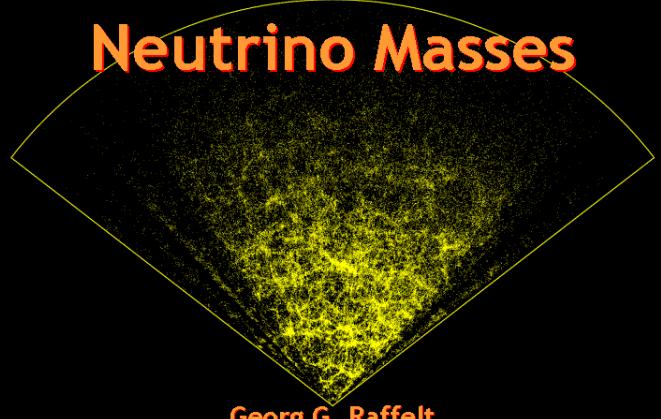


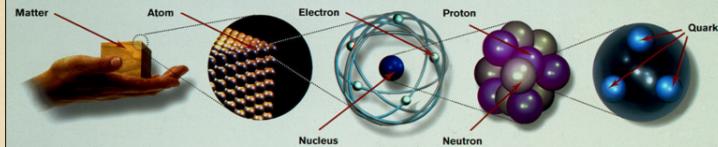
International Heraeus Summer School, 5-16 July 2004, Bad Honnef, Germany

# Neutrino Masses



Georg G. Raffelt  
Max-Planck-Institut für Physik, München, Germany

### Periodic System of Elementary Particles

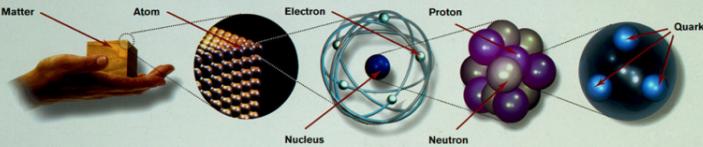


Quarks		Leptons	
Charge	Charge	Charge	Charge
+2/3	-1/3	-1	0
Up u	Down d	Electron e	e-Neutrino $\nu_e$




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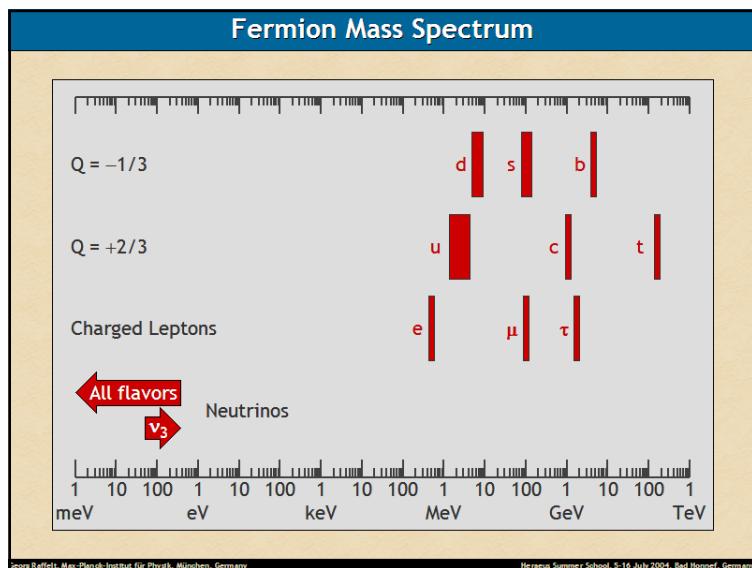
### Periodic System of Elementary Particles



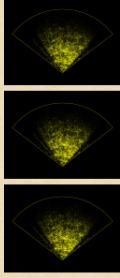
Quarks		Leptons	
Charge	Charge	Charge	Charge
+2/3	-1/3	-1	0
1st Family	Up u	Down d	Electron e
2nd Family	Charm c	Strange s	Muon $\mu$
3rd Family	Top t	Bottom b	Tau $\tau$

Gravitation  
Weak Interaction  
Electromagnetic Interaction  
Strong Interaction

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## Neutrino Masses



**Flavor oscillations and all that**

**Quest for the absolute mass scale  
and cosmological neutrinos**

**Neutrino mass and the  
baryon asymmetry of the universe**

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## Sun Glasses for Neutrinos?



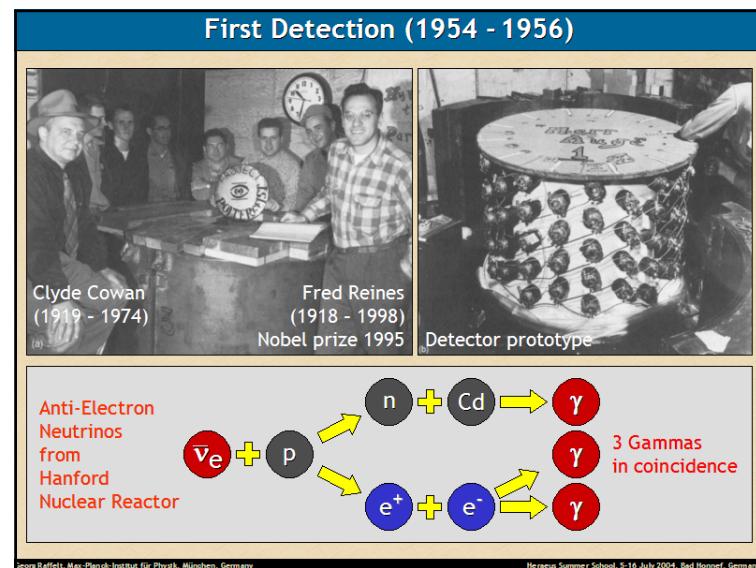
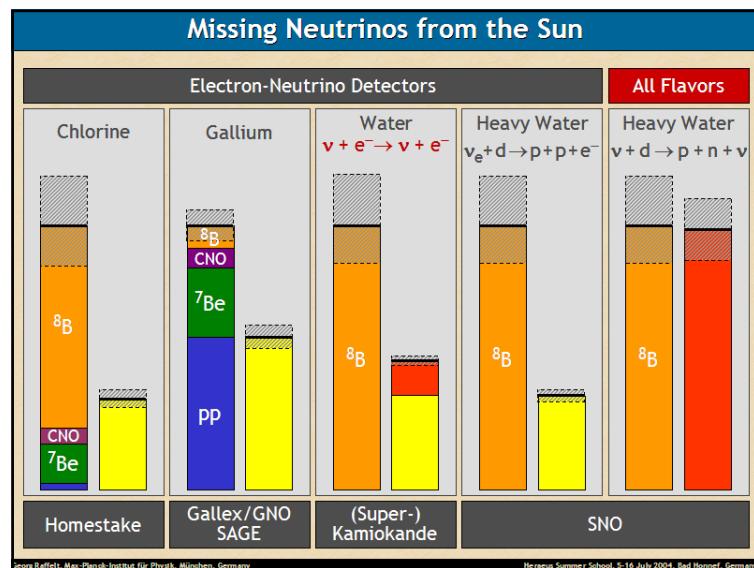
**1000 light years of lead  
needed to shield solar  
neutrinos**

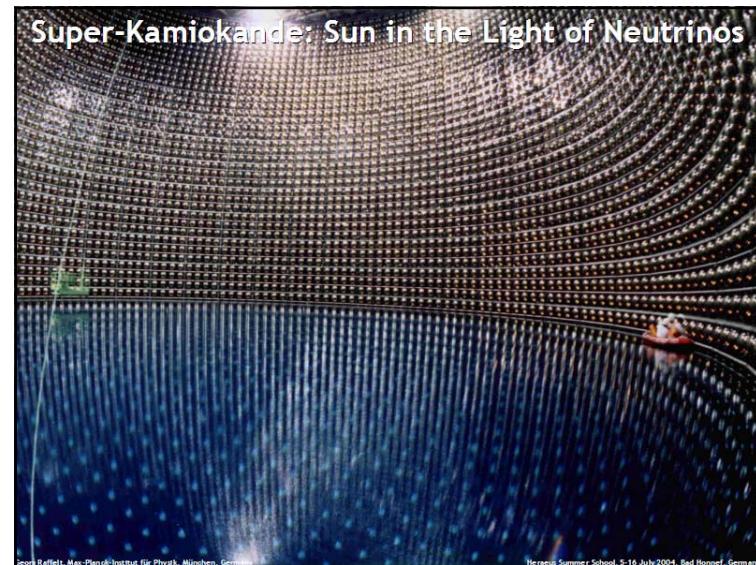
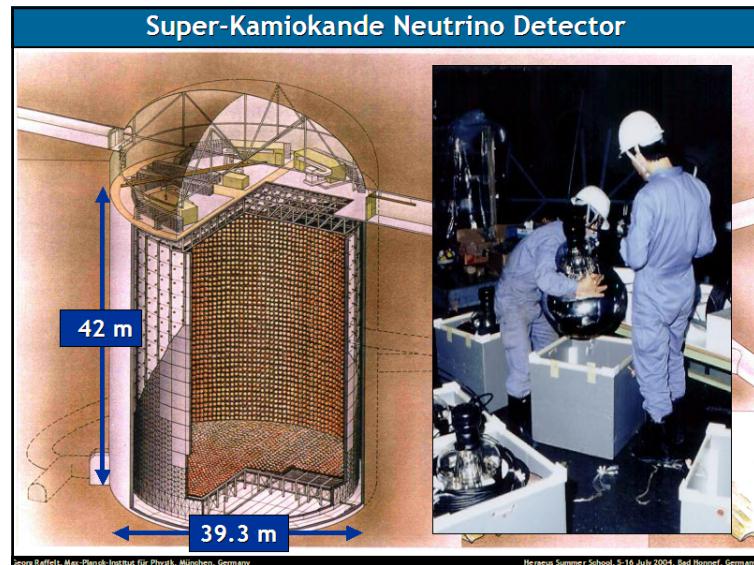
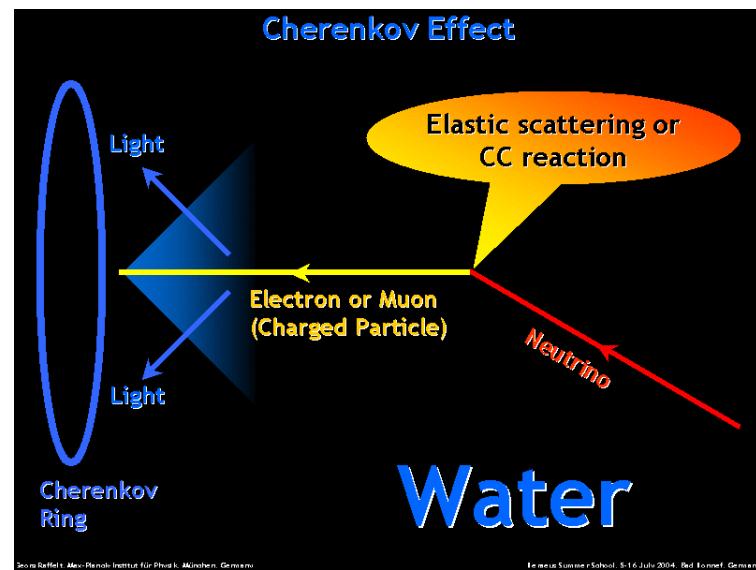
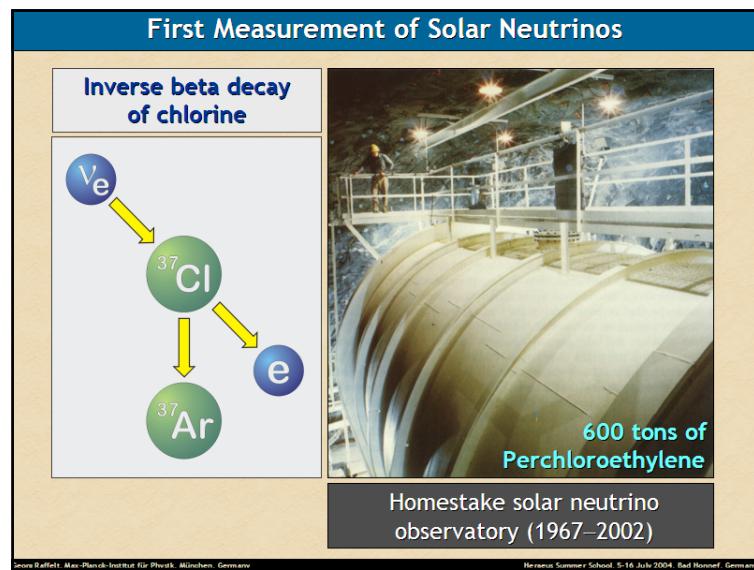
**Bethe & Peierls 1934:**  
*“... this evidently means  
that one will never be able  
to observe a neutrino.”*

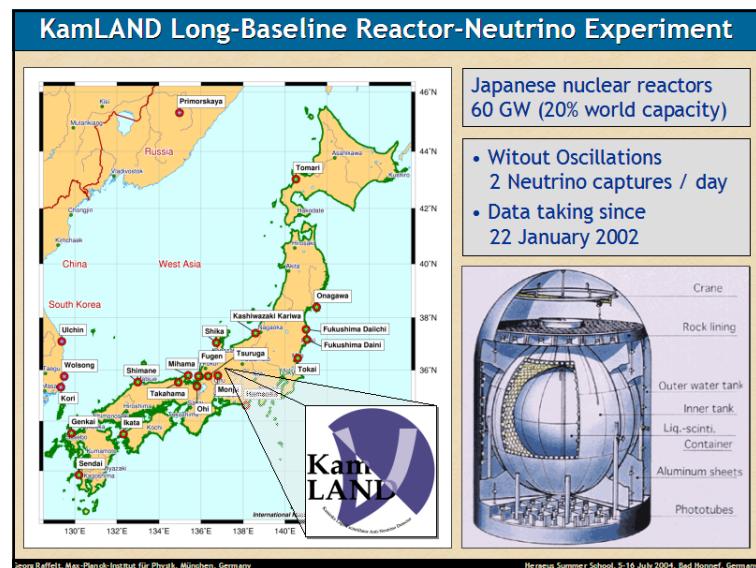
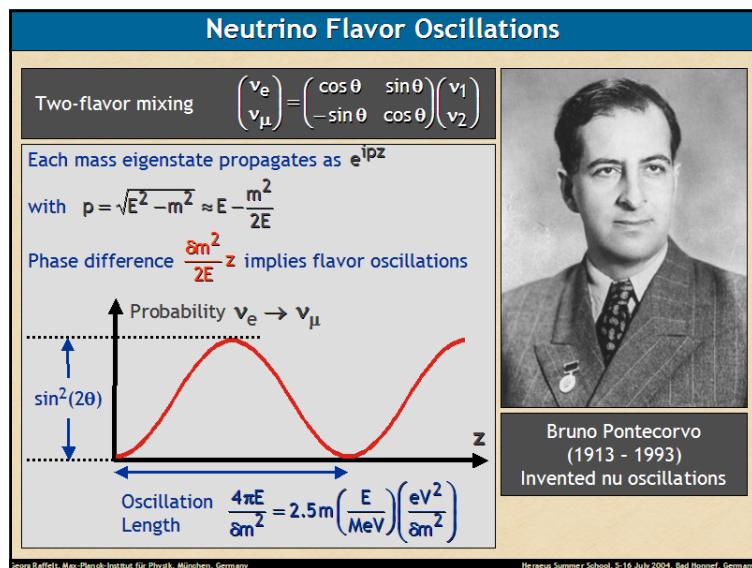
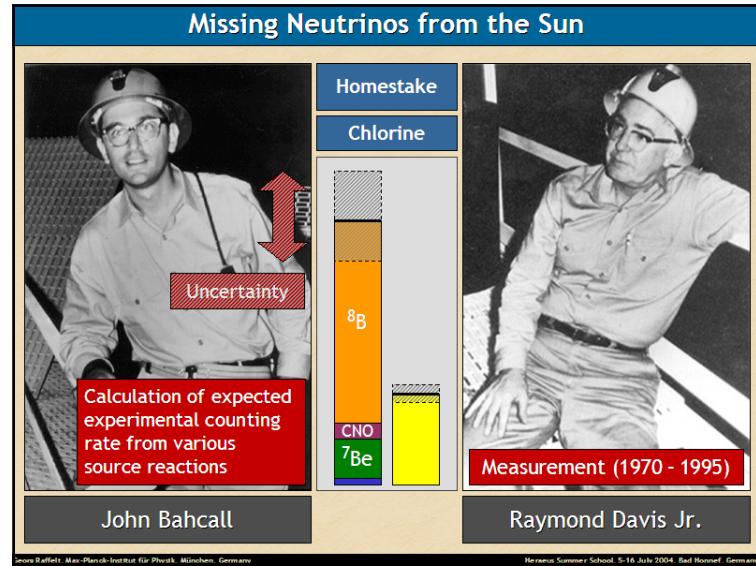
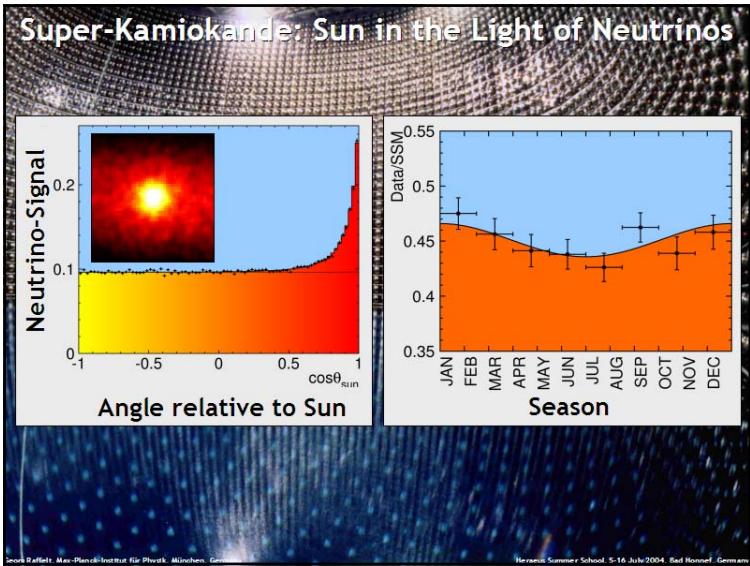


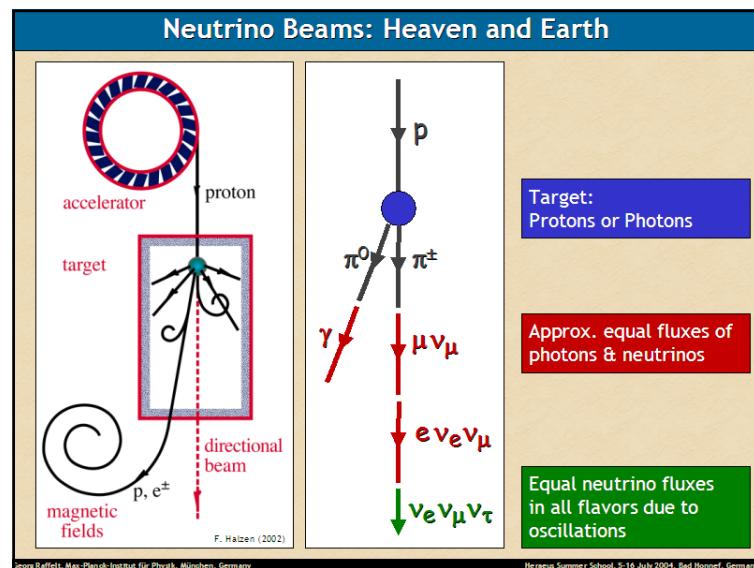
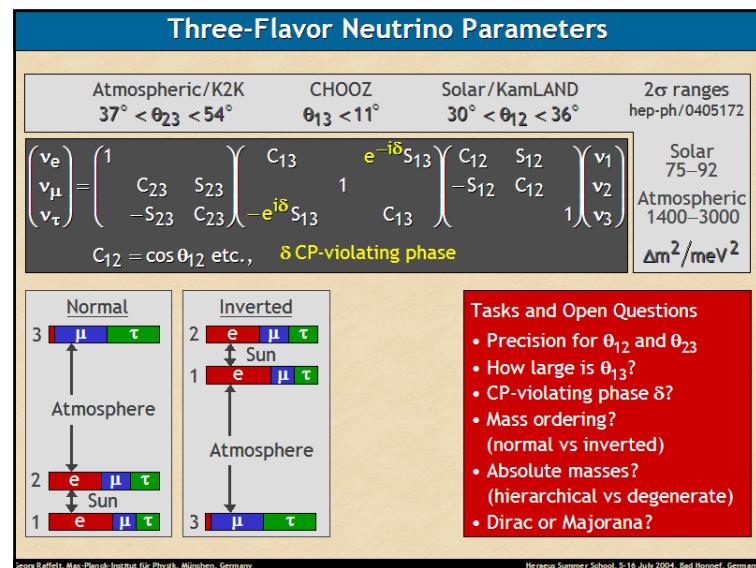
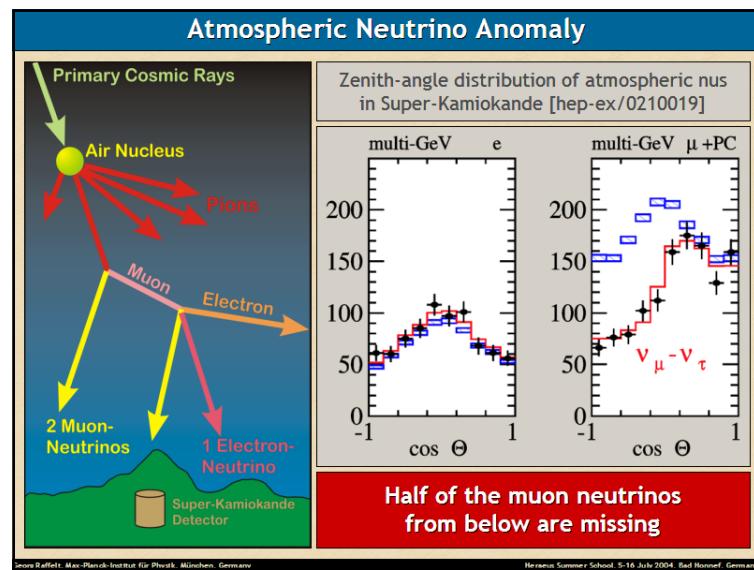
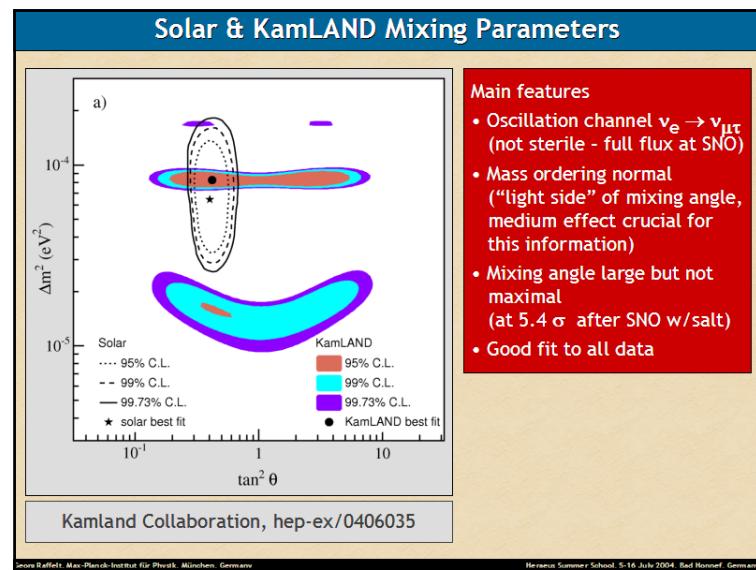
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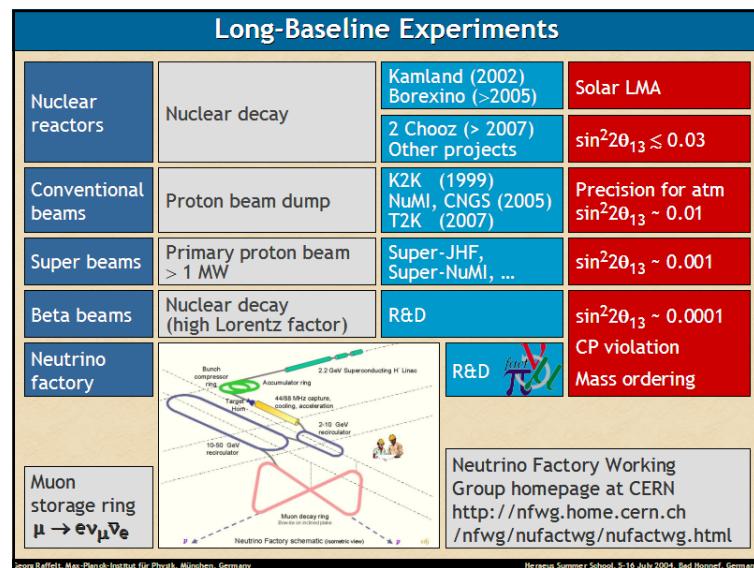
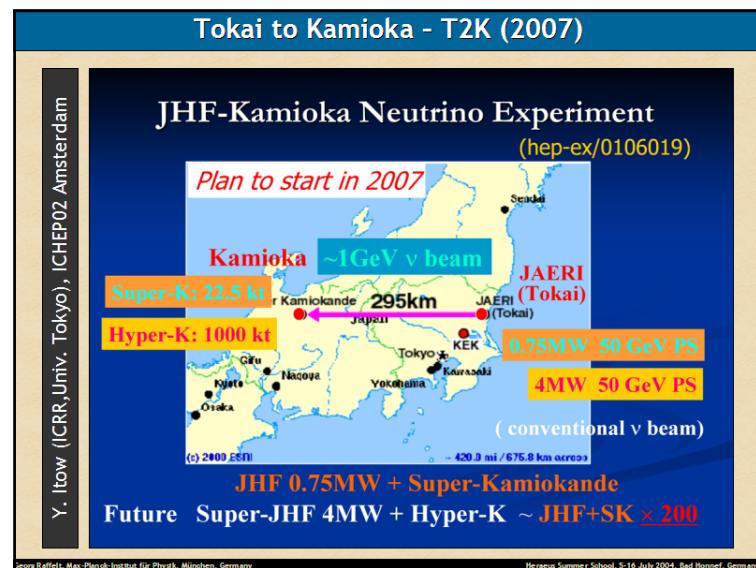
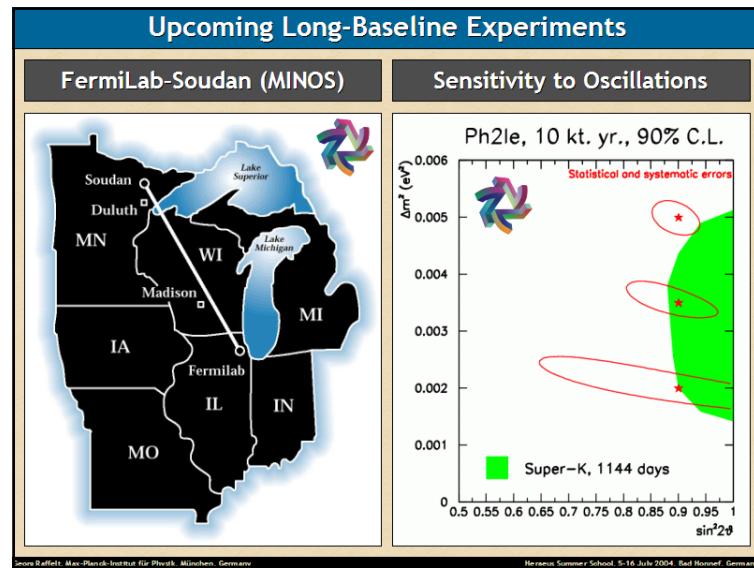
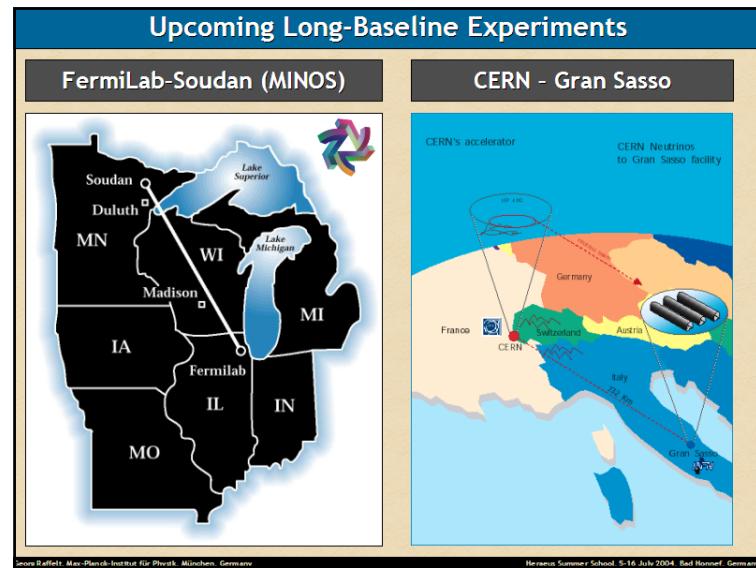
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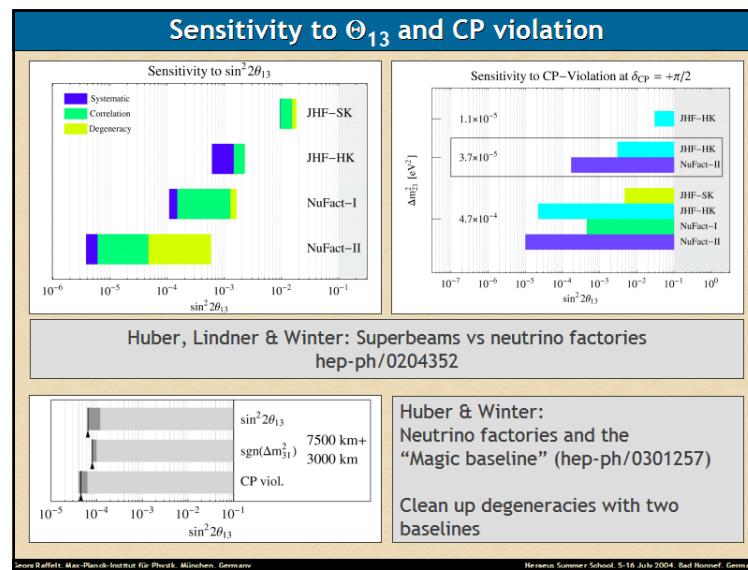




Fiducial Long-Baseline Configurations				
Setup	Baseline	Beam	Detector	Running
JHF-SK	295 km	0.75 MW	Water Cherenkov 23 kt fiducial	v 5 yr
JHF-HK		4 MW	Water Cherenkov 1000 kt fiducial	v 2 yr v 6 yr
NuFact I	3000 km	0.75 MW, $E_\mu = 50$ GeV $10^{20}$ useful $\mu$ decays per year	Magnetized iron calorimeter 10 kt fiducial	v 2.5 yr v 2.5 yr
NuFact II		4 MW, $E_\mu = 50$ GeV $5.3 \times 10^{20}$ useful $\mu$ decays per year	Magnetized iron calorimeter 50 kt fiducial	v 4 yr v 4 yr

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### Three-Neutrino Oscillations with a Long Baseline

The three-flavor oscillation probability at a long-baseline (LBL) experiment can be expanded in the small mixing angle  $\sin 2\theta_{13} < 0.3$  and the small hierarchy parameter  $\alpha = \Delta m_{21}^2 / \Delta m_{31}^2 \approx 0.03$ . Other parameters:  $\xi = \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{23}$ ,  $A = \pm \frac{2\sqrt{2}G_F n_e E}{\Delta m_{31}^2}$ ,  $\Delta = \frac{\Delta m_{31}^2 L}{4E}$ .

Up to  $\alpha^2$  and  $\sin^2(2\theta_{13})$  the transition probability is

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2[(1-A)\Delta]}{(1-A)^2} \quad (0)$$

$$+ \alpha \xi \sin 2\theta_{13} (\cos \delta \cos \Delta \pm \sin \delta \sin \Delta) \frac{\sin(A\Delta) \sin[(1-A)\Delta]}{A (1-A)} \quad (1)$$

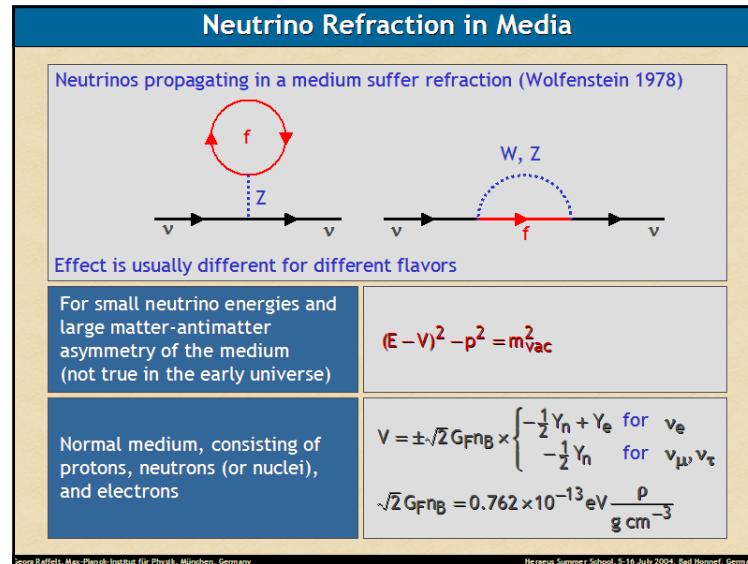
$$+ \alpha^2 \cos^2 \theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(A\Delta)}{A^2} \quad (2)$$

Lower sign for  $P(\nu_\mu \rightarrow \nu_e)$

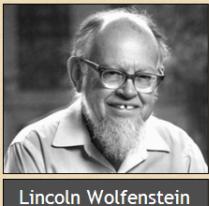
(1,2) disappear for “magic baseline”  $\sim 7500$  km where  $A\Delta \sim 0$ , assuming a neutrino factory with muon energy  $\sim 50$  GeV

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## Neutrino Oscillations in Matter



Neutrinos in a medium suffer flavor-dependent refraction (PRD 17:2369, 1978)

In Earth or Sun weak potential of order  $10^{-13}$  eV

$$i \frac{\partial}{\partial z} \begin{pmatrix} v_e \\ v_\mu \end{pmatrix} = \left[ \frac{M^2}{2E} + \sqrt{2} G_F \begin{pmatrix} n_e - \frac{1}{2} n_n & 0 \\ 0 & -\frac{1}{2} n_n \end{pmatrix} \right] \begin{pmatrix} v_e \\ v_\mu \end{pmatrix}$$

- “Level crossing” possible in a medium with a gradient (MSW effect)
  - For solar nu's large flavor conversion anyway due to large mixing
  - Still important for 13-oscillations in supernova envelope
- Breaks degeneracy between  $\Theta$  and  $\pi/2 - \Theta$  (dark vs light side)
- 12 mass ordering for solar nu's established
- 13 mass ordering (normal vs inverted) at future LBL or SN
- Discriminates against sterile nu's in atmospheric oscillations
- CP asymmetry in LBL, to be distinguished from intrinsic CP violation
- Prevents flavor conversion in a SN core and within shock wave
- Strongly affects sterile nu production in SN or early universe

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## Neutrino Masses



Flavor oscillations and all that

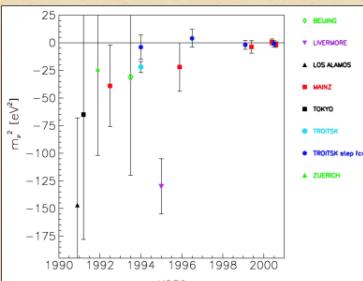
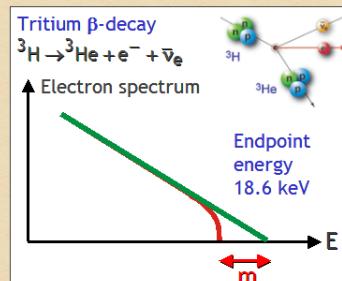
Quest for the absolute mass scale and cosmological neutrinos

Neutrino mass and the baryon asymmetry of the universe

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## Tritium Endpoint Spectrum



Currently best limits from Mainz and Troitsk experiments  
 $m < 2.2$  eV (95% CL)

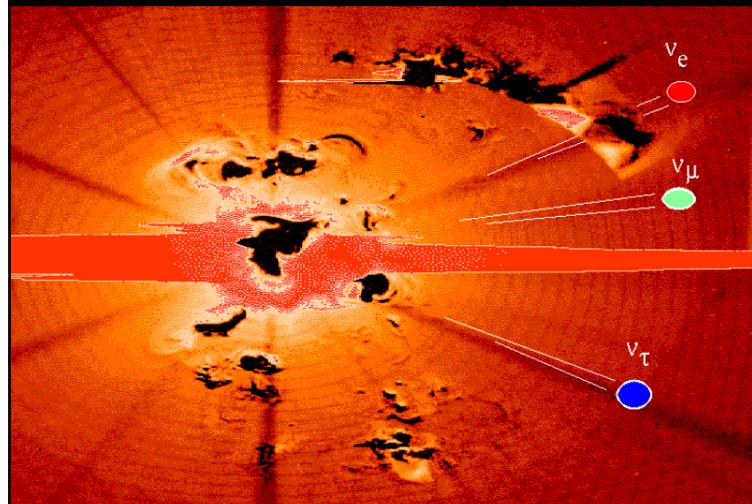
- Scaled-up spectrometer (KATRIN) is expected to reach 0.2 eV
- Currently under construction
- Measurements to begin 2007

<http://ik1au1.fzk.de/~katrin>

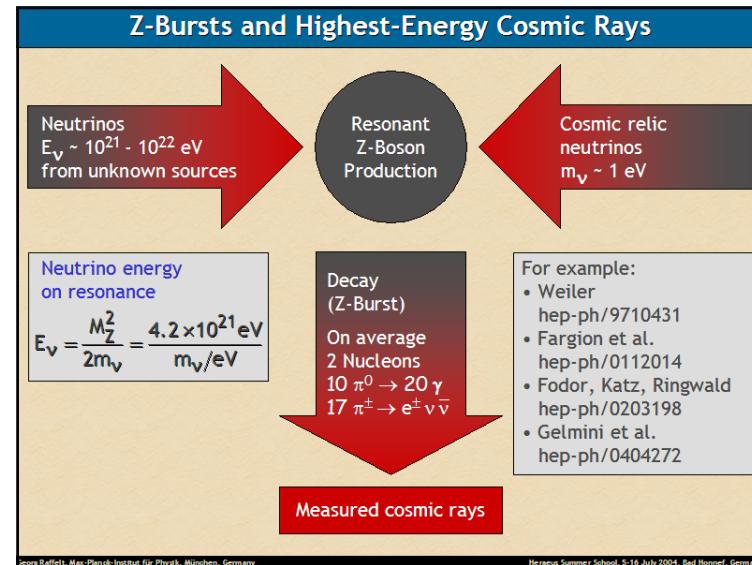
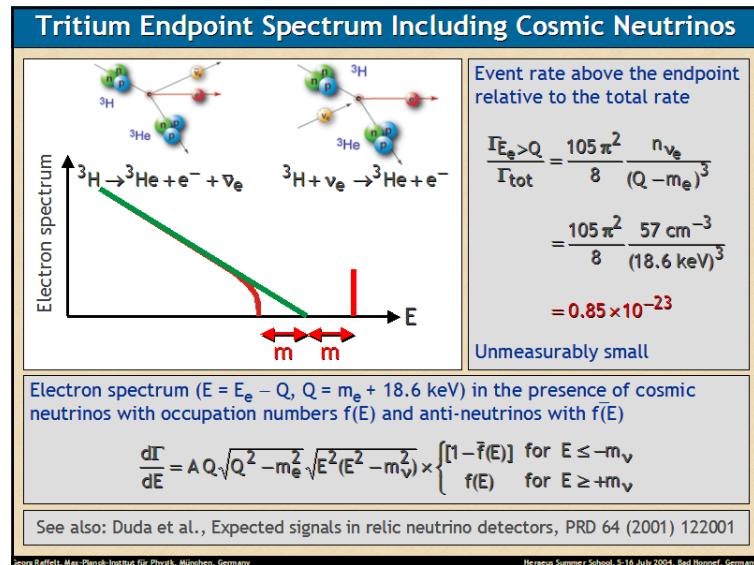
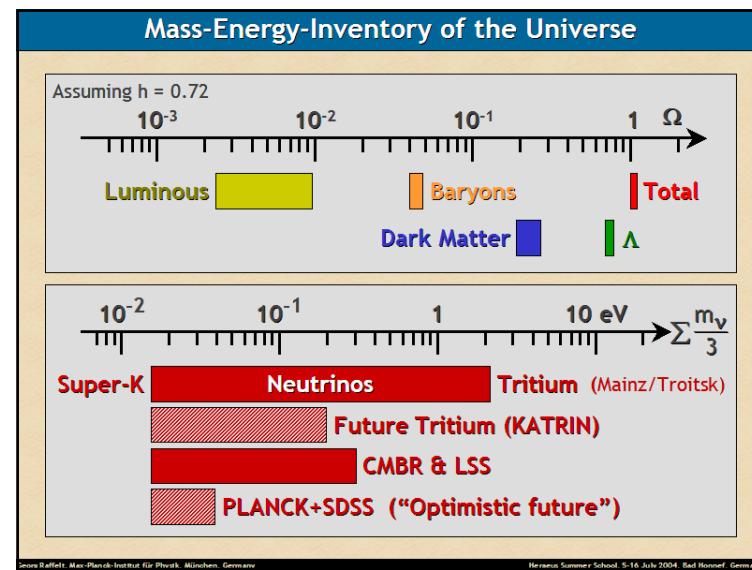
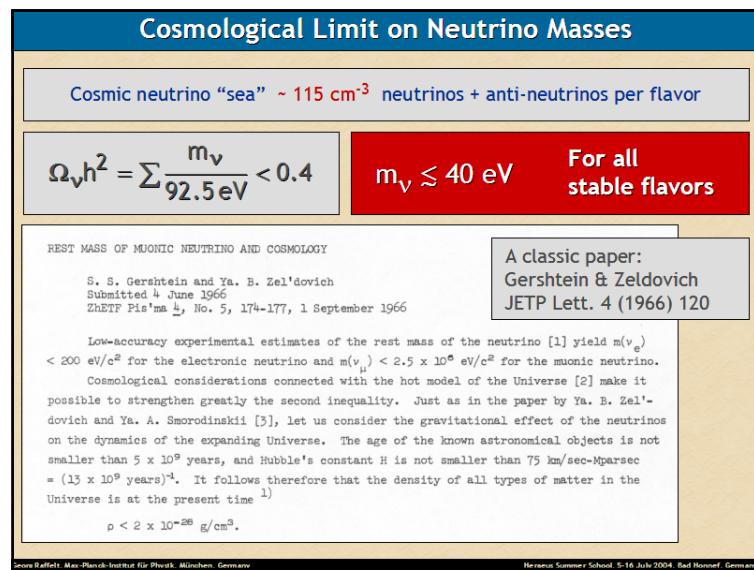
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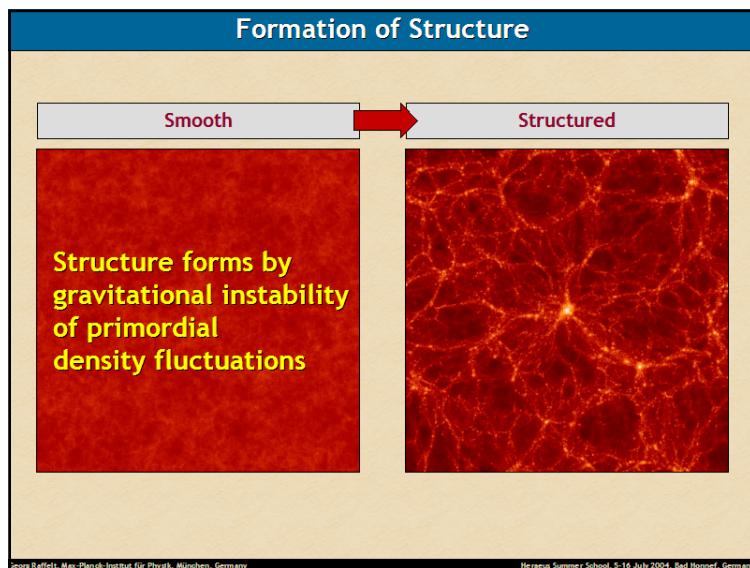
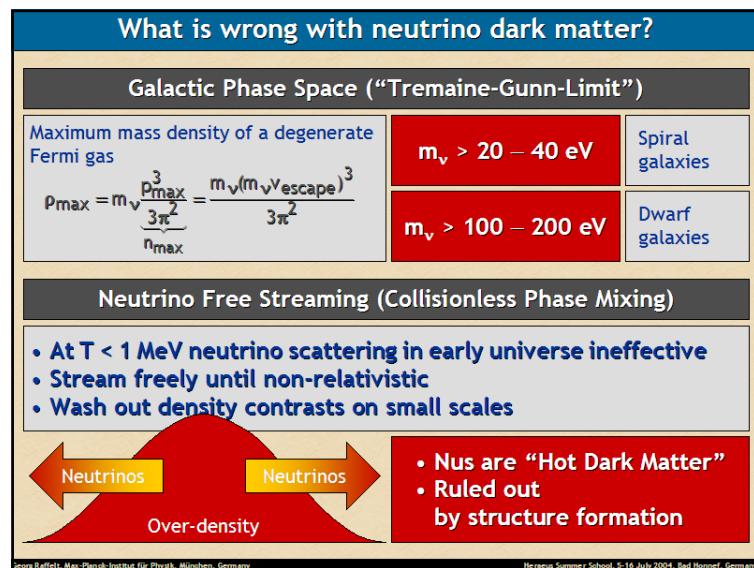
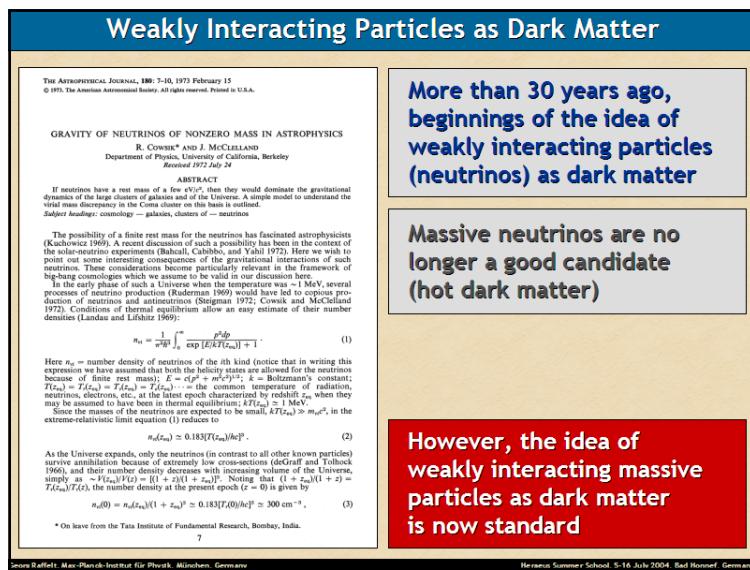
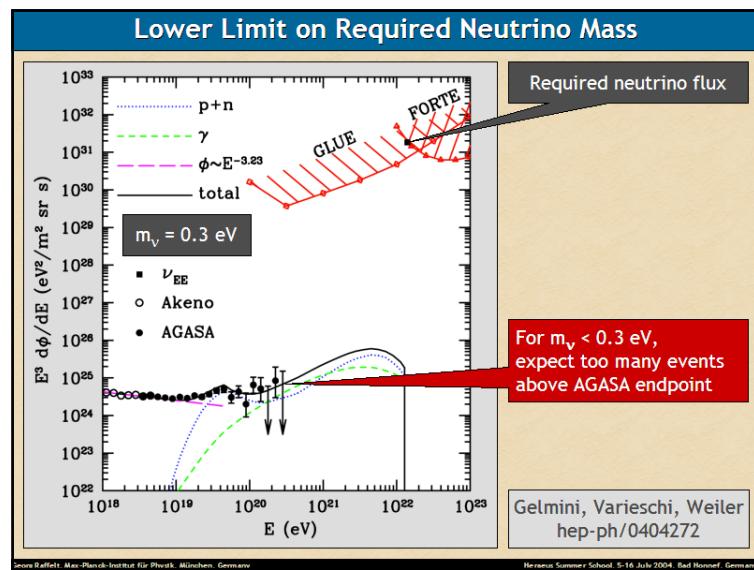
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## Creation of the Universe



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## Power Spectrum of Density Fluctuations

Field of density fluctuations

$$\delta(x) = \frac{\delta p(x)}{p}$$

Fourier transform

$$\delta_k = \int d^3x e^{-ik \cdot x} \delta(x)$$

Power spectrum essentially square of Fourier transformation

$$\langle \delta_k \delta_{k'} \rangle = (2\pi)^3 \delta(k - k') P(k)$$

with  $\delta$  the  $\delta$ -function

Power spectrum is Fourier transform of two-point correlation function ( $x=x_2-x_1$ )

$$\begin{aligned} \xi(x) &= \langle \delta(x_2) \delta(x_1) \rangle = \int \frac{d^3k}{(2\pi)^3} e^{ik \cdot x} P(k) \\ &= \int \frac{d\Omega dk}{4\pi k} e^{ik \cdot x} \frac{k^3 P(k)}{\Delta^2(k)} \end{aligned}$$

Gaussian random field (phases of Fourier modes  $\delta_k$  uncorrelated) is fully characterized by the power spectrum

$$P(k) = |\delta_k|^2$$

or equivalently by

$$\Delta(k) = \left( \frac{k^3 P(k)}{2\pi^2} \right)^{1/2} = \frac{k^{3/2} |\delta_k|}{\sqrt{2\pi}}$$

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## Processed Power Spectrum in Cold Dark Matter Scenario

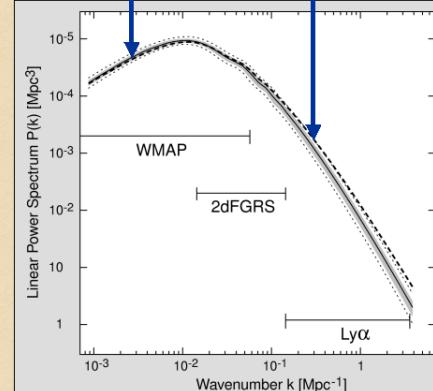
Primordial spectrum usually assumed to be of power-law form

$$P(k) = |\delta_k|^2 \propto k^n$$

Harrison-Zeldovich ("flat") spectrum

$n = 1$   
expected from inflation  
(may be slightly less than 1, depending on details of inflationary phase)

Primordial spectrum  
Suppressed by stagnation during radiation phase



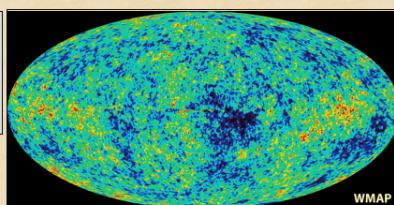
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## Power Spectrum of CMBR Temperature Fluctuations

Sky map of CMBR temperature fluctuations

$$\Delta(\theta, \phi) = \frac{T(\theta, \phi) - \langle T \rangle}{\langle T \rangle}$$

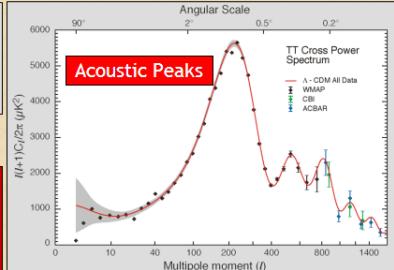


Multipole expansion

$$\Delta(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi)$$

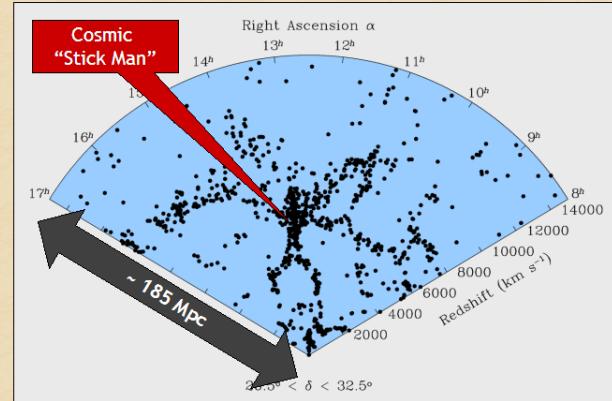
Angular power spectrum

$$C_{\ell} = \langle a_{\ell m}^* a_{\ell m} \rangle = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} a_{\ell m}^* a_{\ell m}$$



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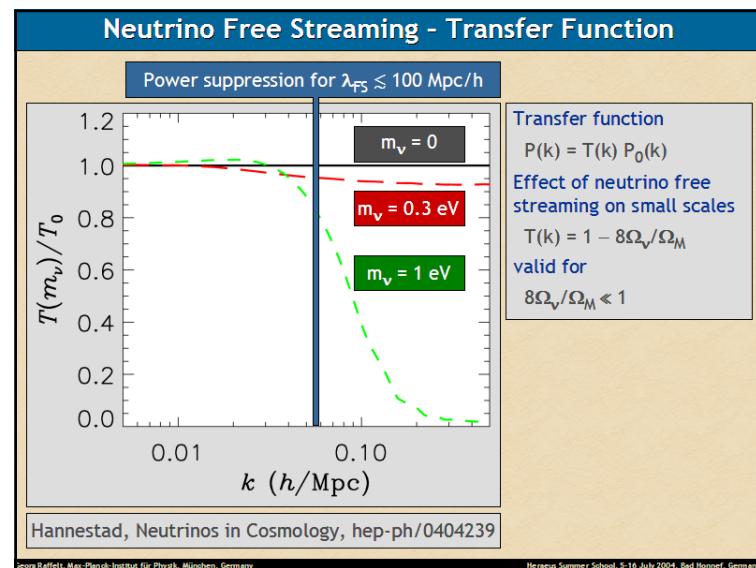
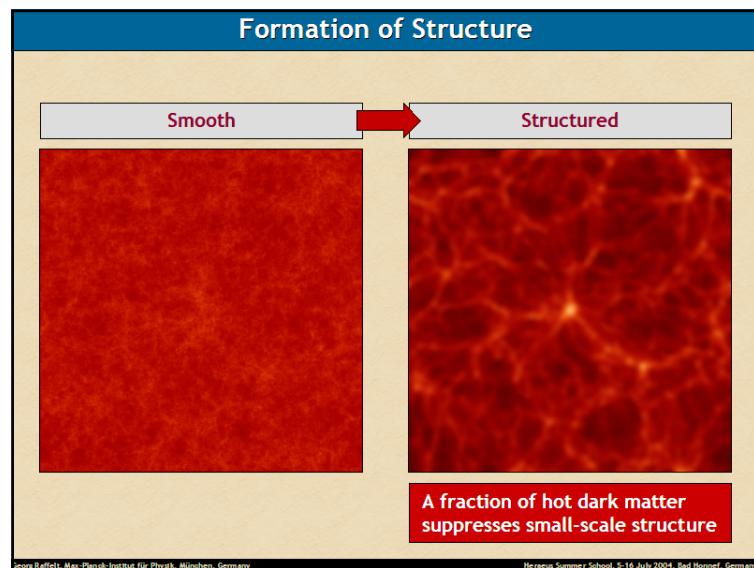
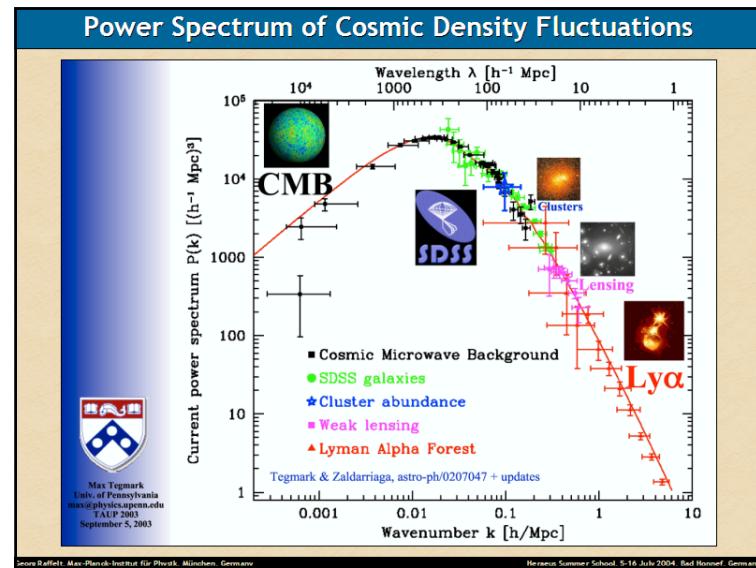
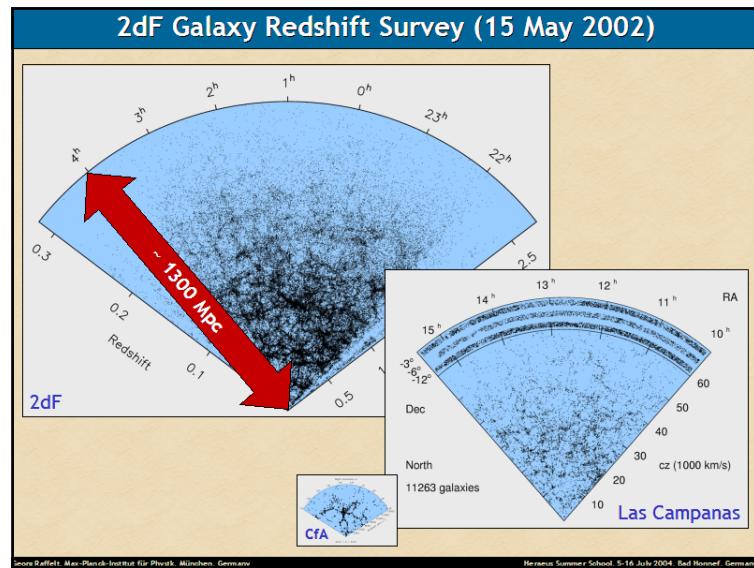
## A Slice of the Universe

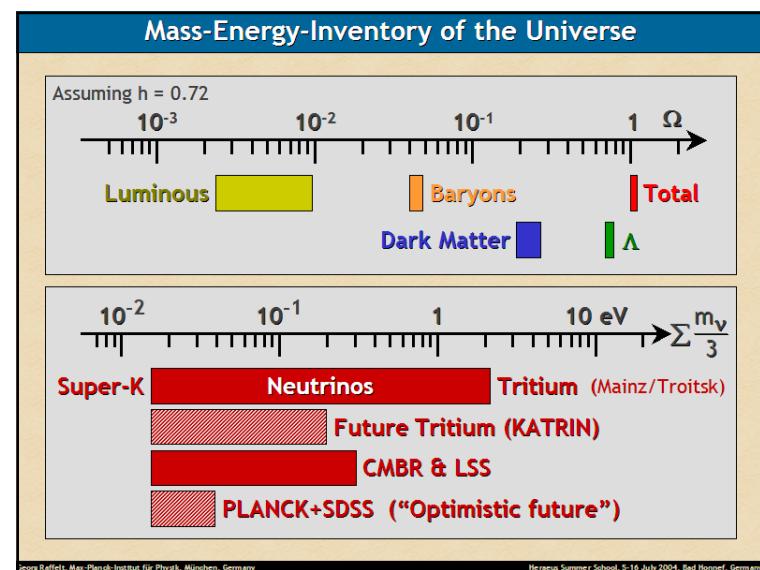
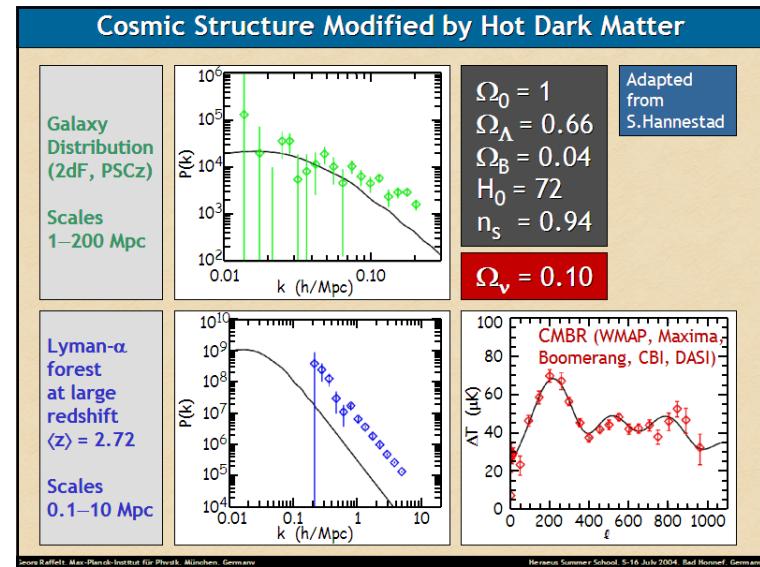
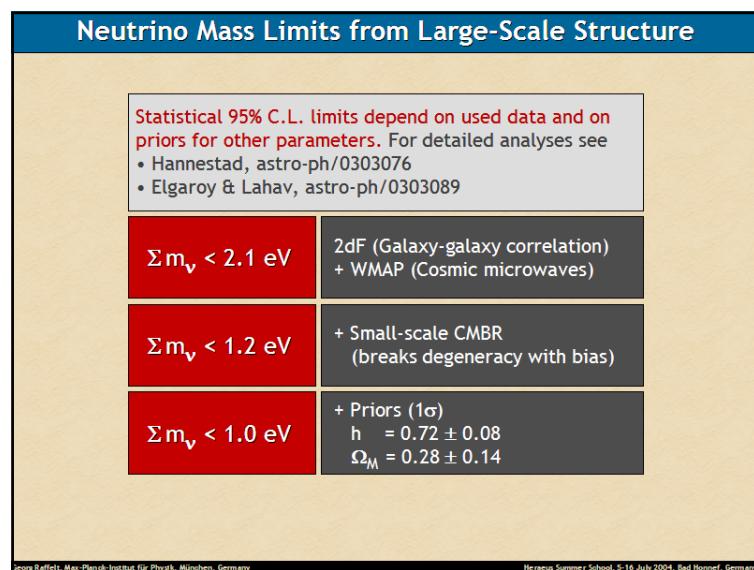
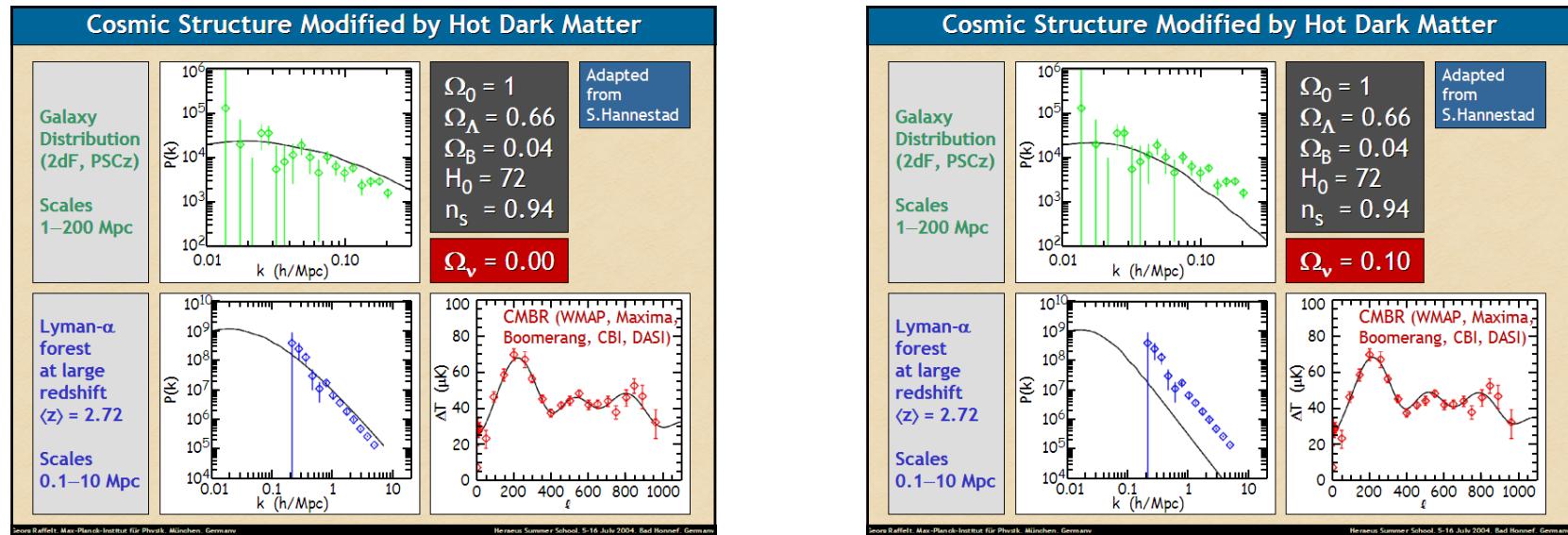


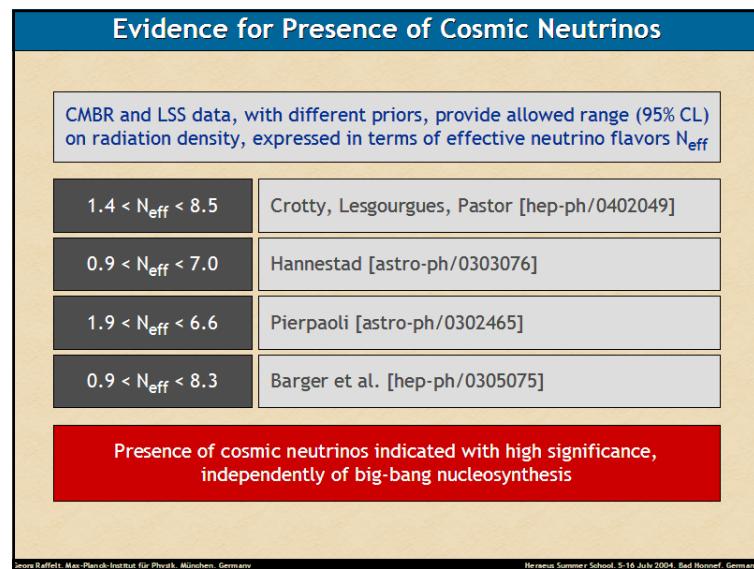
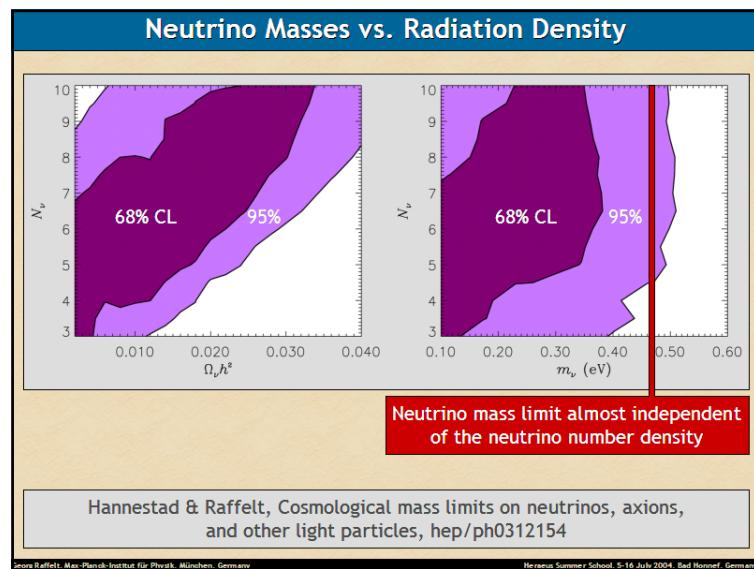
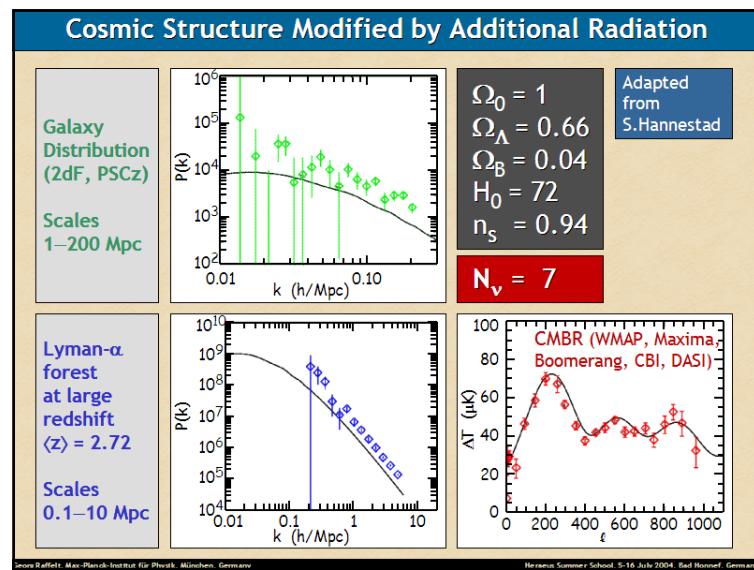
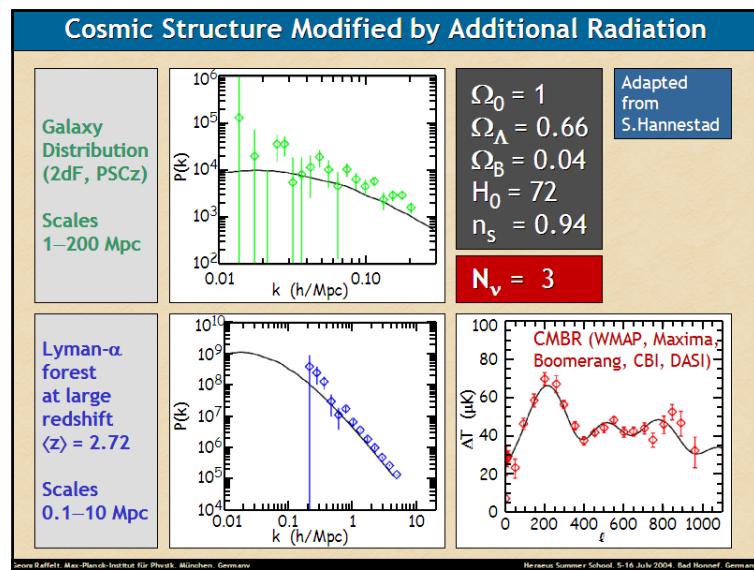
Galaxy distribution from the CfA redshift survey  
[ApJ 302 (1986) L1]

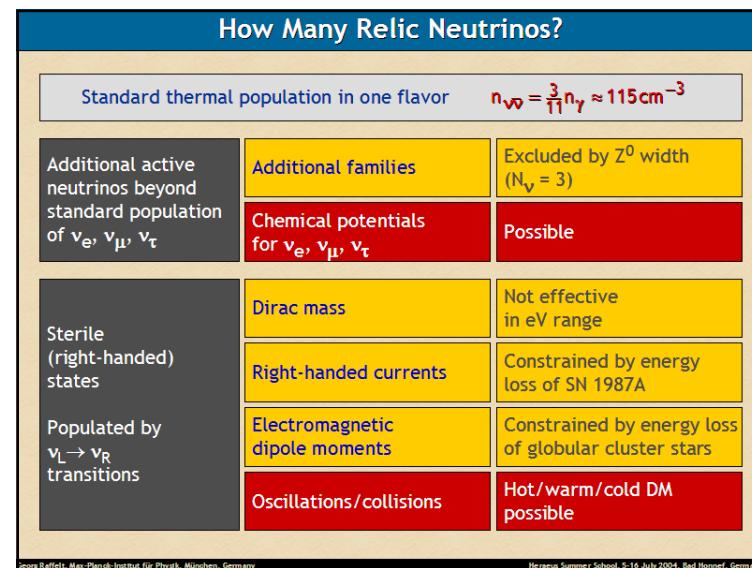
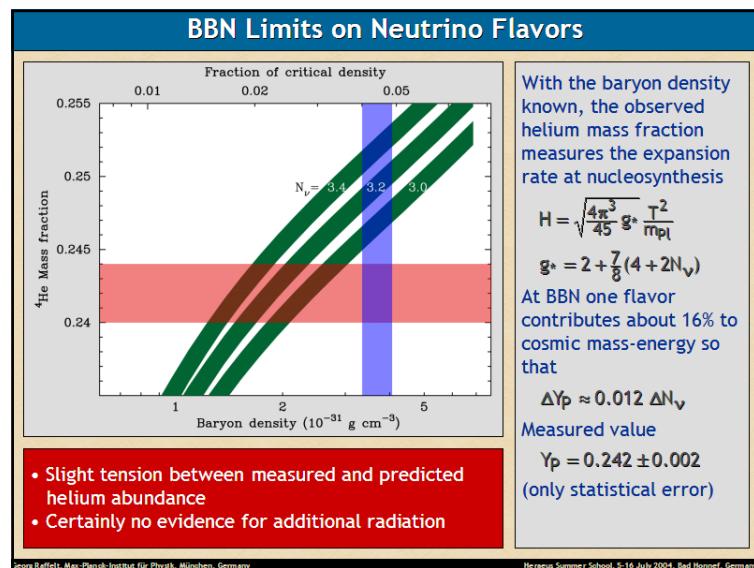
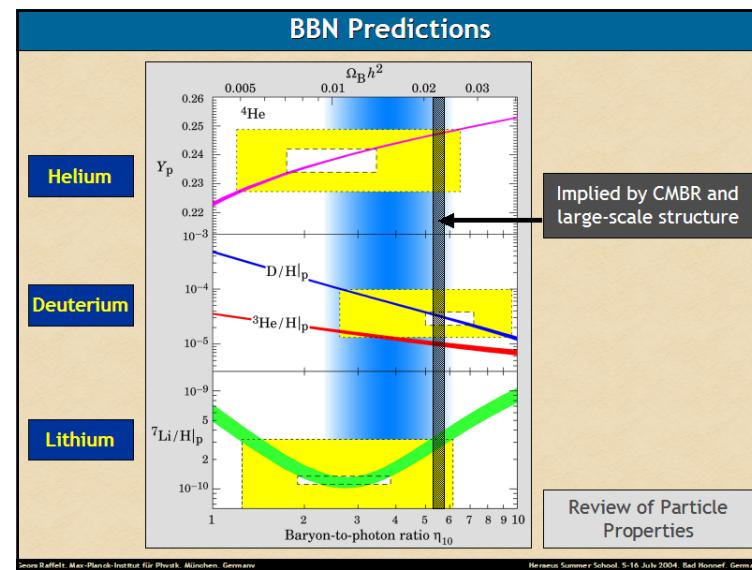
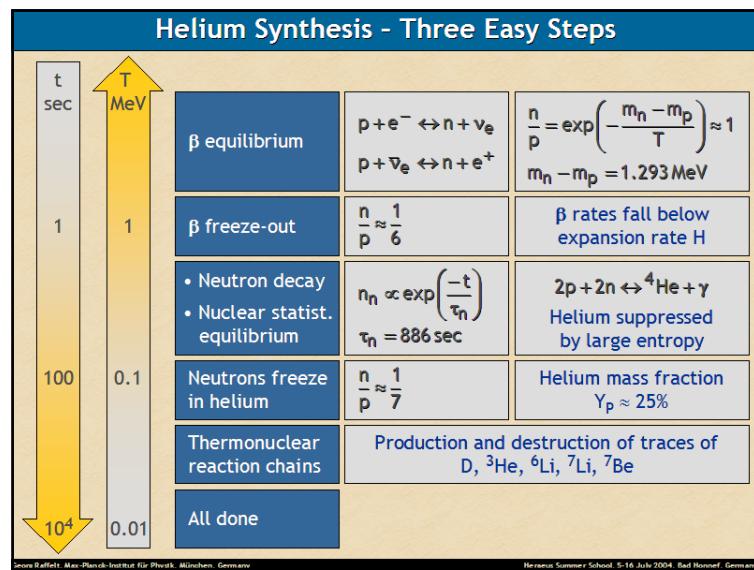
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## Thermal Neutrino Distribution with Chemical Potential

**Fermi-Dirac distribution**

- Temperature  $T$
- Chemical potential  $\mu$
- + $\mu$  Particles
- $\mu$  Anti-particles

$$f_p = \frac{1}{\exp\left(\frac{E-\mu}{T}\right) + 1}$$

**Degeneracy parameter**  $\xi = \frac{\mu}{T}$  Invariant under cosmic expansion

**Number density**

$$n_{\nu\nu} = \int dE \frac{4\pi}{(2\pi)^3} \left( \frac{E^2}{1+\exp(E/T-\xi)} + \frac{E^2}{1+\exp(E/T+\xi)} \right)$$

$$= \frac{3\zeta_3}{2\pi^2} T_v^3 \left[ 1 + \frac{2\ln(2)}{3\zeta_3} \xi^2 + \frac{1}{72\zeta_3} \xi^4 + \dots \right]$$

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## Thermal Neutrino Distribution with Chemical Potential

**Fermi-Dirac distribution**

- Temperature  $T$
- Chemical potential  $\mu$
- + $\mu$  Particles
- $\mu$  Anti-particles

$$f_p = \frac{1}{\exp\left(\frac{E-\mu}{T}\right) + 1}$$

**Degeneracy parameter**  $\xi = \frac{\mu}{T}$  Invariant under cosmic expansion

**Number density**

$$n_{\nu\nu} = \int dE \frac{4\pi}{(2\pi)^3} \left( \frac{E^2}{1+\exp(E/T-\xi)} + \frac{E^2}{1+\exp(E/T+\xi)} \right)$$

$$= \frac{3\zeta_3}{2\pi^2} T_v^3 \left[ 1 + \frac{2\ln(2)}{3\zeta_3} \xi^2 + \frac{1}{72\zeta_3} \xi^4 + \dots \right]$$

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## BBN and Neutrino Chemical Potentials

Expansion Rate Effect (all flavors)	Energy density in one neutrino flavor with degeneracy parameter $\xi = \eta/T$
Beta equilibrium effect for electron flavor $n + v_e \leftrightarrow p + e^-$	$\rho_{\nu\nu} = \frac{7\pi^2}{120} T_v^4 \left[ 1 + \frac{30}{7} \left( \frac{\xi}{\pi} \right)^2 + \frac{15}{7} \left( \frac{\xi}{\pi} \right)^4 \right]$ Helium abundance essentially fixed by n/p ratio at beta freeze-out $\frac{n}{p} = e^{-(m_n - m_p)/T - \xi_{ve}}$ Effect on helium equivalent to $\Delta N_{eff} \sim -18 \xi_{ve}$
<ul style="list-style-type: none"> <li>• <math>v_e</math> beta effect can compensate expansion-rate effect of <math>\nu_{\mu,T}</math></li> <li>• No significant BBN limit on neutrino number density</li> </ul>	

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## Chemical Potentials and Flavor Oscillations

**Flavor mixing (neutrino oscillations)**

- Flavor lepton numbers not conserved
- Only one common neutrino chemical potential
- Stringent  $\xi_{ve}$  limit applies to all flavors  
 $|\xi_{ve,\mu_T}| < 0.07$
- Extra neutrino density  
 $\Delta N_{eff} < 0.0064$
- Cosmic neutrino density close to standard value

**Flavor equilibrium before n/p freeze out ?**

- YES for solar LMA solution
- Our knowledge of the cosmic neutrino density depends on measured oscillation parameters

Lunardini & Smirnov, hep-ph/0012056  
Dolgov, Hansen, Pastor, Petcov, Raffelt & Semikoz, hep-ph/0201287  
Abazajian, Beacom & Bell, astro-ph/0203442  
Wong, hep-ph/0203180

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