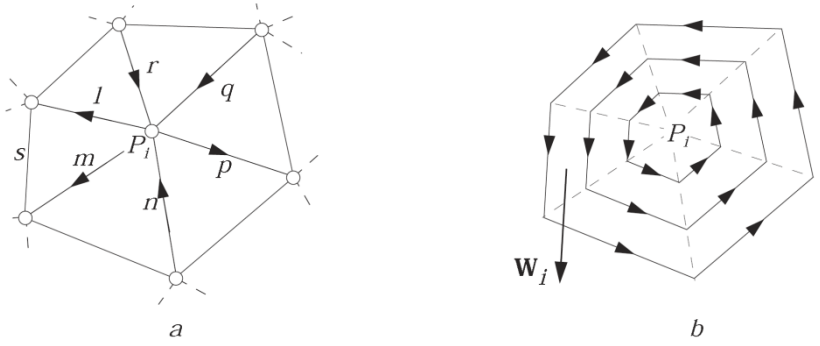


Figure 5.18: *a* - The support of the solenoidal basis functions corresponding to the *i*-th node; *b* - the annular pattern of this function.

$$\mathbf{V}_i = -d_\ell^{-1}\mathbf{f}_\ell - d_m^{-1}\mathbf{f}_m + d_n^{-1}\mathbf{f}_n$$

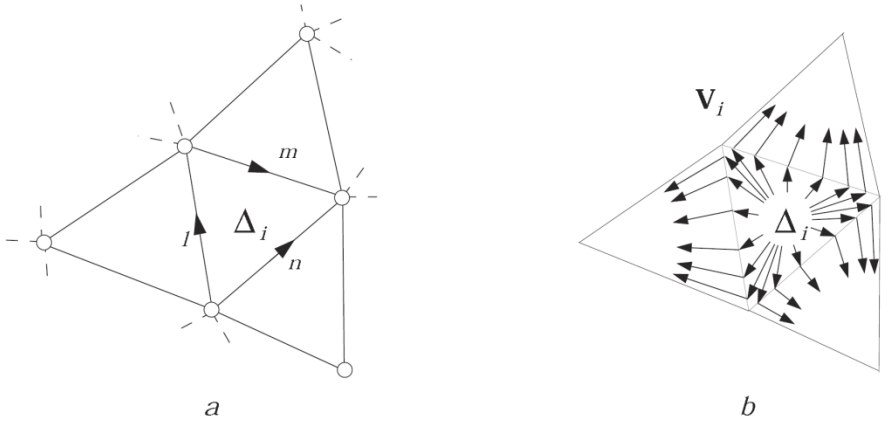
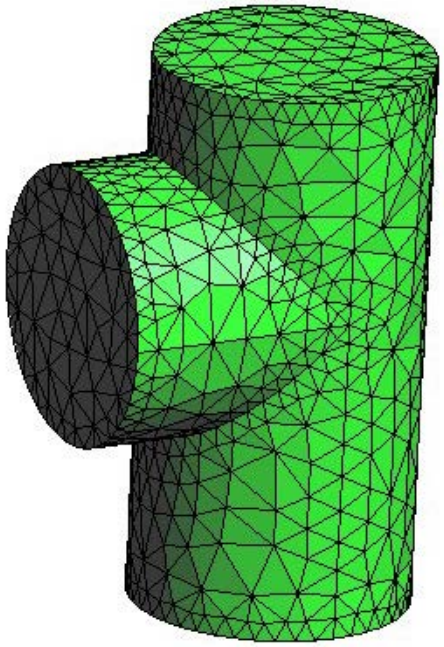


Figure 5.20: *a* - The support of $\mathbf{V}_i$: *b* - the stellate pattern of $\mathbf{V}_i$.

The BI-RME method

In the case of complex structures, we have to use a commercial software for the discretization procedure of the obstacle surface in triangules:

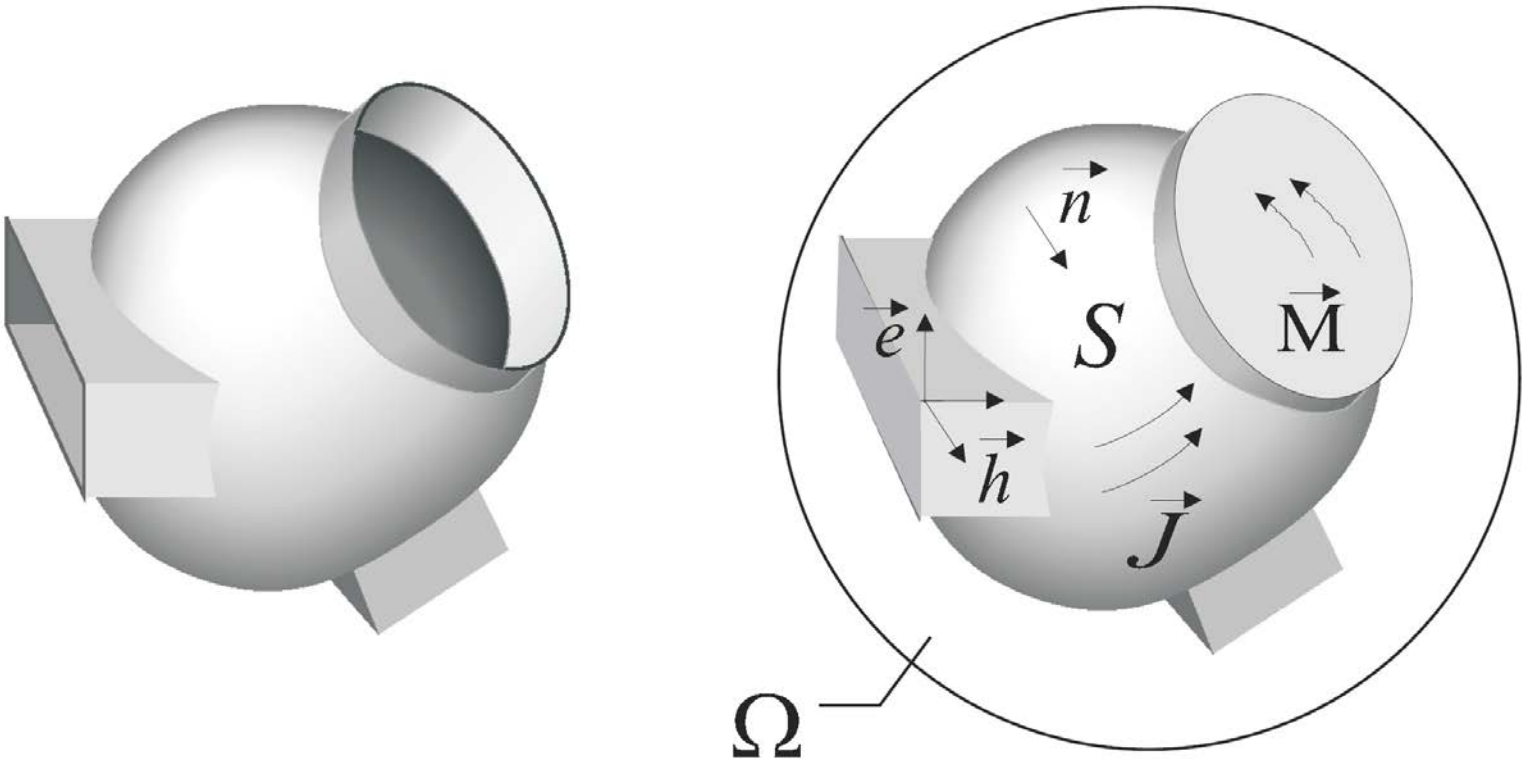


These codes provide the cartesian coordinates of the vertexes of the triangles.

The BI-RME method

- (2) Calculation of the coupling between the closed cavity and the input/ouput waveguide ports.

- We have a microwave cavity (constructed with a metallic wall) connected to several input/output ports:



The BI-RME method

- The formalim is similar to the previos case, but now the tangential electric and magnetic fields exsiting on the input/output waveguide ports are treated as ficticiuos magnetic currents (and charges):

$$\mathbf{E}_{\mathrm{tg}}(\mathbf{r}) = \frac{\eta}{jk}\nabla_S \int_S g^{\mathrm{e}}(\mathbf{r},\mathbf{r}')\nabla'_S \cdot \mathbf{J}(\mathbf{r}')\, dS' - jk\eta \int_S \bar{\mathbf{G}}^{\mathrm{A}}(\mathbf{r},\mathbf{r}') \cdot \mathbf{J}(\mathbf{r}')dS'$$

$$- \int_S \nabla \times \bar{\mathbf{G}}^{\mathrm{F}}(\mathbf{r},\mathbf{r}') \cdot \mathbf{M}(\mathbf{r}')dS' + \frac{1}{2}\,\mathbf{n} \times \mathbf{M}(\mathbf{r}')$$

$$\mathbf{H}_{\mathrm{tg}}(\mathbf{r}) = \frac{1}{jk\eta}\nabla_S \int_S g^{\mathrm{m}}(\mathbf{r},\mathbf{r}')\nabla'_S \cdot \mathbf{M}(\mathbf{r}')\, dS' - \frac{jk}{\eta} \int_S \bar{\mathbf{G}}^{\mathrm{F}}(\mathbf{r},\mathbf{r}') \cdot \mathbf{M}(\mathbf{r}')dS'$$

$$+ \int_S \nabla \times \bar{\mathbf{G}}^{\mathrm{A}}(\mathbf{r},\mathbf{r}') \cdot \mathbf{J}(\mathbf{r}')dS' + \frac{1}{2}\,\mathbf{J}(\mathbf{r}') \times \mathbf{n}$$

The BI-RME method

The magnetic scalar and dyadic Green's functions are described as follows:

$$\nabla^2 g^m(\vec{r},\vec{r'}) = -\delta(\vec{r}-\vec{r'}) \text{ en } V$$

$$\frac{\partial g^m}{\partial n} = 0 \text{ en } S$$

$$g^m(\vec{r},\vec{r'}) = \sum_i \frac{1}{\nu_i^2} w_i(\vec{r}) w_i(\vec{r'}) = g_s^m(\vec{r},\vec{r'}) + g_r^m(\vec{r},\vec{r'}) \qquad g_s^m(\vec{r},\vec{r'}) = \frac{1}{4\pi R}$$

$$\nabla \times \nabla \times \vec{\mathbf{G}}^{\mathrm{F}}(\vec{r},\vec{r'}) - k^2 \vec{\mathbf{G}}^{\mathrm{F}}(\vec{r},\vec{r'}) = \vec{\mathbf{I}}\delta(\vec{r}-\vec{r'}) - \nabla\nabla' g^m(\vec{r},\vec{r'}) \text{ en } V$$

$$\vec{n} \times \nabla \times \vec{\mathbf{G}}^{\mathrm{F}} = 0 \text{ en } S$$

$$\vec{\mathbf{G}}^{\mathrm{F}}(\vec{r},\vec{r'}) = \sum_i \frac{\vec{h}_i(\vec{r})\,\vec{h}_i(\vec{r'})}{k_i^2 - k^2} = \vec{\mathbf{G}}_{\mathbf{s}}^{\mathrm{F}}(\vec{r},\vec{r'}) + \vec{\mathbf{G}}_{\mathbf{r}}^{\mathrm{F}}(\vec{r},\vec{r'})$$

$$\vec{\mathbf{G}}_{\mathbf{s}}^{\mathrm{F}}(\vec{r},\vec{r'}) = \frac{1}{8\pi R}\left(\vec{\mathbf{I}} + \frac{\vec{R}\,\vec{R}}{R^2}\right)$$

$$\vec{\mathbf{G}}_{\mathbf{r}}^{\mathrm{F}}(\vec{r},\vec{r'}) = -k^2 \sum_i \frac{\vec{h}_i(\vec{r})\,\vec{h}_i(\vec{r'})}{k_i^2(k^2 - k_i^2)}$$

The BI-RME method

The surface magnetic current defined on the input/output waveguide ports is expressed as follows:

$$\vec{M} = -\vec{n} \times \sum_n V_n \vec{E}_n = -\sum_n V_n \vec{H}_n$$

where the electric and magnetic normalized modal vector functions of each waveguide port are defined with the following equations:

$$\vec{E}_{trans_{waveguide}} = \sum_n V_n \vec{E}_n \qquad \vec{H}_{trans_{waveguide}} = \sum_n I_n \vec{H}_n$$

$$\vec{n} \times \vec{E}_n = \vec{H}_n \qquad \int_{CS} \vec{E}_n \cdot \vec{E}_m dS = \int_{CS} \vec{H}_n \cdot \vec{H}_m dS = \delta_{n,m}$$

$Z_n = \frac{1}{Y_n} = \frac{V_n}{I_n}$ is the modal characteristic impedance (and admittance).

The BI-RME method

- By imposing the boundary conditions of the tangential electromagnetic field on the surface of the metallic insets, as well as on the input/output waveguide ports, we can obtain the polar expansion of the generalized admittance matrix of the cavity:

$$\mathbf{Y} = \frac{1}{jk\eta}\mathbf{Y}^{\mathrm{A}} + \frac{jk}{\eta}\mathbf{Y}^{\mathrm{B}} + \frac{jk^3}{\eta}\sum_{i=1}^{M'}\frac{\mathbf{y}^{(i)}\,\mathbf{y}_{\mathrm{T}}^{(i)}}{k_i^2 - k^2}$$

where:

$$\mathbf{Y}^{\mathrm{A}} = \mathbf{G} + \mathbf{L''}_{\mathrm{T}}\mathbf{W}^{-1}\mathbf{L''}$$

$$\mathbf{Y}^{\mathrm{B}} = \mathbf{T} - \mathbf{F}_{\mathrm{T}}\mathbf{K}^{-2}\mathbf{F} + \mathbf{C}_{\mathrm{T}}\mathbf{A}^{-1}\mathbf{C}$$

$$\mathbf{y}^{(i)} = k_i^{-1}\mathbf{C}_{\mathrm{T}}\mathbf{x}^{(i)}$$

$$\left(\mathbf{A} - k^2\mathbf{B}\right)\mathbf{x} = \mathbf{C}\,\mathbf{v} \longrightarrow \text{For the resonant modes computation}$$

The BI-RME method

$$\mathbf{A} = \begin{bmatrix} \mathbf{K}^4 & \mathbf{0} \\ \mathbf{0} & \mathbf{S} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} \mathbf{K}^2 - \mathbf{R}''\mathbf{W}^{-1}\mathbf{R}''_{\mathrm{T}} & \mathbf{R}' - \mathbf{R}''\mathbf{W}^{-1}\mathbf{Q}_{\mathrm{T}} \\ \mathbf{R}'_{\mathrm{T}} - \mathbf{Q}\mathbf{W}^{-1}\mathbf{R}''_{\mathrm{T}} & \mathbf{V} - \mathbf{Q}\mathbf{W}^{-1}\mathbf{Q}_{\mathrm{T}} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} -\mathbf{K}\mathbf{F} + \mathbf{R}''\mathbf{W}^{-1}\mathbf{L}'' \\ -\mathbf{L}' + \mathbf{Q}\mathbf{W}^{-1}\mathbf{L}'' \end{bmatrix}$$

$$S_{rp} = \int_S \int_S \nabla_S \cdot \mathbf{V}_r(\mathbf{r}) g^{\mathrm{e}}(\mathbf{r}, \mathbf{r}') \nabla'_S \cdot \mathbf{V}_p(\mathbf{r}') dS dS' \quad V_{rp} = \int_S \int_S \mathbf{V}_r(\mathbf{r}) \cdot \bar{\mathbf{G}}_0^{\mathrm{A}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{V}_p(\mathbf{r}') dS dS'$$

$$W_{sq} = \int_S \int_S \mathbf{W}_s(\mathbf{r}) \cdot \bar{\mathbf{G}}_0^{\mathrm{A}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{W}_q(\mathbf{r}') dS dS' \quad Q_{rq} = \int_S \int_S \mathbf{V}_r(\mathbf{r}) \cdot \bar{\mathbf{G}}_0^{\mathrm{A}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{W}_q(\mathbf{r}') dS dS'$$

$$G_{ln} = \int_S \int_S \nabla_S \cdot \vec{h}_l(\vec{r})\, g^m(\vec{r}, \vec{r}')\, \nabla'_S \cdot \vec{h}_n(\vec{r}')\, dS\, dS'$$

$$T_{ln} = \int_S \int_S \vec{h}_l(\vec{r}) \cdot \vec{\mathbf{G}}_{\mathbf{0}}^{\mathrm{F}}(\vec{r}, \vec{r}') \cdot \vec{h}_n(\vec{r}')\, dS\, dS'$$

The BI-RME method

$$R'_{mp} = \int_S \vec{e}_m(\vec{r}) \cdot \vec{v}_p(\vec{r})\, dS$$

$$R''_{mp} = \int_S \vec{e}_m(\vec{r}) \cdot \vec{W}_p(\vec{r})\, dS$$

$$F_{mn} = \int_S \vec{h}_m(\vec{r}) \cdot \vec{H}_n(\vec{r})\, dS$$

$$L'_{rn} = \int_S \int_S \vec{v}_r(\vec{r}) \cdot \nabla \times \vec{\mathbf{G}}_{\mathbf{0}}^{\mathrm{F}}(\vec{r}, \vec{r}') \cdot \vec{H}_n(\vec{r}')\, dS\, dS' - \frac{1}{2} \int_S \vec{v}_r(\vec{r}) \cdot \vec{E}_n(\vec{r})\, dS$$

$$L''_{sn} = \int_S \int_S \vec{w}_s(\vec{r}) \cdot \nabla \times \vec{\mathbf{G}}_{\mathbf{0}}^{\mathrm{F}}(\vec{r}, \vec{r}') \cdot \vec{H}_n(\vec{r}')\, dS\, dS' - \frac{1}{2} \int_S \vec{w}_s(\vec{r}) \cdot \vec{E}_n(\vec{r})\, dS$$

INDEX

- Introduction
- The rectangular waveguide
- The *empty* rectangular cavity
- Excitation of microwave cavities: coupling
- The BI-RME method
- Examples
- Conclusions

Examples

- Calculations presented throughout this presentation has been computed with a CAD (Computer Aided Design) tool developed by European Space Agency (ESA/ESTEC) and the spin-off company AURORASAT, S.L.

- This software allows the electromagnetic and (automatic) design of complex microwave circuits based on the connections of different canonical and non-canonical waveguides and cavities used in space telecommunicactions subsystems.

- It also contains different modules for the simulations of high-power non-linear phenomena existing in such components under high-power excitation, as multipactor and corona effects.

- web mail address: www.fest3d.com

FEST 3D

Examples

FEST 3D

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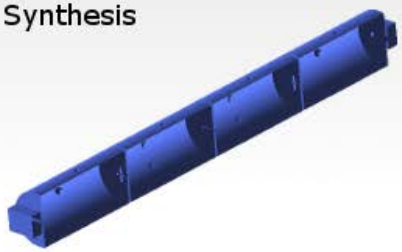
Analysis

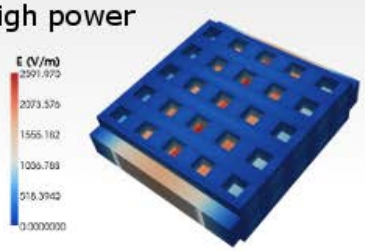
- Combline filters
- Waffle-iron filters
- New excitations!
- Dual-mode filters

Synthesis



- Lowpass filters
- Bandpass filters
- Dual-mode filters
- Transformers

High power



- Multipactor effect
- EM Fields
- Corona discharge effect
- SPARK3D

We don't do mode-matching, **we do better...** Fast

New High power module

SPARK 3D

Multipactor and Corona

IMS 2014

24/02/14

Latest versions of FEST3D/SPARK3D & WAND3D will be showcased in the International Microwave Symposium 2014 to be held in Tampa, Florida (June 1-6 2014). Please visit us at our booth #643 to get more information about the software features.

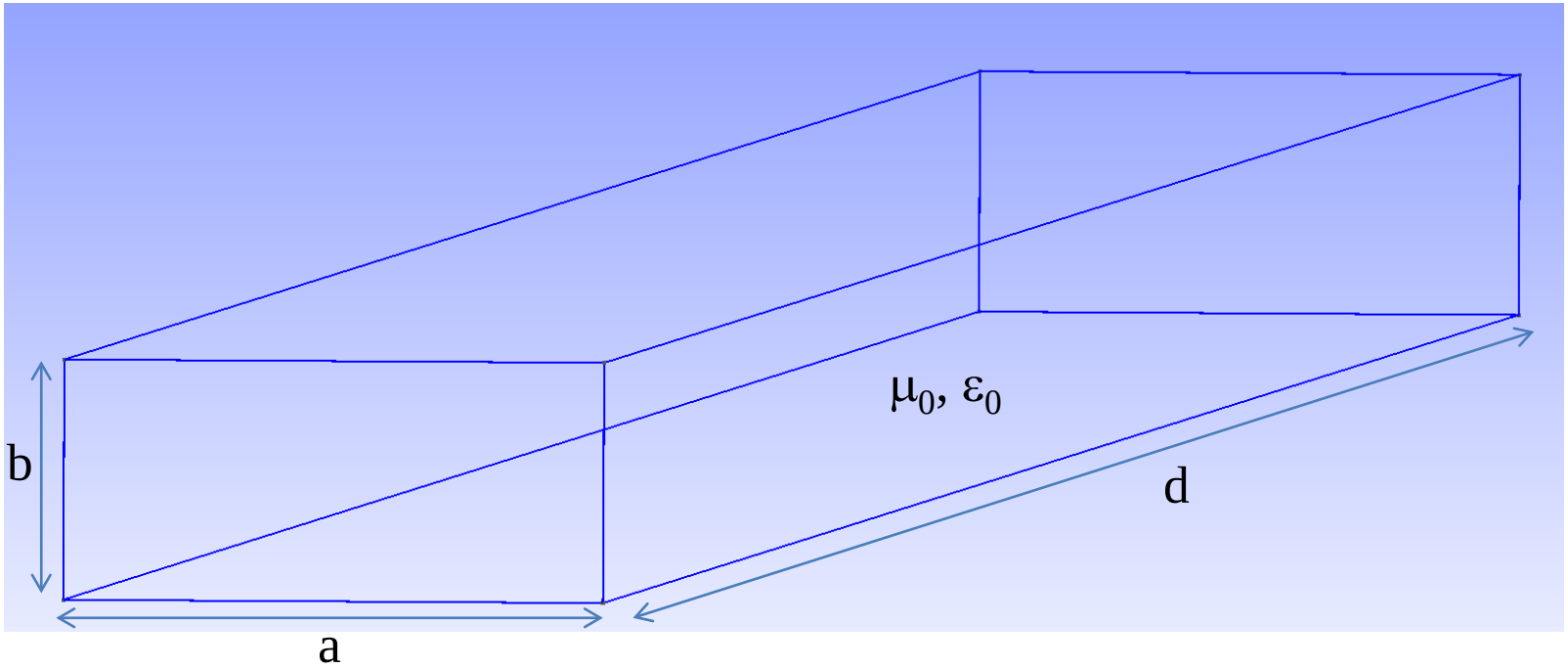
More information...

Aurora Software and Testing, S.L.
contact@aurorasat.es , Tlf: +34 96 371 42 57, Fax: +34 96 371 42 54

Examples

- We are going to analyze the coupling between four different probes and a rectangular cavity based on a standard WR-90 rectangular waveguide:

  a = 22.86 mm, b = 10.16 mm, d = 1 m; conductivity of Cu at 20 ºC: $\sigma = 5.96 \cdot 10^7$ S/m



Examples

- This rectangular resonator supports **703** solenoidal modes in the frequency range of the input coaxial waveguide:

Examples

Modos	m	n	p	frequency (GHz)	Qu
TE	1	0	1	6,558852	6478
TE	1	0	2	6,563989	6485
TE	1	0	3	6,572541	6497
TE	1	0	4	6,584495	6514
TE	1	0	5	6,599833	6535
TE	1	0	6	6,618531	6562
TE	1	0	7	6,640561	6592
TE	1	0	8	6,665889	6628
TE	1	0	9	6,694479	6668
TE	1	0	10	6,726289	6712
TE	1	0	11	6,761273	6761
TE	1	0	12	6,799382	6813
TE	1	0	13	6,840564	6871
TE	1	0	14	6,884764	6932
TE	1	0	15	6,931924	6997
TE	1	0	16	6,981984	7066
TE	1	0	17	7,034883	7138
TE	1	0	18	7,090556	7214
TE	1	0	19	7,148939	7294
TE	1	0	20	7,209967	7377
TE	1	0	21	7,273572	7463
TE	1	0	22	7,339687	7552
TE	1	0	23	7,408246	7644
TE	1	0	24	7,479181	7739
TE	1	0	25	7,552425	7837
TE	1	0	26	7,627911	7937
TE	1	0	27	7,705575	8040
TE	1	0	28	7,78535	8145
TE	1	0	29	7,867172	8252
TE	1	0	30	7,950978	8361
TE	1	0	31	8,036707	8472
TE	1	0	32	8,124297	8584
TE	1	0	33	8,213688	8698
TE	1	0	34	8,304823	8814
TE	1	0	35	8,397645	8931
TE	1	0	36	8,492098	9050
TE	1	0	37	8,588129	9170
TE	1	0	38	8,685685	9290
TE	1	0	39	8,784715	9412
TE	1	0	40	8,885171	9535
TE	1	0	41	8,987004	9658
TE	1	0	42	9,090168	9782
TE	1	0	43	9,194619	9907
TE	1	0	44	9,300312	10032
TE	1	0	45	9,407207	10158
TE	1	0	46	9,515262	10284
TE	1	0	47	9,624438	10411
TE	1	0	48	9,734698	10537
TE	1	0	49	9,846006	10664
TE	1	0	50	9,958326	10791
TE	1	0	51	10,071624	10918
TE	1	0	52	10,185868	11045
TE	1	0	53	10,301026	11173
TE	1	0	54	10,417068	11299
TE	1	0	55	10,533964	11426
TE	1	0	56	10,651688	11553
TE	1	0	57	10,770211	11679
TE	1	0	58	10,889507	11806
TE	1	0	59	11,009551	11932
TE	1	0	60	11,130319	12057
TE	1	0	61	11,251788	12182
TE	1	0	62	11,373936	12307
TE	1	0	63	11,49674	12432
TE	1	0	64	11,620179	12556
TE	1	0	65	11,744235	12680
TE	1	0	66	11,868887	12803
TE	1	0	67	11,994117	12926
TE	1	0	68	12,119906	13048
TE	1	0	69	12,246239	13170
TE	1	0	70	12,373097	13291
TE	1	0	71	12,500466	13412
TE	1	0	72	12,628329	13532
TE	1	0	73	12,756672	13651
TE	1	0	74	12,885481	13770
TE	1	0	75	13,014741	13889
TE	2	0	1	13,115135	9159
TE	2	0	2	13,117704	9162
TE	2	0	3	13,121986	9166
TE	2	0	4	13,127978	9172
TE	2	0	5	13,135677	9179
TE	1	0	76	13,144439	14007
TE	2	0	6	13,145082	9189
TE	2	0	7	13,156187	9200
TE	2	0	8	13,16899	9212
TE	2	0	9	13,183485	9227
TE	2	0	10	13,199666	9243
TE	2	0	11	13,217527	9260
TE	2	0	12	13,237062	9280
TE	2	0	13	13,258263	9301
TE	1	0	77	13,274562	14124
TE	2	0	14	13,281122	9323
TE	2	0	15	13,30563	9348
TE	2	0	16	13,331779	9373
TE	2	0	17	13,359558	9401
TE	2	0	18	13,388958	9430
TE	1	0	78	13,405099	14241
TE	2	0	19	13,419969	9460
TE	2	0	20	13,452578	9492
TE	2	0	21	13,486774	9526

Examples

TE	2	0	22	13,522545	9561
TE	1	0	79	13,536037	14357
TE	2	0	23	13,559879	9598
TE	2	0	24	13,598763	9636
TE	2	0	25	13,639184	9676
TE	1	0	80	13,667364	14473
TE	2	0	26	13,681128	9717
TE	2	0	27	13,72458	9759
TE	2	0	28	13,769528	9803
TE	2	0	29	13,815955	9848
TE	2	0	30	13,863848	9895
TE	2	0	31	13,913191	9943
TE	2	0	32	13,963969	9992
TE	2	0	33	14,016165	10043
TE	2	0	34	14,069766	10095
TE	2	0	35	14,124753	10148
TE	2	0	36	14,181112	10203
TE	2	0	37	14,238826	10259
TE	2	0	38	14,297879	10315
TE	2	0	39	14,358254	10374
TE	2	0	40	14,419934	10433
TE	2	0	41	14,482903	10493
TE	2	0	42	14,547143	10555
TE	2	0	43	14,61264	10617
TE	2	0	44	14,679374	10681
TE	2	0	45	14,74733	10745
TE	0	1	1	14,754324	7507
TE	0	1	2	14,756609	7509
TE	0	1	3	14,760415	7513
TE	0	1	4	14,765742	7519
TE	0	1	5	14,772588	7527
TE	0	1	6	14,780951	7536
TE	0	1	7	14,790828	7546
TE	0	1	8	14,802217	7559
TE	0	1	9	14,815114	7572
TE	2	0	46	14,816492	10811
TE	0	1	10	14,829515	7588
TE	0	1	11	14,845415	7605
TE	0	1	12	14,862811	7624
TE	0	1	13	14,881695	7645
TE	2	0	47	14,886841	10878
TE	0	1	14	14,902064	7667
TE	0	1	15	14,923911	7691
TE	0	1	16	14,947229	7716
TE	2	0	48	14,958361	10945
TE	0	1	17	14,972011	7743
TE	0	1	18	14,998251	7772
TE	0	1	19	15,02594	7802
TE	2	0	49	15,031036	11014
TE	0	1	20	15,055071	7834
TE	0	1	21	15,085635	7867
TE	2	0	50	15,104849	11084
TE	0	1	22	15,117624	7903
TE	0	1	23	15,151028	7939
TE	2	0	51	15,179783	11154
TE	0	1	24	15,185838	7978
TE	0	1	25	15,222045	8018
TE	2	0	52	15,255822	11225
TE	0	1	26	15,259639	8059
TE	0	1	27	15,298609	8103
TE	2	0	53	15,33295	11297
TE	0	1	28	15,338944	8147
TE	0	1	29	15,380635	8194
TE	2	0	54	15,411149	11370
TE	0	1	30	15,42367	8242
TE	0	1	31	15,468038	8292
TE	2	0	55	15,490404	11444
TE	0	1	32	15,513728	8343
TE	0	1	33	15,560727	8396
TE	2	0	56	15,570699	11519
TE	0	1	34	15,609024	8450
TE	2	0	57	15,652018	11594
TE	0	1	35	15,658607	8506
TE	0	1	36	15,709465	8564
TE	2	0	58	15,734344	11670
TE	0	1	37	15,761583	8623
TE	0	1	38	15,814951	8684
TE	2	0	59	15,817662	11747
TE	0	1	39	15,869555	8746
TE	2	0	60	15,901957	11824
TE	0	1	40	15,925383	8810
TE	0	1	41	15,982421	8875
TE	2	0	61	15,987212	11902
TE	0	1	42	16,040658	8942
TE	2	0	62	16,073414	11981
TE	0	1	43	16,10008	9011
TM	1	1	0	16,145083	10506
TE	1	1	1	16,145778	6644
TM	1	1	1	16,145778	10449
TE	1	1	2	16,147866	6646
TM	1	1	2	16,147866	10449
TE	1	1	3	16,151344	6648
TM	1	1	3	16,151344	10450
TE	1	1	4	16,156212	6652
TM	1	1	4	16,156212	10452
TE	2	0	63	16,160546	12060
TE	0	1	44	16,160673	9081
TE	1	1	5	16,162469	6656
TM	1	1	5	16,162469	10454
TE	1	1	6	16,170114	6662

Examples

TM	1	1	6	16,170114	10456
TE	1	1	7	16,179143	6668
TM	1	1	7	16,179143	10459
TE	1	1	8	16,189555	6676
TM	1	1	8	16,189555	10463
TE	1	1	9	16,201348	6684
TM	1	1	9	16,201348	10467
TE	1	1	10	16,214518	6694
TM	1	1	10	16,214518	10471
TE	0	1	45	16,222425	9152
TE	1	1	11	16,229061	6704
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TE	1	1	12	16,244975	6716
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TE	1	1	13	16,262255	6728
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TE	1	1	14	16,280897	6741
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TE	0	1	46	16,285323	9225
TE	1	1	15	16,300895	6756
TM	1	1	15	16,300895	10499
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TE	2	0	65	16,337542	12221
TE	1	1	17	16,344944	6788
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TE	1	1	18	16,368983	6805
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TE	1	1	19	16,394357	6823
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TE	0	1	48	16,414503	9376
TE	1	1	20	16,421061	6842
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TE	1	1	21	16,449087	6862
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TE	0	1	49	16,480758	9454
TE	1	1	23	16,50908	6906
TM	1	1	23	16,50908	10565
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TE	1	1	24	16,541033	6929
TM	1	1	24	16,541033	10576
TE	0	1	50	16,548106	9533
TE	1	1	25	16,574279	6953
TM	1	1	25	16,574279	10586
TE	1	1	26	16,608812	6978

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TE	2	0	68	16,609646	12465
TE	0	1	51	16,616533	9614
TE	1	1	27	16,644624	7003
TM	1	1	27	16,644624	10609
TE	1	1	28	16,681705	7030
TM	1	1	28	16,681705	10621
TE	0	1	52	16,686026	9696
TE	2	0	69	16,702053	12548
TE	1	1	29	16,720048	7058
TM	1	1	29	16,720048	10633
TE	0	1	53	16,756571	9780
TE	1	1	30	16,759644	7086
TM	1	1	30	16,759644	10645
TE	2	0	70	16,79529	12631
TE	1	1	31	16,800484	7116
TM	1	1	31	16,800484	10658
TE	0	1	54	16,828156	9865
TE	1	1	32	16,84256	7146
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TE	1	1	33	16,885861	7178
TM	1	1	33	16,885861	10685
TE	2	0	71	16,889342	12715
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TM	1	1	34	16,930378	10699
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TE	0	1	57	17,049017	10129
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TE	2	0	73	17,079839	12883
TE	1	1	38	17,120419	7347
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TE	0	1	58	17,124629	10220
TE	1	1	39	17,170872	7384
TM	1	1	39	17,170872	10775
TE	2	0	74	17,176258	12968
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TE	2	0	75	17,273439	13053
TE	1	1	41	17,275239	7460
TM	1	1	41	17,275239	10808
TE	0	1	60	17,27876	10407
TE	1	1	42	17,329131	7499

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TE	2	0	77	17,470038	13225
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TE	0	1	63	17,517037	10697
TE	1	1	46	17,555849	7663
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TE	1	1	47	17,615261	7706
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TE	0	1	69	18,017822	11314
TE	1	1	54	18,060535	8030
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TE	0	1	70	18,104284	11422
TE	1	1	55	18,128211	8079
TM	1	1	55	18,128211	11071
TE	0	1	71	18,191569	11531
TE	1	1	56	18,19687	8129
TM	1	1	56	18,19687	11092
TE	1	1	57	18,266501	8180
TM	1	1	57	18,266501	11114
TE	0	1	72	18,279668	11642
TE	1	1	58	18,337093	8231
TM	1	1	58	18,337093	11135
TE	0	1	73	18,368566	11753
TE	1	1	59	18,408635	8283
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TE	0	1	74	18,458254	11866
TE	1	1	60	18,481115	8336
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TE	0	1	75	18,54872	11981
TE	1	1	61	18,554524	8389
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TE	1	1	62	18,628849	8444
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TE	1	1	63	18,704081	8498
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TE	1	1	70	19,255159	8899
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TE	1	1	71	19,337251	8959
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TE	1	1	72	19,420152	9019
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TE	1	1	73	19,503854	9080
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TE	3	0	1	19,671988	11217
TE	1	1	75	19,673614	9203
TM	1	1	75	19,673614	11534
TE	3	0	2	19,673702	11218
TE	3	0	3	19,676557	11221
TE	3	0	4	19,680553	11224
TE	3	0	5	19,68569	11228
TE	3	0	6	19,691966	11233
TE	3	0	7	19,699382	11239
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TE	3	0	9	19,717623	11254
TE	3	0	10	19,728445	11263
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TE	2	1	16	19,884767	7429
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TE	2	1	17	19,903402	7440
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TE	2	1	19	19,944001	7463
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TE	3	0	22	19,945917	11438
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TE	2	1	24	20,064745	7532
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TE	2	1	25	20,092162	7547
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TE	2	1	27	20,150229	7580
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TE	2	1	47	20,959142	8038
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TE	2	1	48	21,010002	8067
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TM	2	1	56	21,450275	14461
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TM	2	1	60	21,691931	14543
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TM	2	1	61	21,754507	14563
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TE	3	0	76	22,732023	13632
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TE	2	1	78	22,942374	9144
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TE	2	1	79	23,019125	9187
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TM	3	1	11	24,644493	18114
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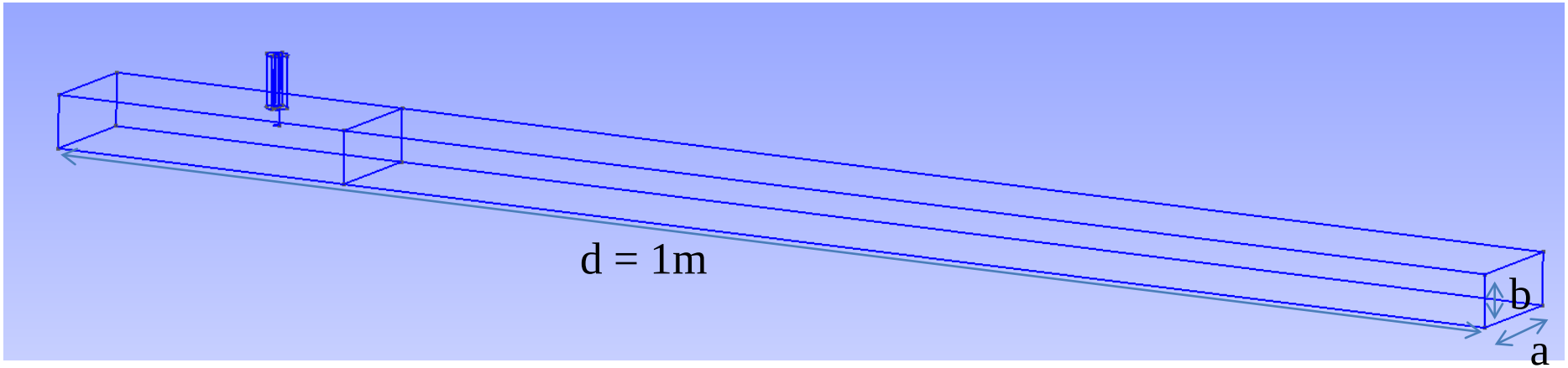
TE	3	1	14	24,678658	8240
TM	3	1	14	24,678658	18126
TE	3	1	15	24,691856	8247
TM	3	1	15	24,691856	18131
TE	3	1	16	24,705957	8253
TM	3	1	16	24,705957	18136
TE	3	1	17	24,720958	8260
TM	3	1	17	24,720958	18142
TE	3	1	18	24,736859	8267
TM	3	1	18	24,736859	18148
TE	3	1	19	24,753657	8275
TM	3	1	19	24,753657	18154
TE	3	1	20	24,771351	8283
TM	3	1	20	24,771351	18160
TE	3	1	21	24,789939	8292
TM	3	1	21	24,789939	18167
TE	3	1	22	24,809418	8300
TM	3	1	22	24,809418	18174
TE	3	1	23	24,829787	8310
TM	3	1	23	24,829787	18182
TE	3	1	24	24,851043	8320
TM	3	1	24	24,851043	18189
TE	3	1	25	24,873185	8330
TM	3	1	25	24,873185	18198
TE	3	1	26	24,896209	8340
TM	3	1	26	24,896209	18206
TE	3	1	27	24,920114	8351
TM	3	1	27	24,920114	18215
TE	3	1	28	24,944897	8362
TM	3	1	28	24,944897	18224
TE	3	1	29	24,970555	8374
TM	3	1	29	24,970555	18233
TE	3	1	30	24,997085	8386
TM	3	1	30	24,997085	18243

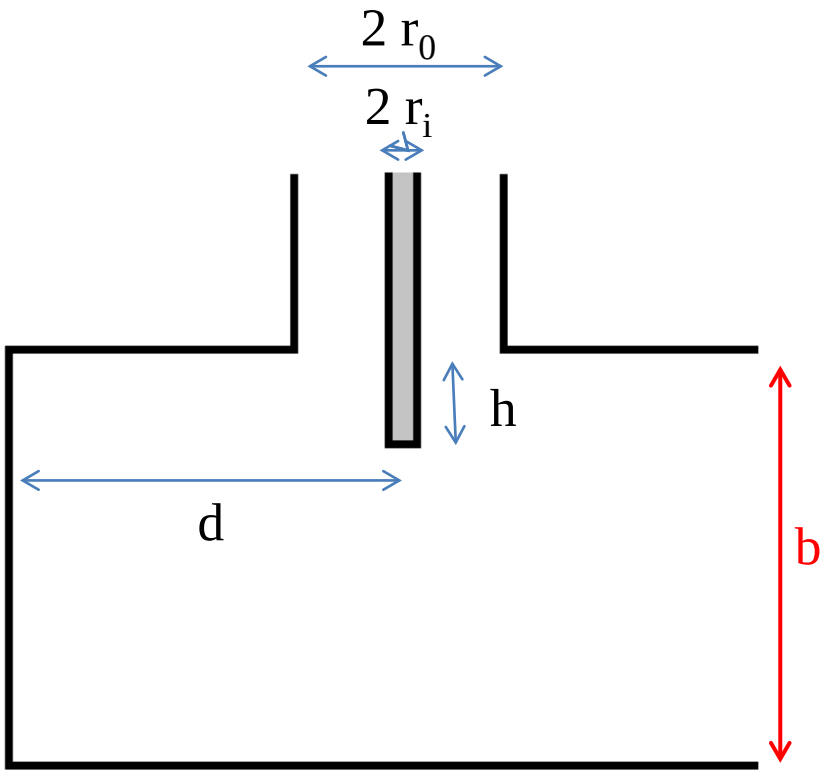
Examples

- Example1: Simple electric probe exciting a rectangular cavity



WR-90: a=22.86 mm, b=10.16 mm

Examples



h=b/3=3.387 mm

d=λ_g/4=39.707 mm

$\beta_{TE10}=((\omega/c)^2-(\pi/a)^2)^{1/2}$

$\beta_{TE10}=2\pi/\lambda_g$

$\omega=2\pi f$

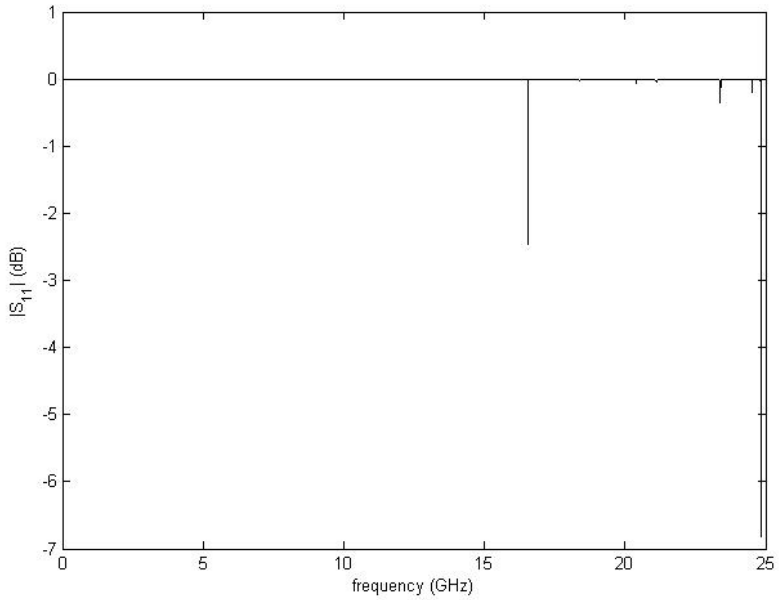
f = 10 GHz

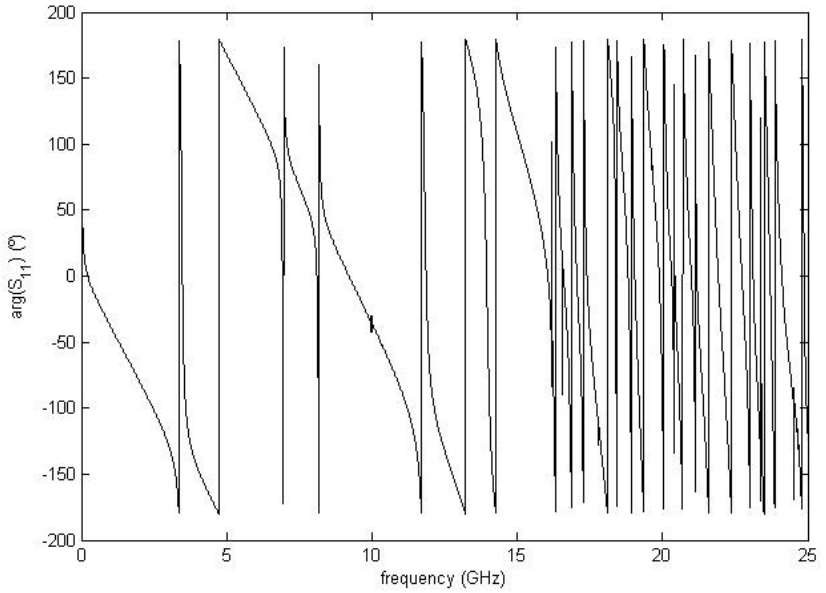
INPUT COAXIAL WAVEGUIDE:

r_i=0.635 mm, r_o=2.11, ε_r=2.08; Z_0=50 Ω

Examples

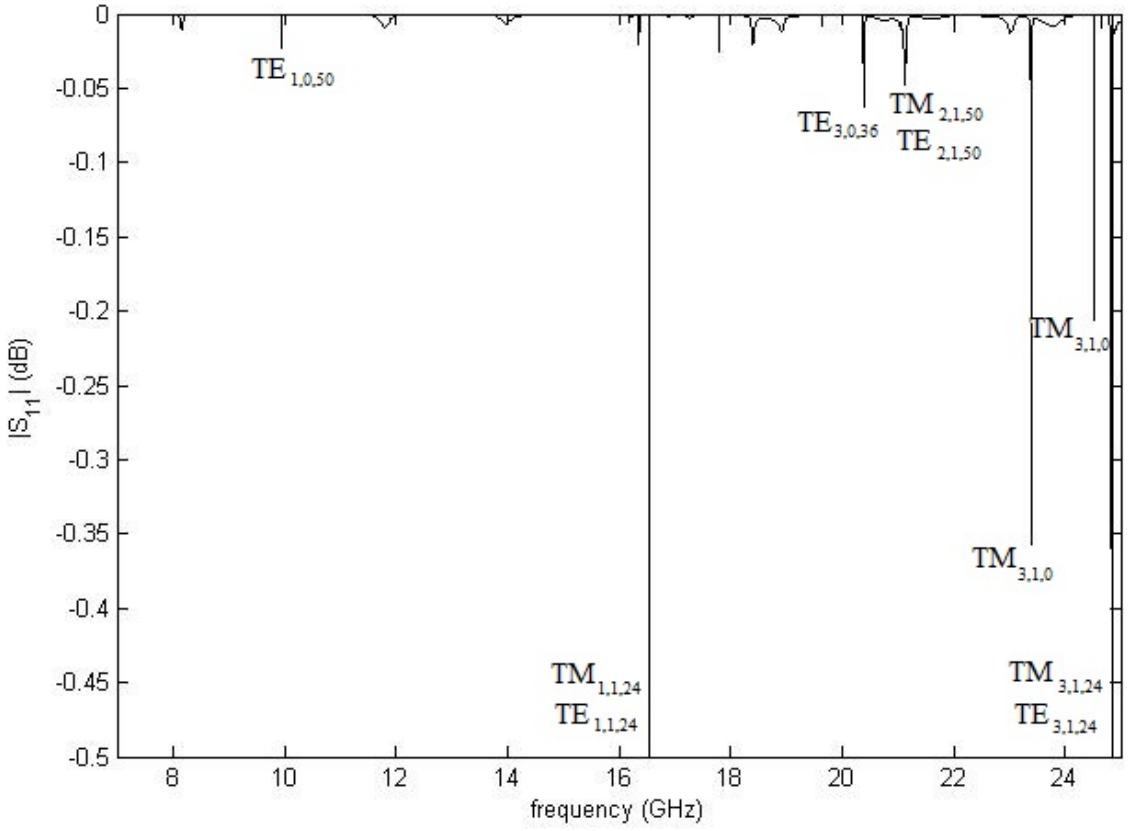
Wide frequency band simulations of the simple electric probe:





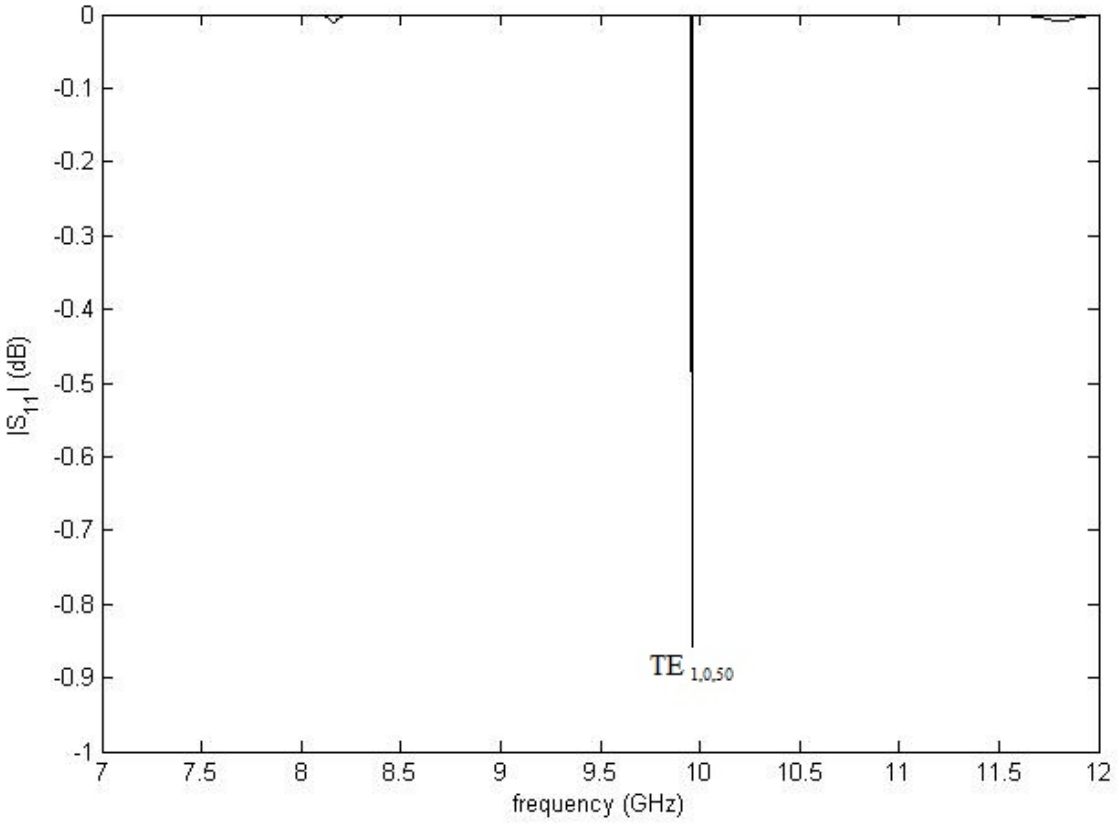
Examples

Identification of the most relevant 'detected' resonances of the simple electric probe:



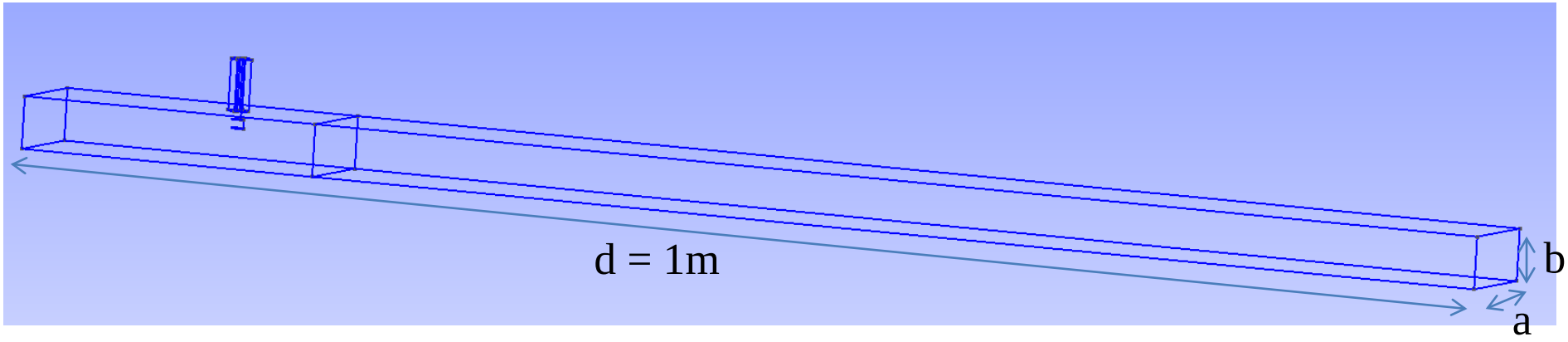
Examples

Identification of the most relevant 'detected' resonances of the simple electric probe in the bandwidth of the rectangular waveguide (WR-90):



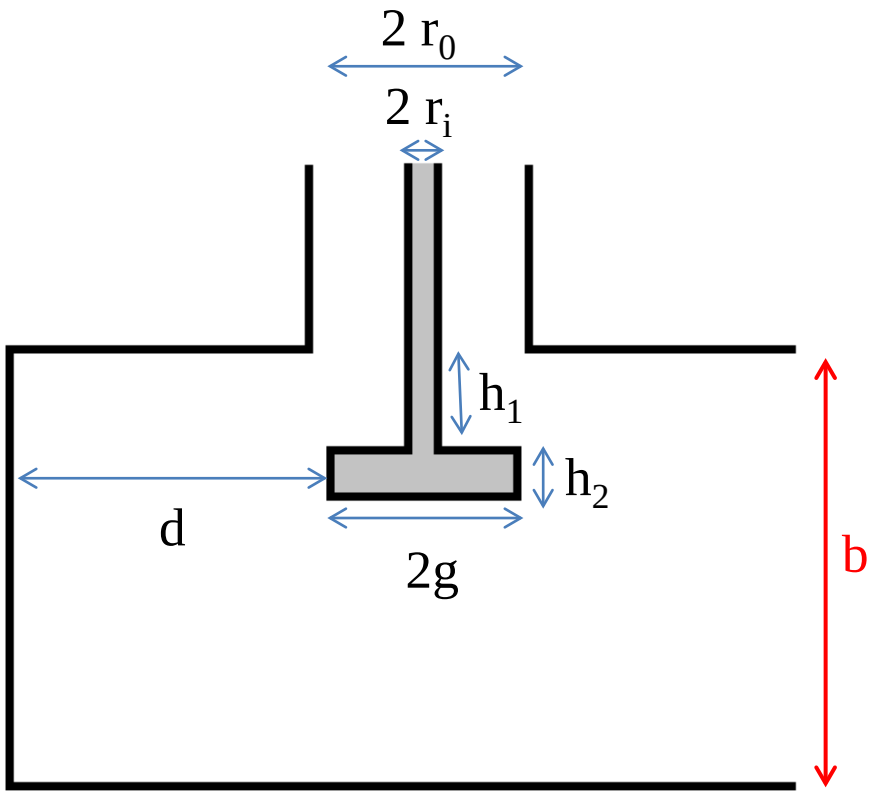
Examples

- Example2: *Mushroom* electric probe exciting a rectangular cavity



WR-90: a=22.86 mm, b=10.16 mm

Examples



$h_1=h_2=(b/3)/2=1.694$ mm

$h=h_1+h_2=b/3=3.387$mm

$g=2r_i=1.27$ mm

$d=\lambda_g/4=39.707$ mm

$\beta_{TE10}=((\omega/c)^2-(\pi/a)^2)^{1/2}$

$\beta_{TE10}=2\pi/\lambda_g$

$\omega=2\pi f$

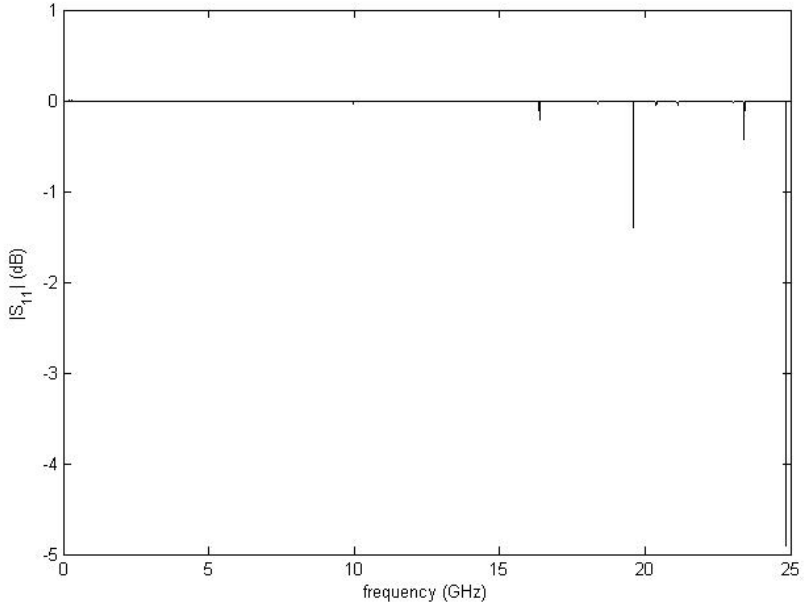
f = 10 GHz

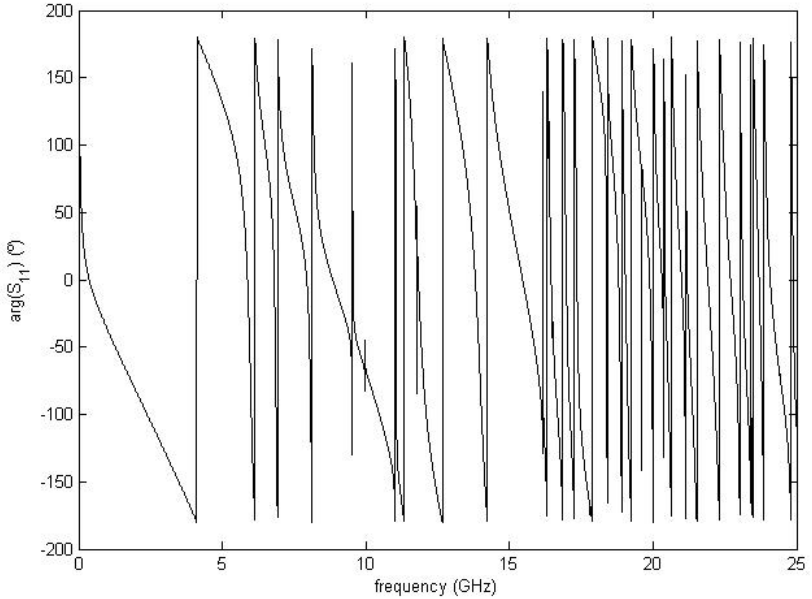
INPUT COAXIAL WAVEGUIDE:

$r_i=0.635$ mm, $r_o=2.11$, $\varepsilon_r=2.08$; $Z_0=50\ \Omega$

Examples

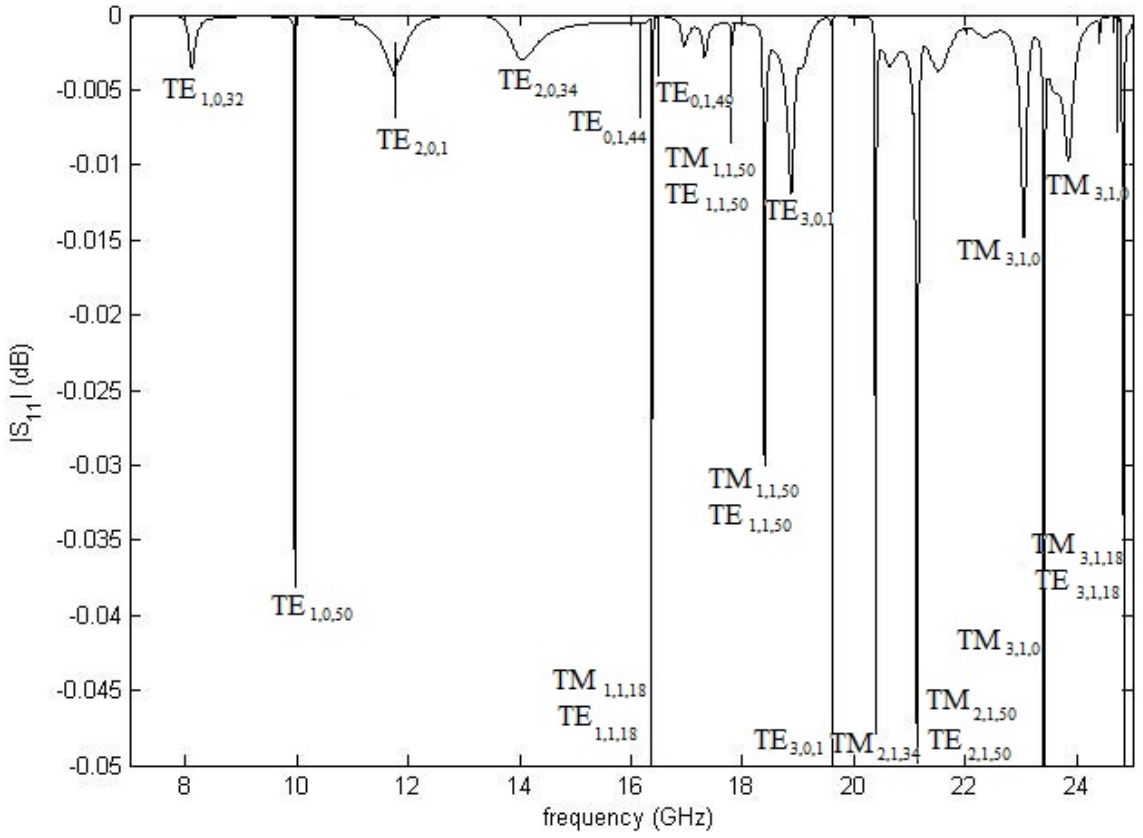
Wide frequency band simulations of the *mushroom* electric probe:





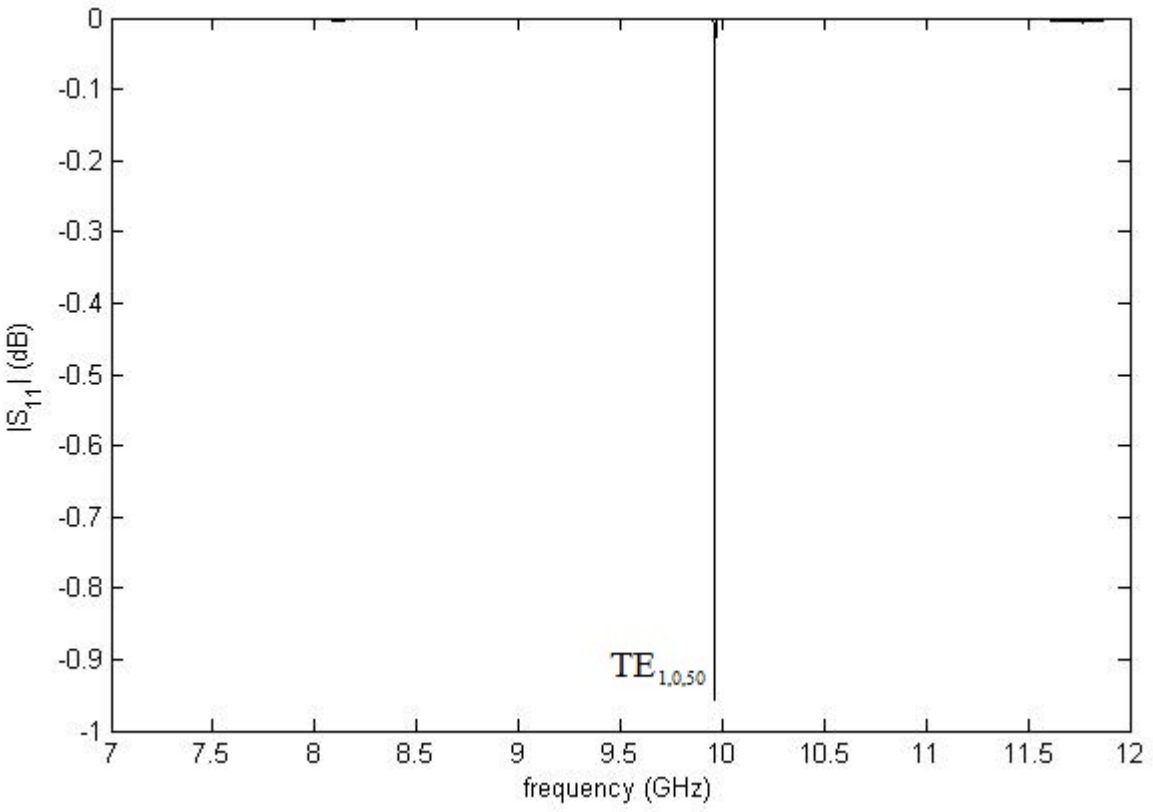
Examples

Identification of the most relevant 'detected' resonances of the *mushroom* electric probe:



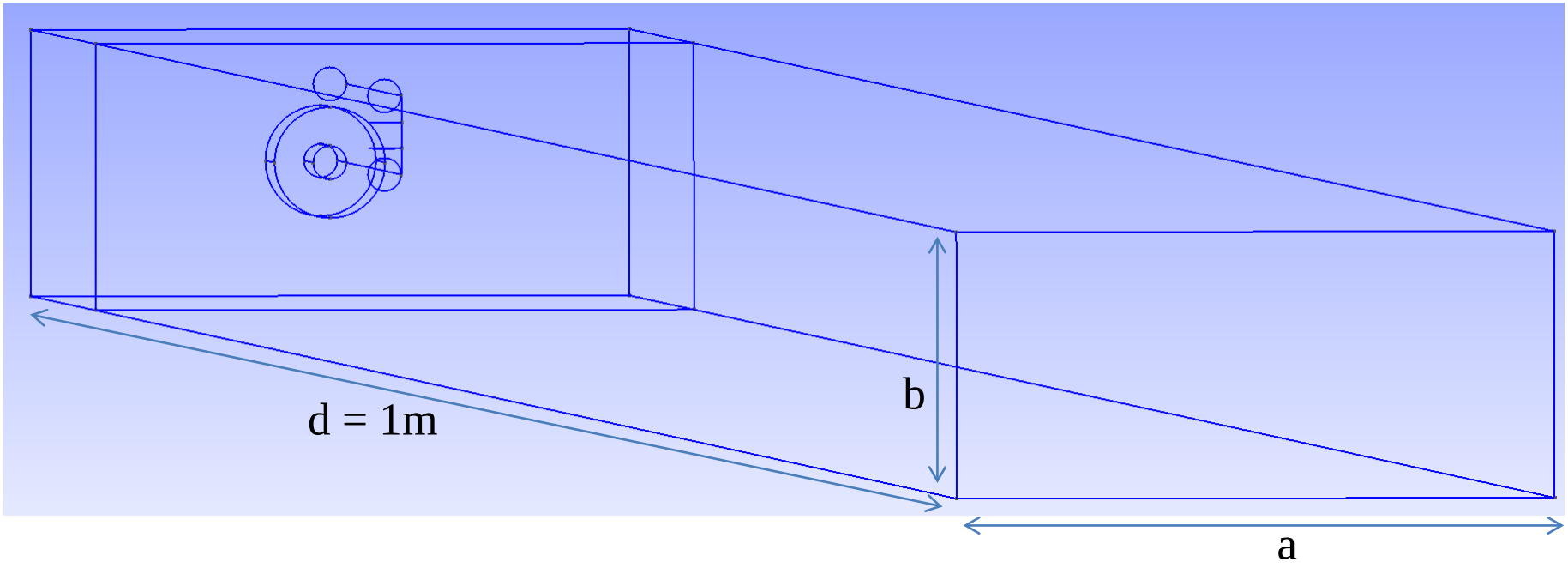
Examples

Identification of the most relevant 'detected' resonances of the *mushroom* electric probe in the bandwidth of the rectangular waveguide (WR-90):



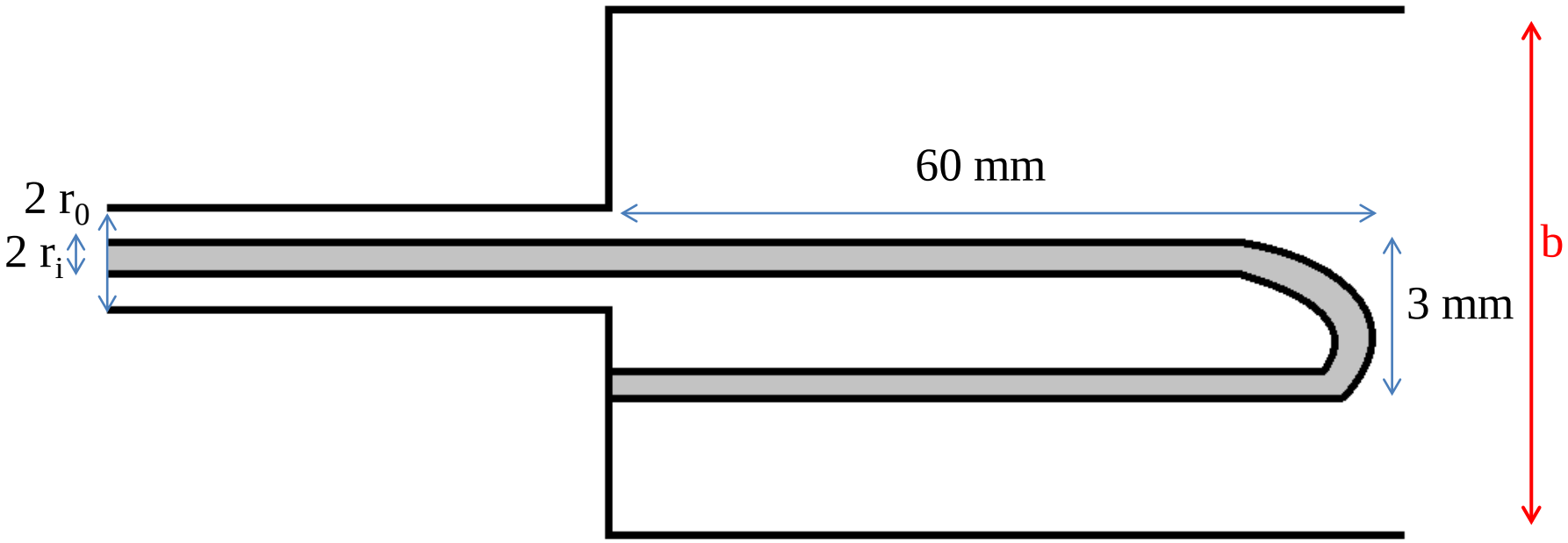
Examples

- Example3: Vertical current loop exciting a rectangular cavity



WR-90: a=22.86 mm, b=10.16 mm

Examples

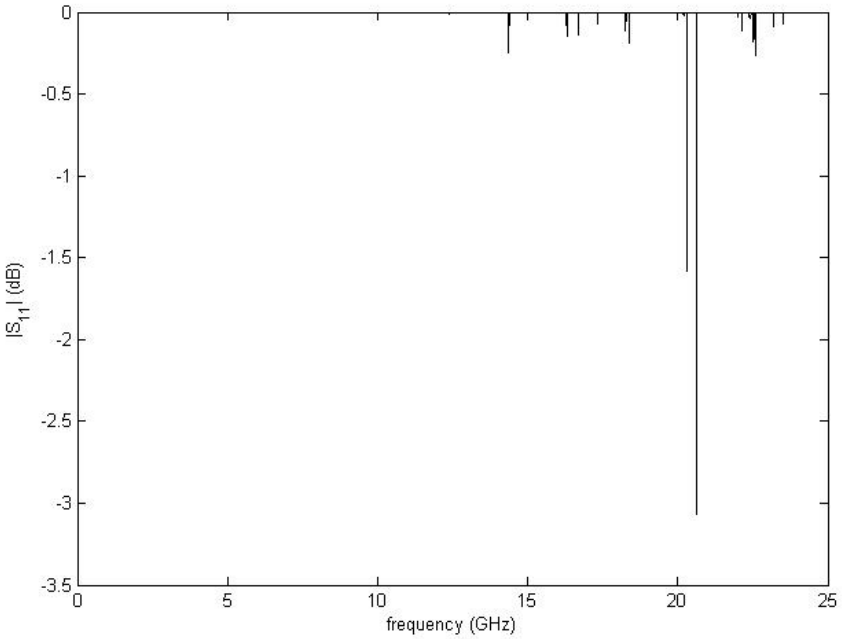


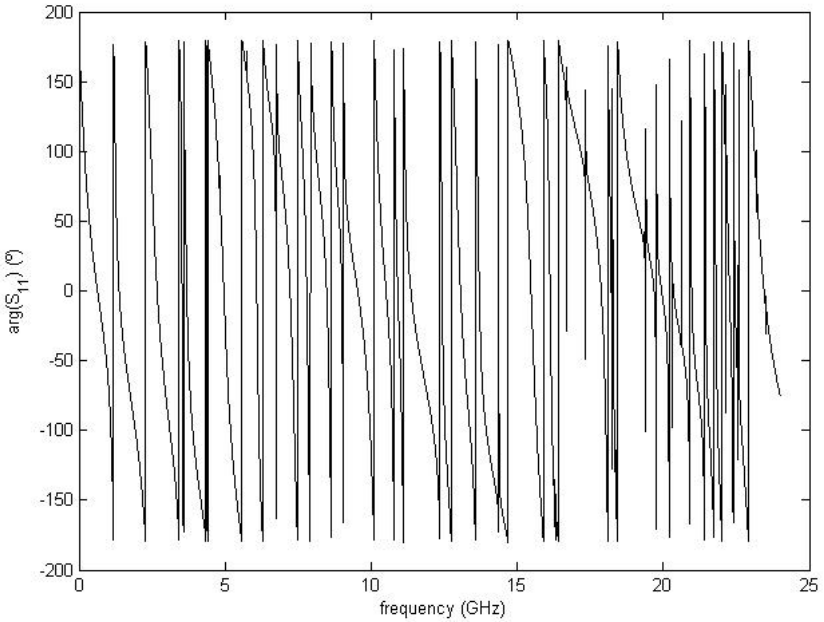
INPUT COAXIAL WAVEGUIDE:

r_i=0.635 mm, r_o=2.11, ε_r=2.08; Z_0=50 Ω

Examples

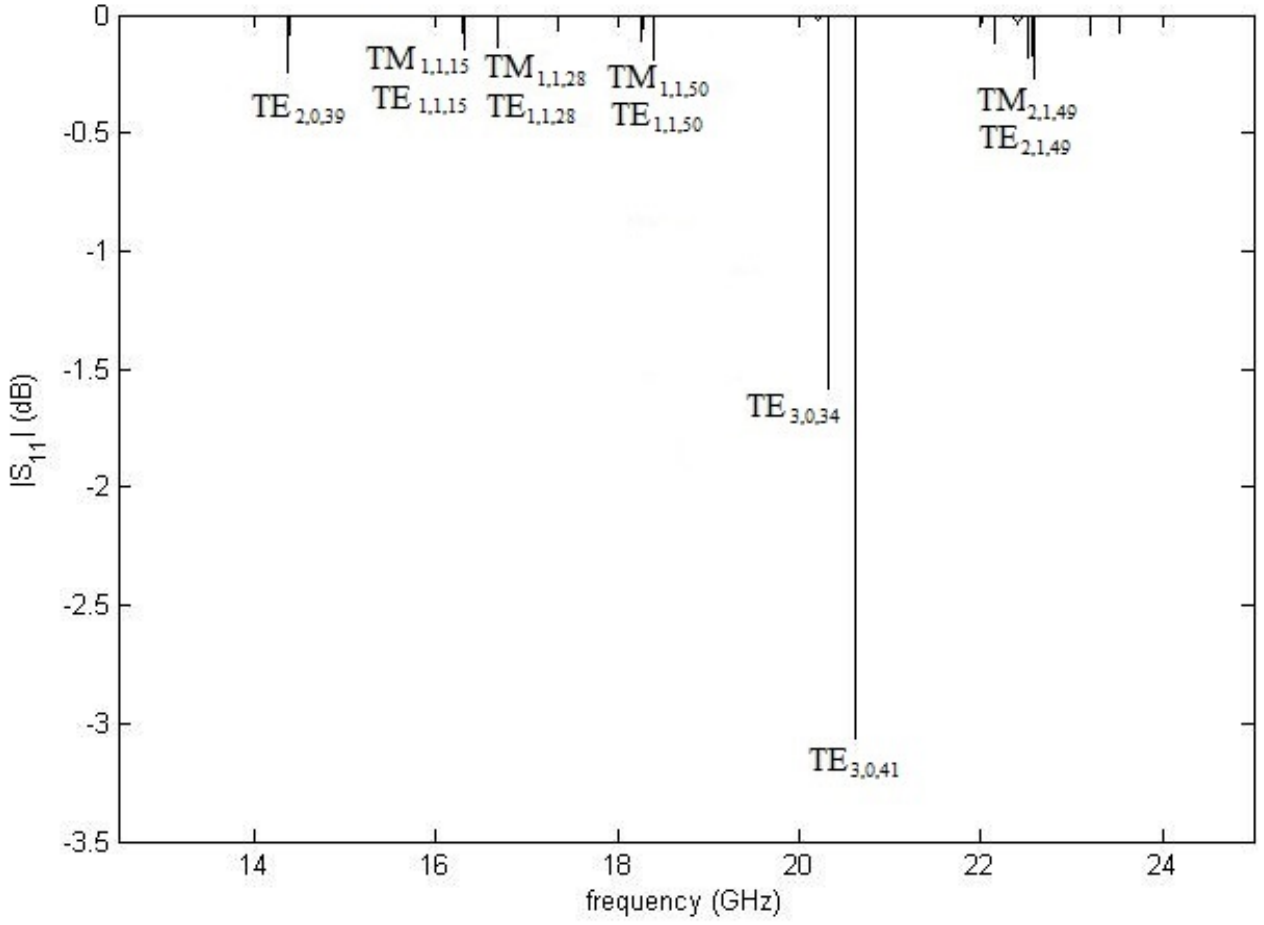
Wide frequency band simulations of the vertical current loop:





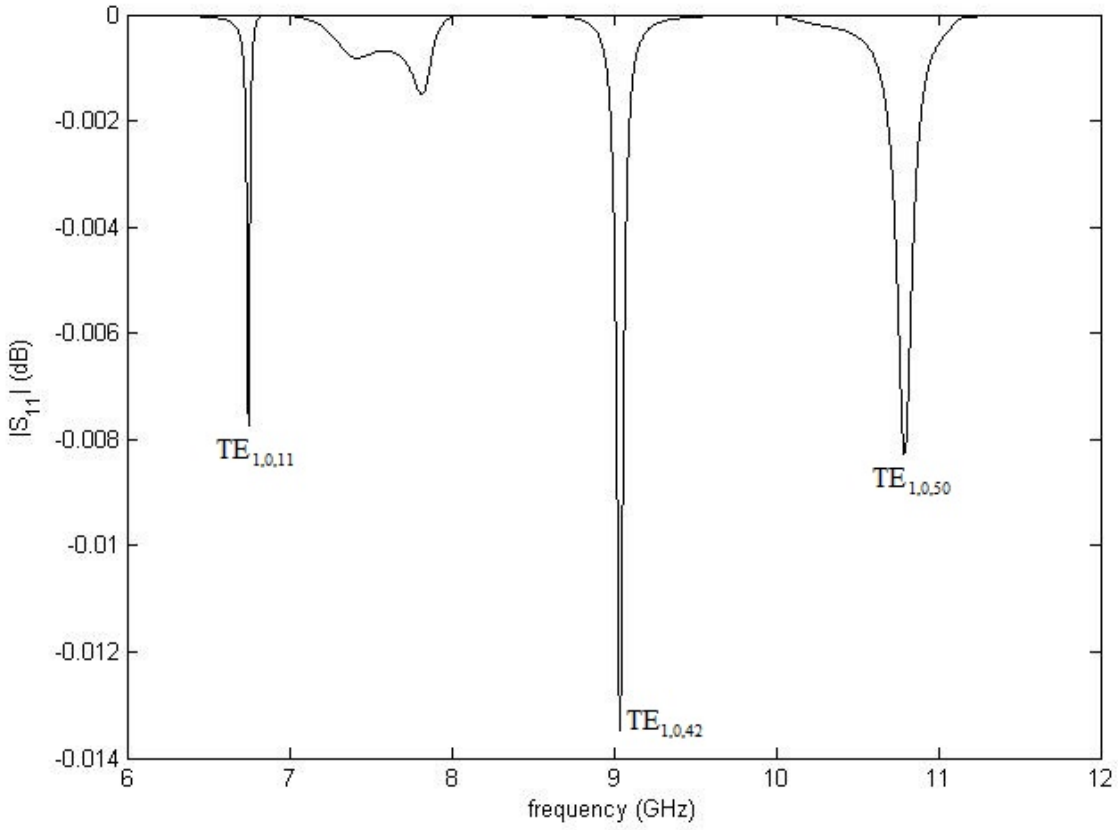
Examples

Identification of the most relevant 'detected' resonances of the vertical current loop:



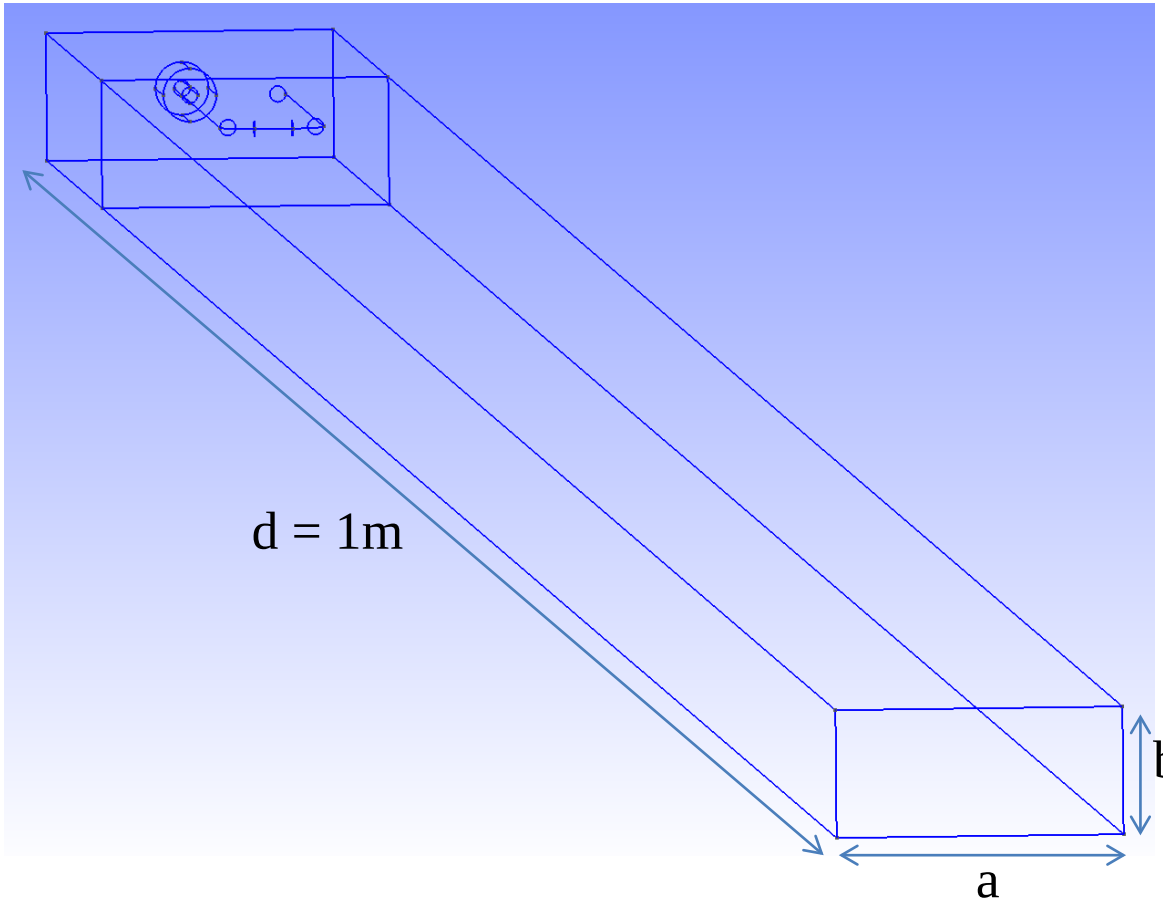
Examples

Identification of the most relevant 'detected' resonances of the vertical current loop in the bandwidth of the rectangular waveguide (WR-90):



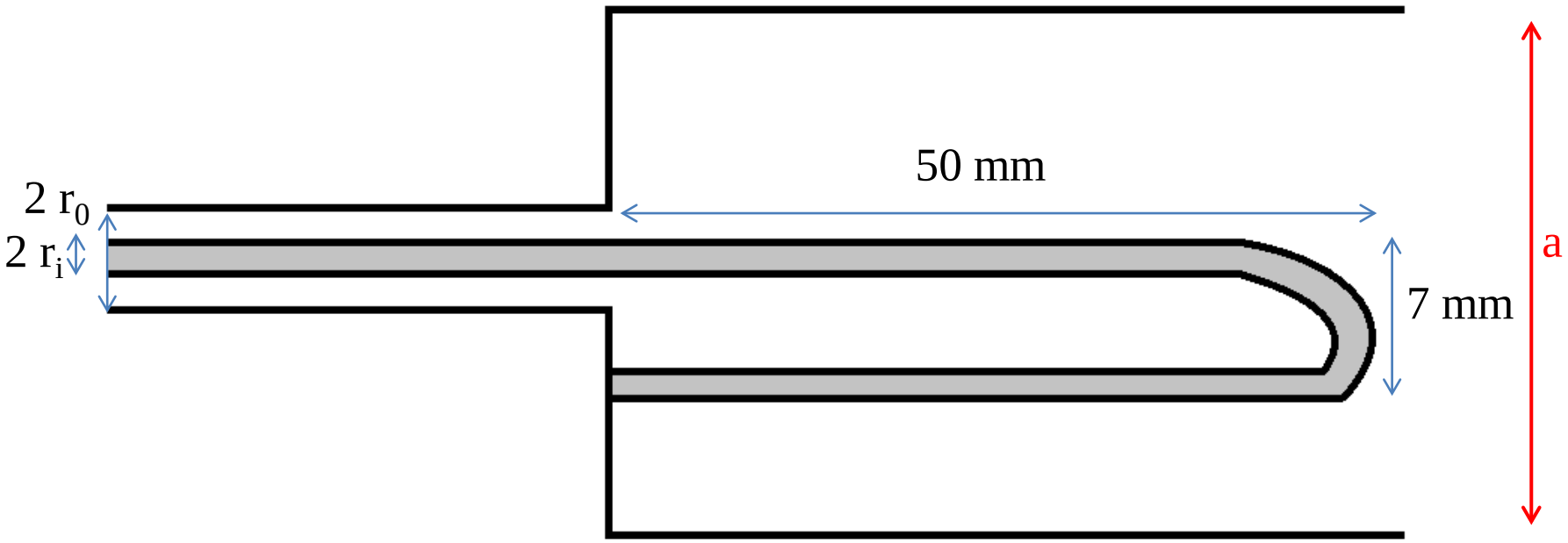
Examples

- Example4: Horizontal current loop exciting a rectangular cavity



WR-90: a=22.86 mm, b=10.16 mm

Examples

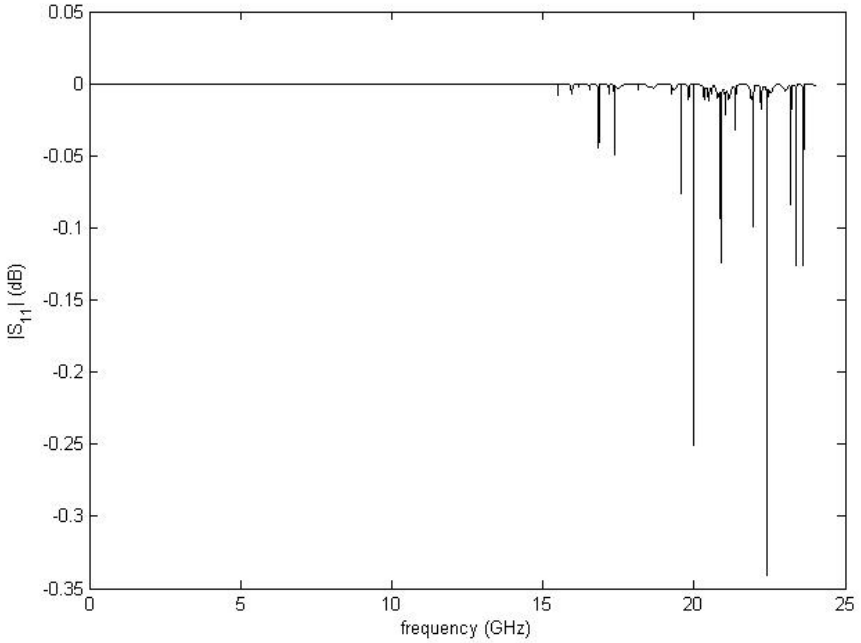


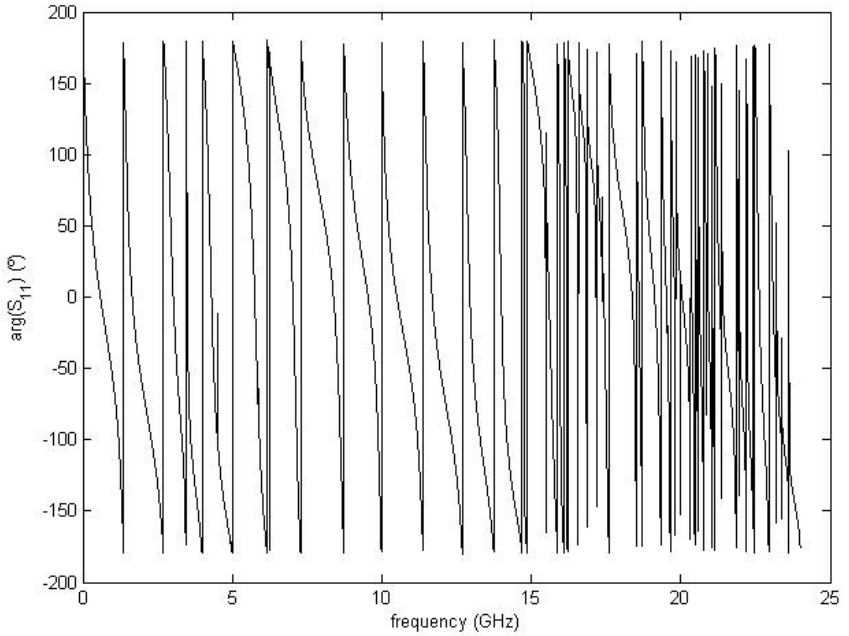
INPUT COAXIAL WAVEGUIDE:

r_i=0.635 mm, r_o=2.11, ε_r=2.08; Z_0=50 Ω

Examples

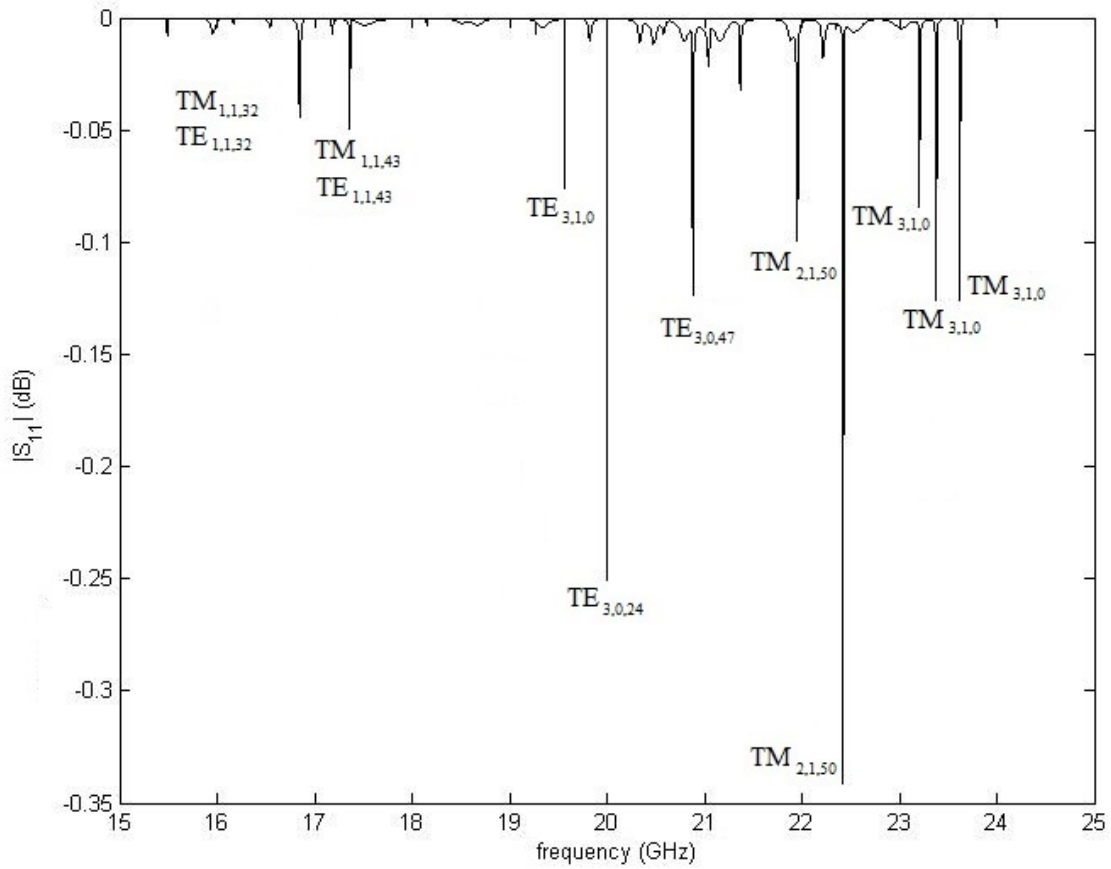
Wide frequency band simulations of the horizontal current loop:





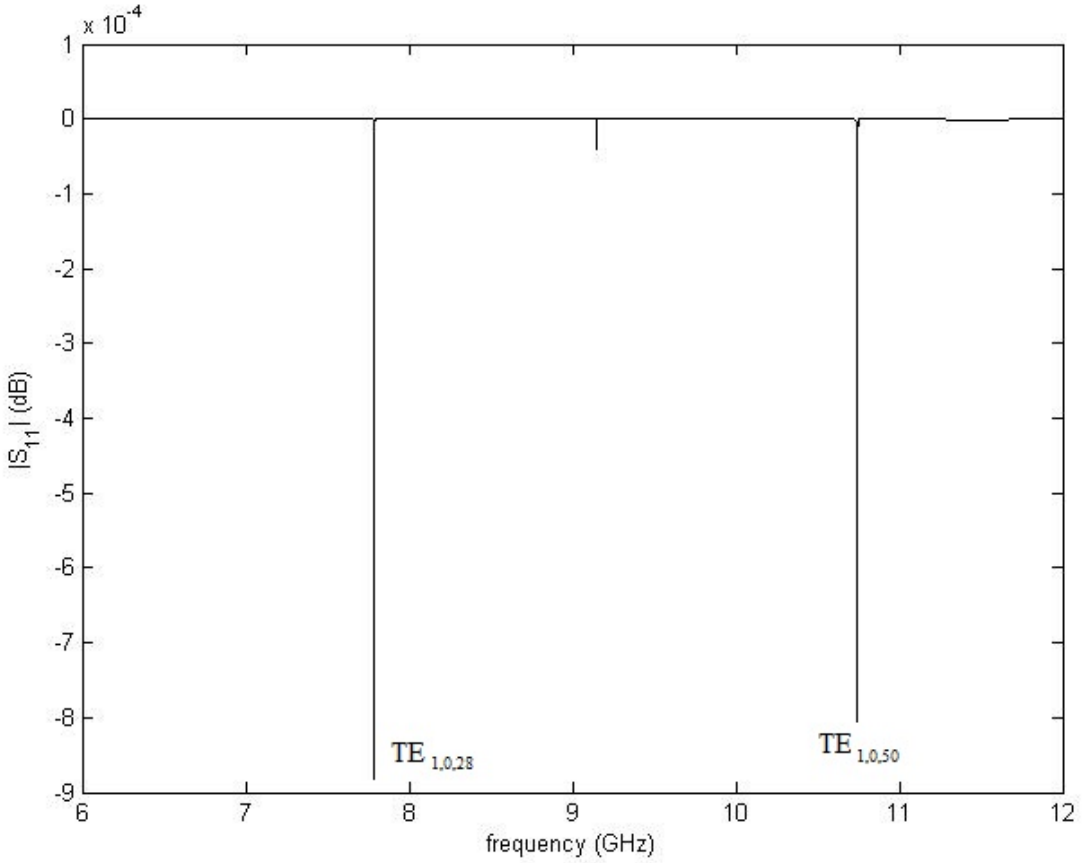
Examples

Identification of the most relevant 'detected' resonances of the horizontal current loop:



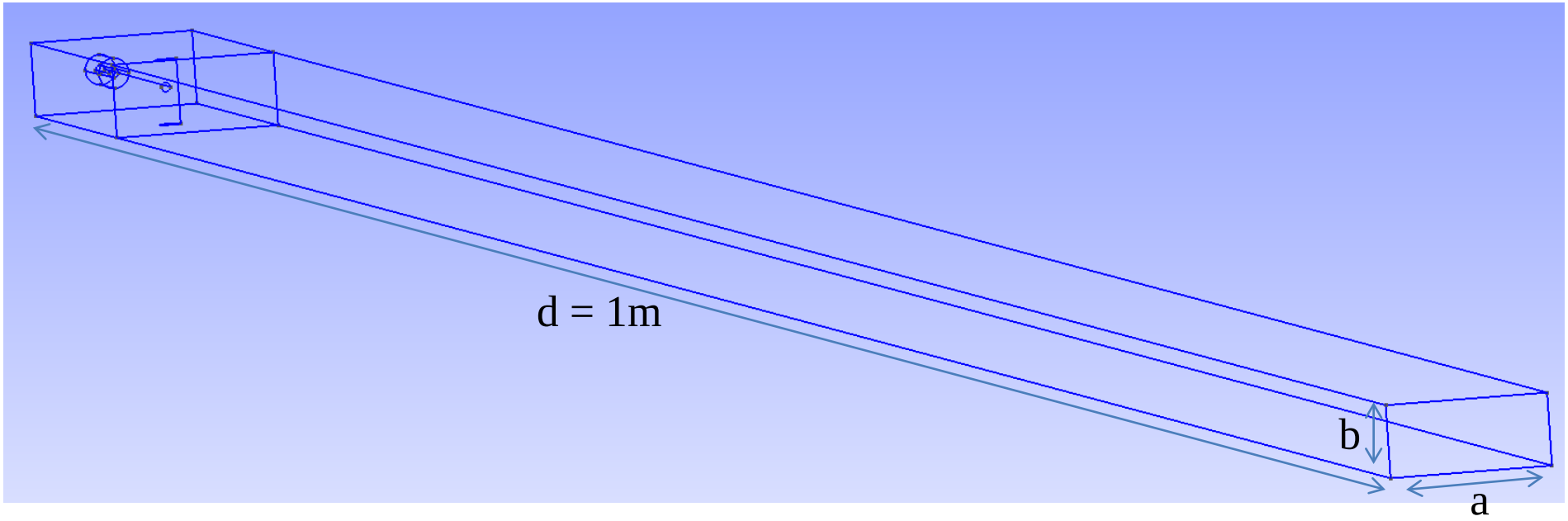
Examples

Identification of the most relevant 'detected' resonances of the horizontal current loop in the bandwidth of the rectangular waveguide (WR-90):



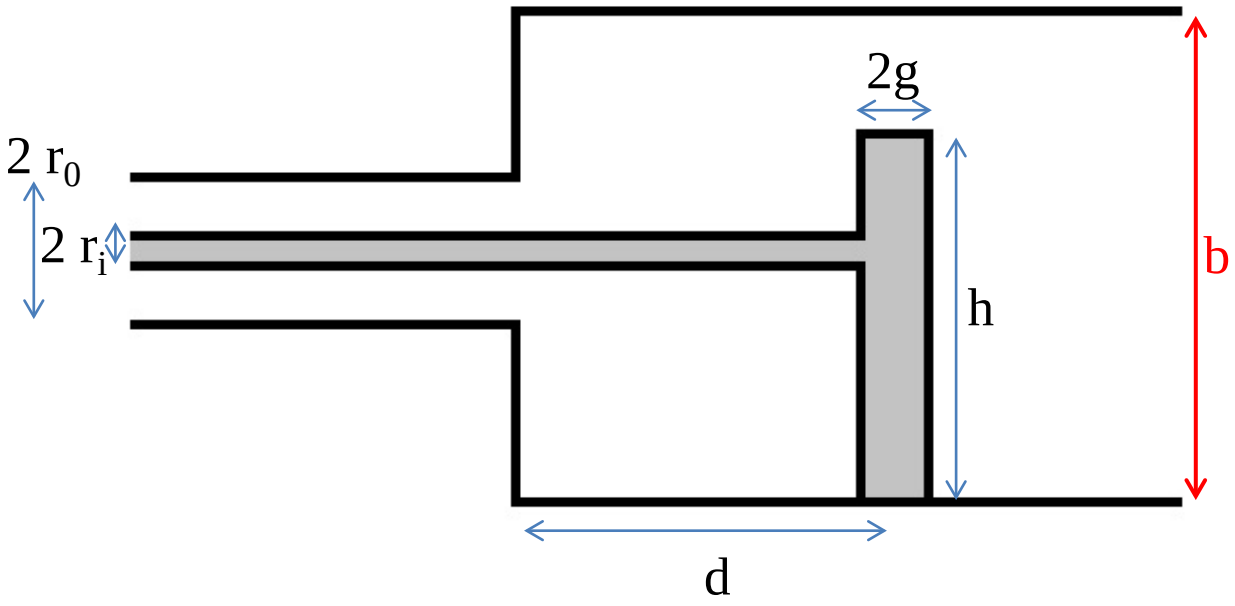
Examples

- Example5: Magnetic probe with vertical post exciting a rectangular cavity



WR-90: a=22.86 mm, b=10.16 mm

Examples



h=9.0 mm

g=1.5 mm

$d=\lambda_g/4=39.707$ mm

$\beta_{TE10}=((\omega/c)^2-(\pi/a)^2)^{1/2}$

$\beta_{TE10}=2\pi/\lambda_g$

$\omega=2\pi f$

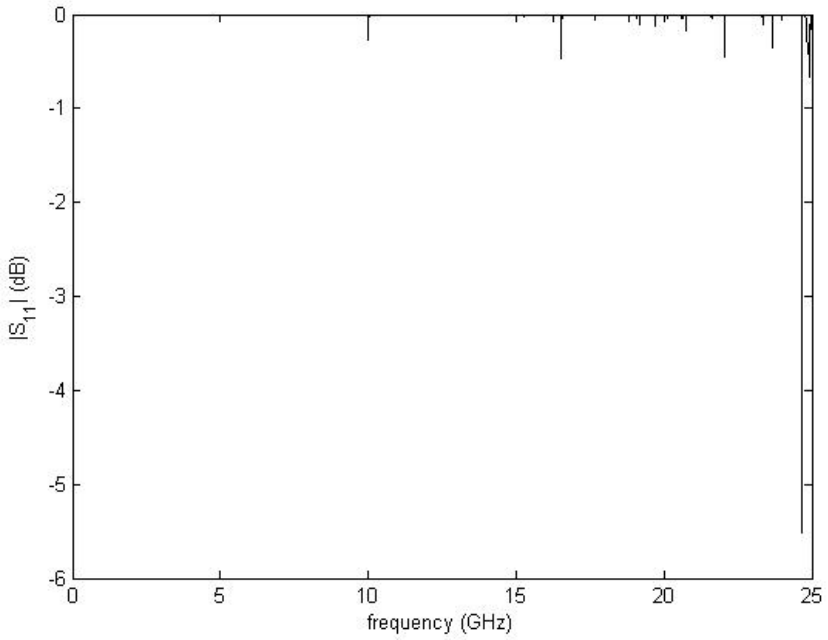
f = 10 GHz

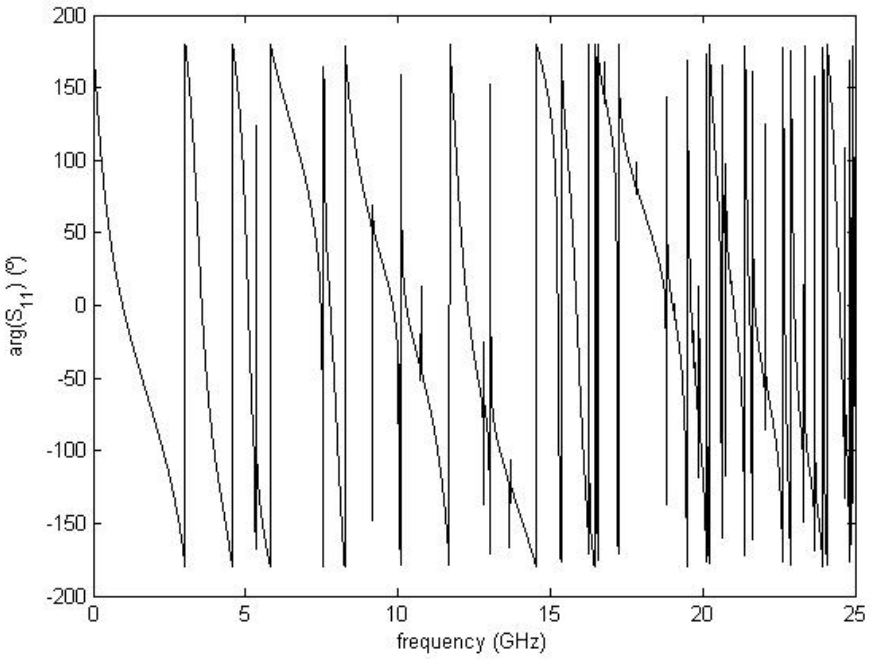
INPUT COAXIAL WAVEGUIDE:

$r_i=0.635$ mm, $r_o=2.11$, $\varepsilon_r=2.08$; $Z_0=50\ \Omega$

Examples

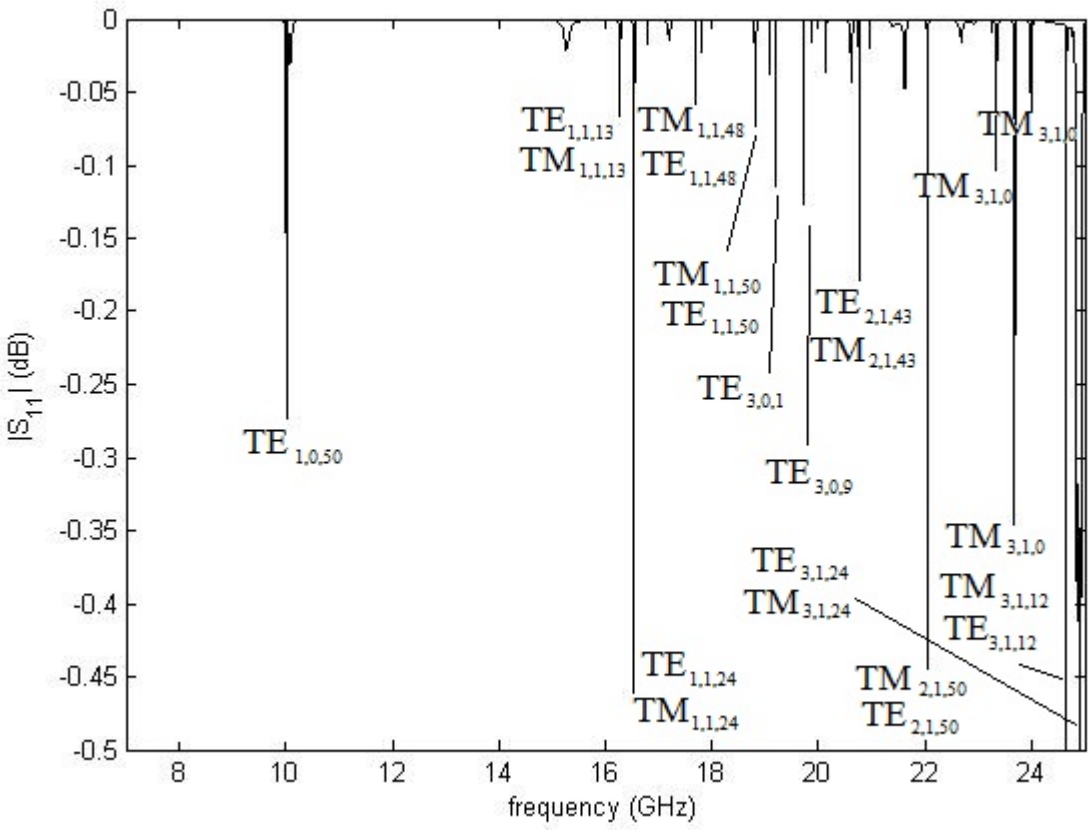
Wide frequency band simulations of the magnetic probe with vertical post:





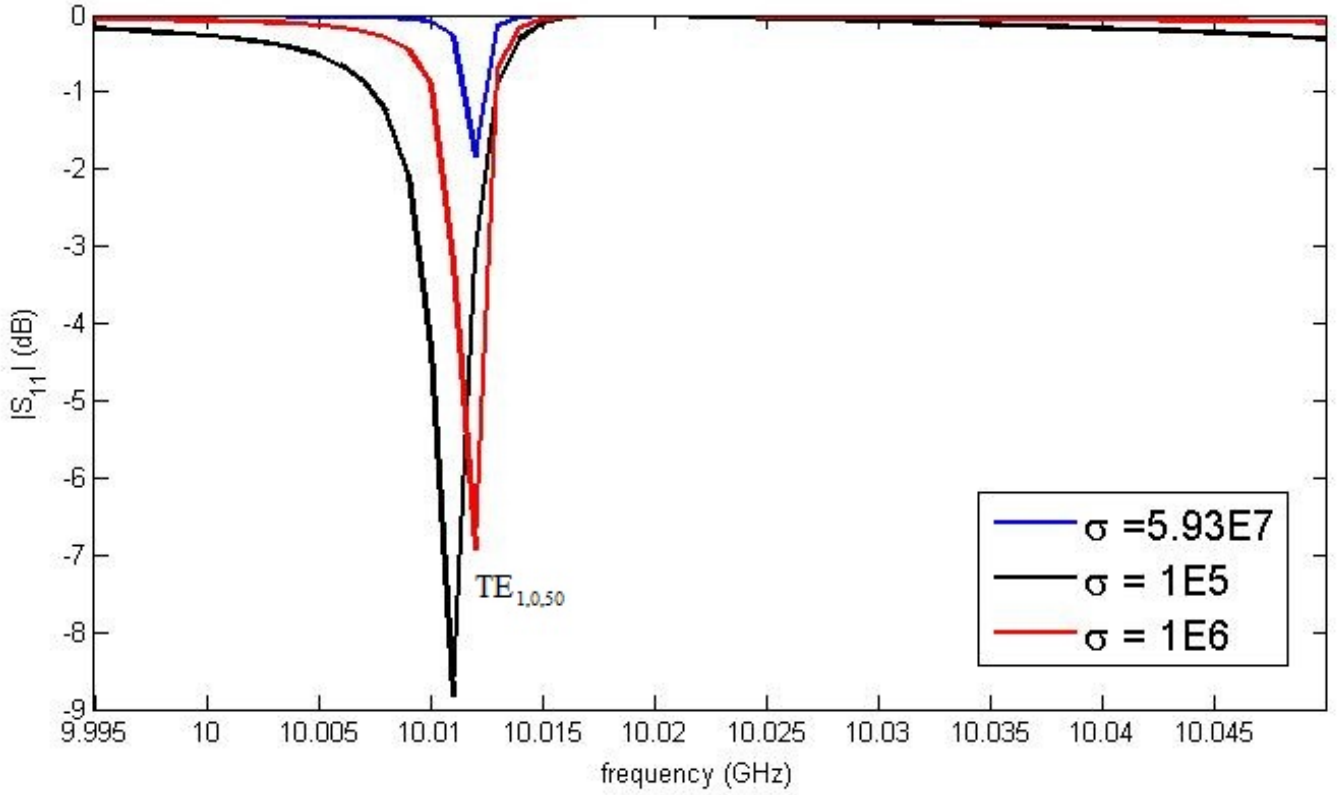
Examples

Identification of the most relevant 'detected' resonances of the magnetic probe with vertical post:



Examples

Identification of the most relevant 'detected' resonances of the magnetic probe with vertical post around the bandwidth of the rectangular waveguide (WR-90): the electrical conducticity of the metallic walls has been modified



INDEX

- Introduction
- The rectangular waveguide
- The *empty* rectangular cavity
- Excitation of microwave cavities: coupling
- The BI-RME method
- Examples
- Conclusions

Conclusions

- We have presented and overview of rectangular waveguides and cavities, presenting the most relevant relationships and concepts.
- A summary of the most important techniques to feed *empty* microwave cavities has been summarized.
- BI-RME 3D formulation has been introduced, showing the most relevant issues of the theory.
- Several coaxial probes have been explored for rectangular cavity excitation.

WE ARE OPEN TO COOPERATE WITH ALL OF YOU.

Thanks a lot for your attention.