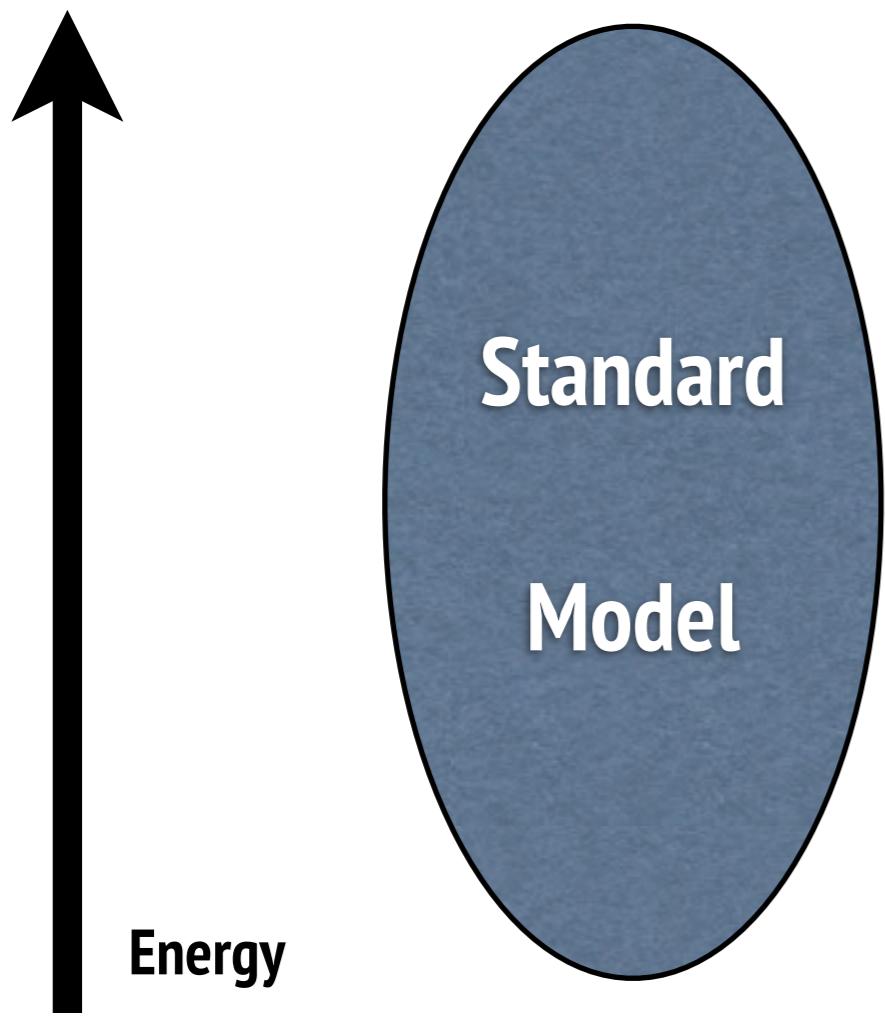


The quest for axions and ALPs

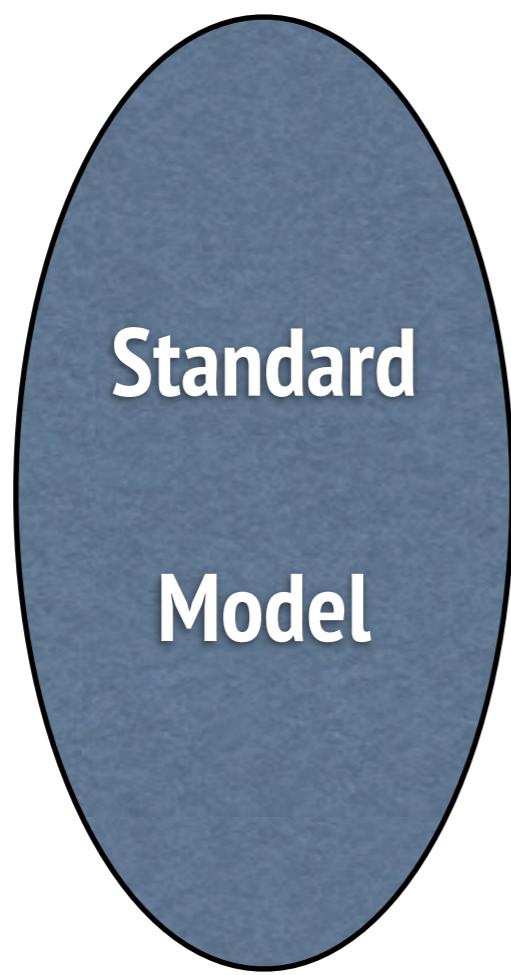
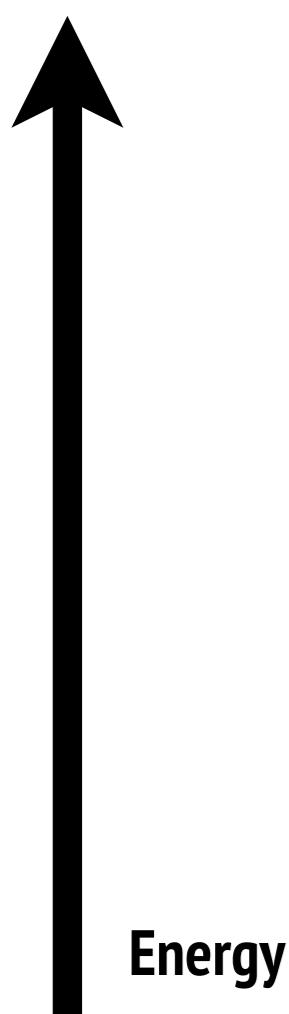
Javier Redondo
(Zaragoza U. & MPP)

Outline

- 1 big picture
- 2 types of ALPs
- 3 types of interactions
- 4 ~ hints of existence
- 5 ... Experiments to find them
- 6 Conclusions



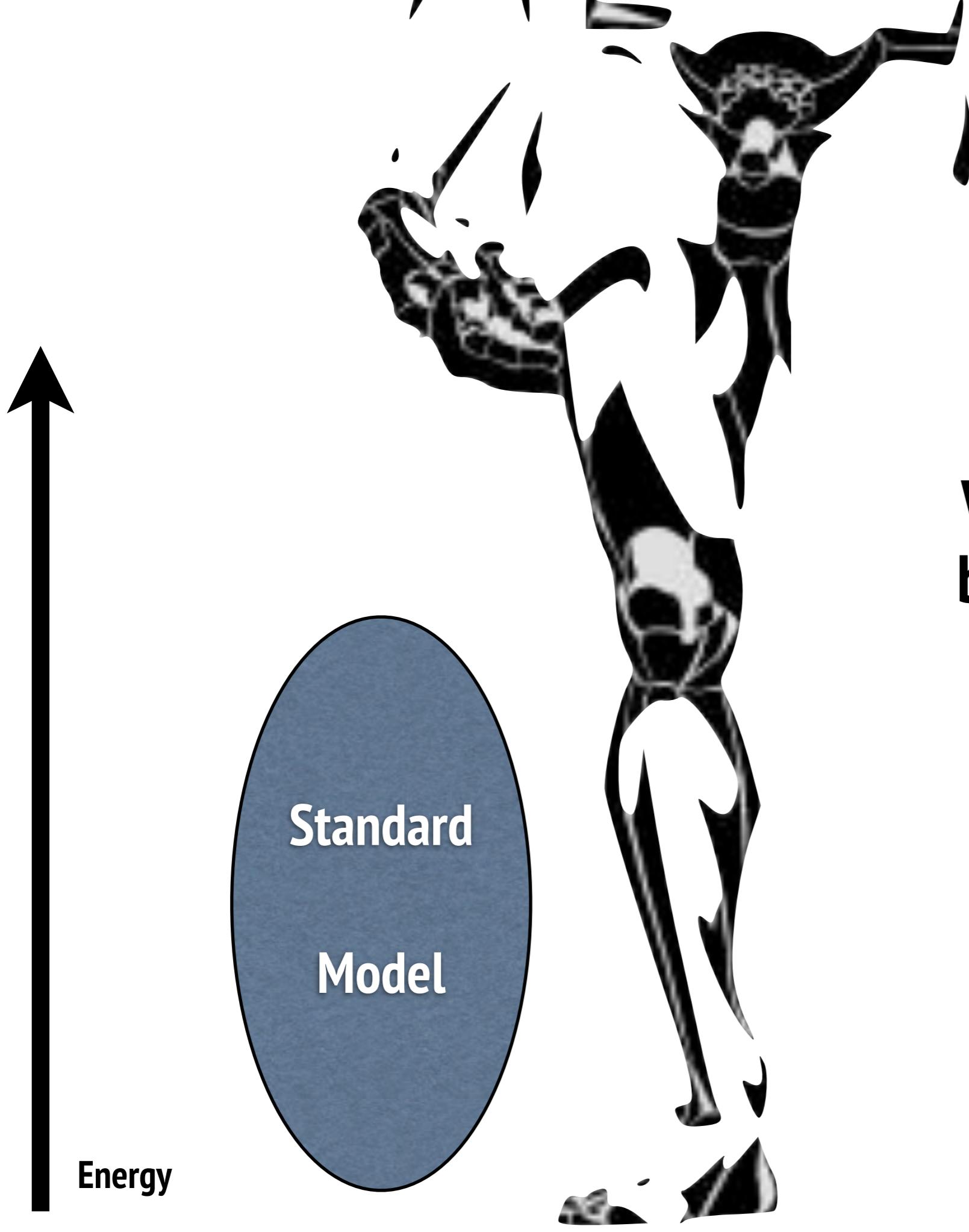
**Describes extremely well
particle physics
(at low energies)**



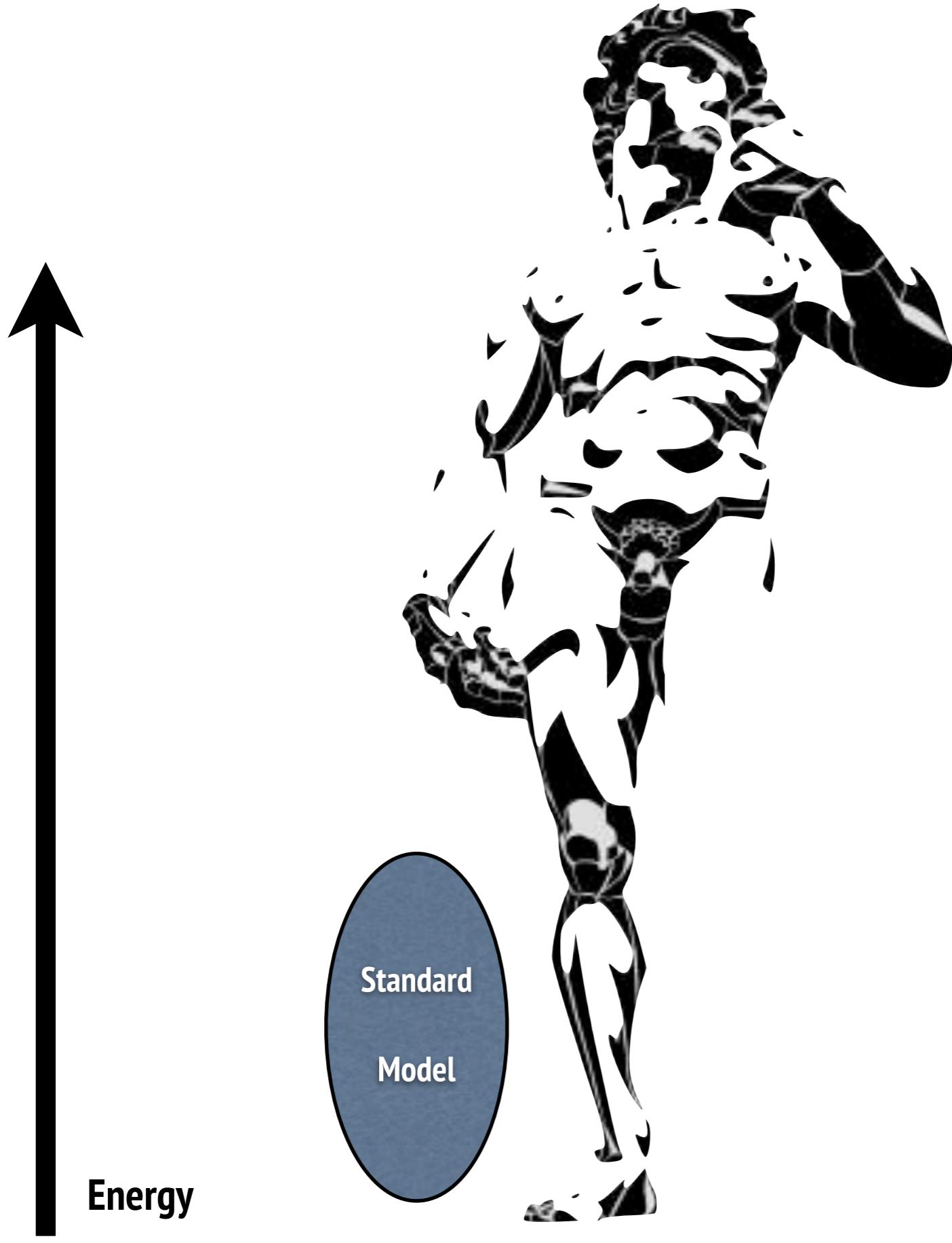
**Describes extremely well
particle physics
(at low energies)**

but it is certainly ...

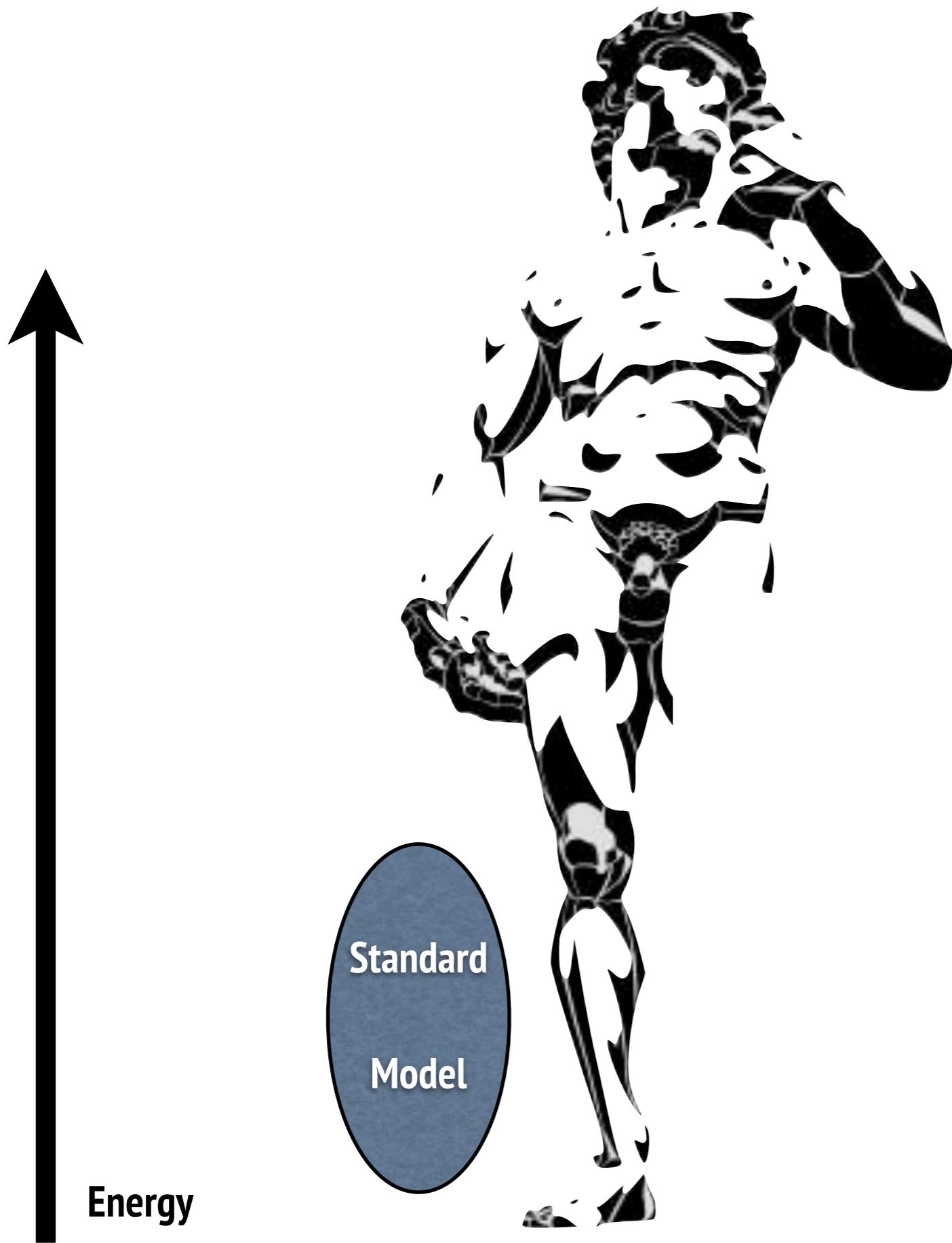
INCOMPLETE



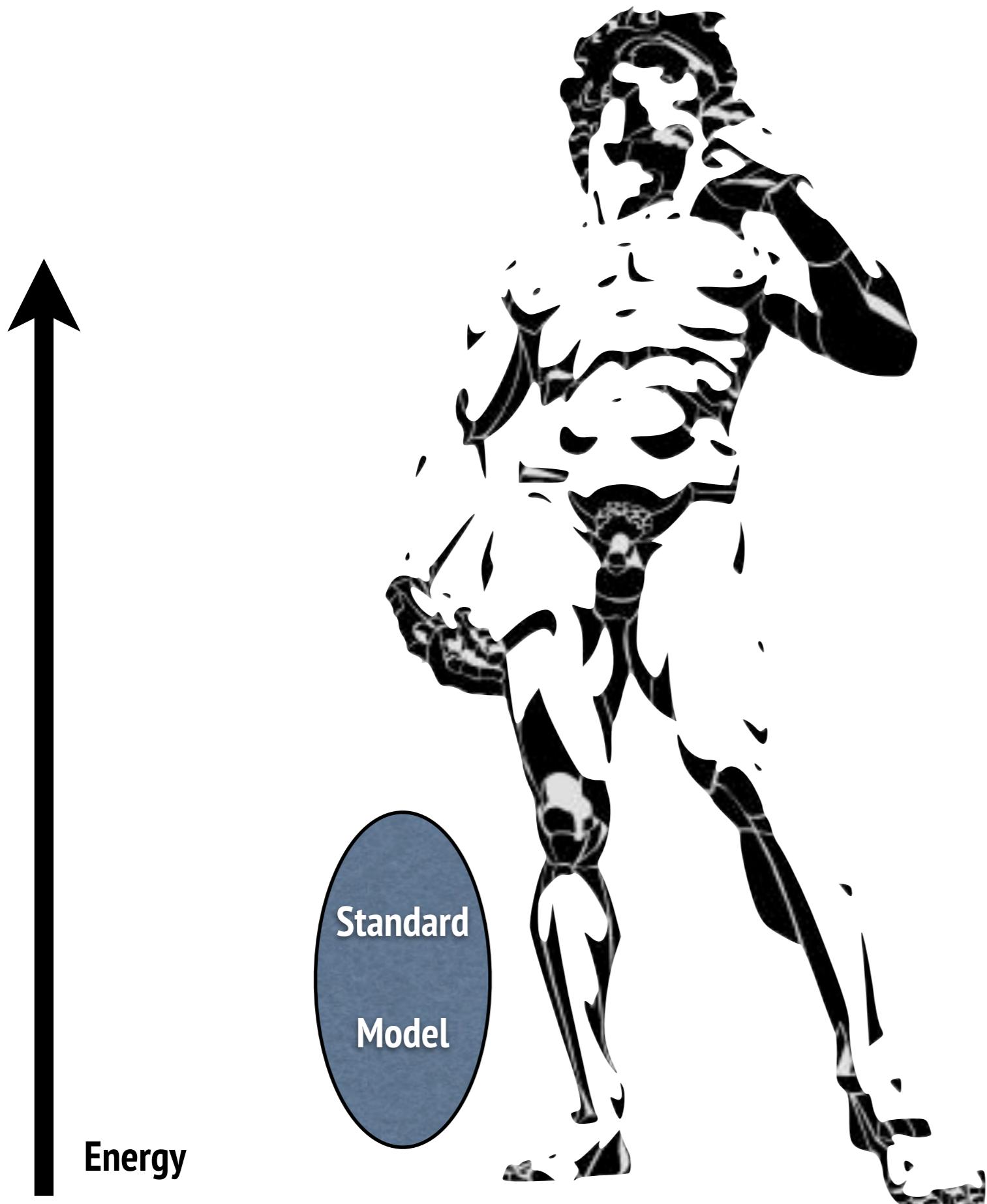
**Answers wait in the
high energy frontier
where more symmetric
beautiful theories arise**



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**Answers wait in the
high energy frontier
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... often implying**

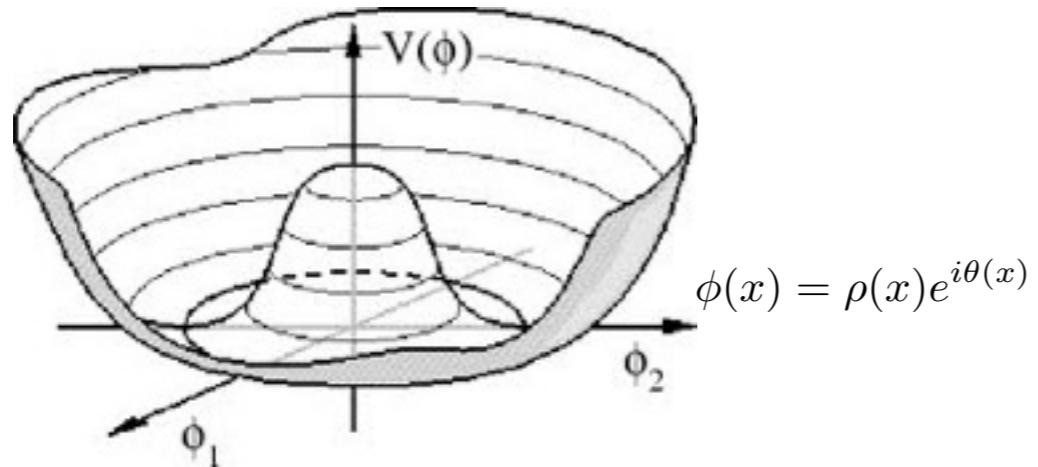


**Answers wait in the
high energy frontier
where more symmetric
beautiful theories arise
... often implying
new low energy physics!**

Axion-like particles (ALPs)

pseudo Goldstone Bosons

- Global symmetry spontaneously broken



- massless Goldstone Boson @ Low Energy

shift symmetry $\theta(x) \rightarrow \theta(x) + \alpha$

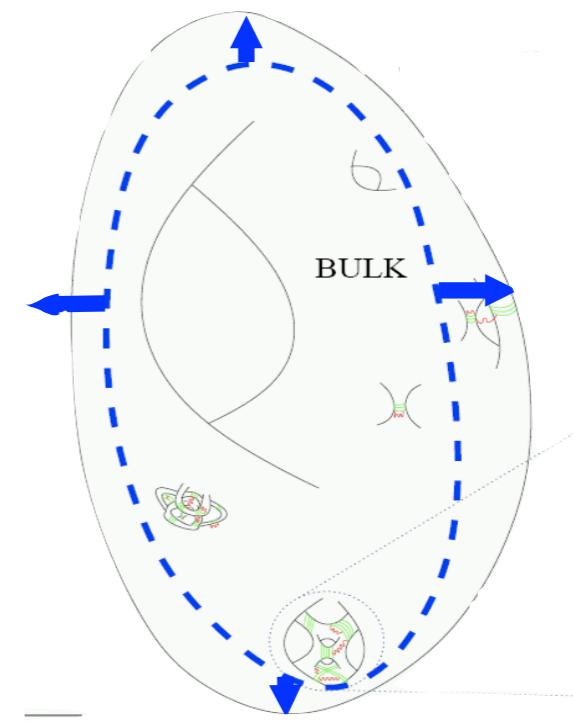
$$\mathcal{L}_{\text{kin}} = \frac{1}{2}(\partial_\mu \theta)(\partial^\mu \theta)f^2$$

- HE decay constant, $f = \langle \rho \rangle$

- small symmetry breaking \longrightarrow small mass

stringy axions

- Im parts of moduli fields (control sizes)



- 0(100) candidates in compactification

- “decay constant”, string scale M_s

- masses from non-perturbative effects

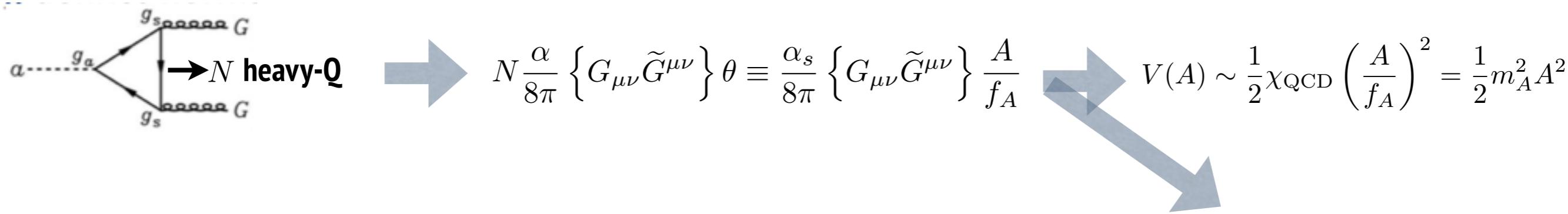
Low-energy effective action

- Shift symmetry allows some generic types of interactions

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu \theta)(\partial^\mu \theta) f^2 + \sum_f c_f [\bar{f} \gamma^\mu \gamma_5 f] \partial_\mu \theta - E \frac{\alpha}{8\pi} F_{\mu\nu} \tilde{F}^{\mu\nu} \theta$$

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) + \sum_f g_{af} [\bar{f} \gamma_5 f] a - \frac{g_{a\gamma}}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} a \quad (\text{canonically normalised})$$

- SS breaking terms induce mass + new interactions (one example ...)



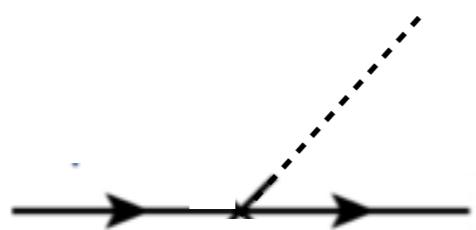
photon coupling

$$-\frac{g_{a\gamma}}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} a$$



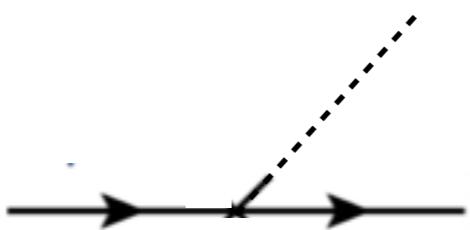
electron coupling

$$g_{ef} [\bar{e} \gamma_5 e] a$$



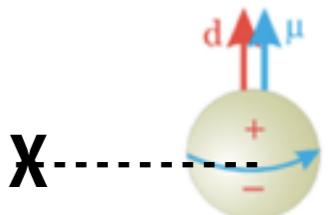
nucleon coupling

$$g_{Nf} [\bar{N} \gamma_5 N] a$$



\not{CP} Neutron electric dipole

$$\propto \frac{1}{m_n} [F_{\mu\nu} \bar{n} \sigma^{\mu\nu} \gamma_5 n] \frac{A}{f_A}$$



Strong CP problem / PQ solution

$$\left\{ G_{\mu\nu} \tilde{G}^{\mu\nu} \right\} \theta_{\text{SM}} \rightarrow d_n \sim \frac{e}{m_n} \theta_{\text{SM}} < 5 \times 10^{-12} \frac{e}{m_n}$$

why!! $\theta_{\text{SM}} < 10^{-11} !!$

Strong CP problem / PQ solution

$$\left\{ G_{\mu\nu} \tilde{G}^{\mu\nu} \right\} \left(\theta_{\text{SM}} + \frac{A}{f_A} \right) \rightarrow d_n \propto \left(\theta_{\text{SM}} + \frac{\langle A \rangle}{f_A} \right)$$



$$V(A) \sim \frac{1}{2} \chi \left(\theta_{\text{SM}} + \frac{A}{f_A} \right)^2$$

potential min.

$$\langle A \rangle / f_A = -\theta_{\text{SM}}$$

The QCD Axion cancels the effect of any constant θ_{SM}

4 hints

Strong CP problem / PQ solution

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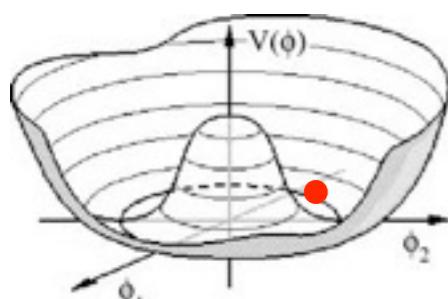
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Dark matter / vacuum realignment



pick up a vacuum when quasi-degenerate

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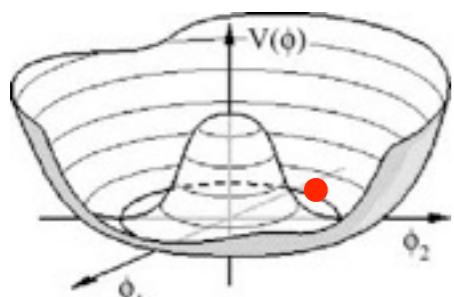
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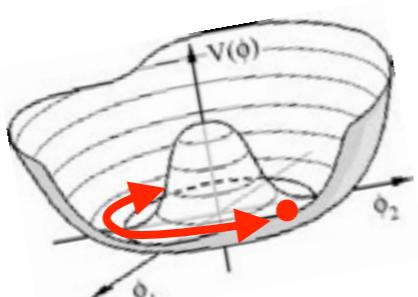
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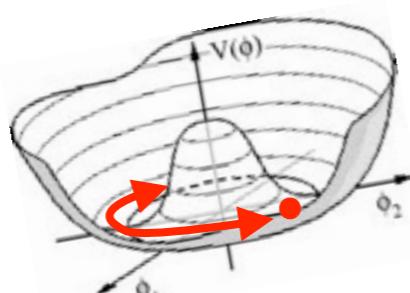
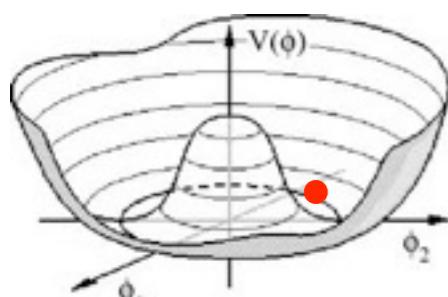
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cold DM in oscillations [cosmology dependent]

$$\Omega h_c^2 \simeq 0.12 \sqrt{\frac{m_a}{\text{meV}}} \left(\frac{a_i}{3 \times 10^{12} \text{ GeV}} \right)^2$$

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↓

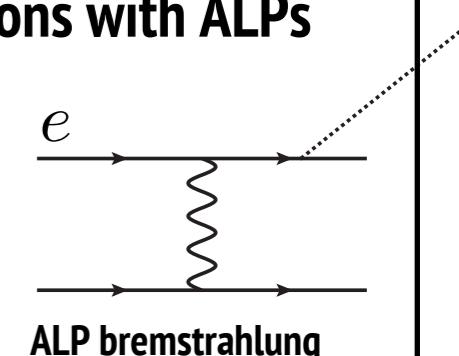
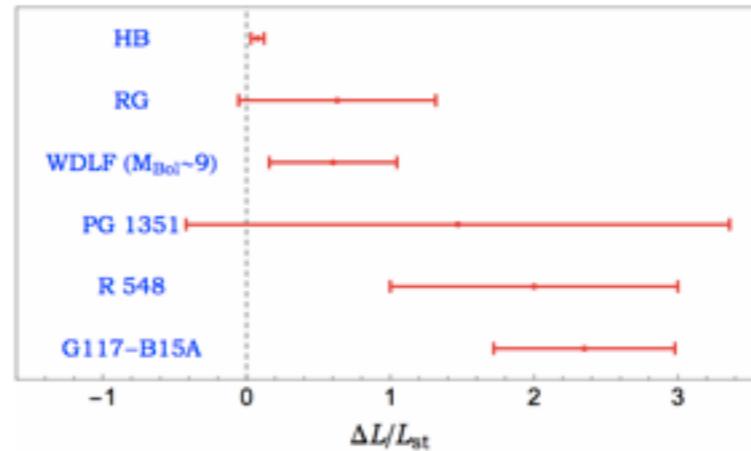
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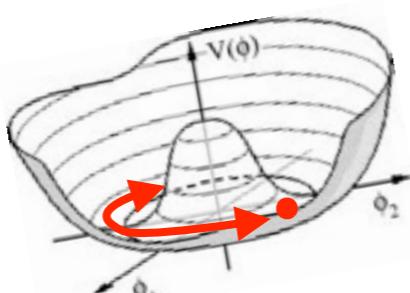
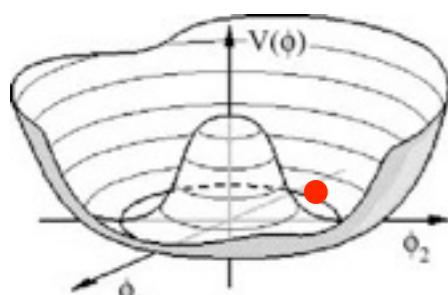
Anomalous Star cooling / ALP emission

Theory fits better some observations with ALPs



Giannotti 2016

Dark matter / vacuum realignment



pick up a vacuum when quasi-degenerate ups! not the lowest ... oscillate!

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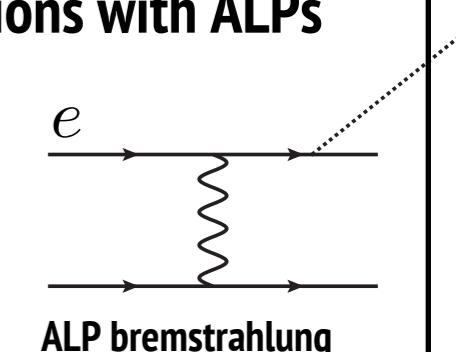
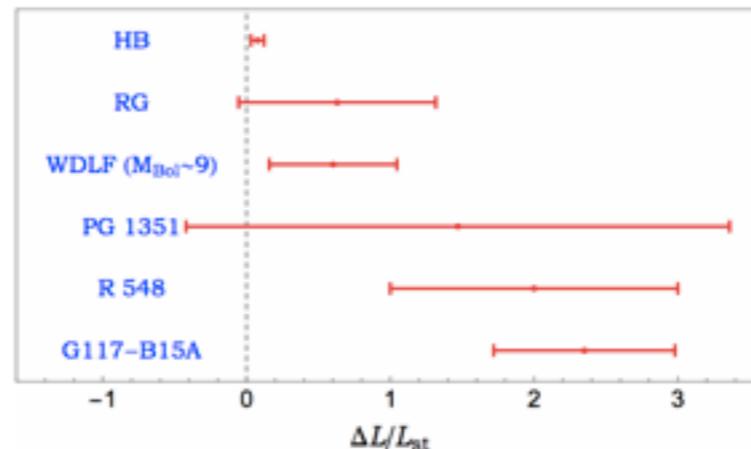
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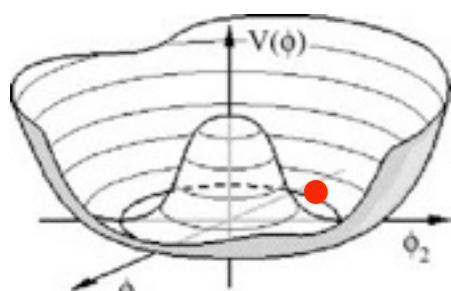
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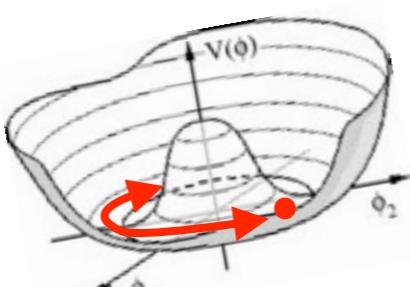


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γ-ray transparency / photon regeneration

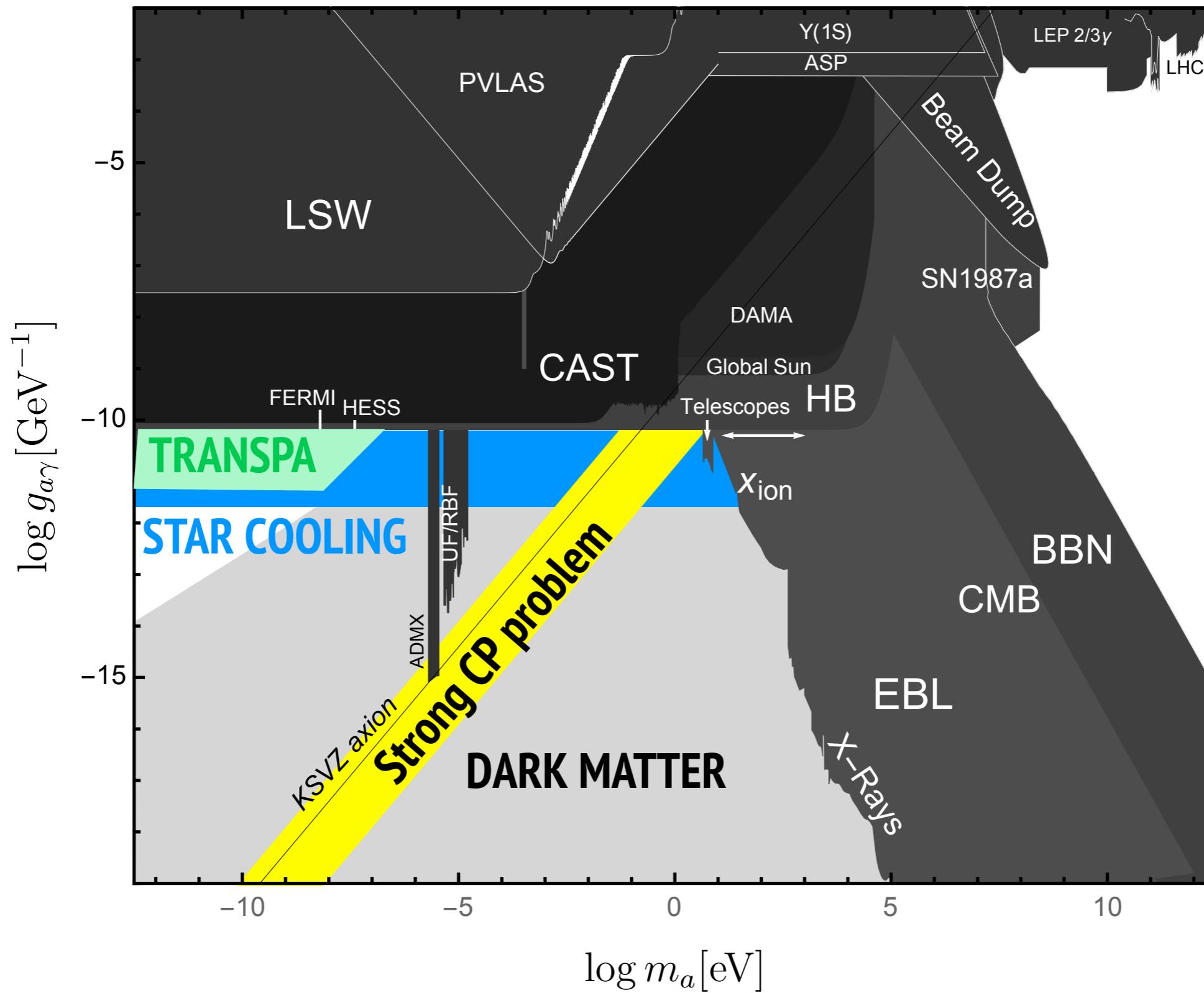
Too many gamma-rays from far away sources?



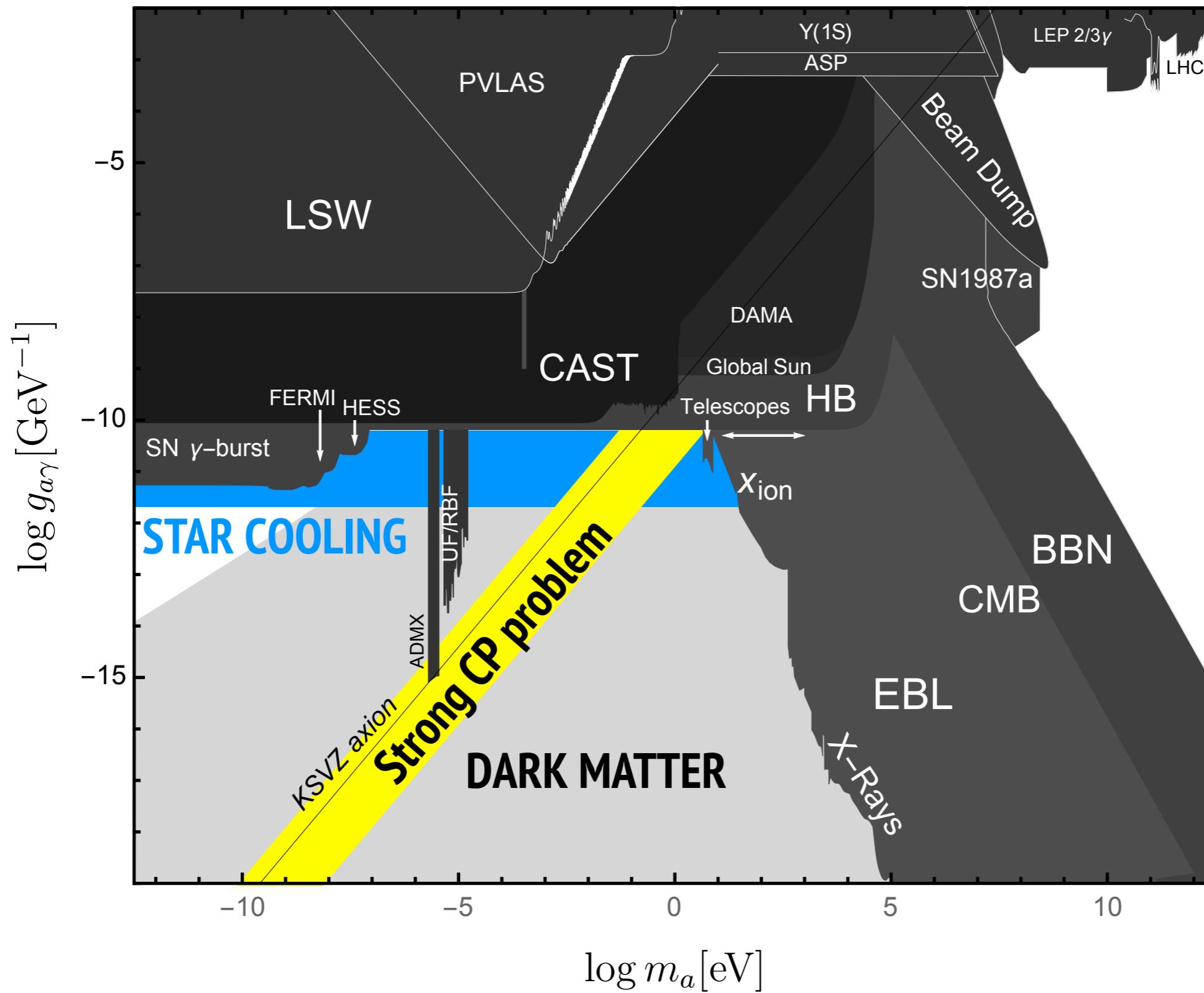
low estimate of opacity vs ALP-mediated regeneration

Trostski 2017

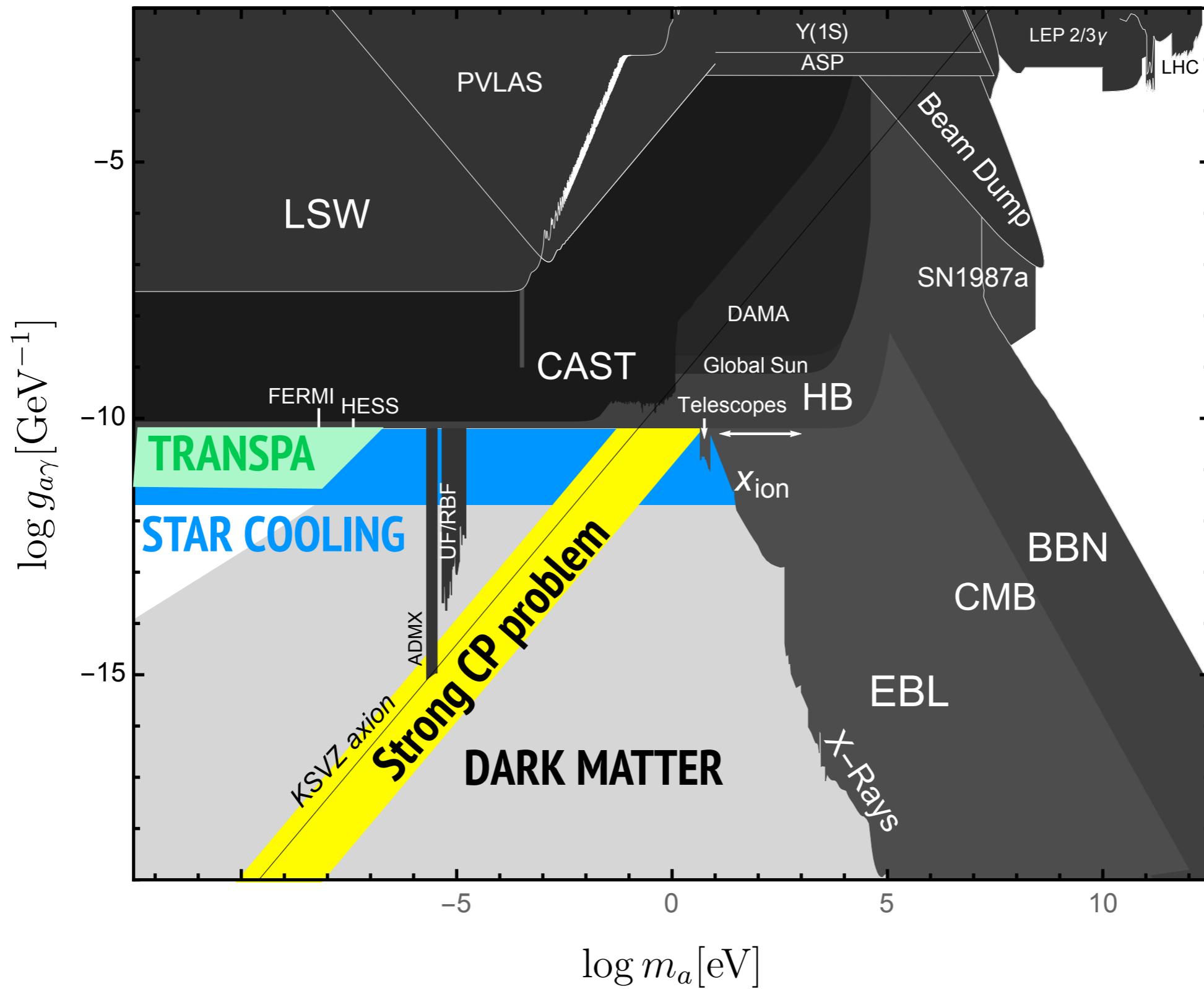
Hints and constraints (example)



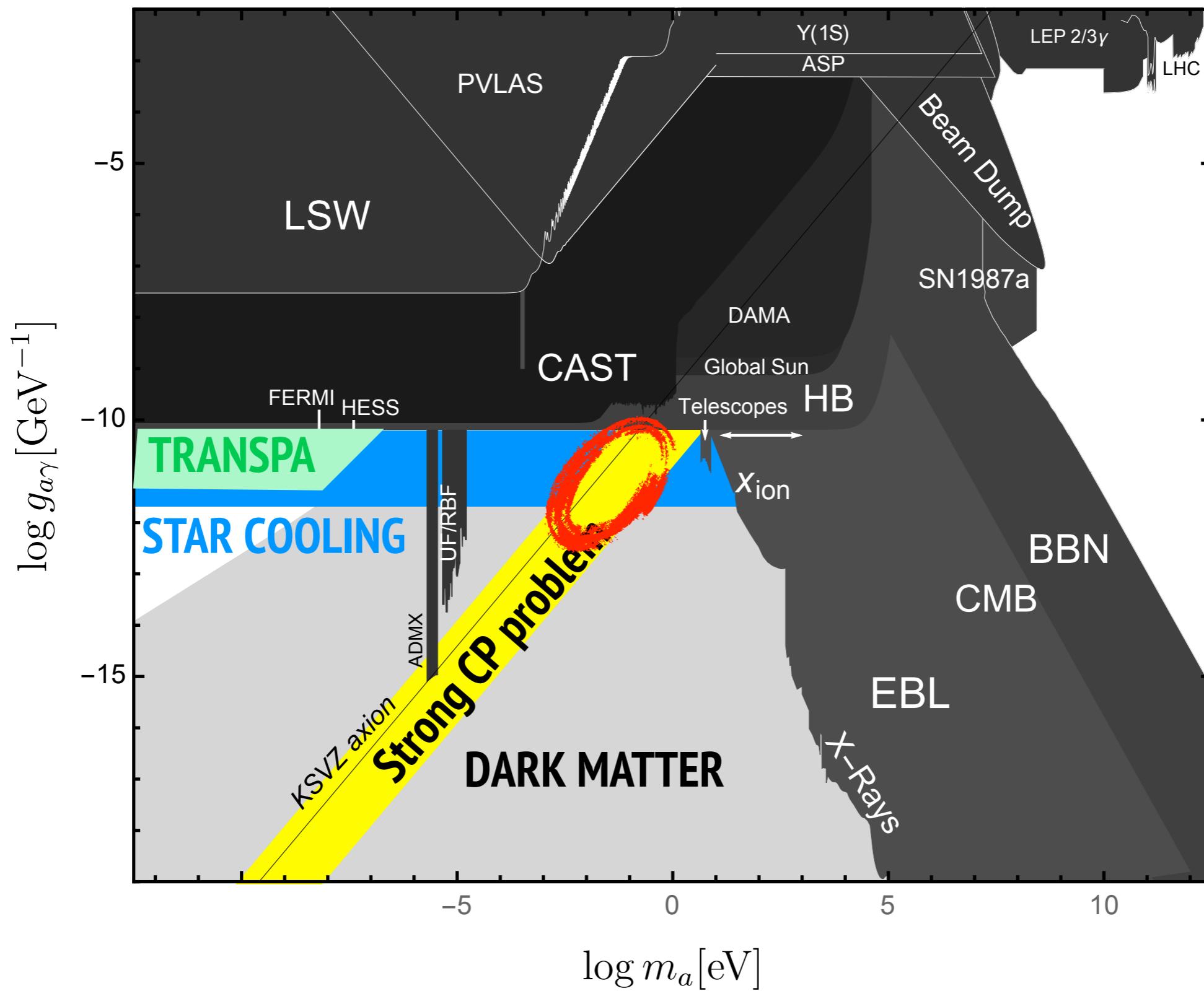
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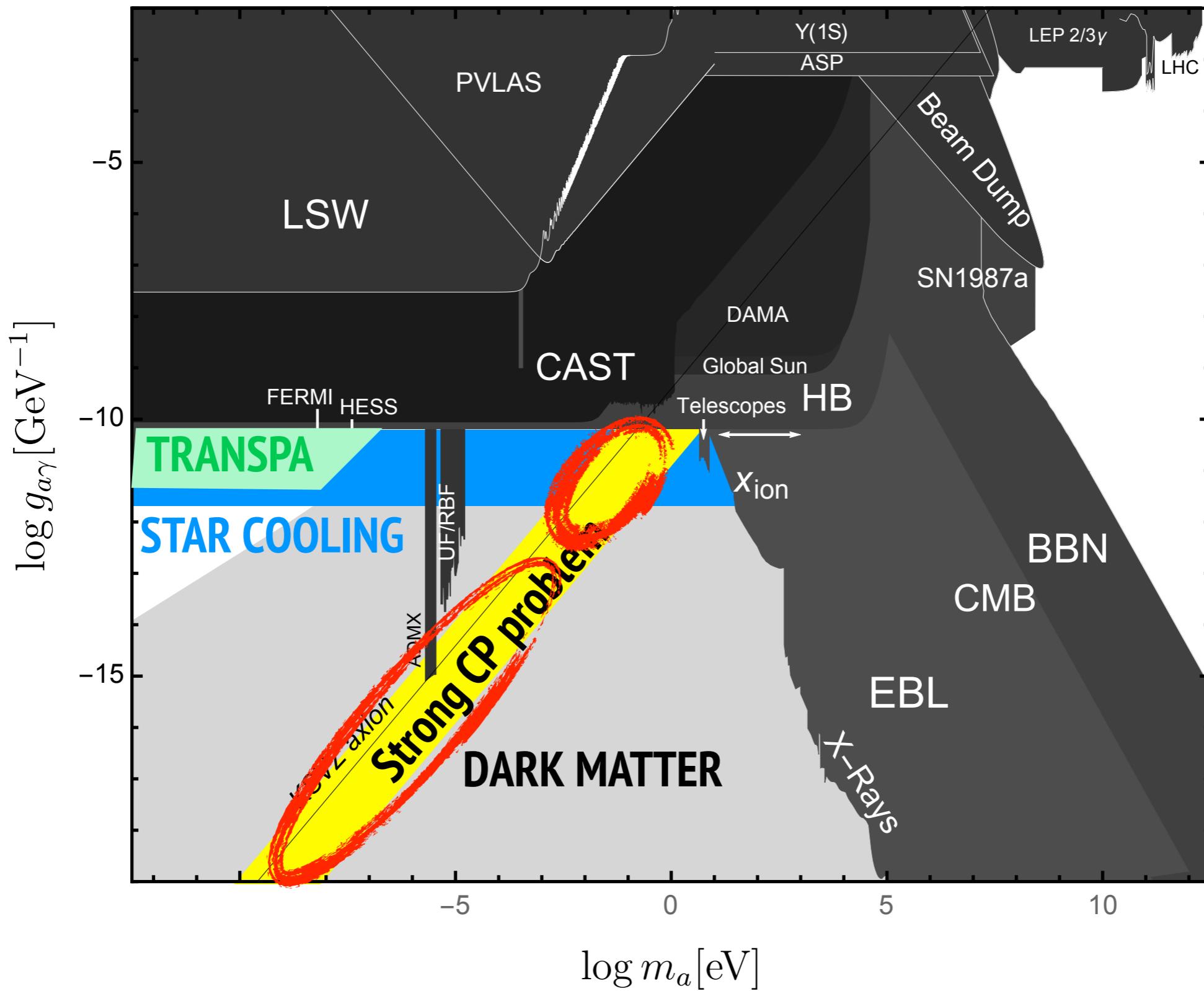
birds and stones ...



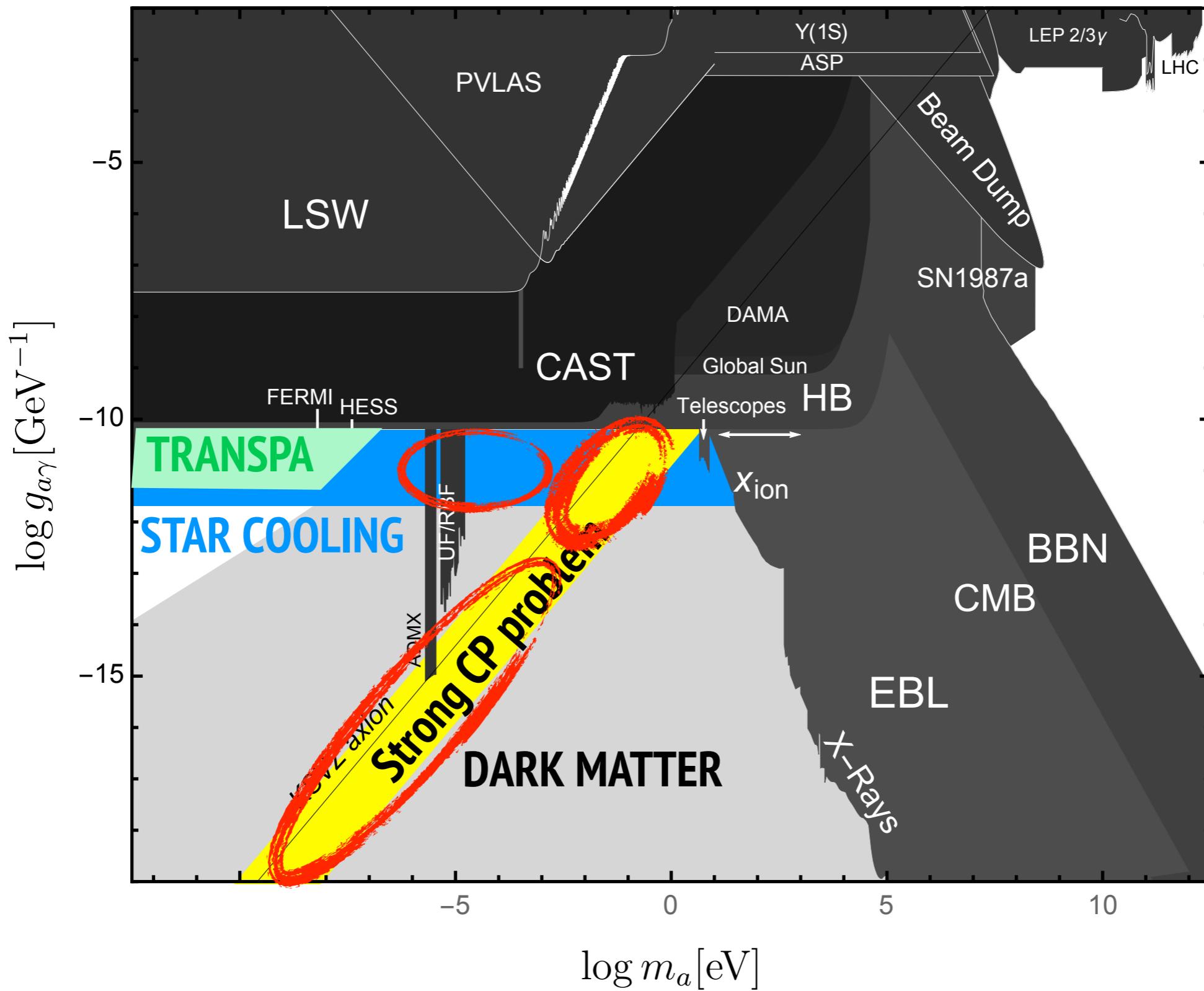
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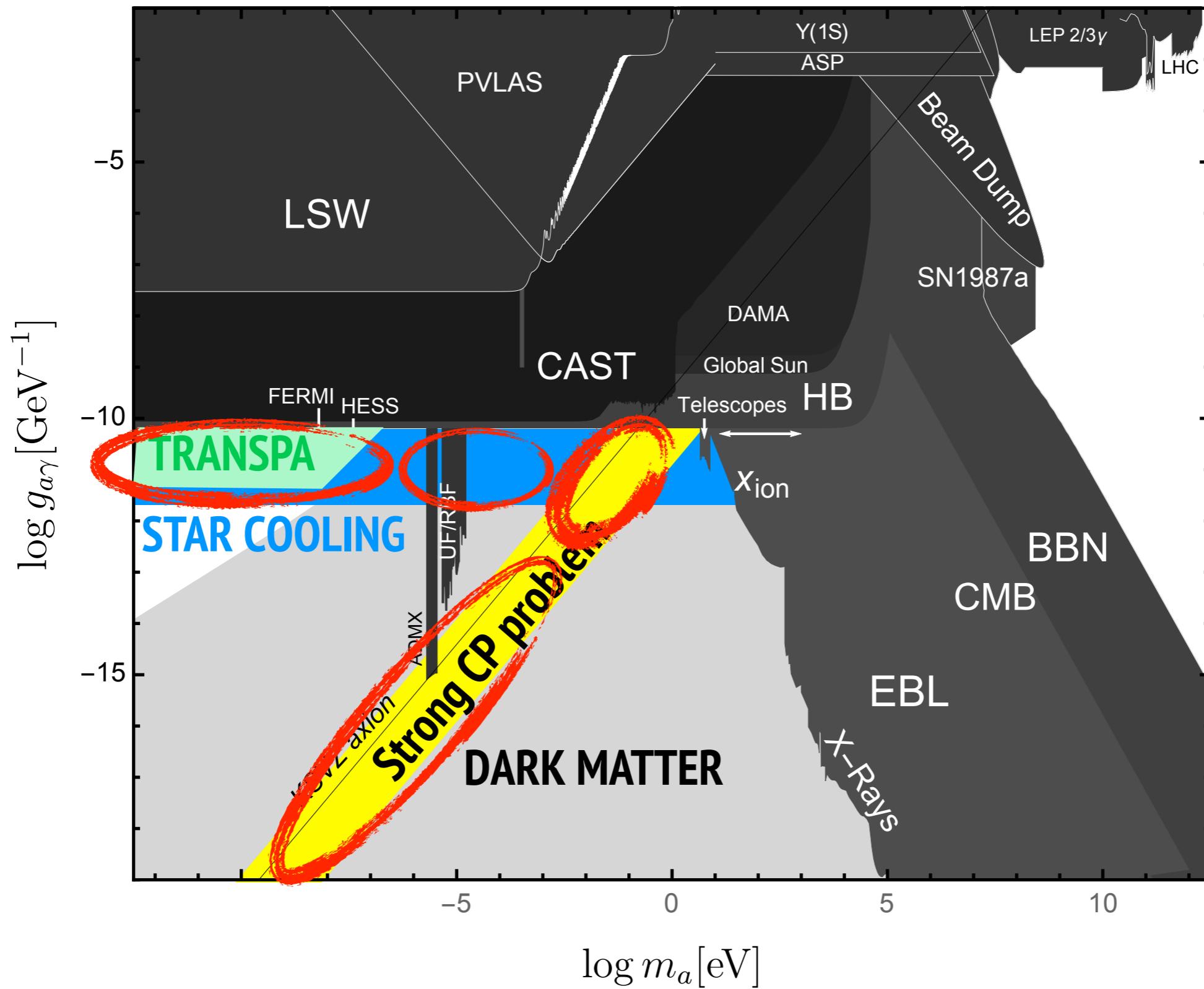
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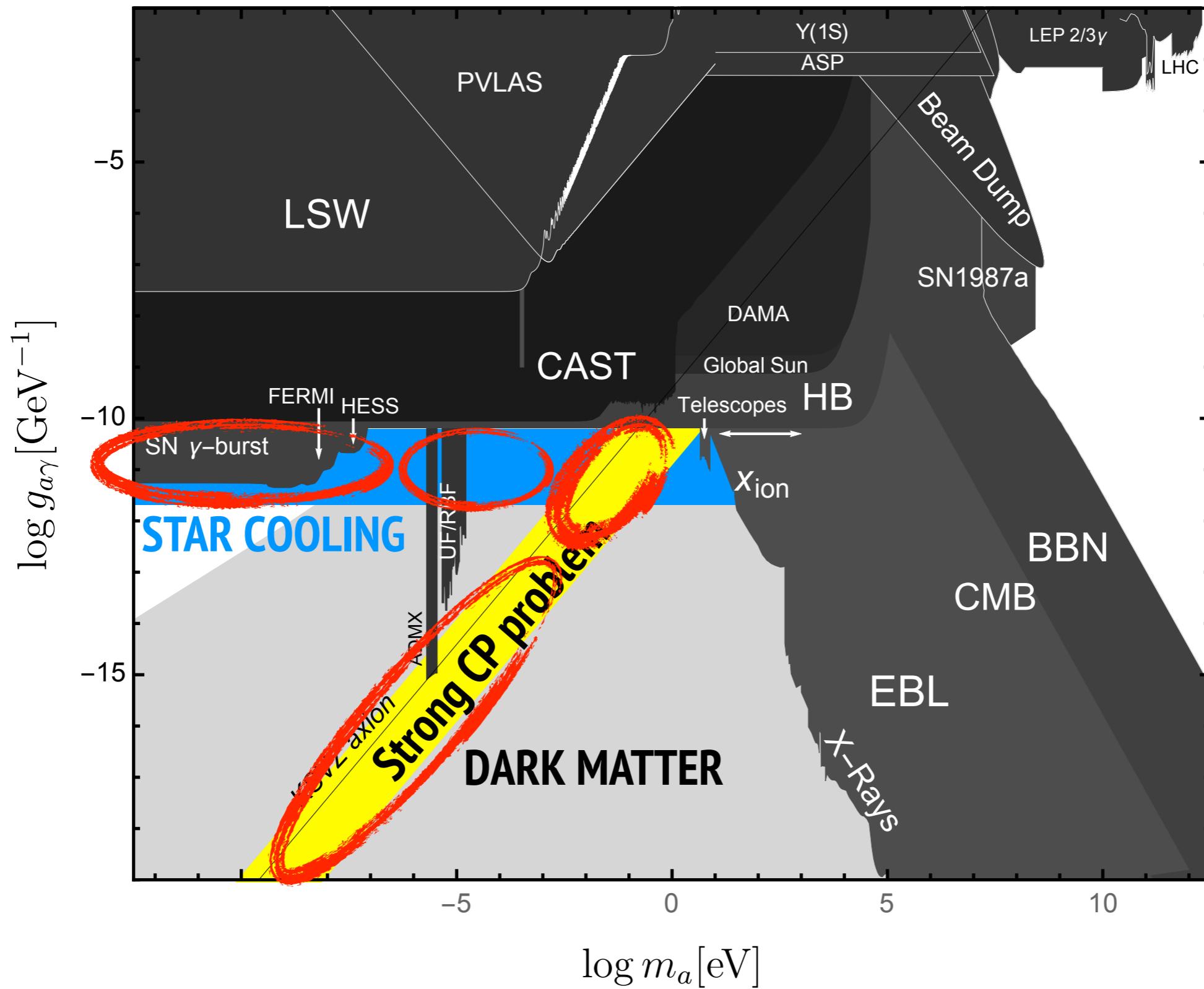
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birds and stones ...



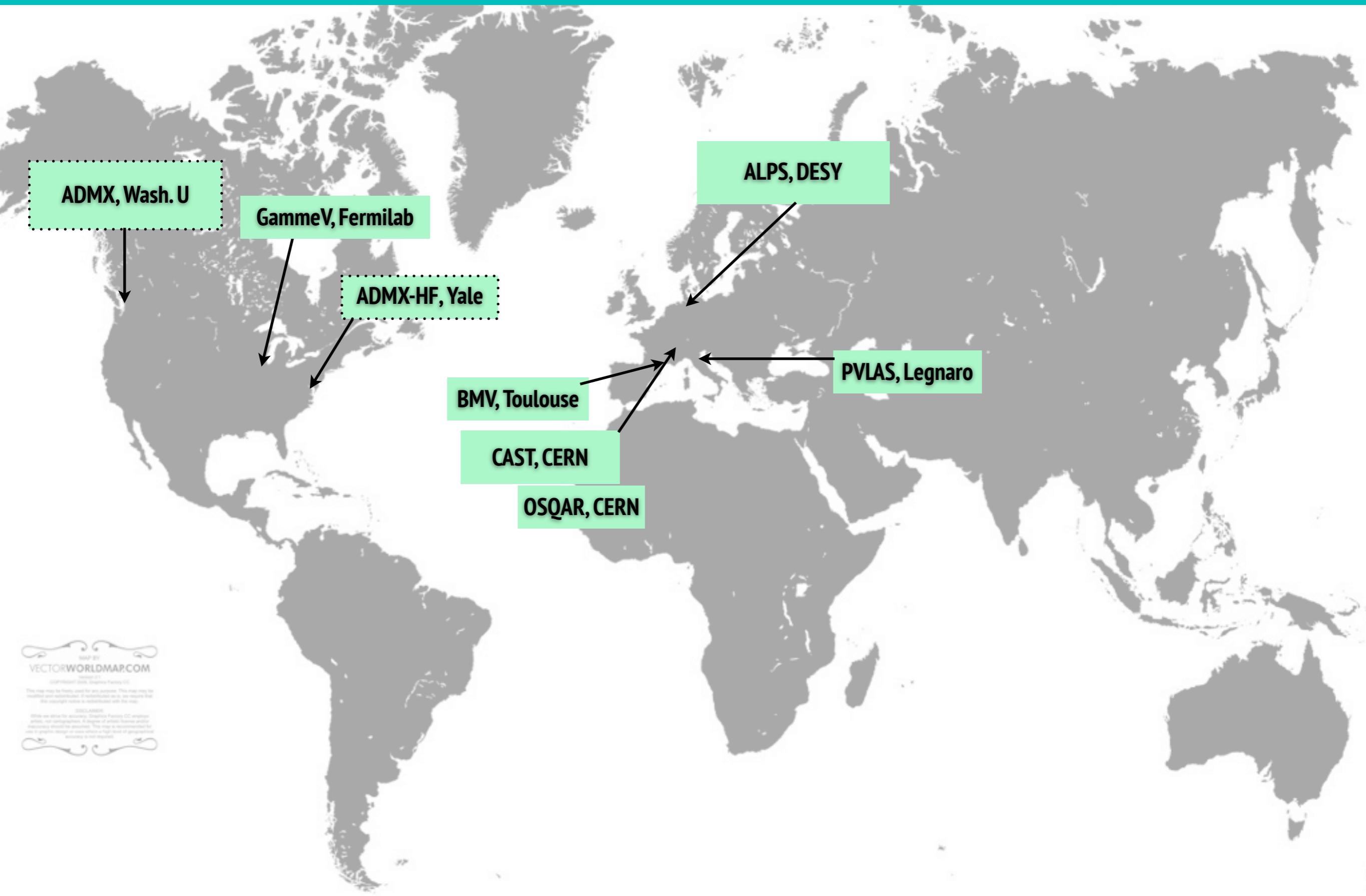
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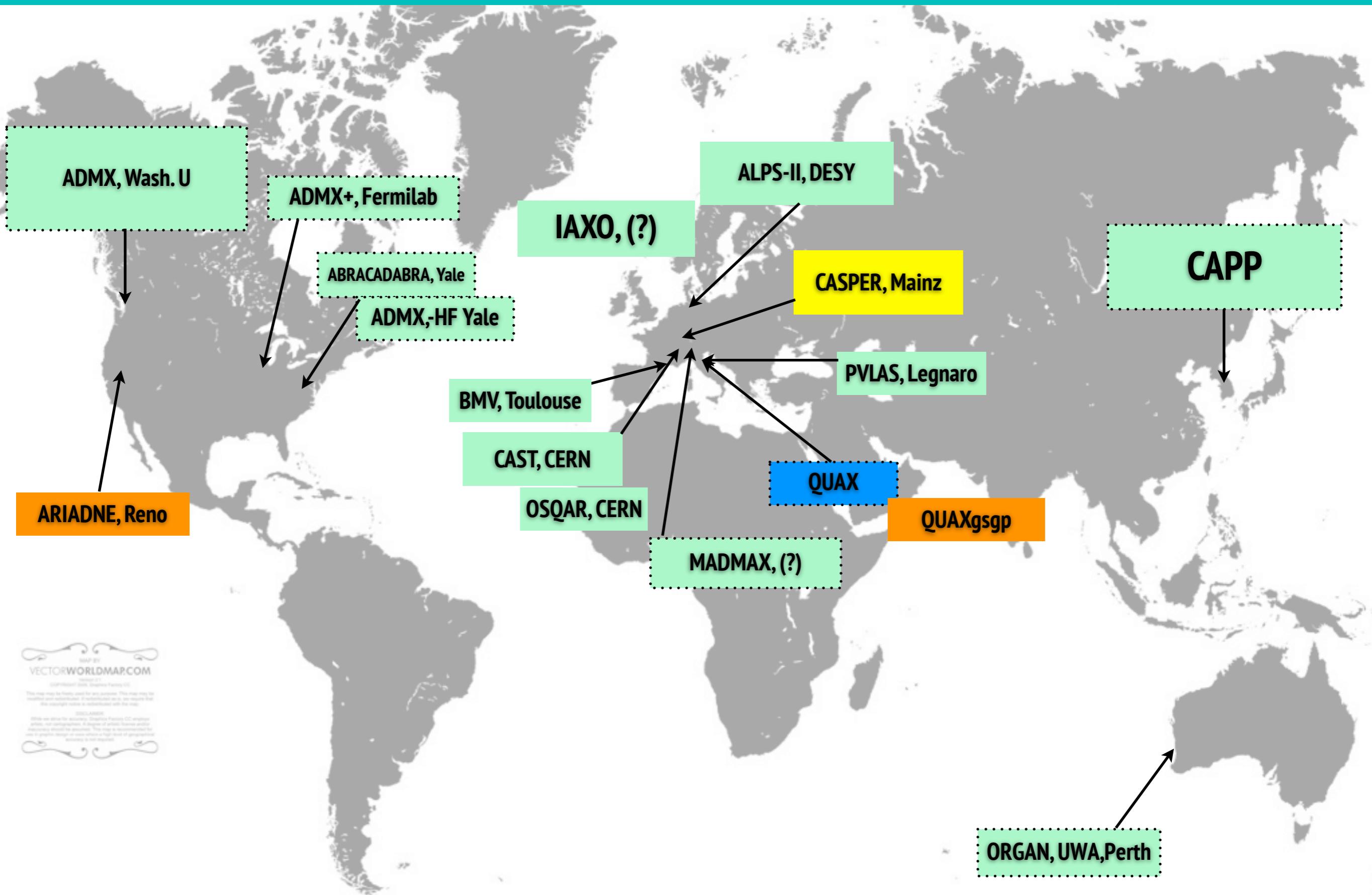
Direct Detection of ALPs



Lab experiments 2011



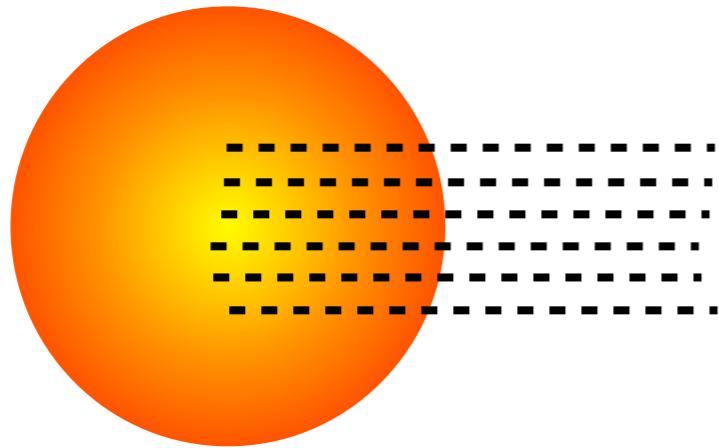
Lab experiments 2017



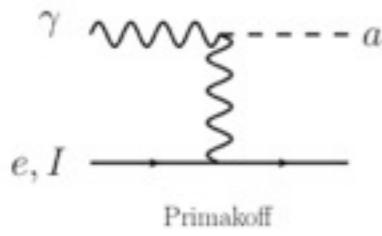
Helioscopes (search solar ALPs)

Sikivie PRL 1983

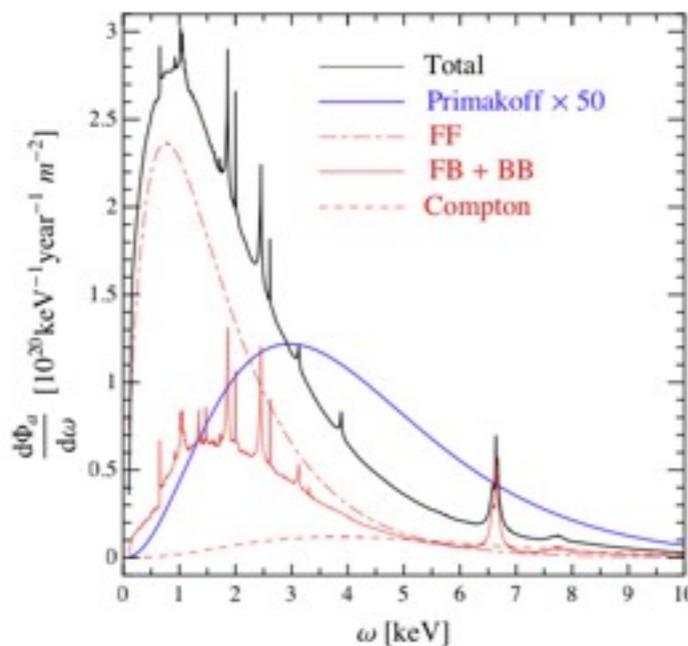
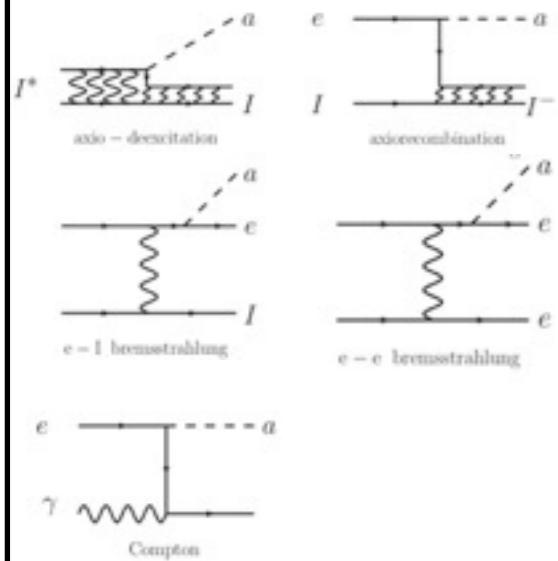
The Sun is a copious emitter of ALPs!



photon coupling



electron coupling

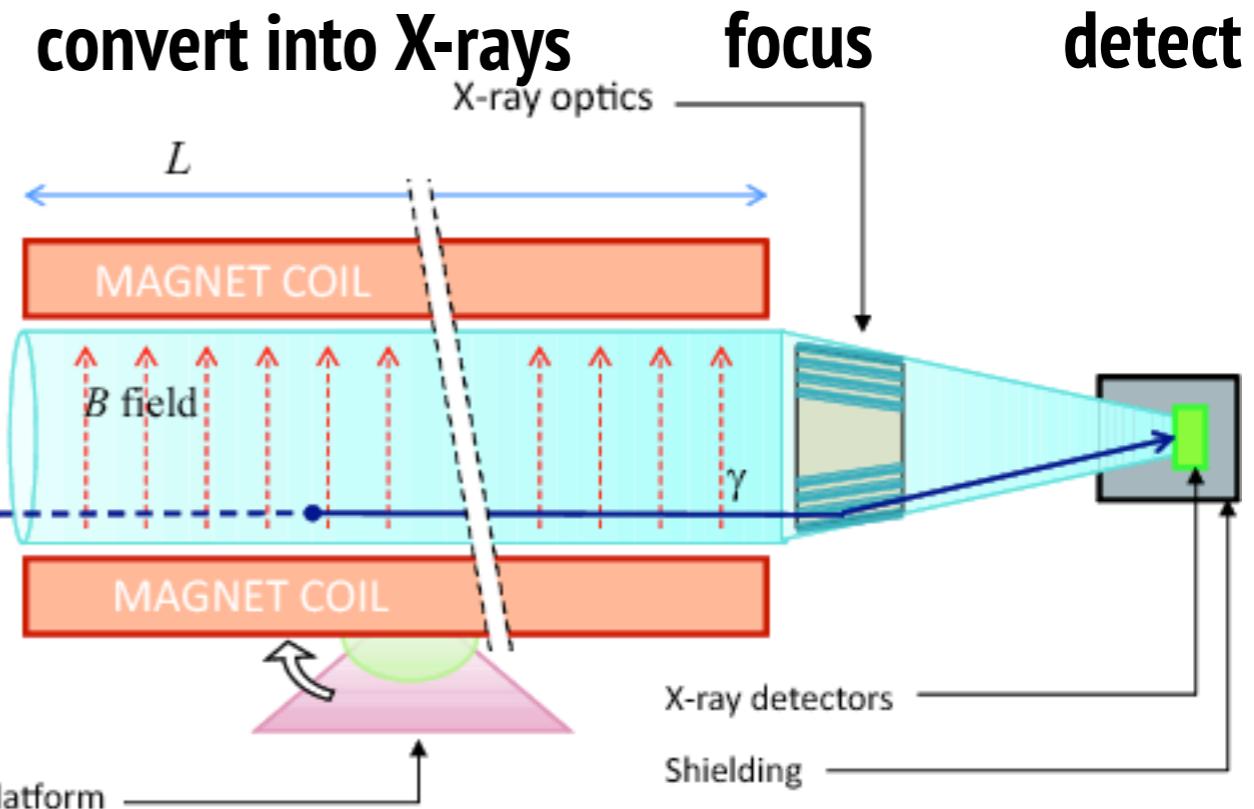
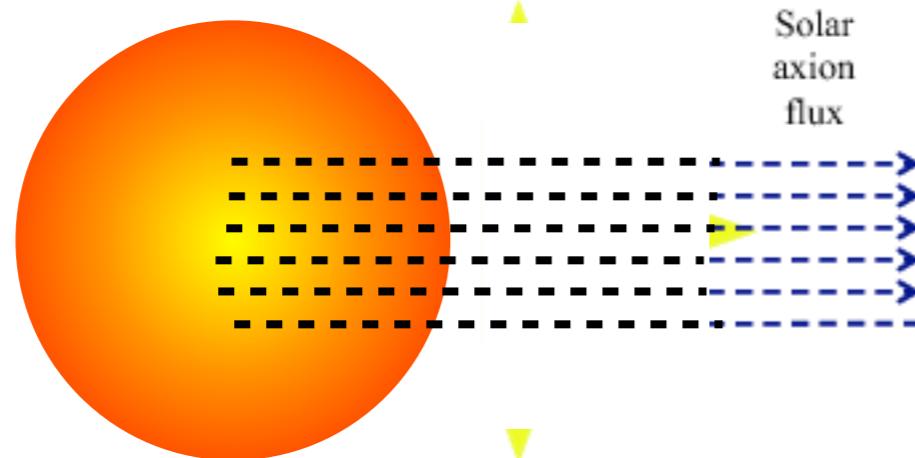


$$g_{ae} = 10^{-13}$$
$$g_{a\gamma} = 10^{-12}$$

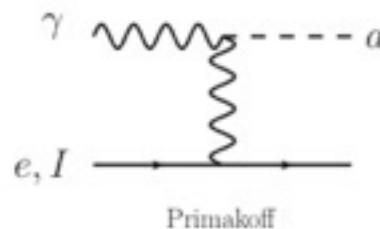
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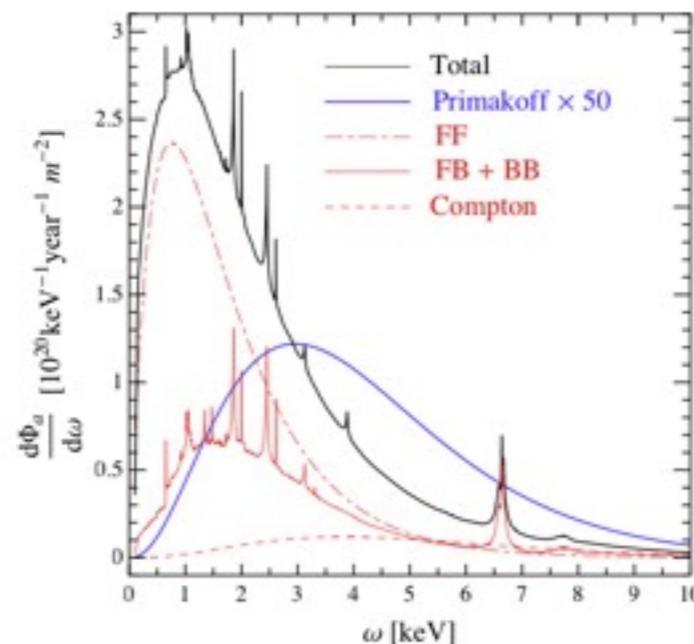
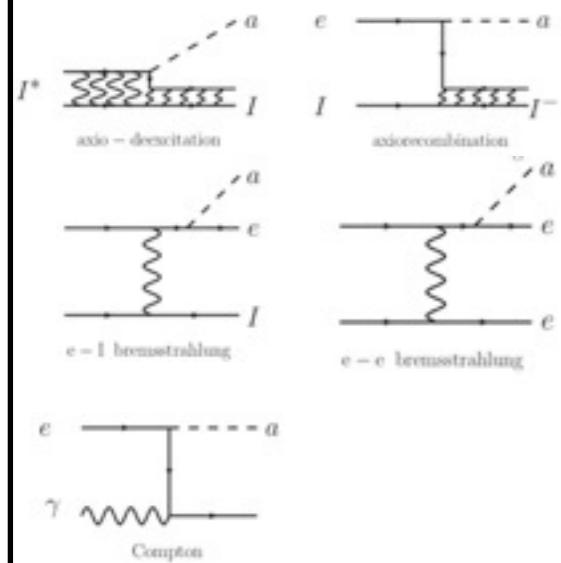
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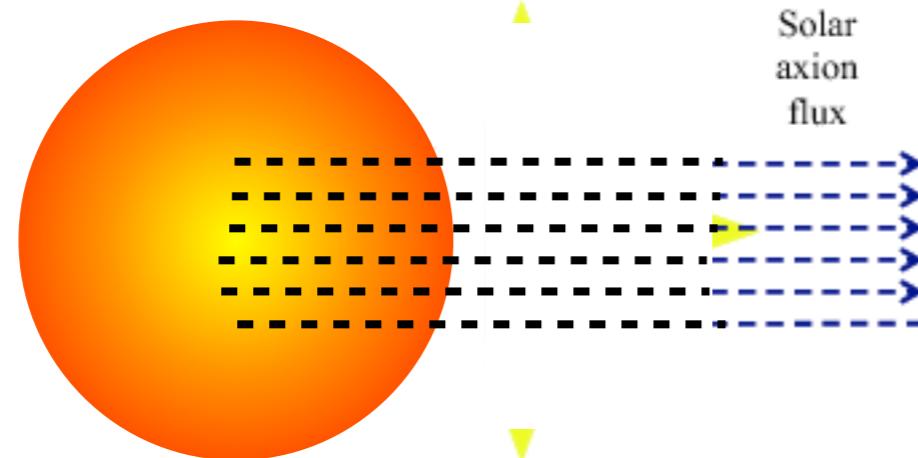
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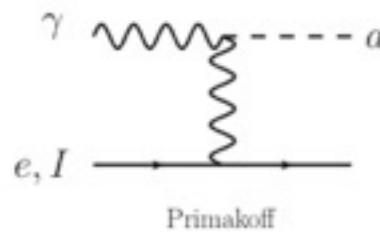
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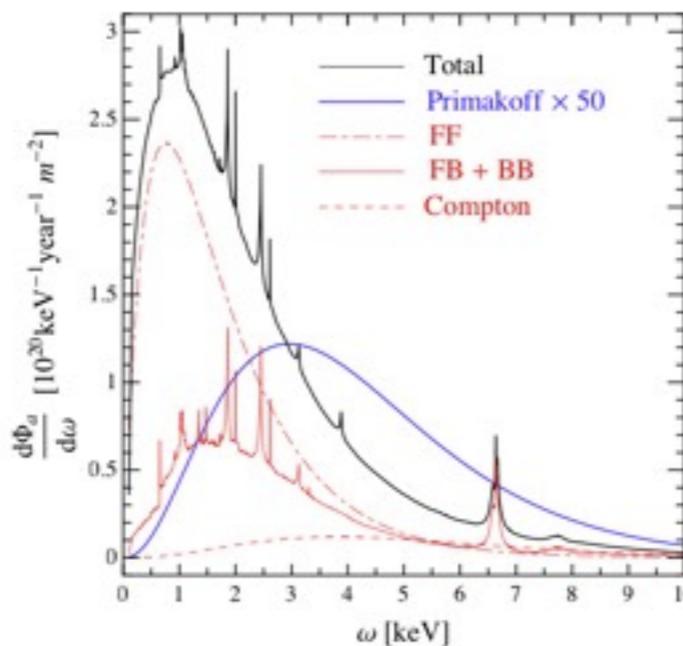
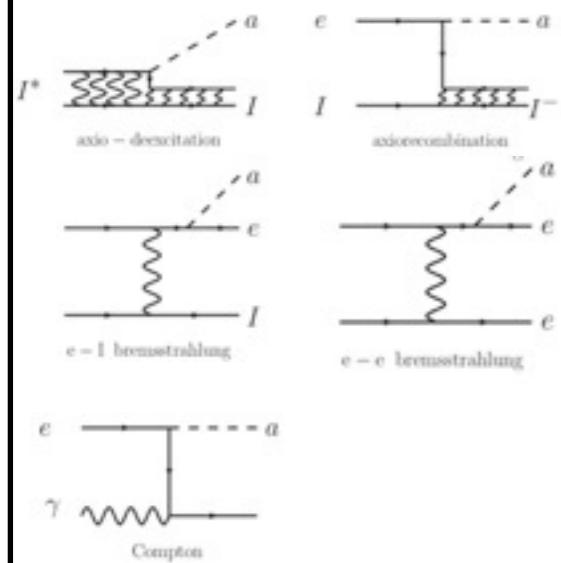
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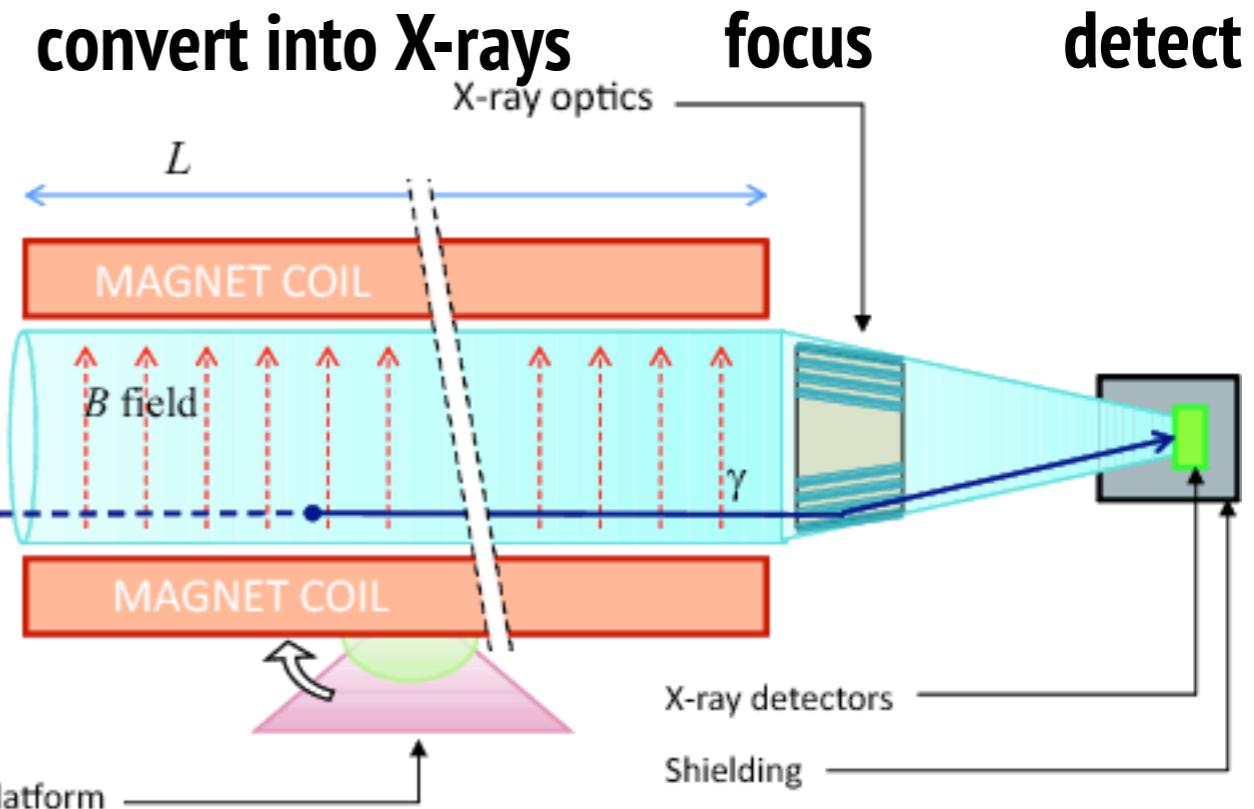
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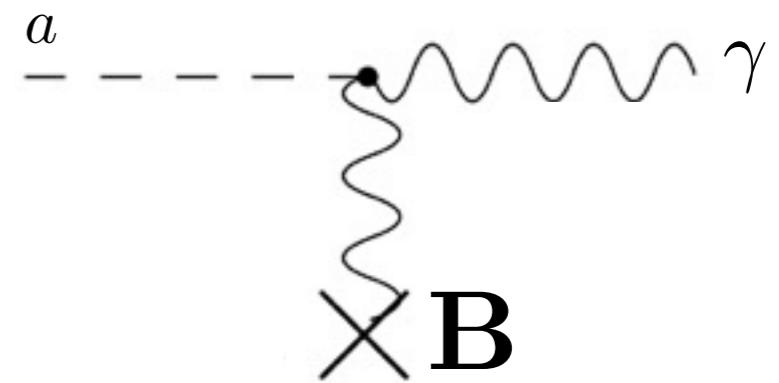


$g_{ae} = 10^{-13}$
 $g_{a\gamma} = 10^{-12}$



Coherent Conversion along the B-field

$$P(a \leftrightarrow \gamma) = \left(\frac{2g_{a\gamma} B_T \omega}{m_a^2} \right)^2 \sin^2 \left(\frac{m_a^2 L}{4\omega} \right)$$



International AXion Observatory

Large toroidal 8-coil magnet $L = \sim 20$ m

8 bores: 600 mm diameter each

8 x-ray optics + 8 detection systems

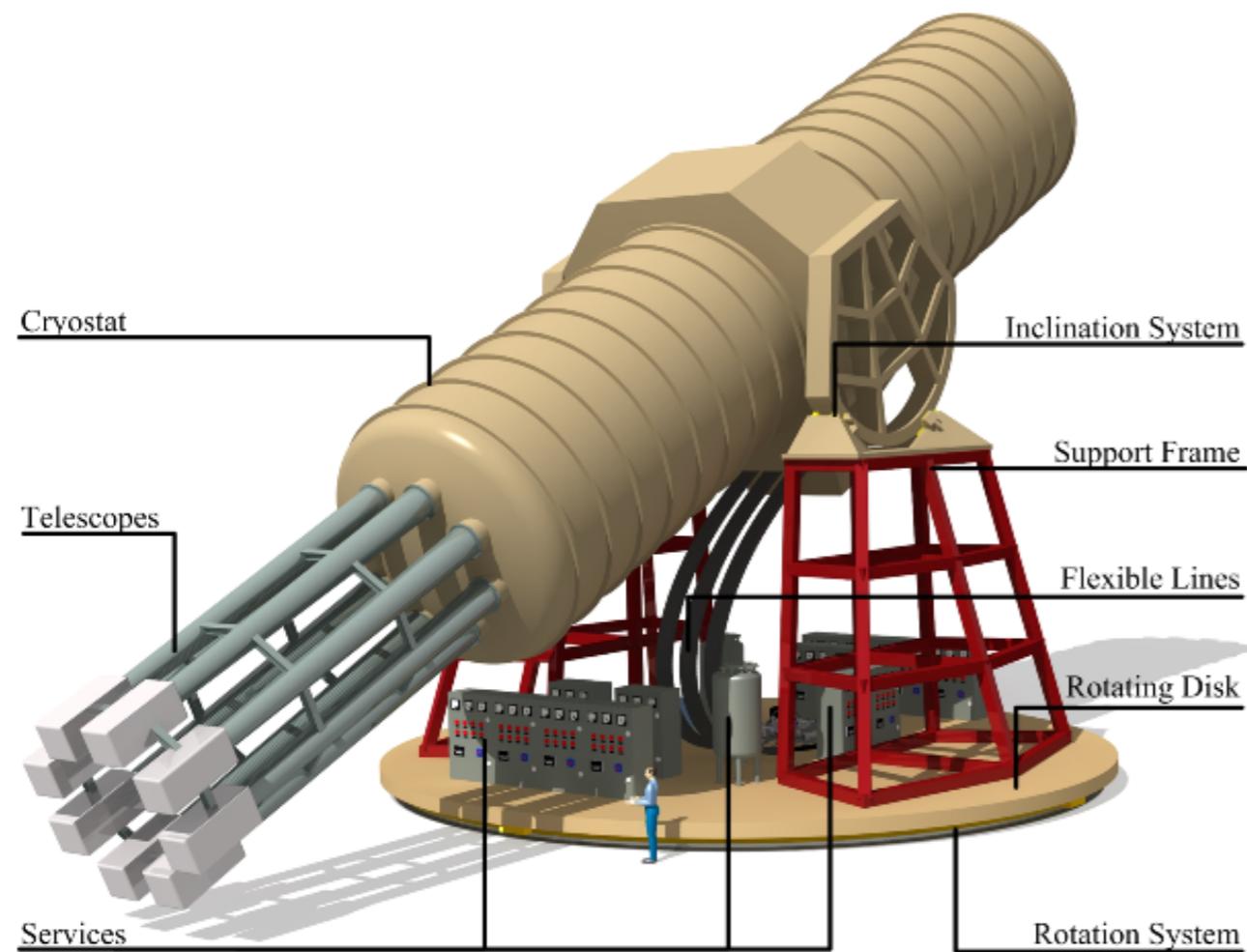
Rotating platform with services

-NGAG paper JCAP 1106:013,2011

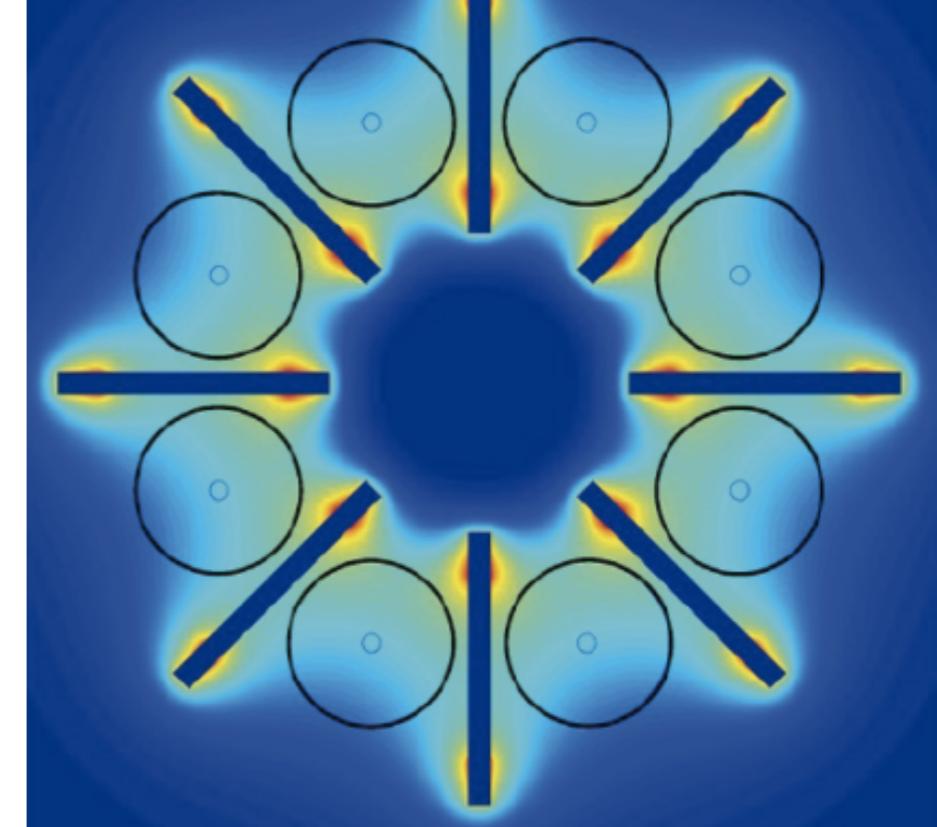
-Conceptual design report IAXO 2014 JINST 9 T05002

-LOI submitted to CERN, TDR in preparation

-Possibility of Direct Axion DM experiments (cavities,ABRACA)



Transverse B-field (peak 5T, average 2.5T)



IAXO detectors

Goal background level for IAXO:

$$\frac{10^{-7} \rightarrow 10^{-8}}{\text{keV cm}^2 \text{ s}}$$

- Small Micromegas-TPC chambers:

Shielding

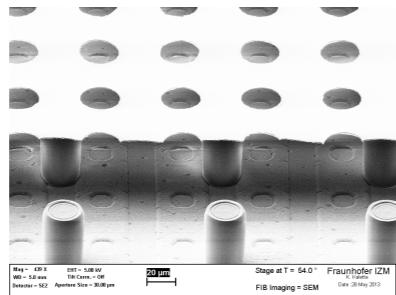
Radiopure components

Offline discrimination

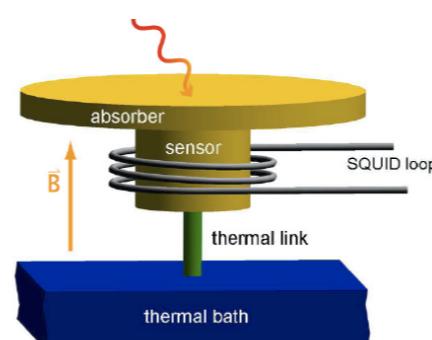


Already demonstrated: $\frac{8 \times 10^{-7}}{\text{keV cm}^2 \text{ s}}$ **(in CAST 2014 result)** $\frac{10^{-7}}{\text{keV cm}^2 \text{ s}}$ **(underground at LSC)**

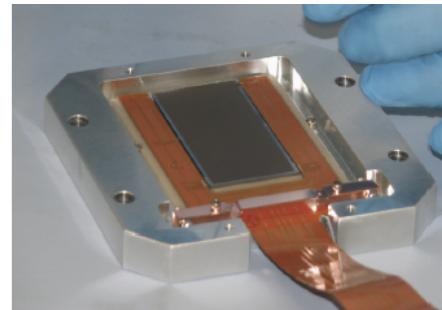
- Gridpix/InGrid,



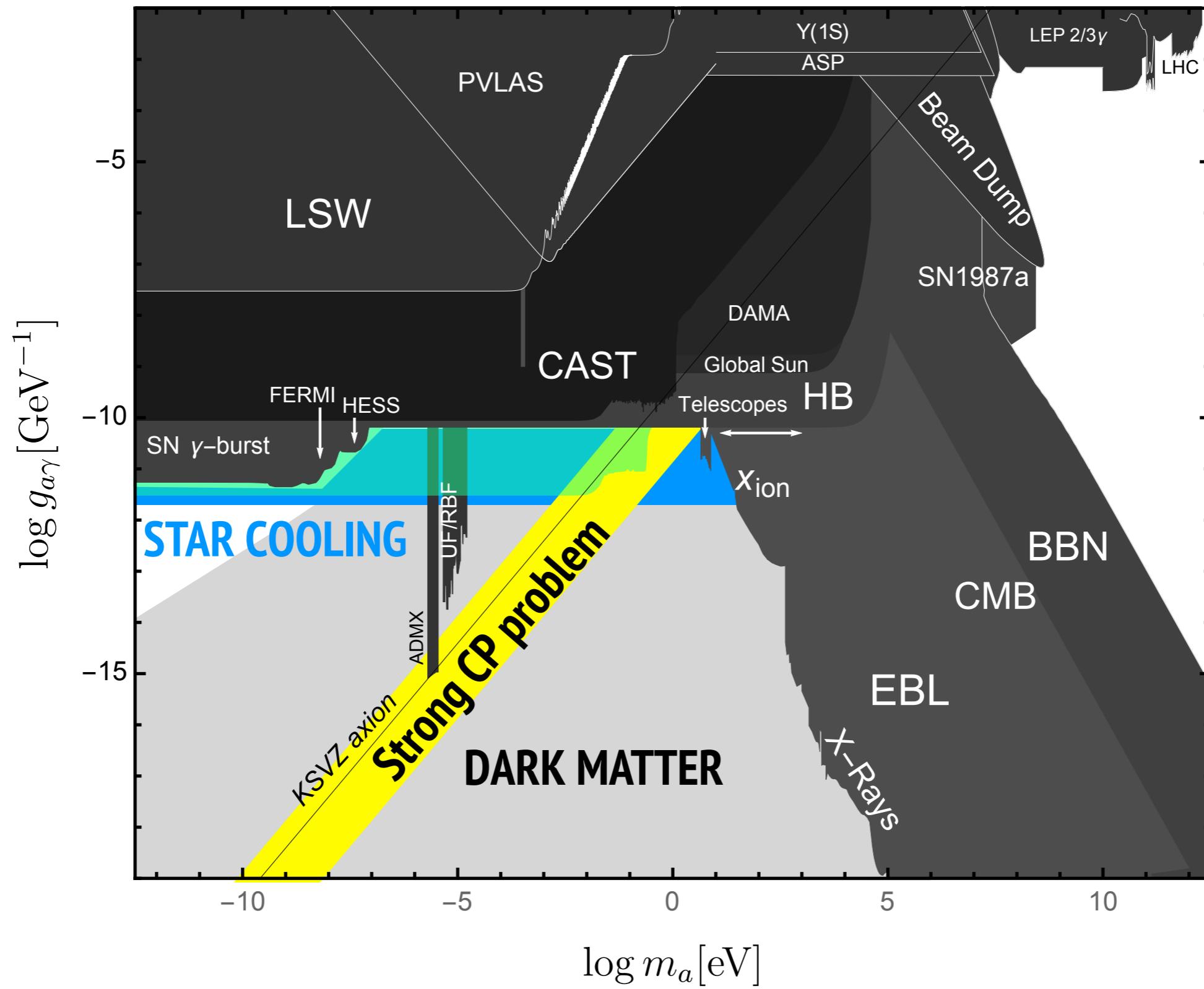
- MMC



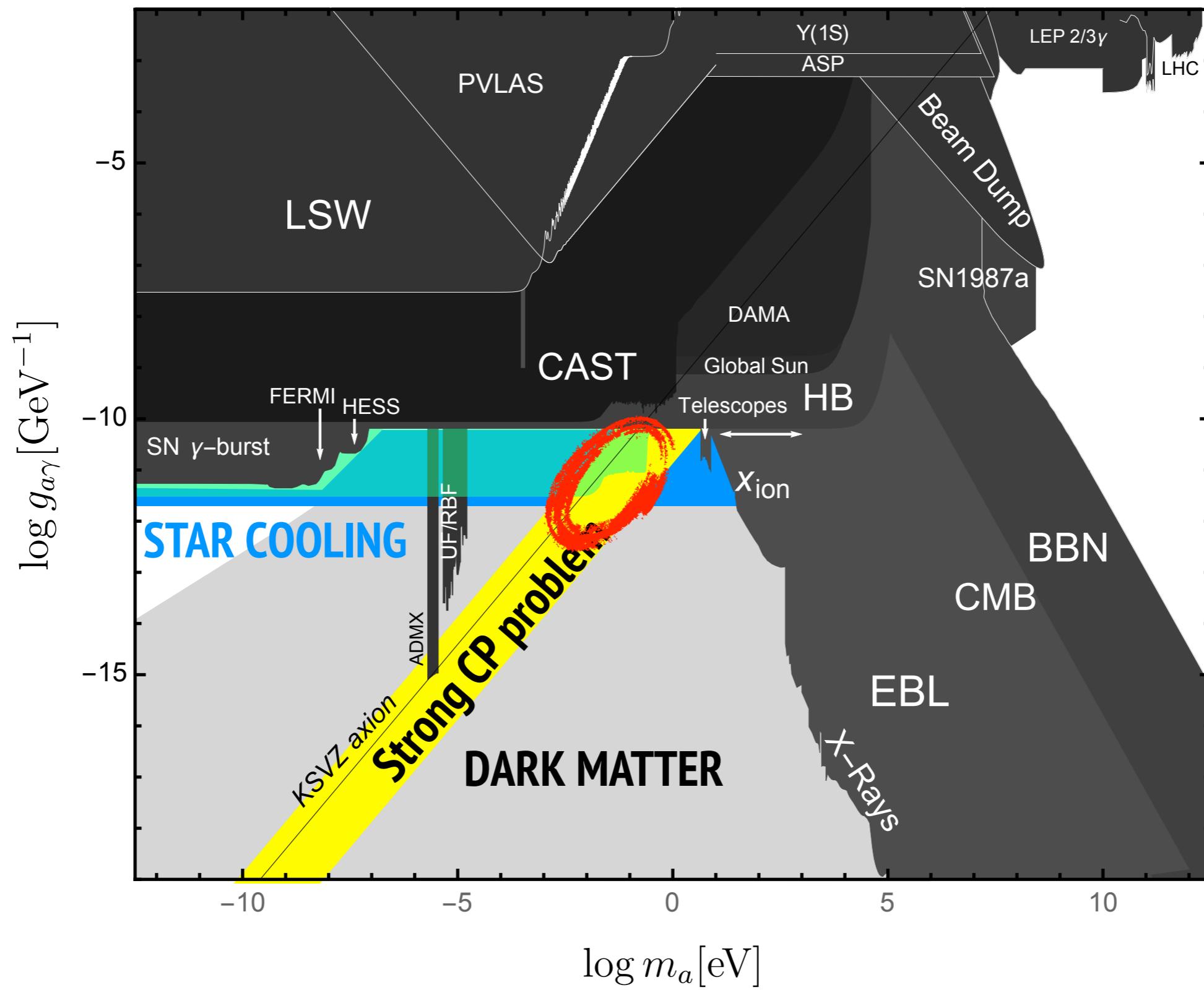
- Low noise CCDs



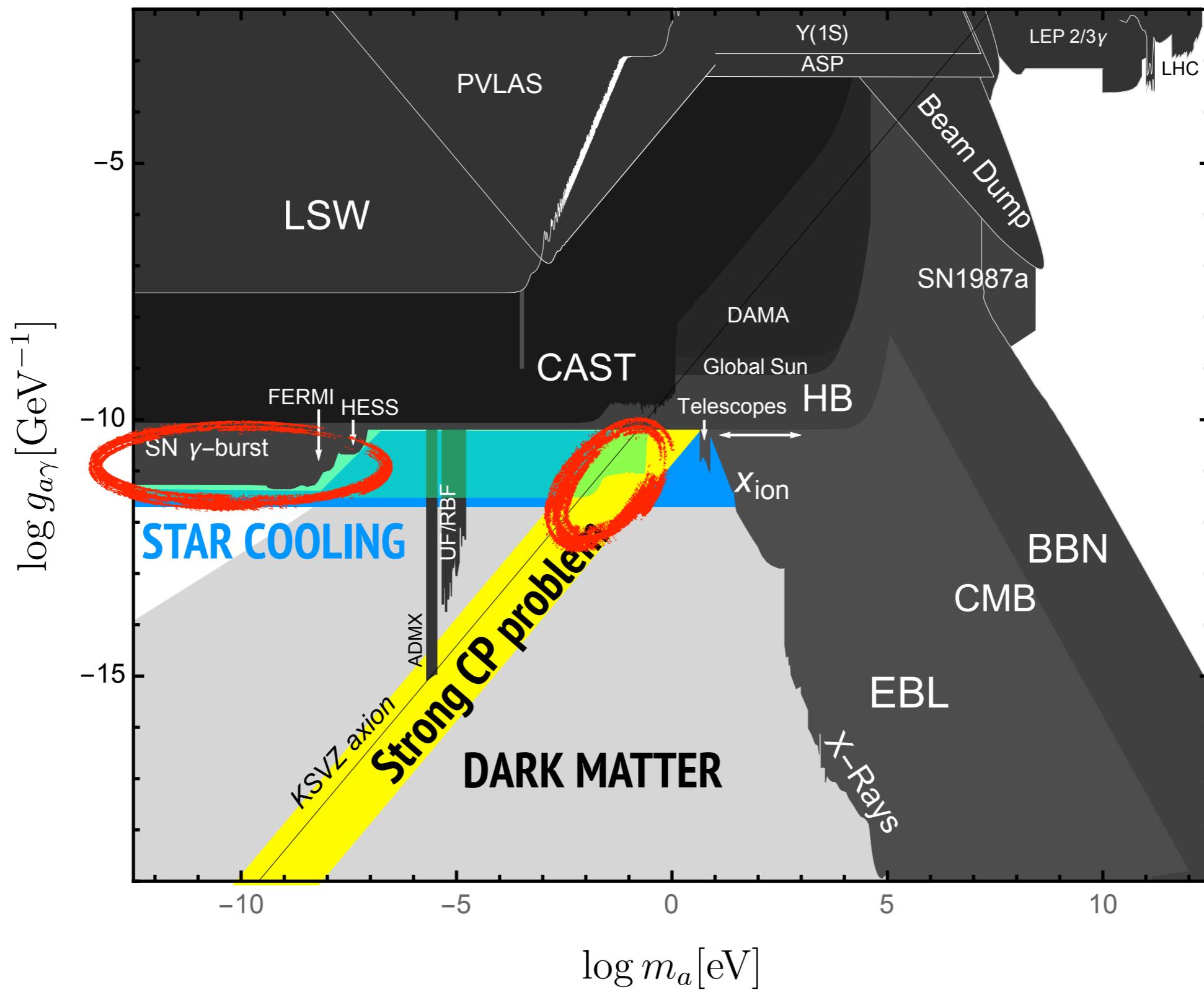
IAXO reach



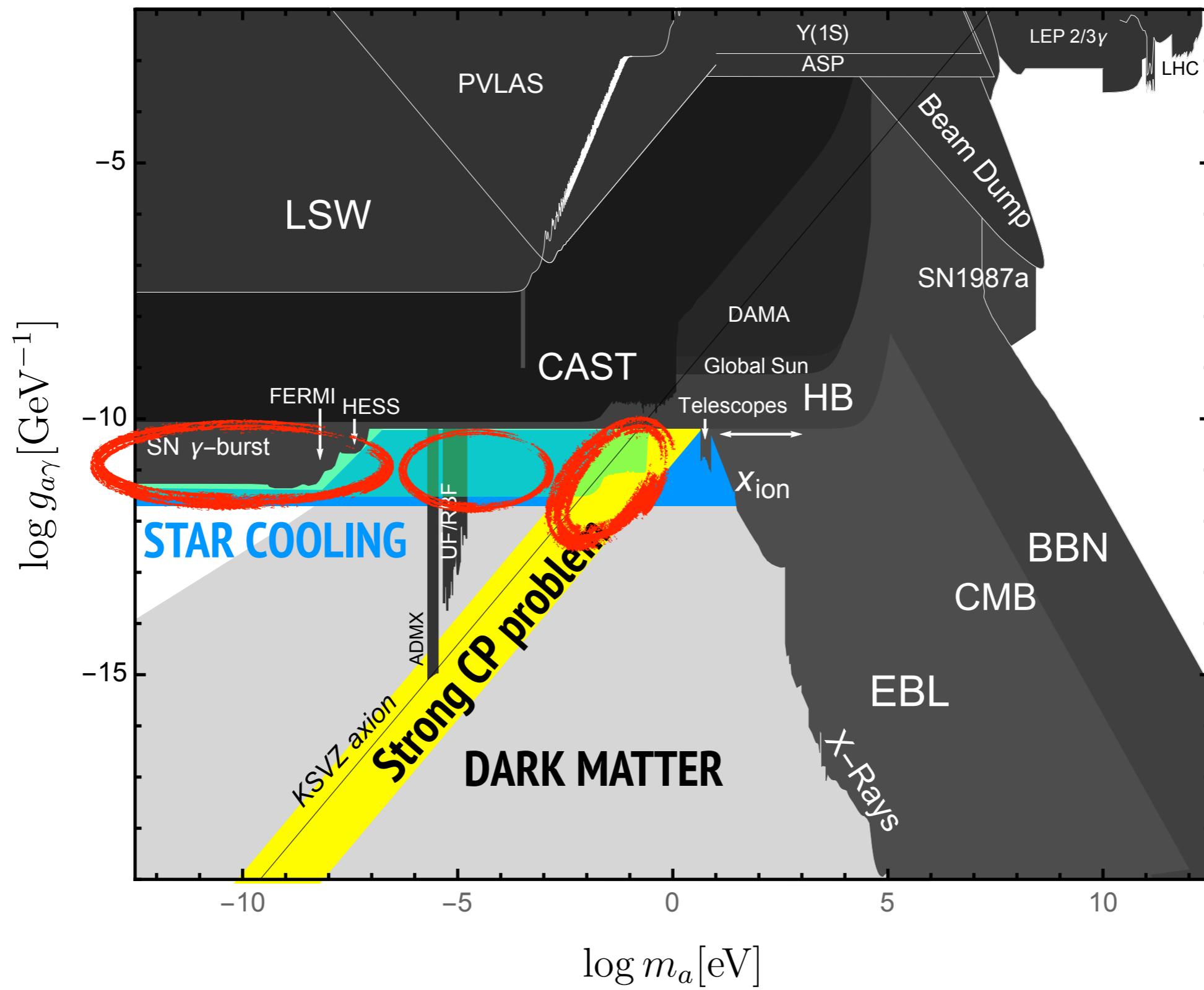
IAXO reach



IAXO reach

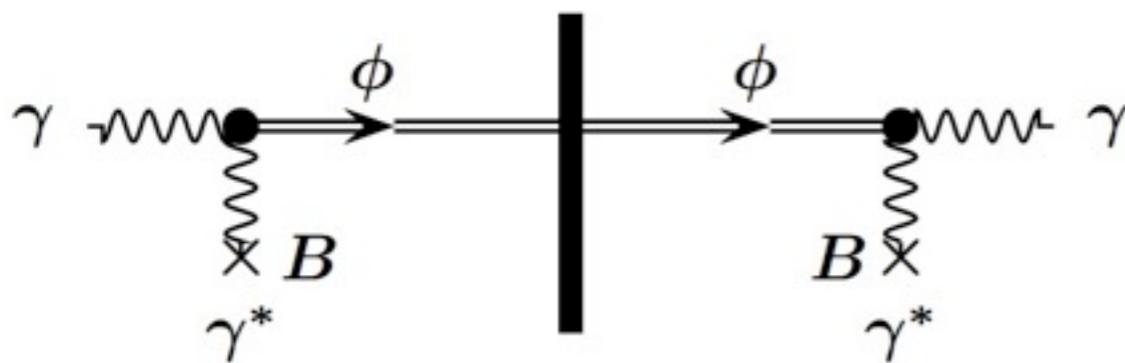


IAXO reach

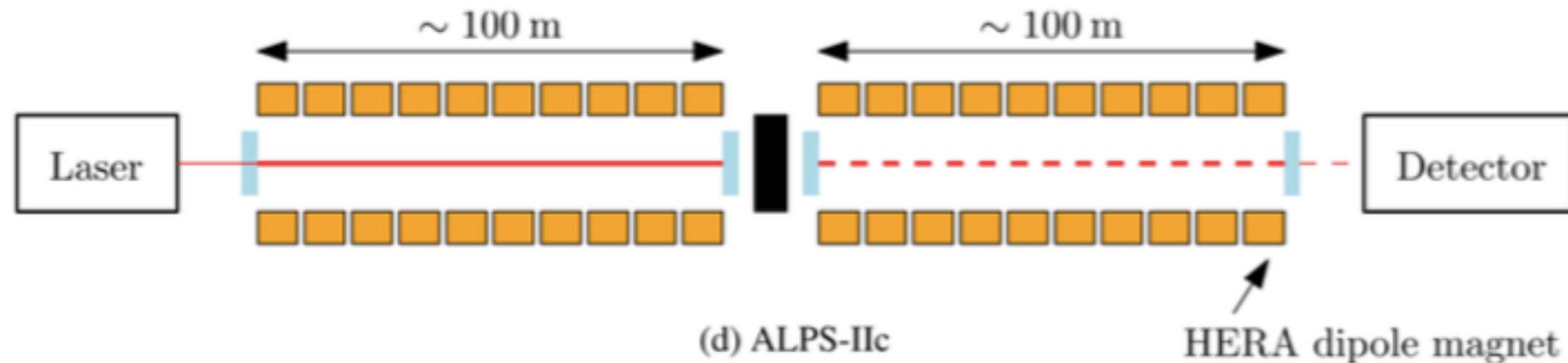


the ANY-Light-Particle-Search

Light shining through walls



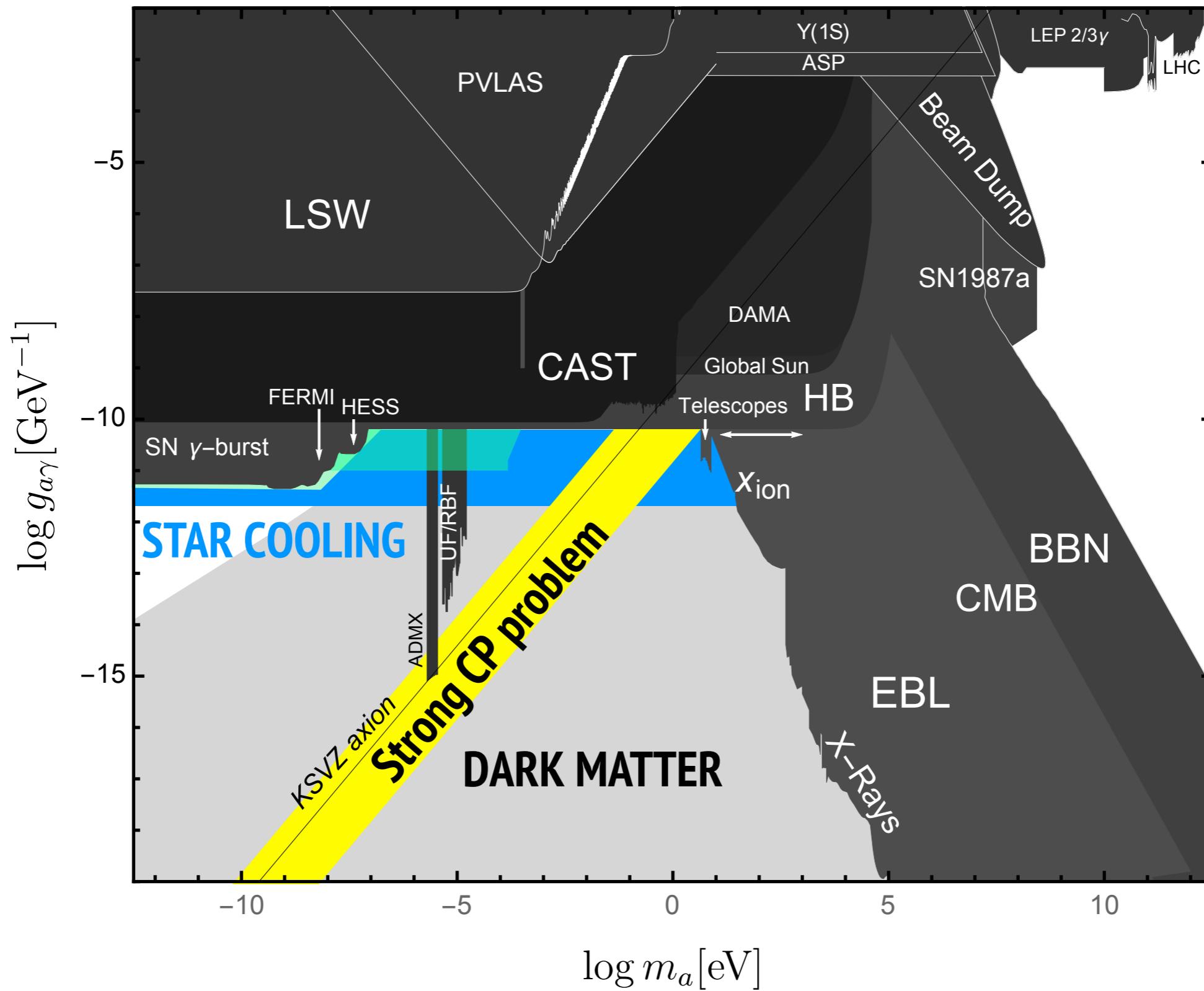
Resonant regeneration in the receiving cavity (see later)



Exp.	Photon flux (1/s)	Photon E (eV)	B (T)	L (m)	B·L (Tm)	PB reg.cav.	Sens. (rel.)
ALPS I	$3.5 \cdot 10^{21}$	2.3	5.0	4.4	22	1	0.0003
ALPS II	$1 \cdot 10^{24}$	1.2	5.3	106	468	40,000	1
"ALPS III"	$3 \cdot 10^{25}$	1.2	13	400	5200	100,000	27

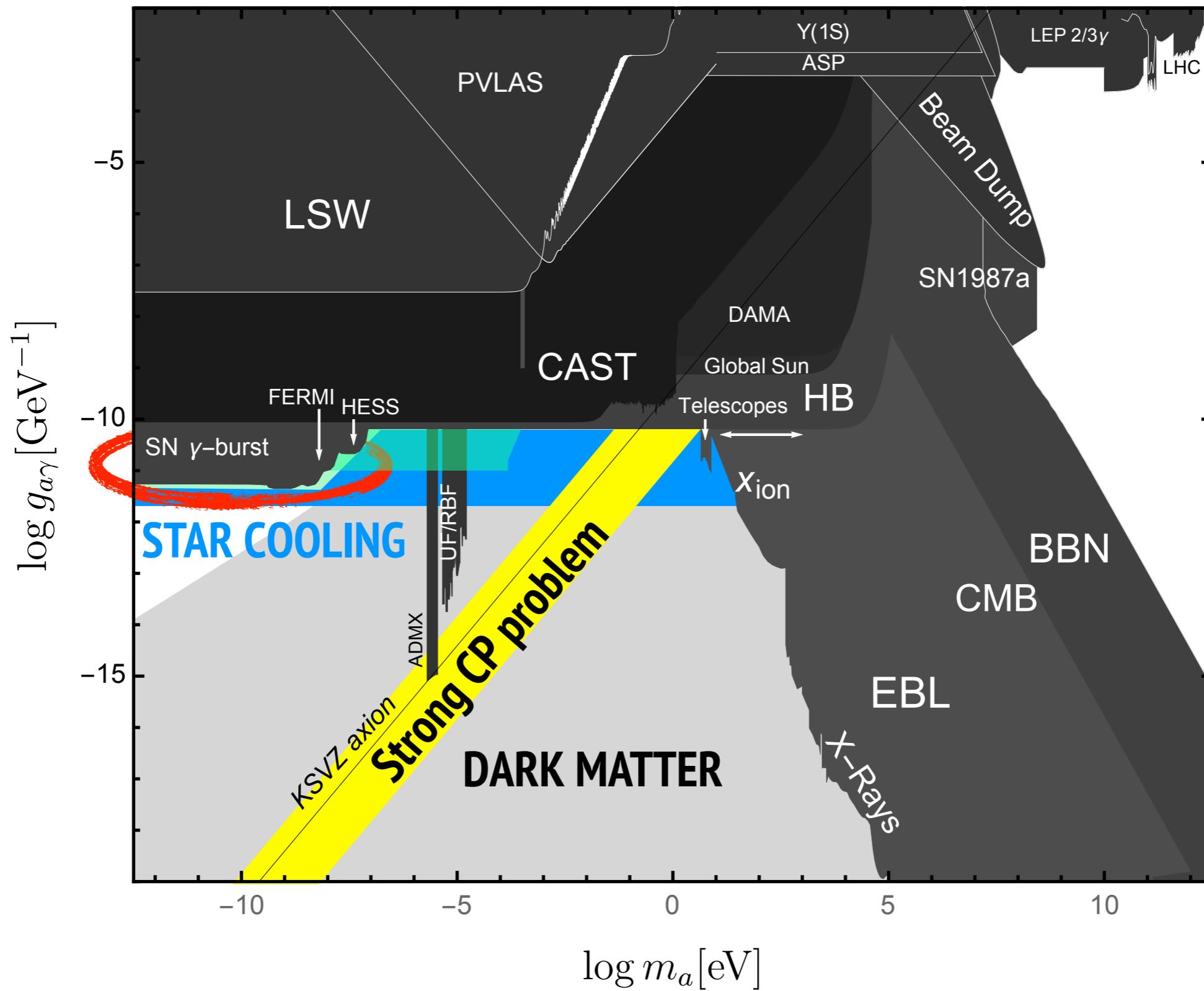


ALPS IIc reach

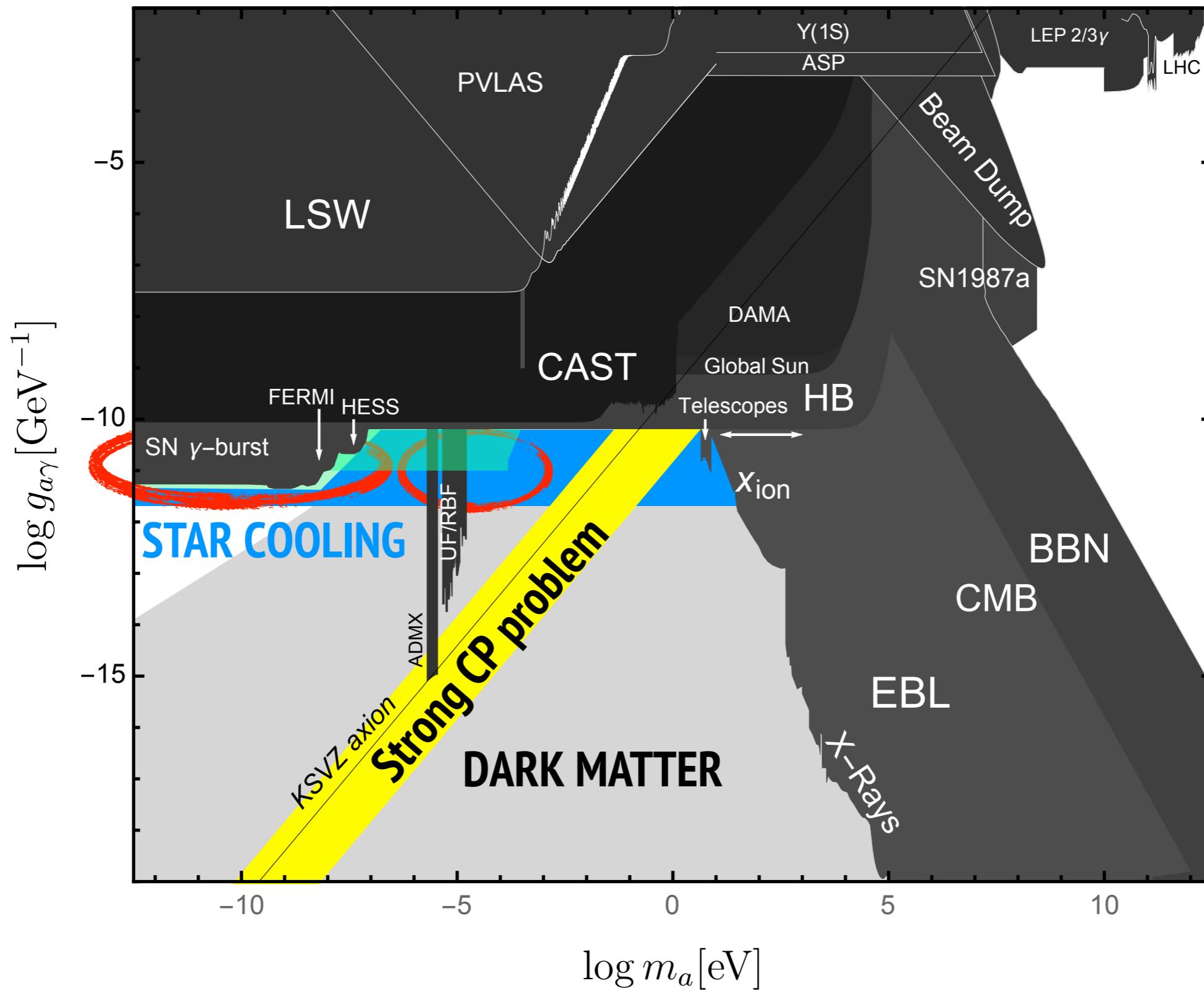


but much earlier than IAXO ...

ALPS IIc reach



ALPS IIc reach

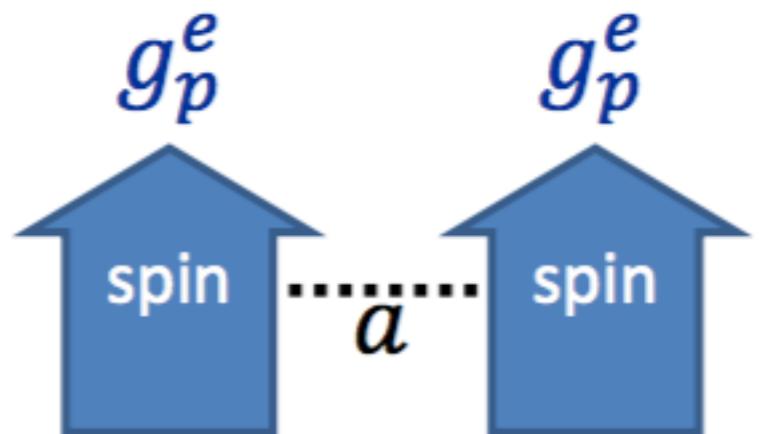


Long-range forces

Wilzcek '84, Geraci 14

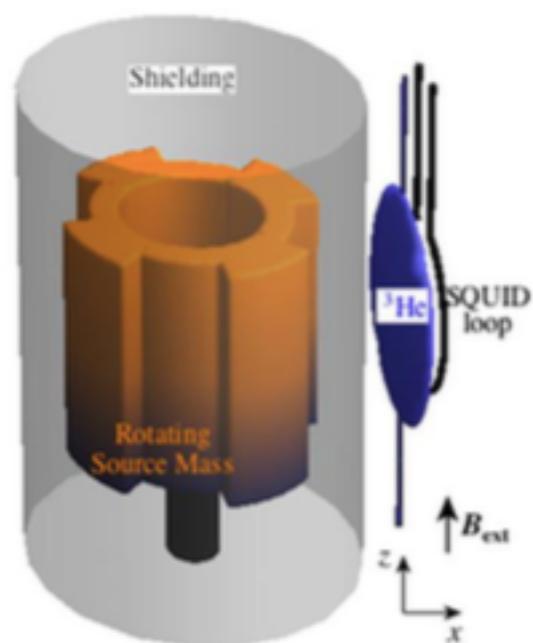
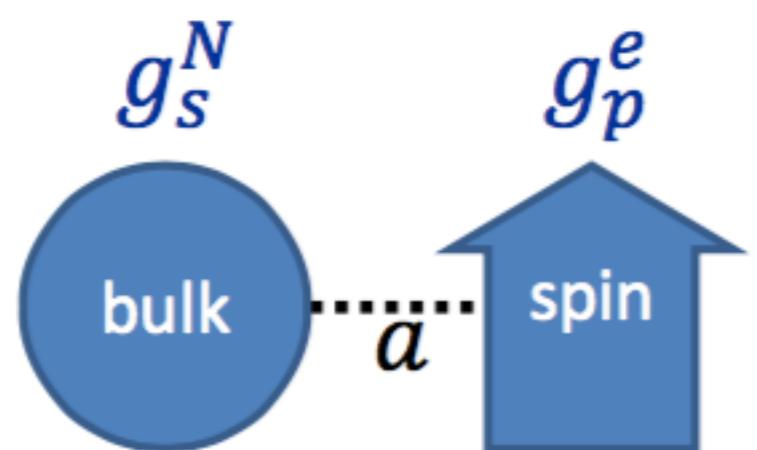
Long-range forces between macroscopic bodies

p-p forces are spin-spin ... very hard to measure!



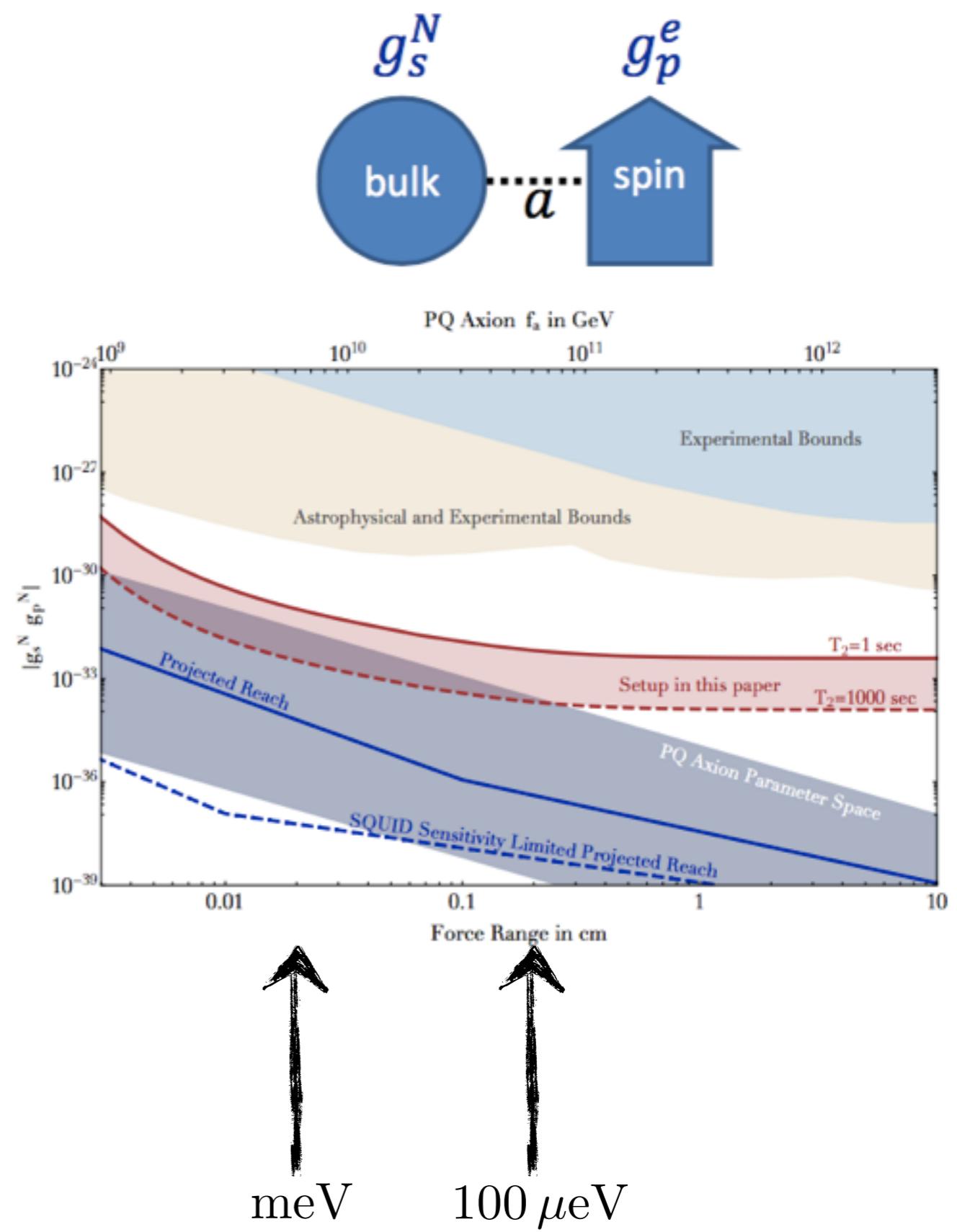
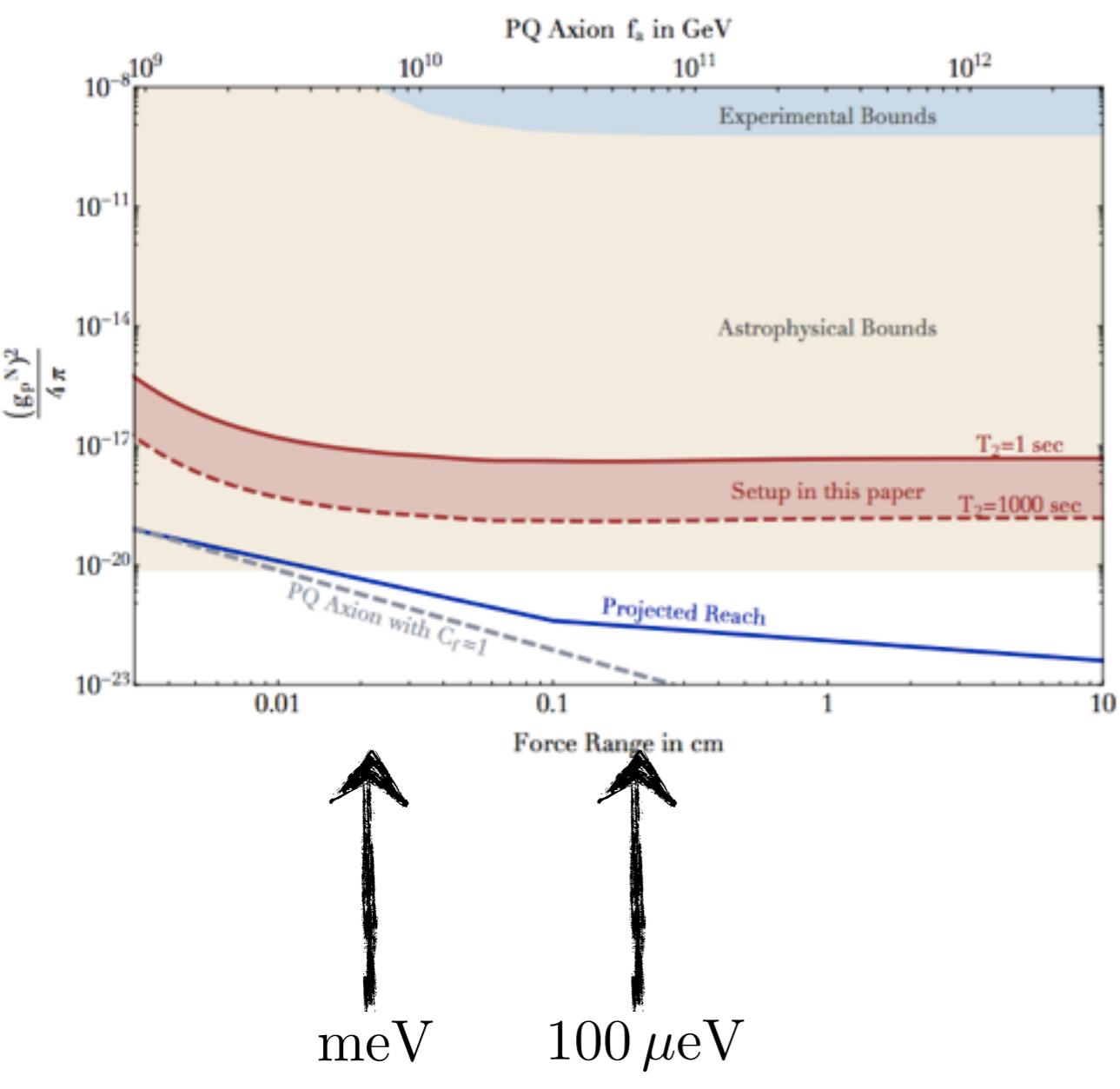
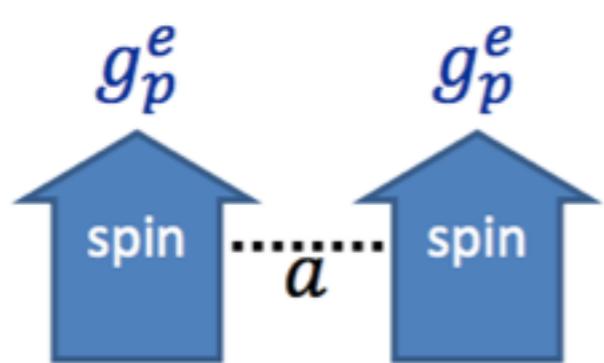
In some case a tiny s-coupling can lead to a larger effect

s-p forces are number-spin ... much easier



ARIADNE reach

Arvanitaki, Geraci 14



Detecting Dark Matter



Detecting Dark Matter

Imperfect Vacuum realignment $\theta(t) = \theta_0 \cos(m_a t)$

$$\rho_{\text{CDM}} = 0.3 \frac{\text{GeV}}{\text{cm}^3} \equiv \frac{1}{2}(\dot{a})^2 + \frac{1}{2}m_a^2 a^2 = \frac{1}{2}m_a^2 f_a^2 \theta_0^2$$

$$\xrightarrow[m_A^2 f_A^2 = \chi_{\text{QCD}}]{\text{QCD axion}} \theta_0 \sim 3.6 \times 10^{-19}$$

$\sim 10^{-6}$

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Non-zero velocity in galaxy -> finite width

$$\omega \simeq m_a(1 + v^2/2 + \dots)$$

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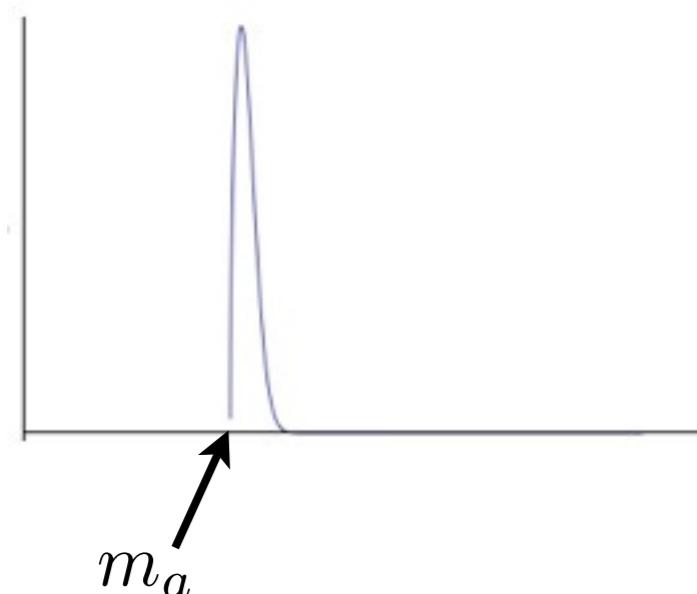
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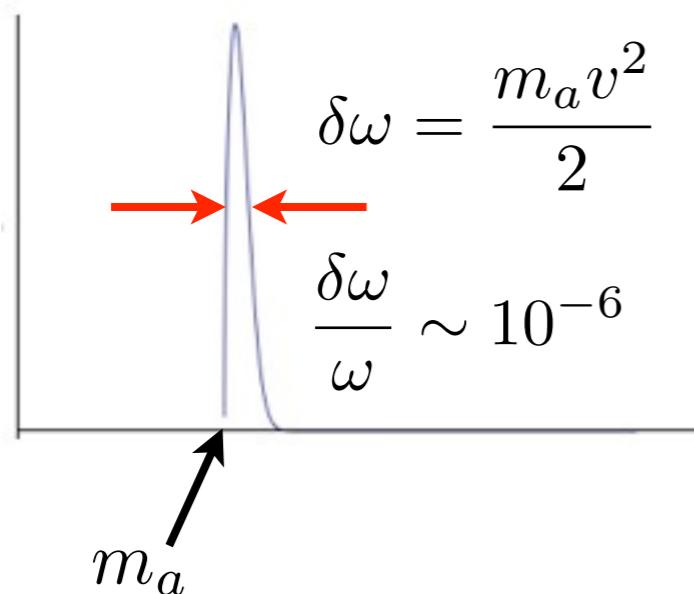
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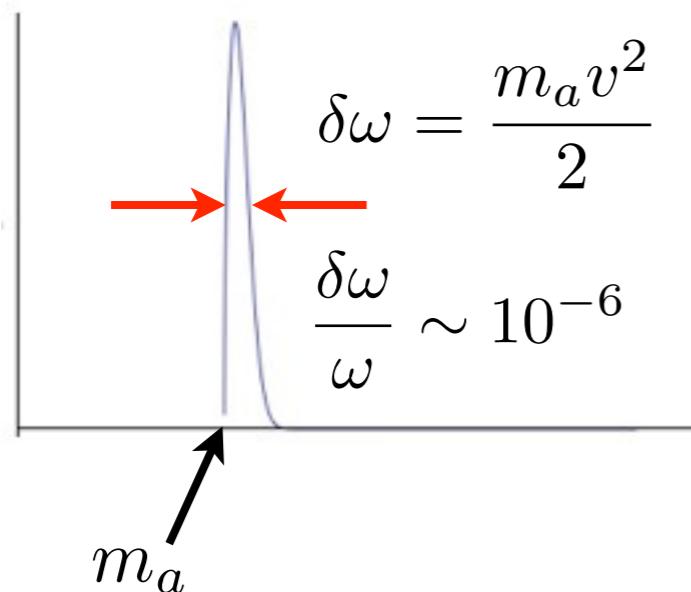
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coherence time

$$\delta t \sim \frac{1}{\delta\omega} \sim 0.13\text{ms} \left(\frac{10^{-5}\text{eV}}{m_a} \right)$$



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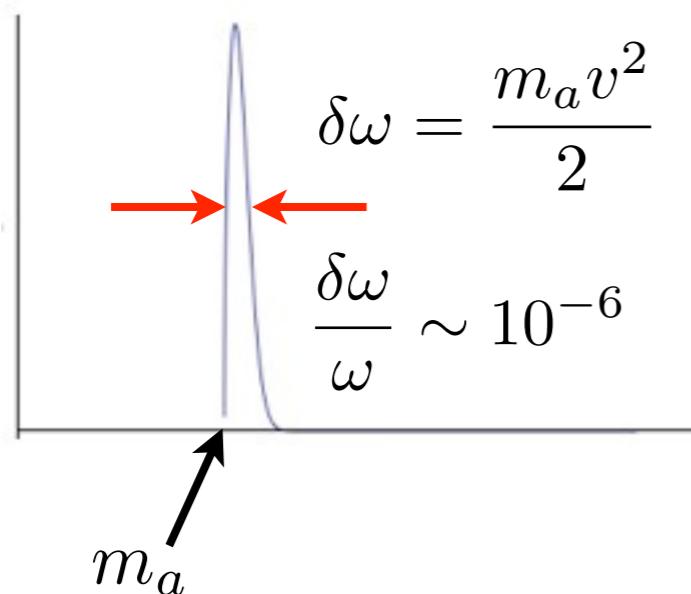
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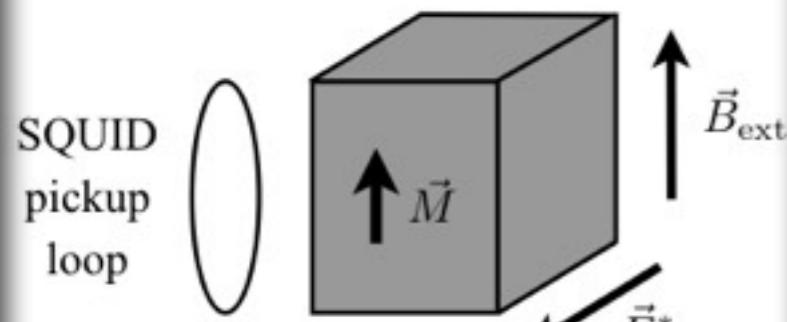
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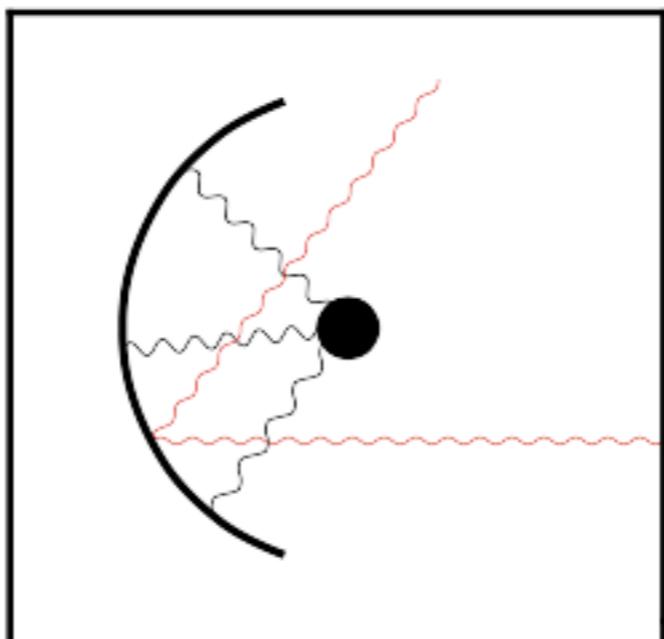
coherence length

$$\delta L \sim \frac{1}{\delta p} \sim 20\text{m} \left(\frac{10^{-5}\text{eV}}{m_a} \right)$$

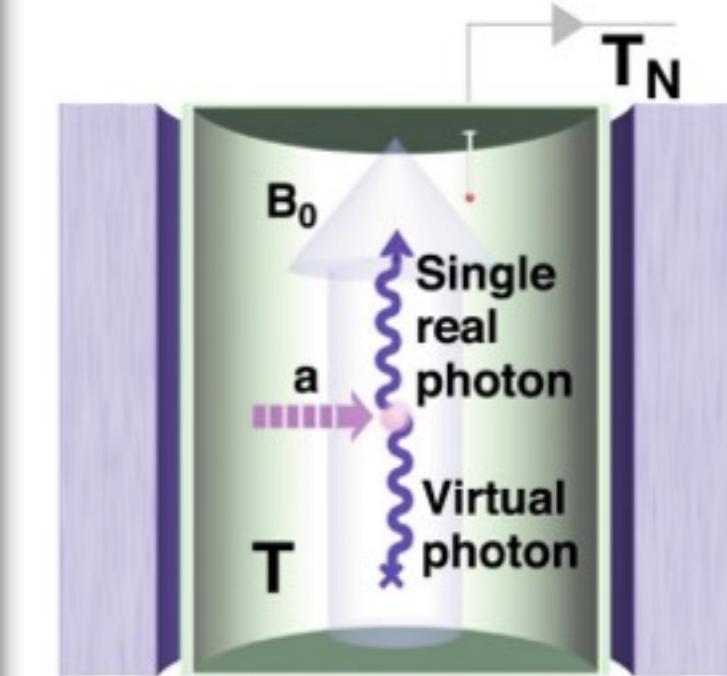
Spin precession



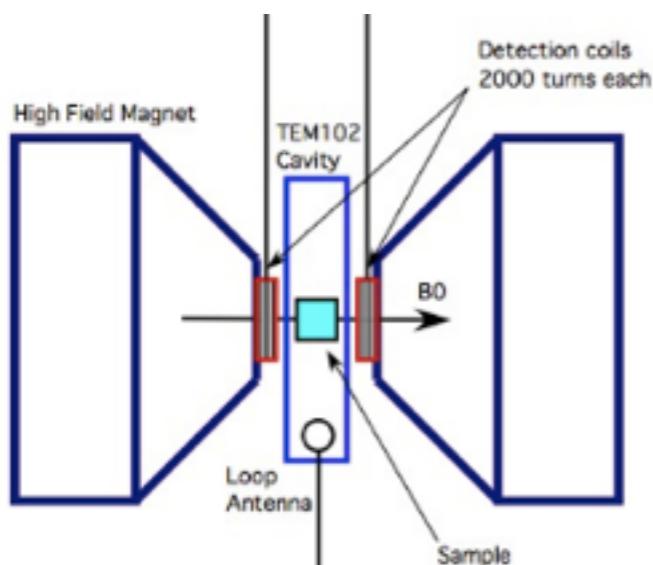
Mirrors+



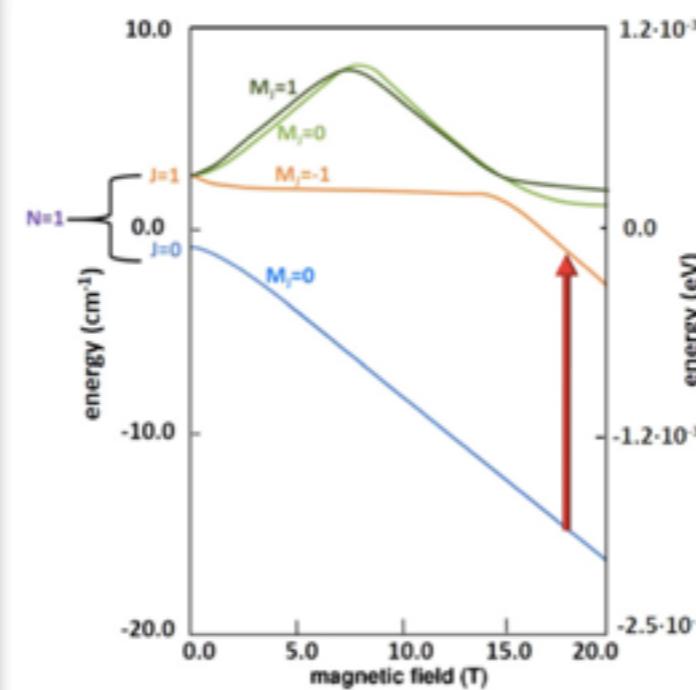
Cavities



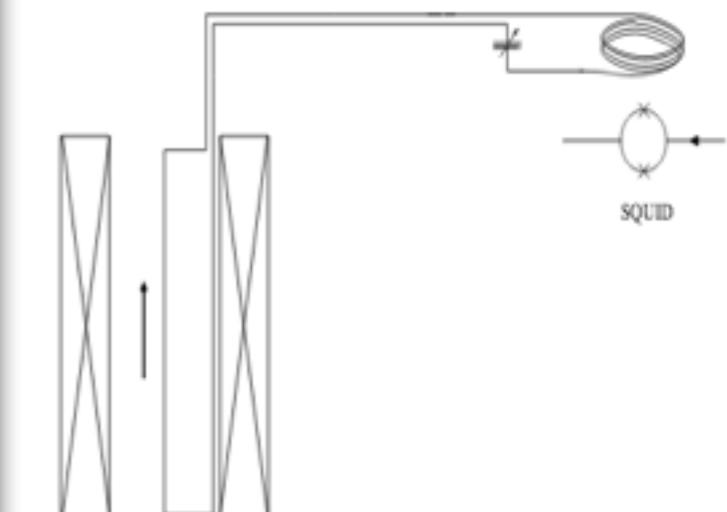
e-spin precession



Atomic transitions

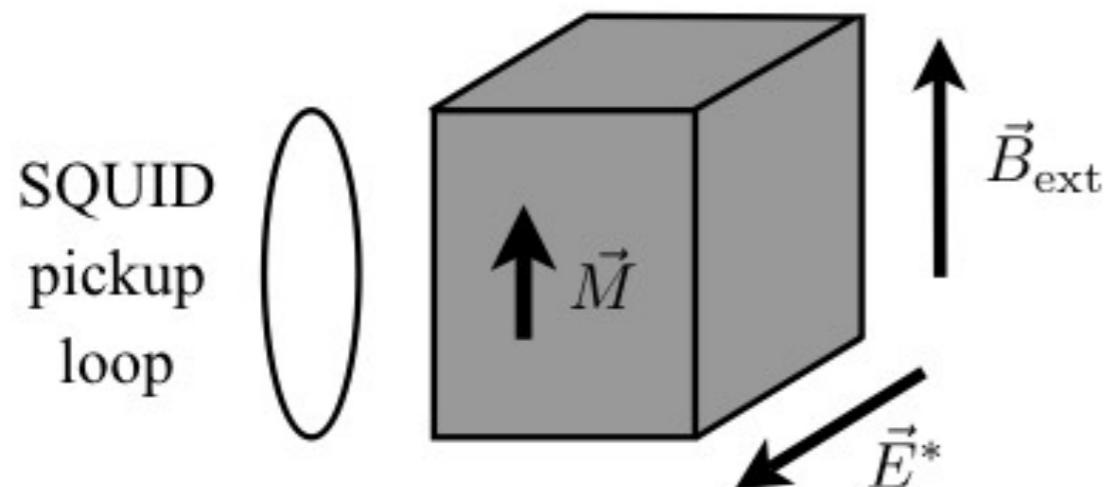


LC-circuit



CASPER at Mainz

Graham 2012



$$\text{magnetic signal} \propto np\varepsilon_S dE^* T_2$$

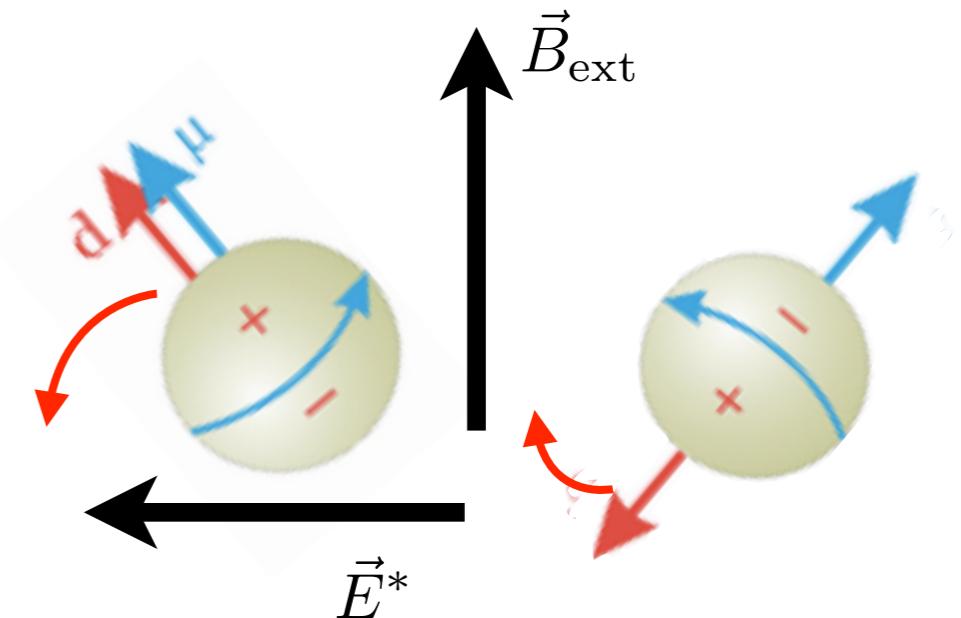
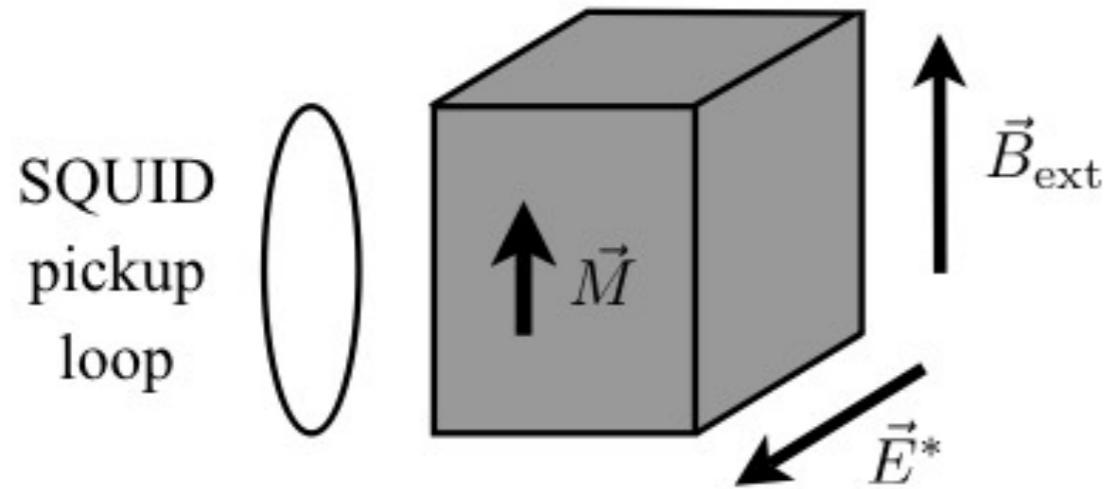
number density nuclear spin polarization Schiff suppression nuclear spin coherence time

Arrows point from "number density" and "nuclear spin polarization" to the term $np\varepsilon_S$. Arrows point from "Schiff suppression" and "nuclear spin coherence time" to the term $dE^* T_2$.

- EDM + Large E-fields in PbTiO₃
- Mainz (D. Budker's group) & Berkeley
- B-field, coherence time, sensitivity to $m < \text{neV}$
- Mass range limited by B-field strength

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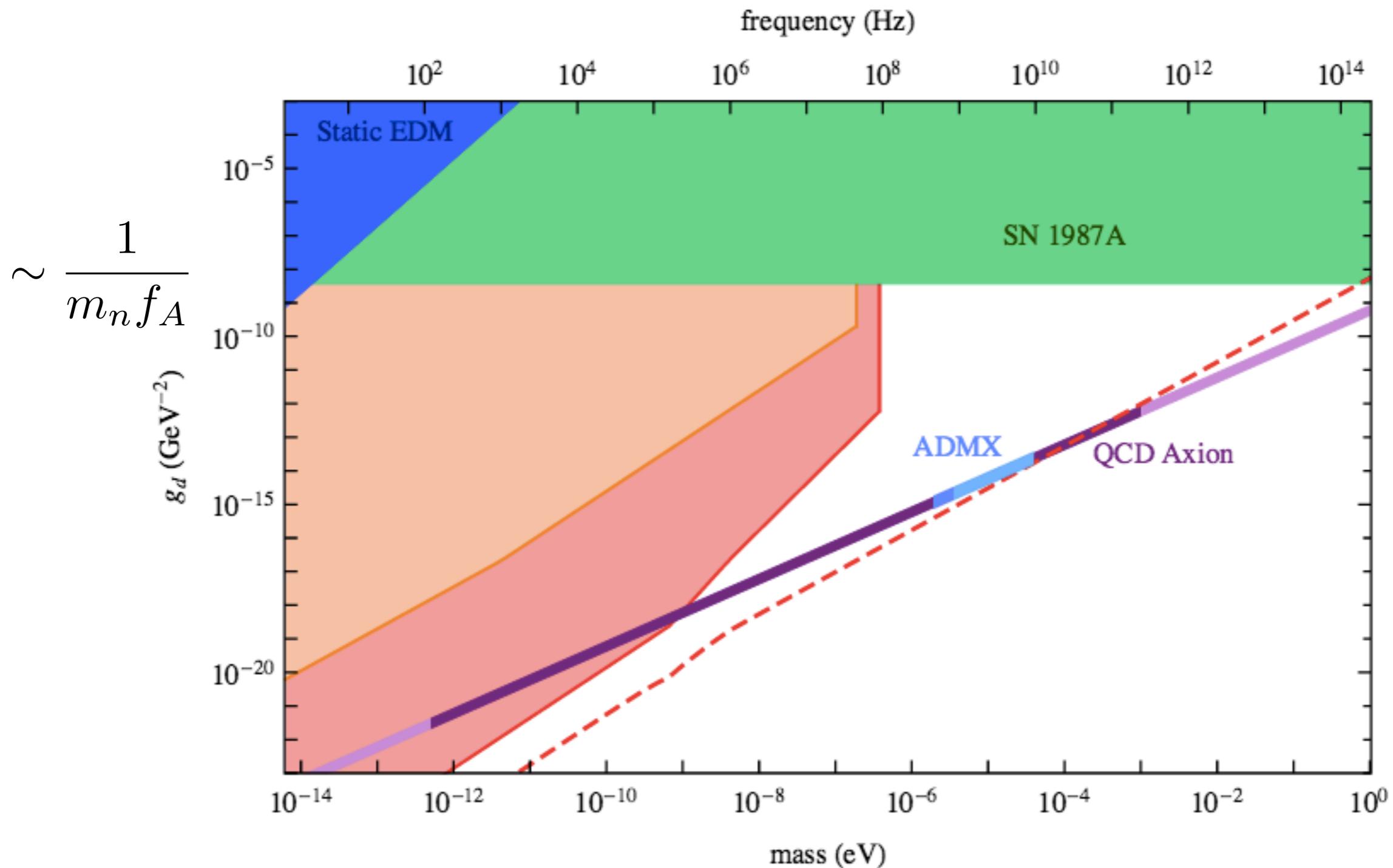
number density nuclear spin polarization Schiff suppression nuclear spin coherence time

**Oscillating EDM, effects add up,
transverse magnetisation grows
if** $m_a = \omega = \mu |\vec{B}_{\text{ext}}|$

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CASPER reach

Graham 2012



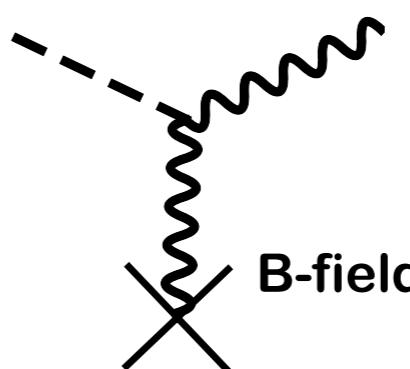
Axion DM in a B-field

$$\mathcal{L}_I = -C_{a\gamma} \frac{\alpha}{2\pi} \frac{a}{f_a} \mathbf{B} \cdot \mathbf{E}$$

- In a static magnetic field, the oscillating axion field generates EM-fields

$$\mathcal{L}_I = -C_{a\gamma} \frac{\alpha}{2\pi} \theta(t) \mathbf{B}_{\text{ext}} \cdot \mathbf{E}$$

source



The diagram illustrates the interaction between an axion field and a magnetic field. A wavy line, representing the axion field, is shown interacting with a dashed line, representing the magnetic field vector. The label "B-field" is placed near the dashed line.

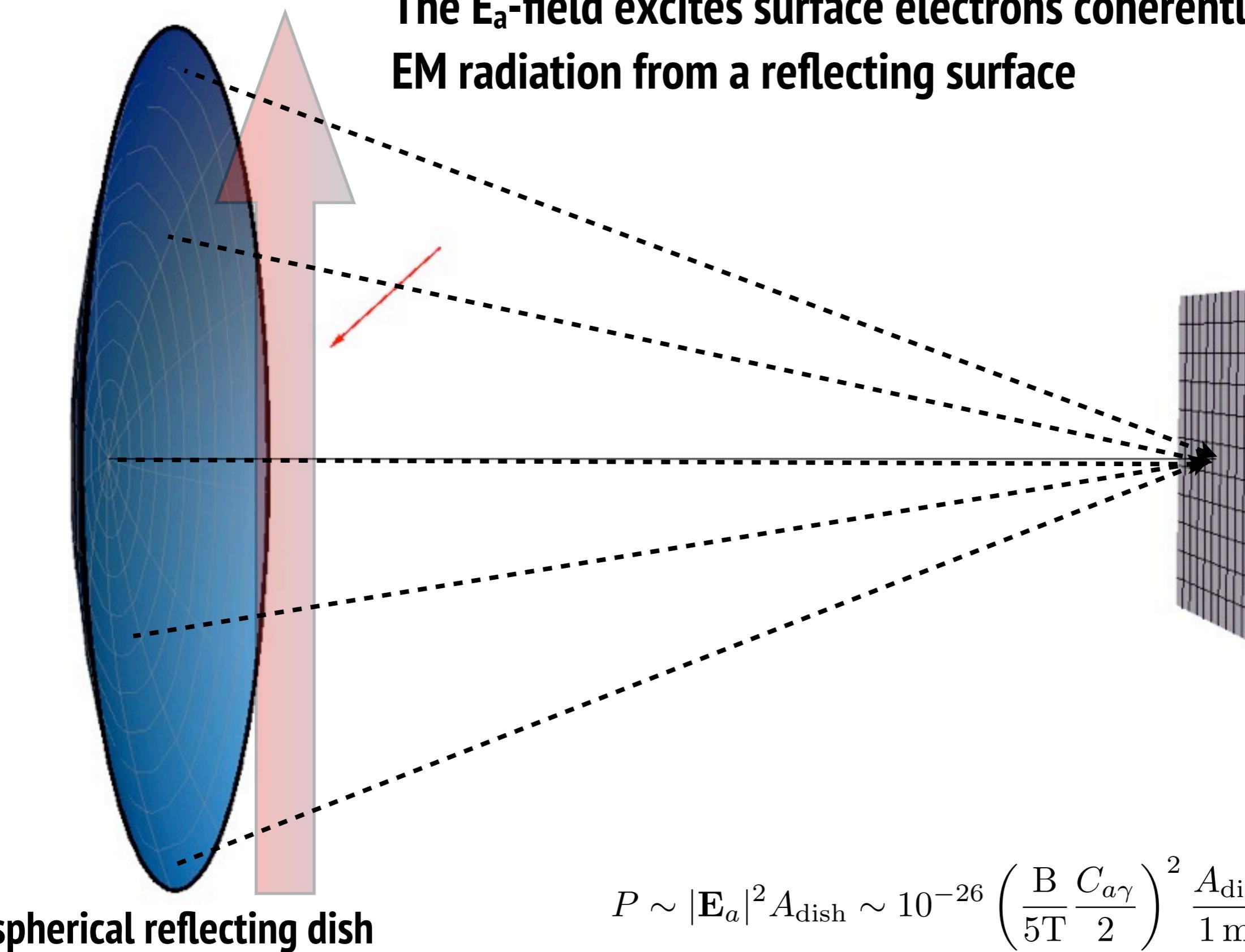
- Electric fields $\mathbf{E}_a = C_{a\gamma} \frac{\alpha \mathbf{B}_{\text{ext}}}{2\pi} \theta_0 \cos(m_a t)$
- Oscillating at a frequency $\omega \simeq m_a$

- B-fields $\propto \nabla \theta$ $|\mathbf{B}_a| \sim \langle v \rangle |\mathbf{E}_a|$

Dish antenna experiment?

Horns 2012

The E_a -field excites surface electrons coherently
EM radiation from a reflecting surface



Cavity resonators (Haloscopes)

- Haloscope (Sikivie 83)

$$P \sim Q|\mathbf{E}_a|^2(Vm_a)\mathcal{G}_K \quad (\text{ON ReSONANCE})$$

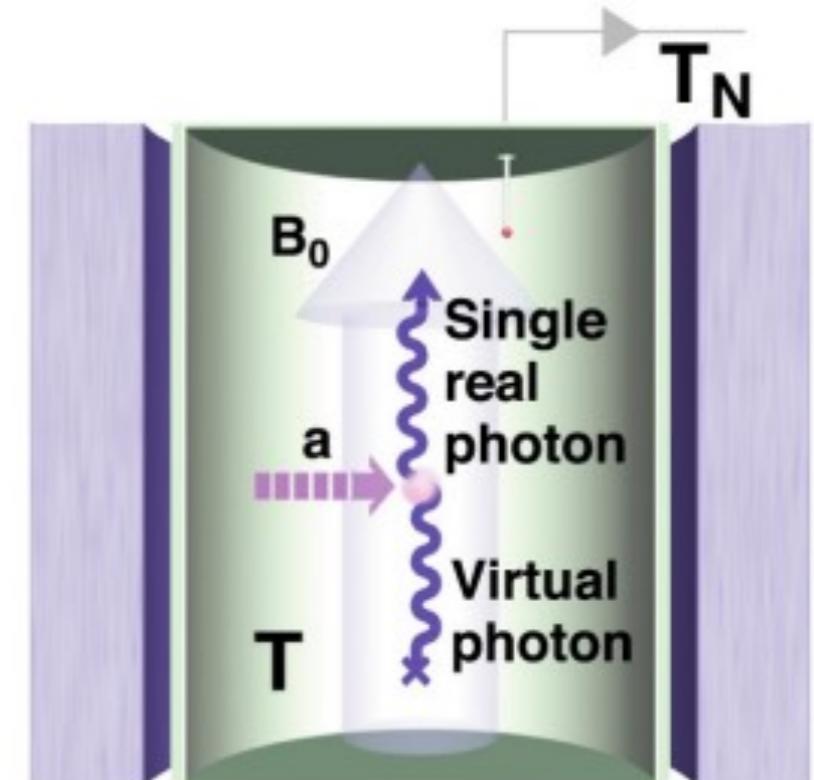
- Naive ADMX scaling (e.g. an ADMX every octave)

- Signal $(V \propto m_a^{-3}) \quad P_{\text{out}} \propto Vm_a \sim \frac{1}{m_a^2}$

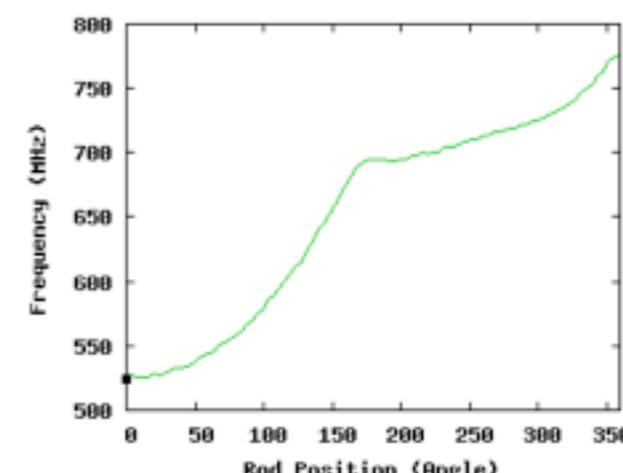
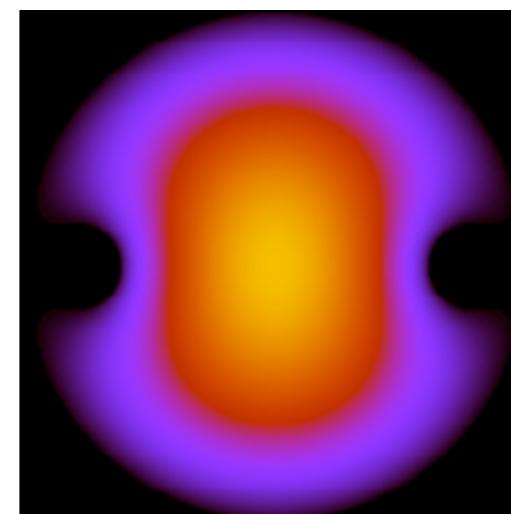
- Noise $P_{\text{noise}} = T_{\text{sys}}\Delta\nu_a \propto m_a^2$

- Signal/noise in $\Delta\nu_a$ of time, t, $\frac{S}{N} = \frac{P_{\text{out}}}{P_{\text{noise}}} \sqrt{\Delta\nu_a t}$

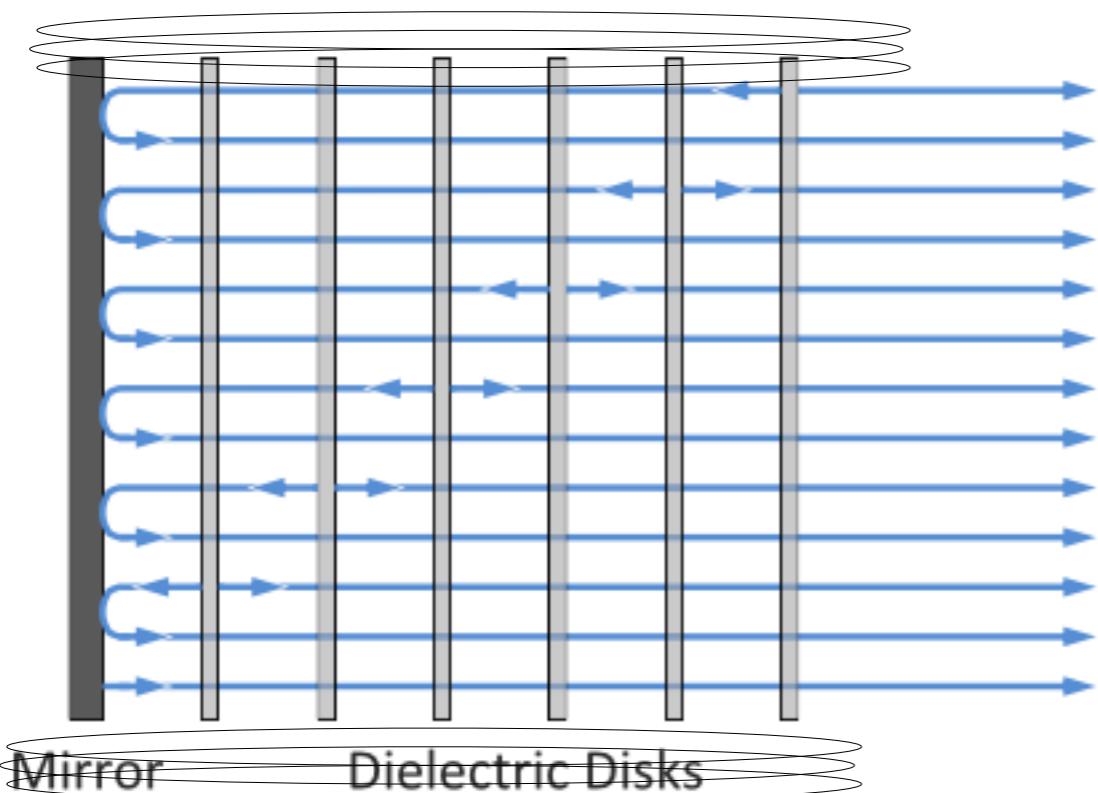
- Scanning rate $\frac{1}{m_a} \frac{d\Delta m_a}{dt} \propto \frac{C_{A\gamma}^4}{m_a^7}$



Scanning over frequencies



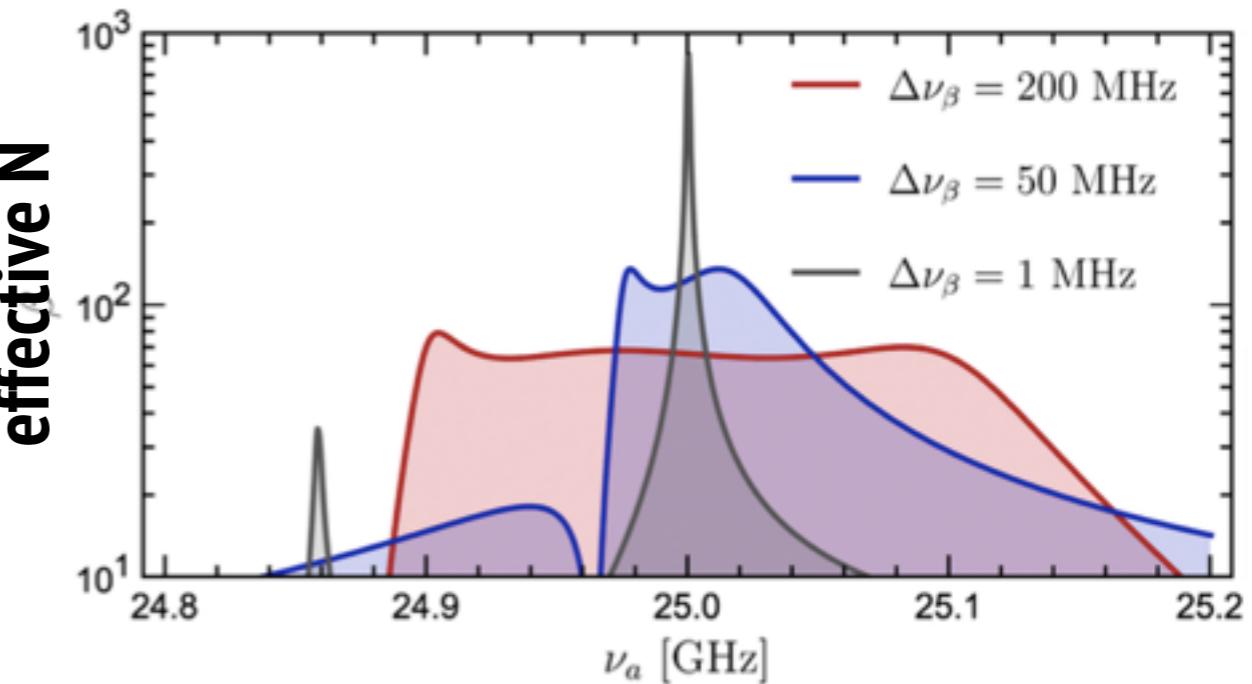
MADMAX: MAgnetised Disk and Mirror Axion eXperiment



Emitted EM-waves from each interface
+ internal reflections ...

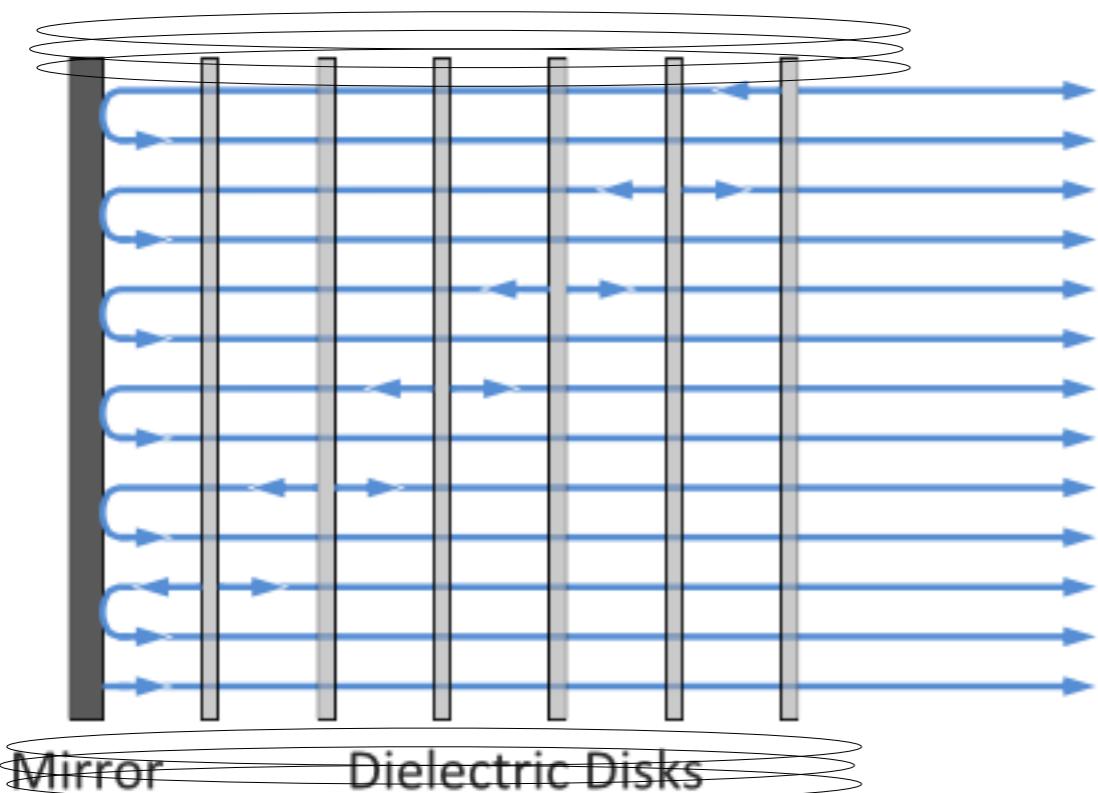
$$P \sim |\mathbf{E}_a|^2 \text{Area} \times \mathcal{O}(N^2)$$

Receiver



Caldwell 2017

MADMAX: MAgnetised Disk and Mirror Axion eXperiment

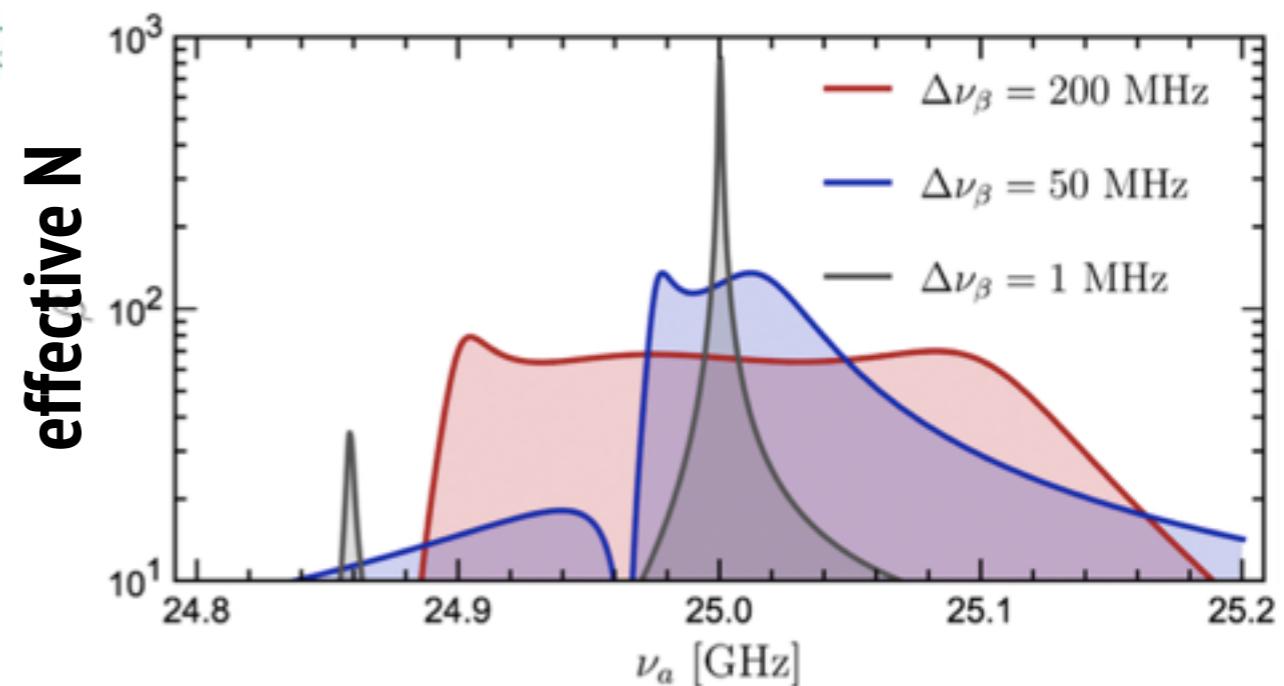
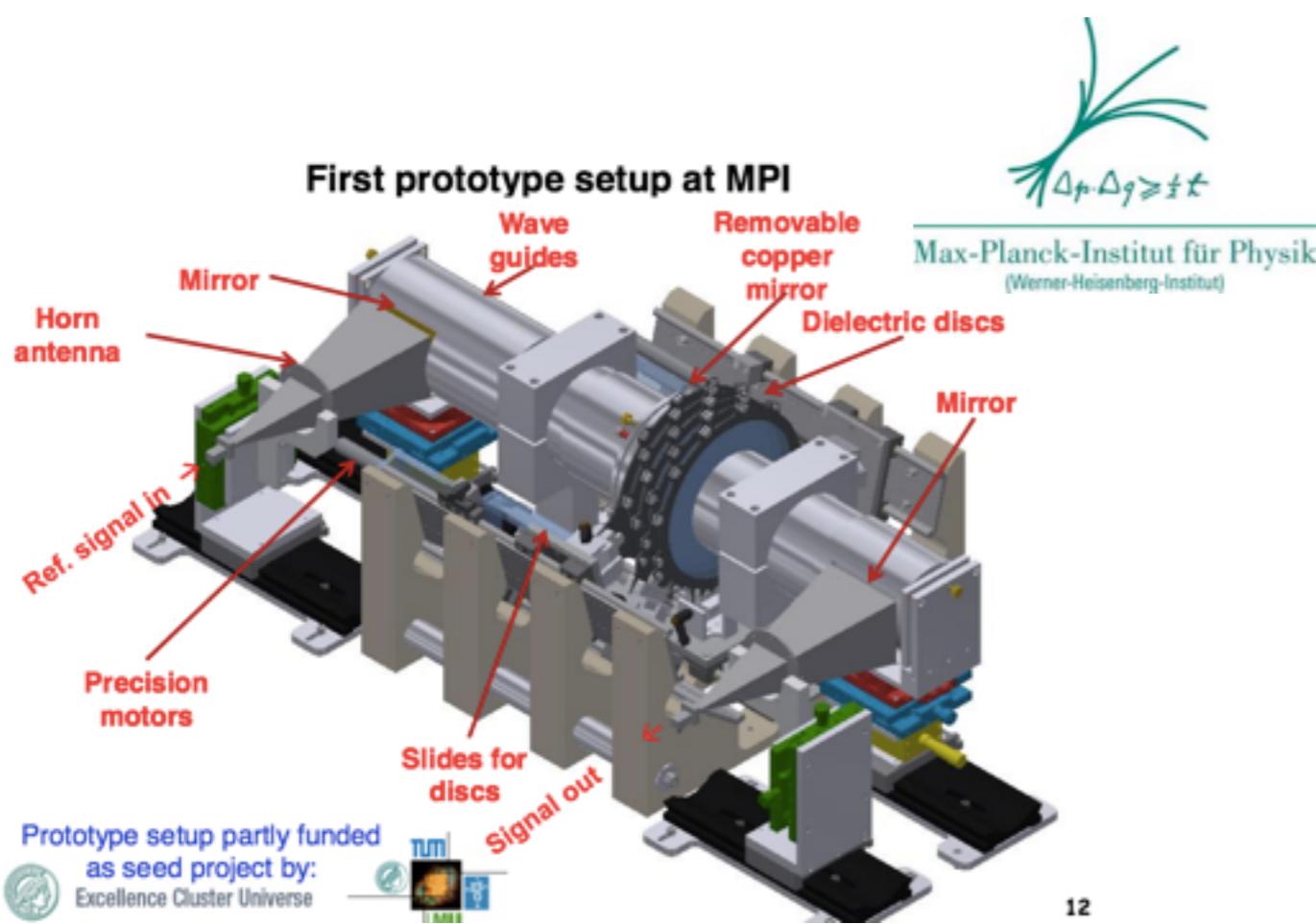


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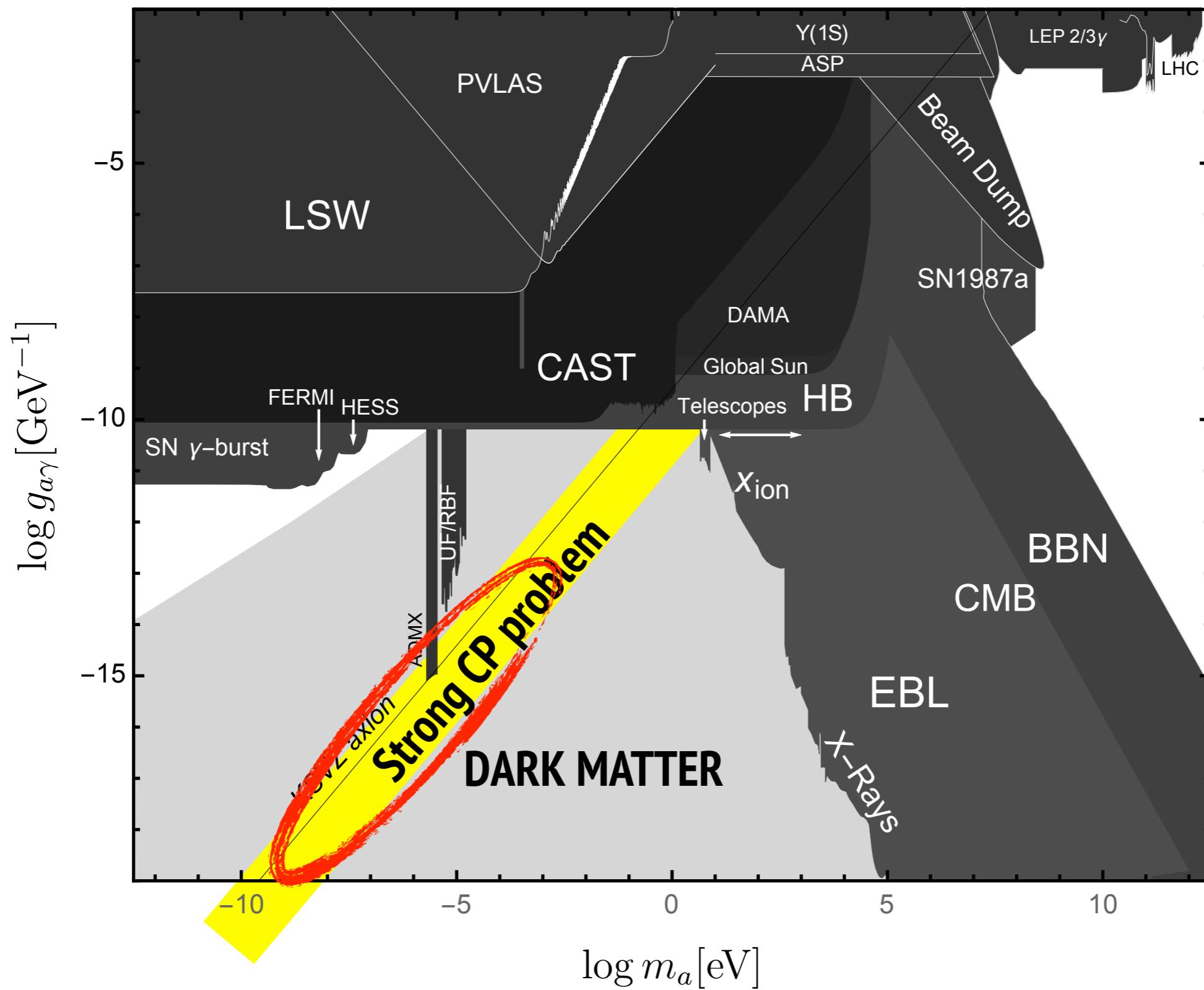
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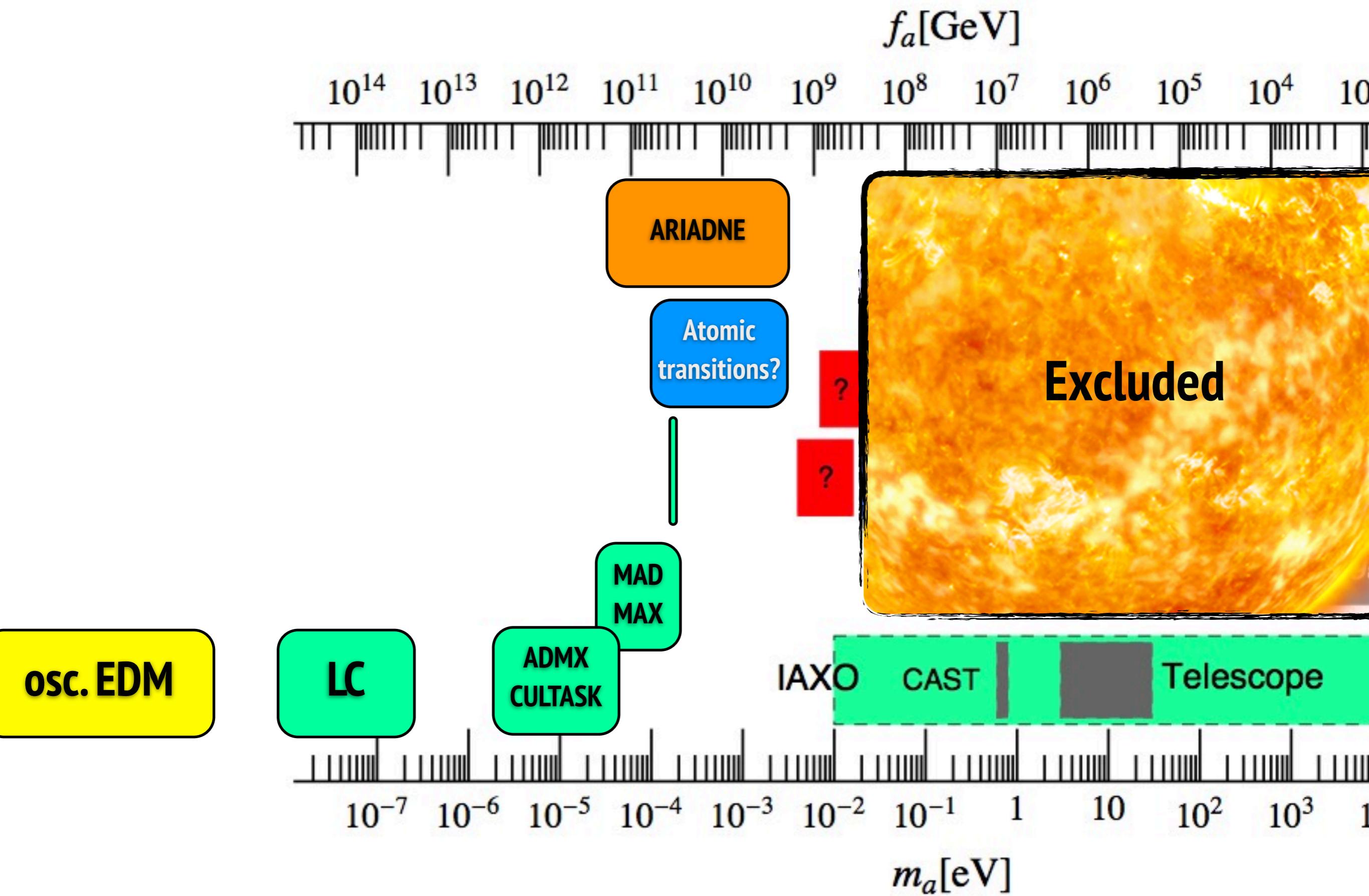
Receiver



Summary plot



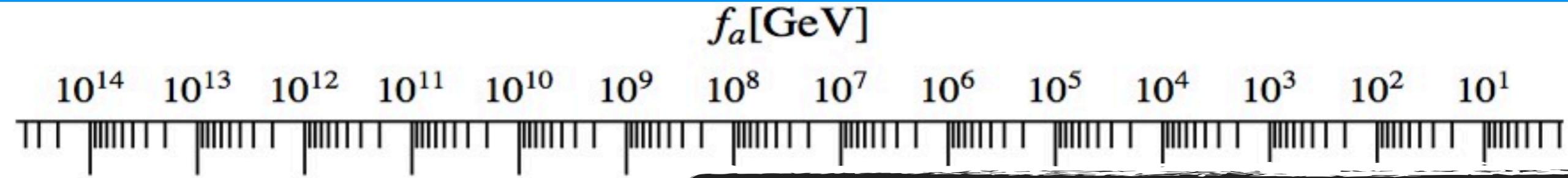
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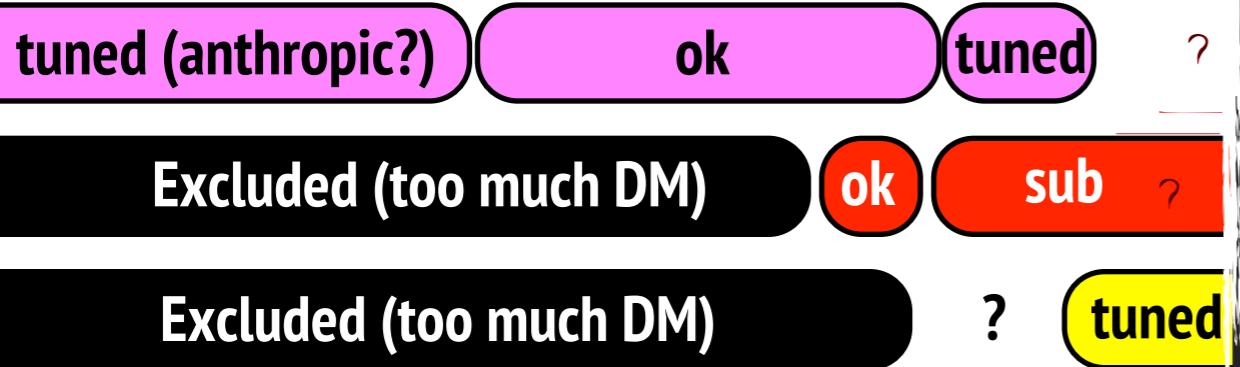
Conclusions

- Beyond the SM with extremely low energies
- Detect an ALP, new energy scale!
- Generic interactions
- hints: Strong CP problem, DM, Stellar evolution, Transparency of Gamma's
- Good Experimental ideas
- Still a lot of parameter space to explore!

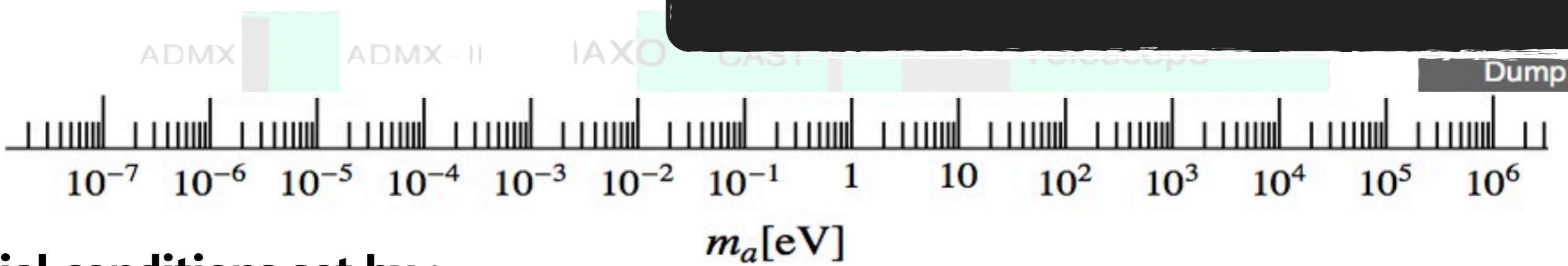
Axion dark matter scenarios



- Axion DM scenarios



Excluded



Initial conditions set by :

Inflation smooth

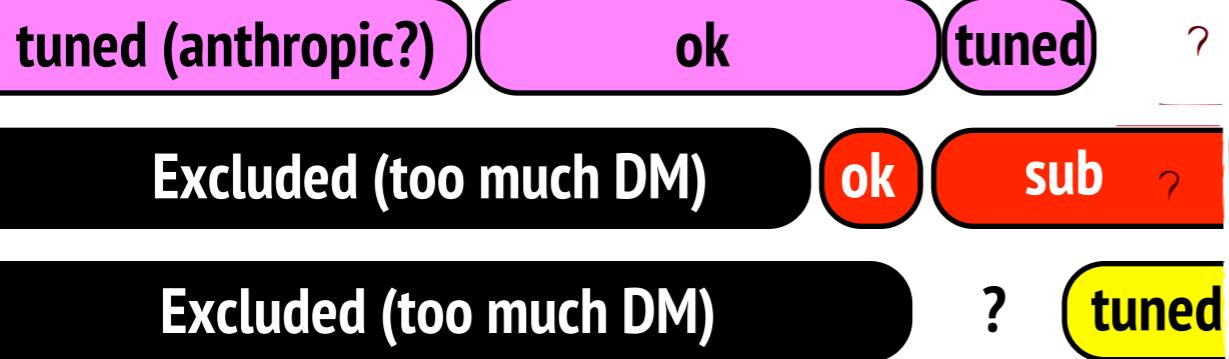
$$\Omega_{a\text{DM}} h^2 \simeq \theta_I^2 \left(\frac{80 \mu\text{eV}}{m_a} \right)^{1.19}$$

Phase transition ($N=1$)
strings+unstable DW's

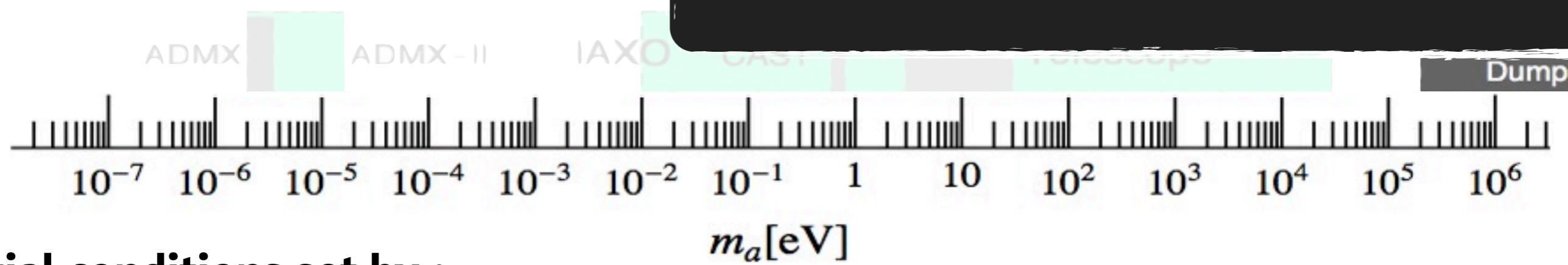
Phase transition ($N>1$)
strings+long-lived DWs

Axion dark matter scenarios

**Dark Matter
huge parameter space!**



Excluded



Initial conditions set by :

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