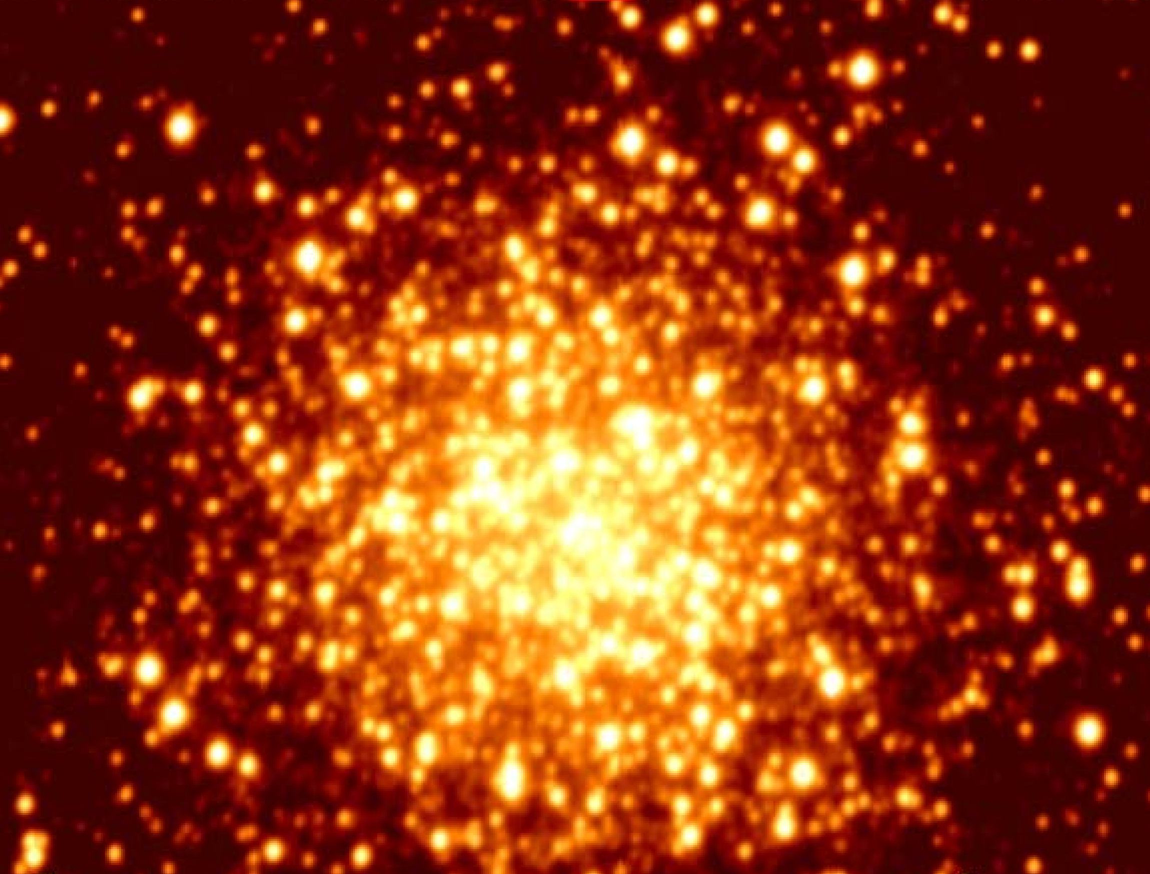
Stellar Evolution



- Role of neutrinos in normal stars
- Limits on "exotic" neutrino properties
- Neutrino interactions in a medium

Equations of Stellar Structure

Assume spherical symmetry and static structure (neglect kinetic energy)
Excludes: Rotation, convection, magnetic fields, supernova-dynamics, ...

Hydrostatic equilibrium	$\frac{dP}{dr} = -\frac{G_N M_r \rho}{r^2}$
Energy conservation	$\frac{dL_r}{dr} = 4\pi r^2 \varepsilon \rho$
Energy transfer	$L_r = \frac{4\pi r^2}{3\kappa\rho}\frac{d(aT^4)}{dr}$

r	Radius from center
P	Pressure
G_N	Newton's constant
ρ	Mass density
M_r	Integrated mass up to r
L_r	Luminosity (energy flux)
ε	Local rate of energy generation [erg/g/s] $\varepsilon = \varepsilon_{nuc} + \varepsilon_{grav} - \varepsilon_{\nu}$
κ	Opacity $\kappa^{-1} = \kappa_{\gamma}^{-1} + \kappa_{c}^{-1}$
κ_{γ}	Radiative opacity $\kappa_{\gamma}\rho = \langle \lambda_{\gamma} \rangle_{\text{Rosseland}}^{-1}$
κ_c	Electron conduction

Literature

- Clayton: Principles of stellar evolution and nucleosynthesis (Univ. Chicago Press 1968)
- Kippenhahn & Weigert: Stellar structure and evolution (Springer 1990)

Convection in Main-Sequence Stars

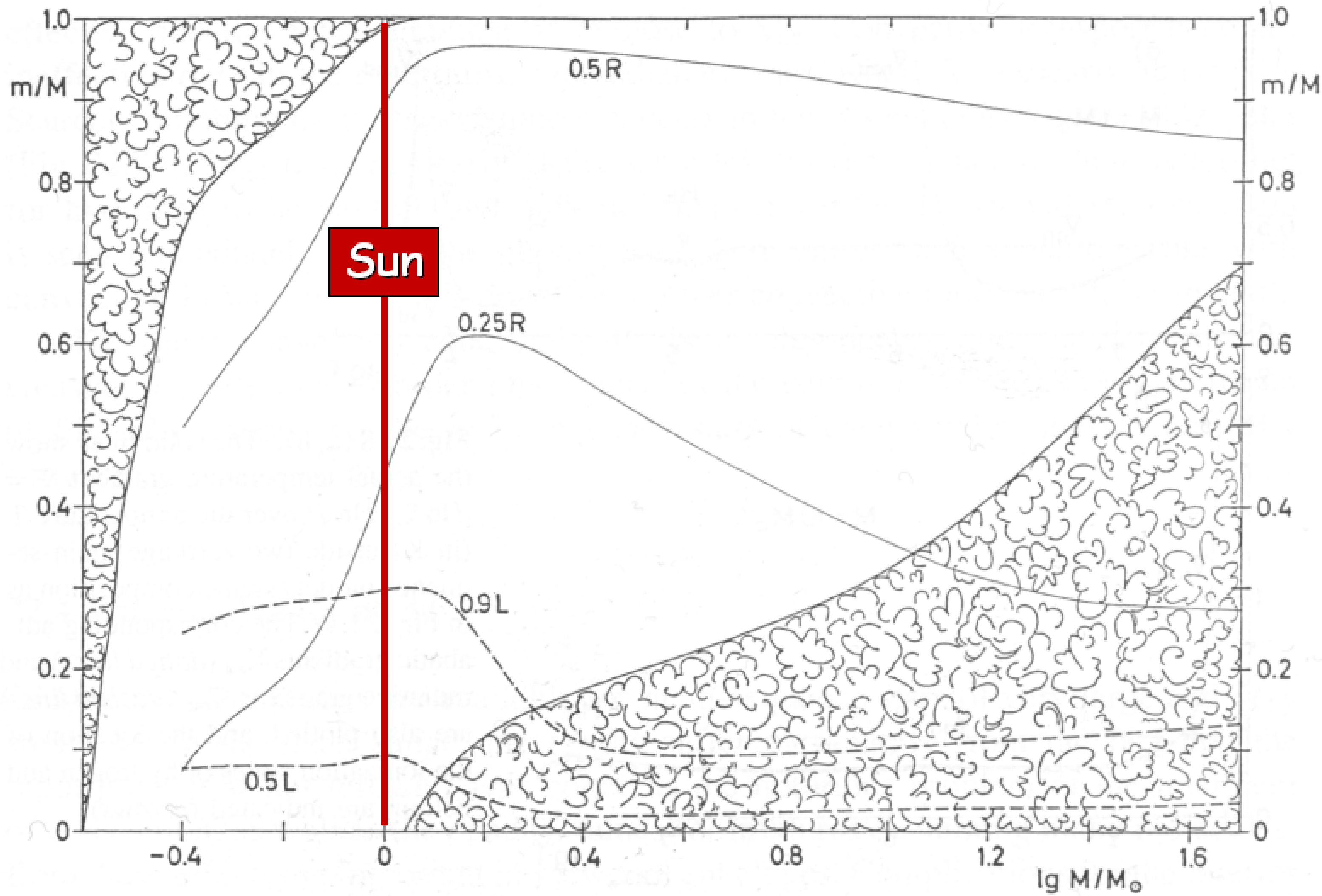


Fig. 22.7. The mass values m from centre to surface are plotted against the stellar mass M for the same zero-age main-sequence models as in Fig. 22.1. "Cloudy" areas indicate the extension of convective zones inside the models. Two solid lines give the m values at which r is 1/4 and 1/2 of the total radius R. The dashed lines show the mass elements inside which 50% and 90% of the total luminosity L are produced

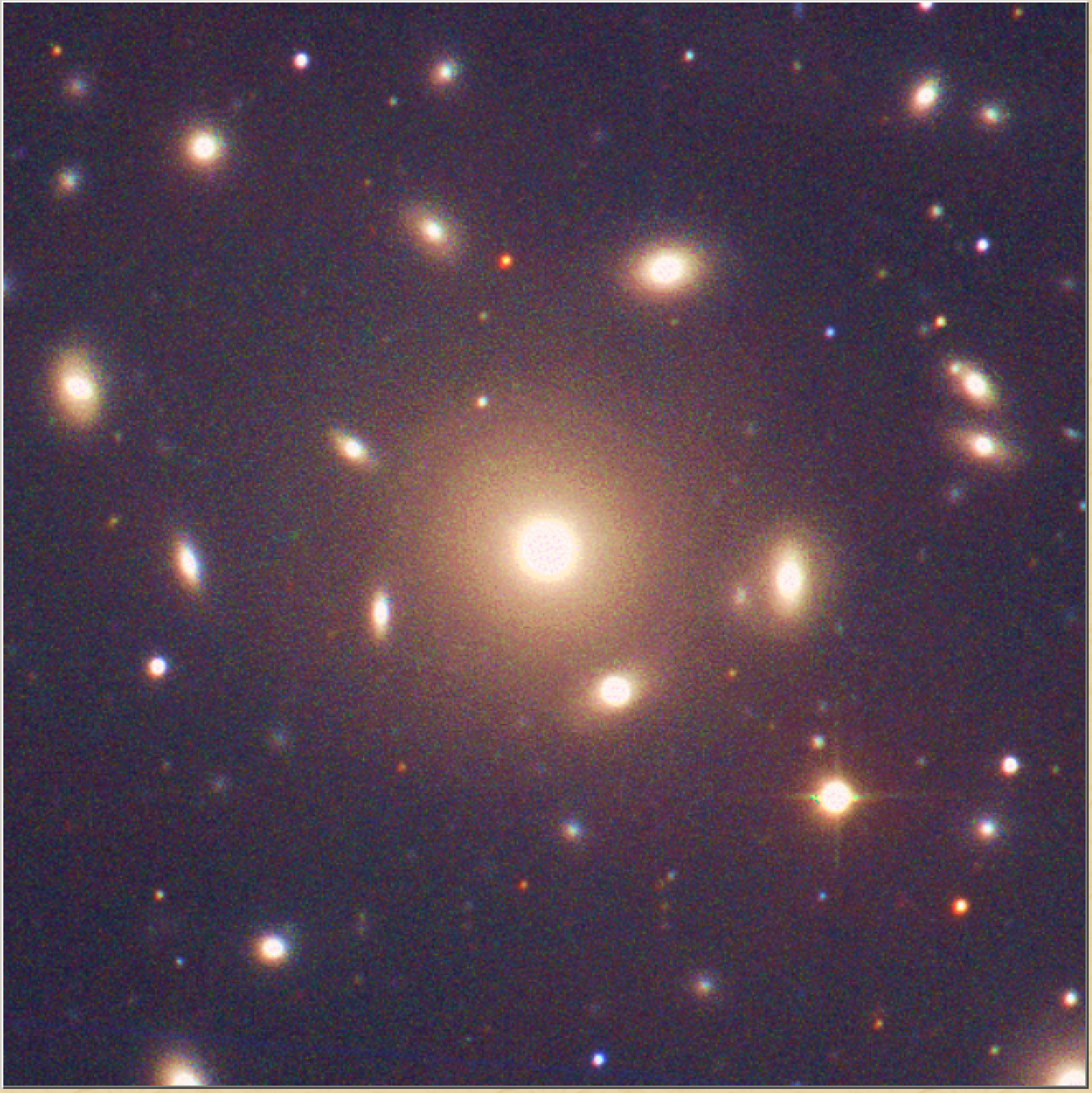
Kippenhahn & Weigert, Stellar Structure and Evolution (Springer 1990)

Virial Theorem and Hydrostatic Equilibrium

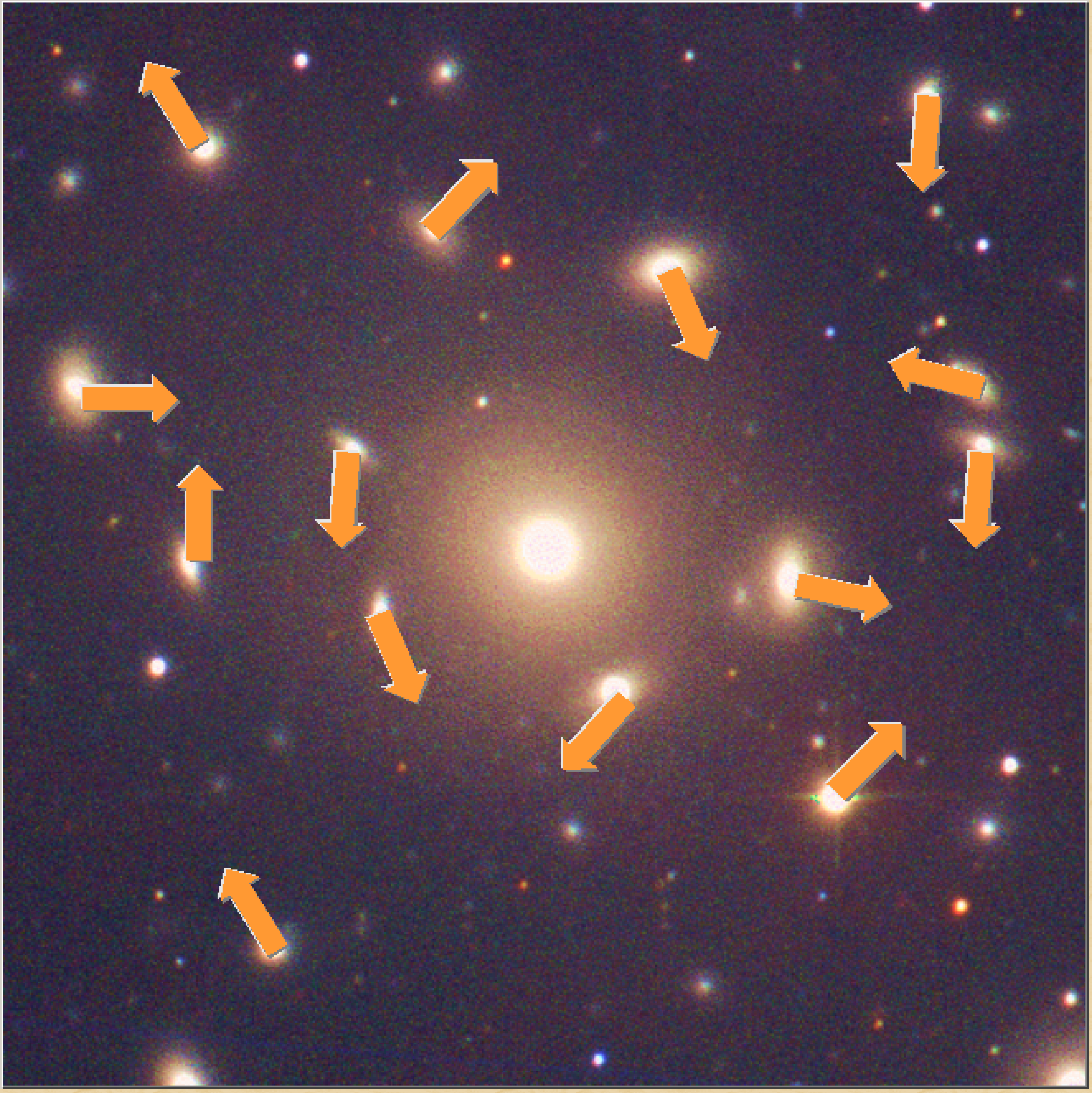
Hydrostatic equilibrium	$\frac{dP}{dr} = -\frac{G_N M_r \rho}{r^2}$
Integrate both sides	$\int_0^R dr\, 4\pi r^3 P' = -\int_0^R dr\, 4\pi r^3 \frac{G_N M_r \rho}{r^2}$
L.h.s. partial integration with $P = 0$ at surface R	$-3\int_0^R dr\, 4\pi r^2 P = E_{grav}^{tot}$
Classical monatomic gas: $P = \frac{2}{3}U$ (U density of internal energy)	$U^{tot} = -\frac{1}{2} E_{grav}^{tot}$
Average energy of single "atoms" of the gas	$\langle E_{kin} \rangle = -\frac{1}{2} \langle E_{grav} \rangle$ Virial Theorem

Most important tool to understand self-gravitating systems

Coma Cluster of Galaxies



Coma Cluster of Galaxies

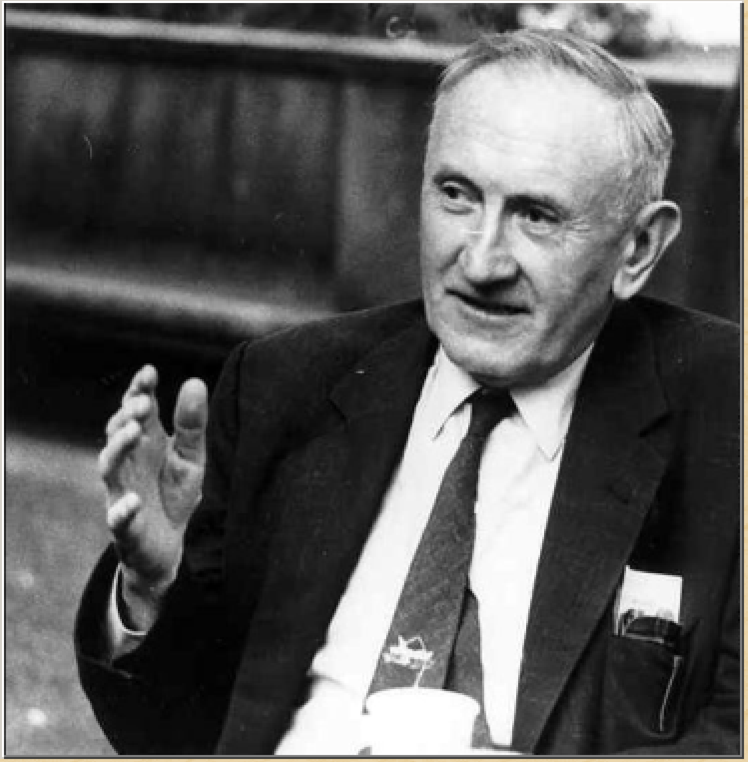


Velocities from Doppler shifts of spectral lines

Estimate mass of galaxy cluster

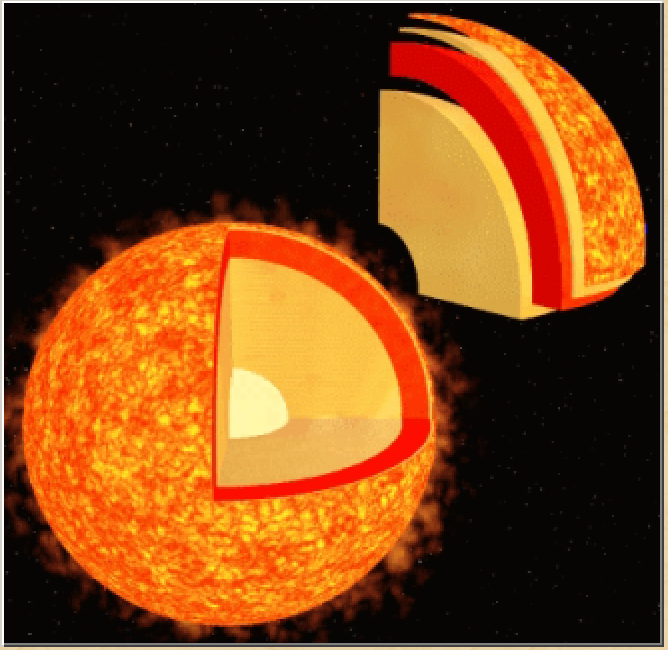
Dark Matter in Galaxy Clusters

Fritz Zwicky
"Die Rotverschiebung von Extragalaktischen Nebeln"
[Helv. Phys. Acta 6 (1933) 110]



[...] In order to obtain the observed average Doppler effect of 1000 km/s or more, the average density of the Coma cluster would have to be at least 400 times larger than what is found from observations of the luminous matter. Should this be confirmed one would find the surprising result that dark matter is far more abundant than luminous matter. [...]

Virial Theorem Applied to the Sun



$$\langle E_{kin} \rangle = -\frac{1}{2} \langle E_{grav} \rangle$$

Virial Theorem

Approximate Sun as a homogeneous sphere with

Mass $M_\odot = 1.99 \times 10^{33}\,\mathrm{g}$

Radius $R_\odot = 6.96 \times 10^{10}\,\mathrm{cm}$

Gravitational potential energy of a proton near center of the sphere

$$\langle E_{grav} \rangle = -\frac{3}{2} \frac{G_N M_\odot m_p}{R_\odot} = -3.2\,\mathrm{keV}$$

Thermal velocity distribution

$$\langle E_{kin} \rangle = \frac{3}{2} k_B T = -\frac{1}{2} \langle E_{grav} \rangle$$

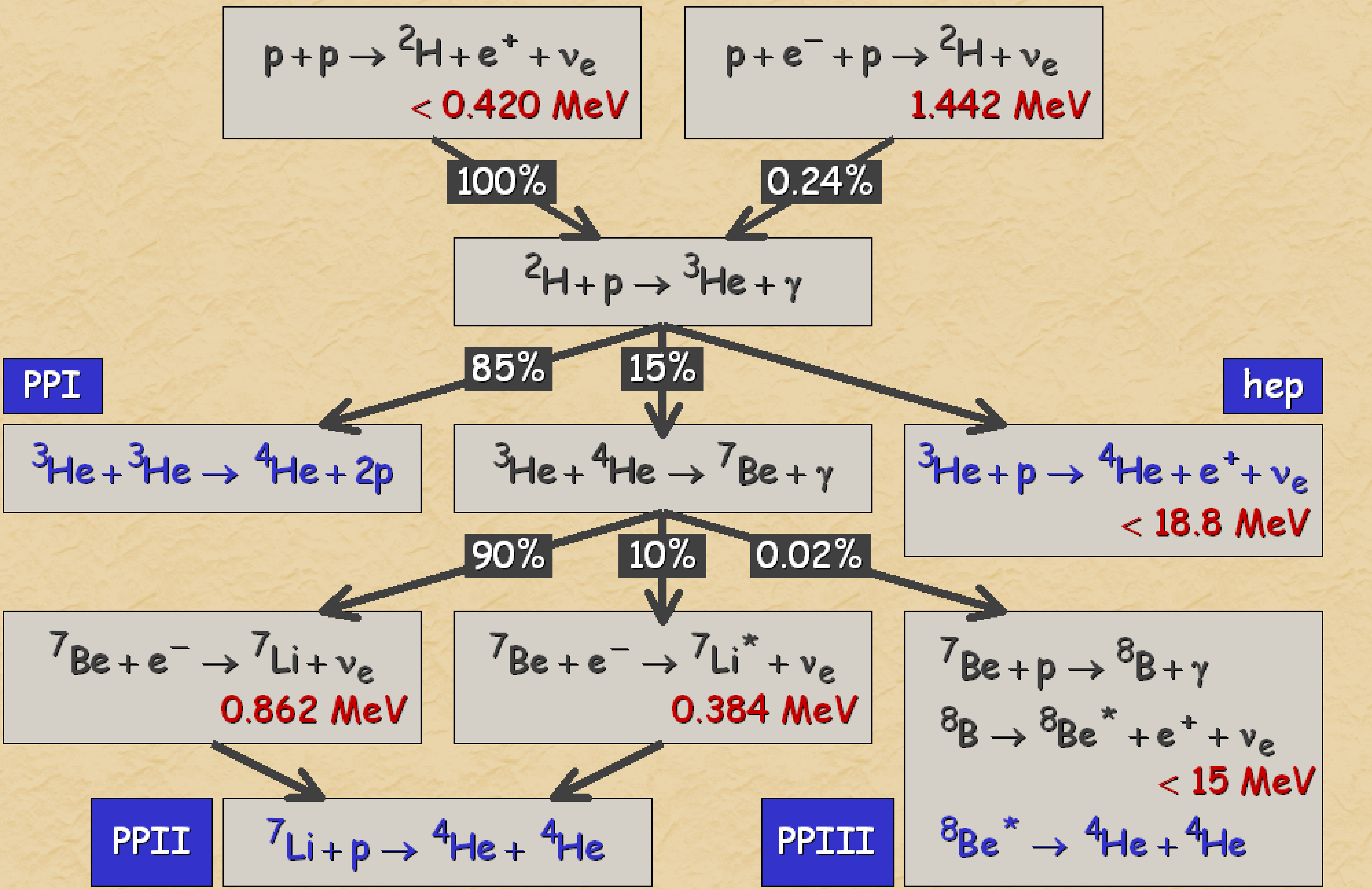
Estimated temperature

$$T = 1.1\,\mathrm{keV}$$

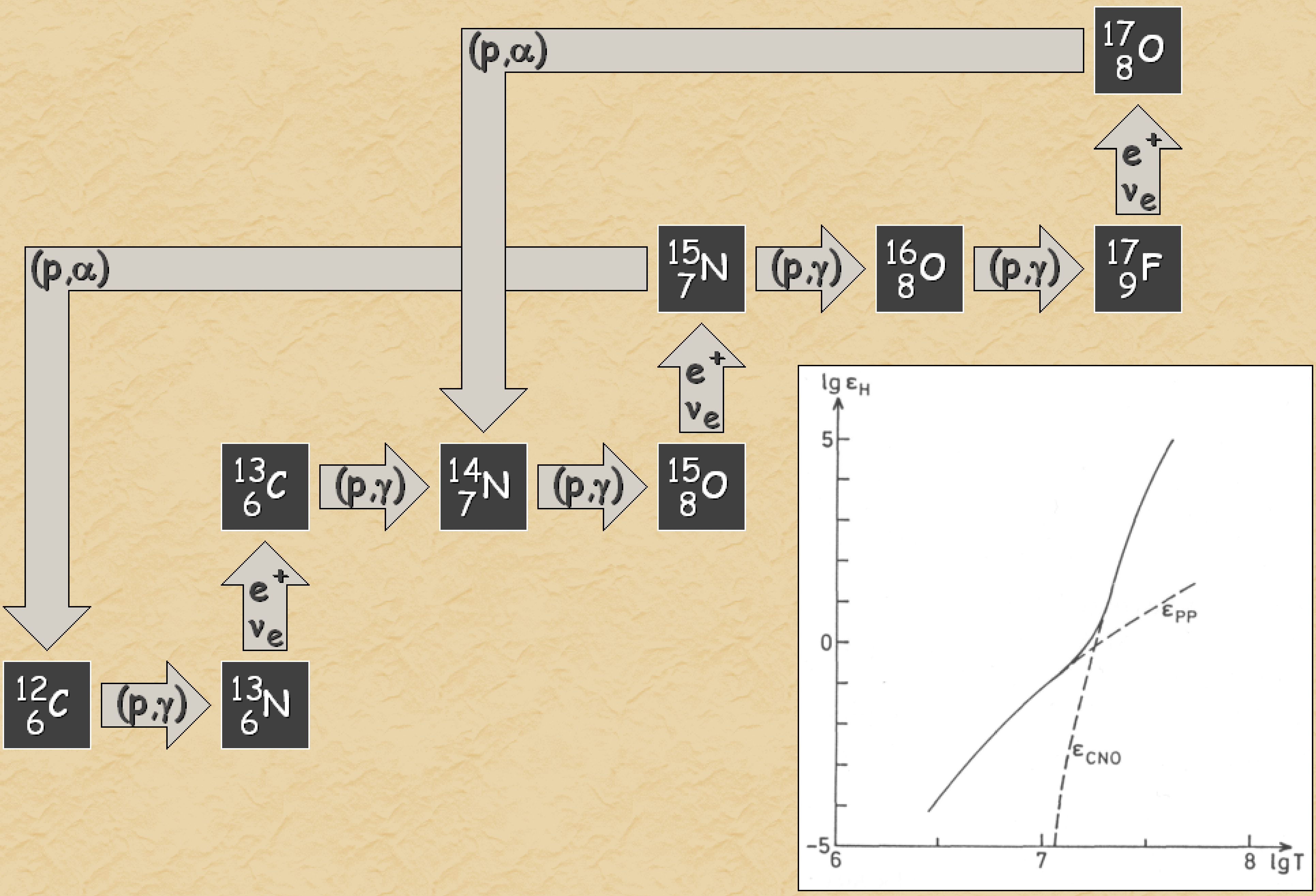
Central temperature from standard solar models

$$T_c = 1.56 \times 10^7\,\mathrm{K}$$
$$= 1.34\,\mathrm{keV}$$

Hydrogen burning: Proton-Proton Chains



Hydrogen Burning: CNO Cycle



Thermonuclear Reactions and Gamow Peak

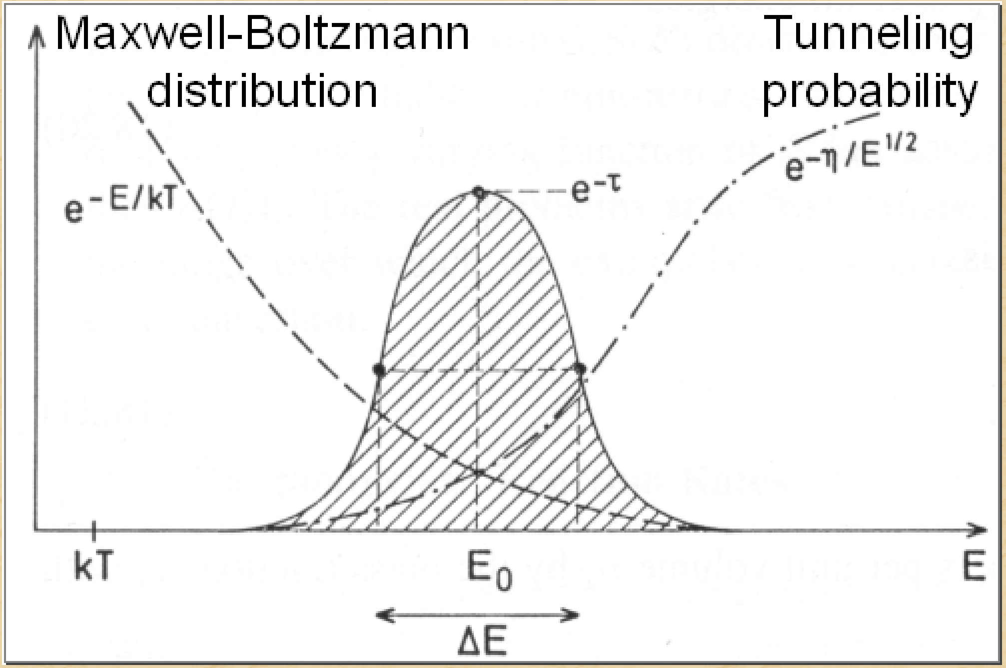
Coulomb repulsion prevents nuclear reactions, except for Gamow tunneling

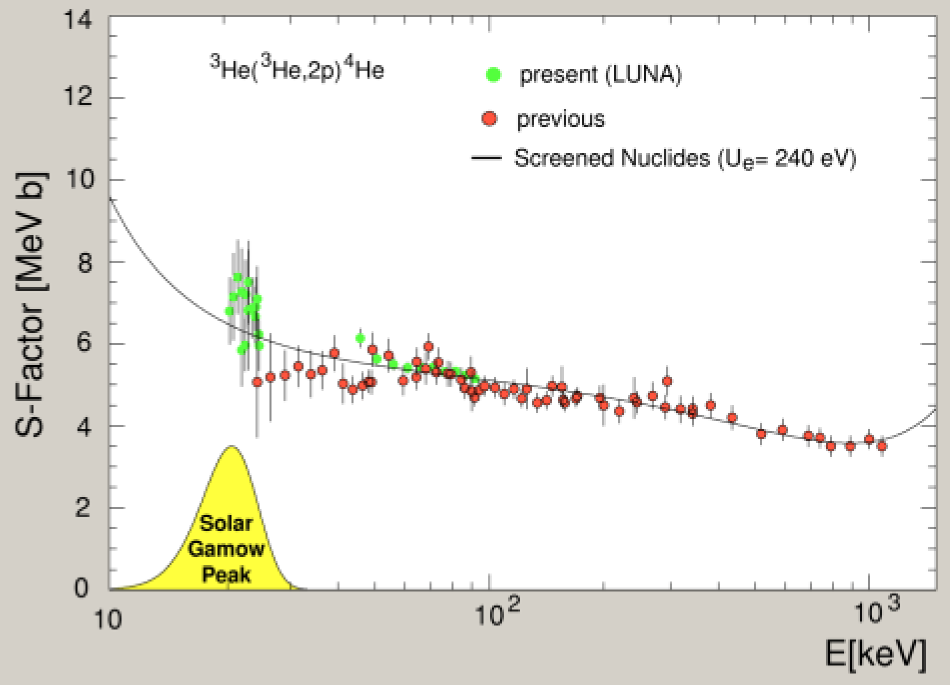
Tunneling probability $\propto E^{-1/2} e^{-2\pi\eta}$ with Sommerfeld parameter

$$\eta = \left(\frac{m}{2}\right)^{1/2} \frac{Z_1 Z_2 e^2}{E^{1/2}}$$

Parameterize cross section with astrophysical S-factor

$$S(E) = \sigma(E)\, E\, e^{2\pi\eta(E)}$$





Main Nuclear Burnings

Hydrogen burning $4p + 2e^- \rightarrow {}^4He + 2\nu_e$

- Proceeds by pp chains and CNO cycle
- No higher elements are formed because no stable isotope with mass number 8
- Neutrinos from $p \rightarrow n$ conversion
- Typical temperatures: 10^7 K (~1 keV)

Helium burning

$${}^4He + {}^4He + {}^4He \leftrightarrow {}^8Be + {}^4He \rightarrow {}^{12}C$$

"Triple alpha reaction" because ^8Be unstable, builds up with concentration ~ 10^{-9}

$${}^{12}C + {}^4He \rightarrow {}^{16}O$$

$${}^{16}O + {}^4He \rightarrow {}^{20}Ne$$

Typical temperatures: 10^8 K (~10 keV)

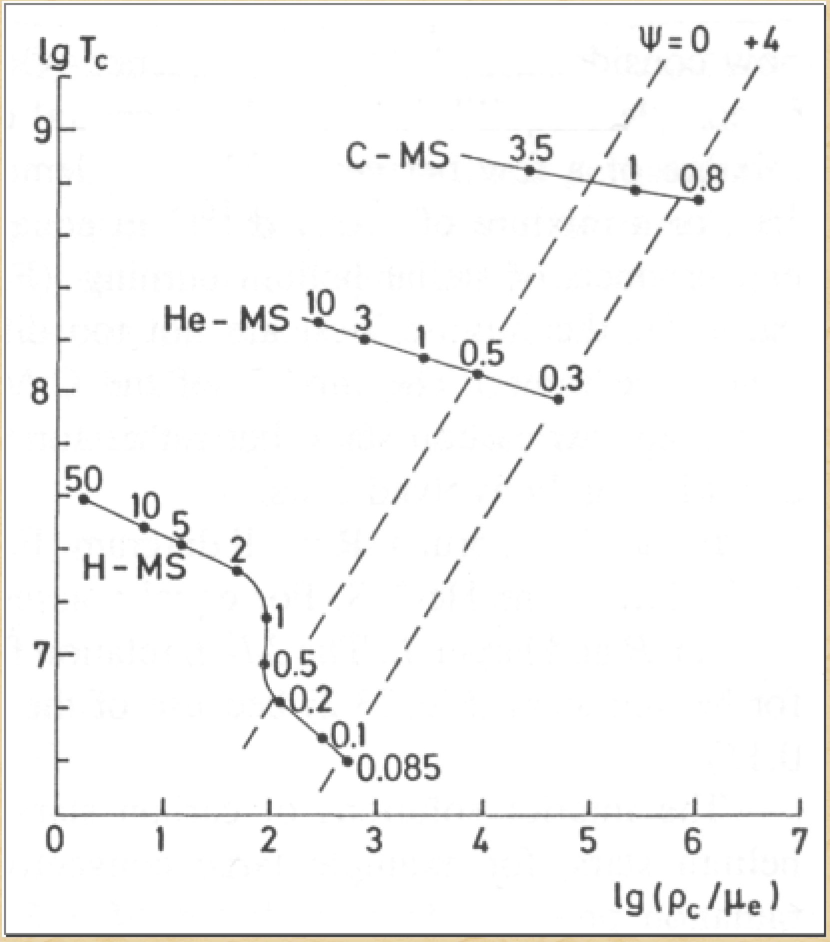
Carbon burning

Many reactions, for example

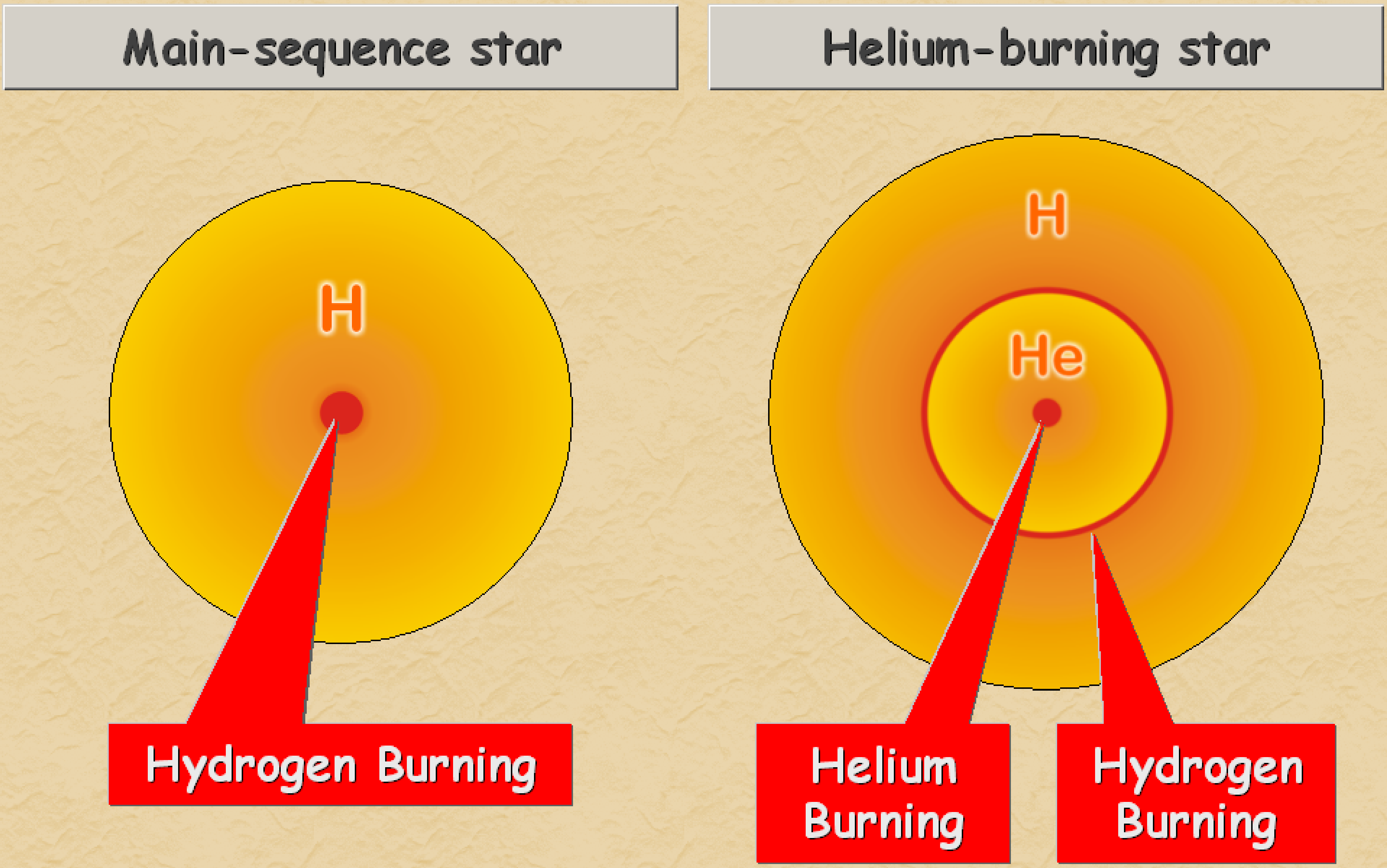
$${}^{12}C + {}^{12}C \rightarrow {}^{23}Na + p \quad \text{or} \quad {}^{20}Ne + {}^4He \quad \text{etc}$$

Typical temperatures: 10^9 K (~100 keV)

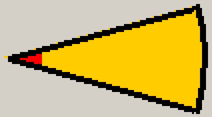
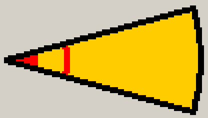
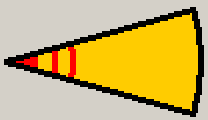
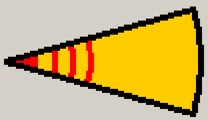
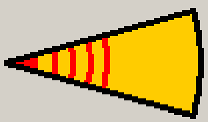
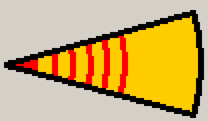
- Each type of burning occurs at very different T but broad range of density
- Never co-exist in same location



Hydrogen Exhaustion



Burning Phases of a 15 Solar-Mass Star

Burning Phase		Dominant Process	T_c [keV]	ρ_c [g/cm³]	L_γ [$10^4\ L_{sun}$]	L_ν/L_γ	Duration [years]
	Hydrogen	H → He	3	5.9	2.1	-	1.2×10^7
	Helium	He → C, O	14	1.3×10^3	6.0	1.7×10^{-5}	1.3×10^6
	Carbon	C → Ne, Mg	53	1.7×10^5	8.6	1.0	6.3×10^3
	Neon	Ne → O, Mg	110	1.6×10^7	9.6	1.8×10^3	7.0
	Oxygen	O → Si	160	9.7×10^7	9.6	2.1×10^4	1.7
	Silicon	Si → Fe, Ni	270	2.3×10^8	9.6	9.2×10^5	6 days

Neutrinos from Thermal Plasma Processes

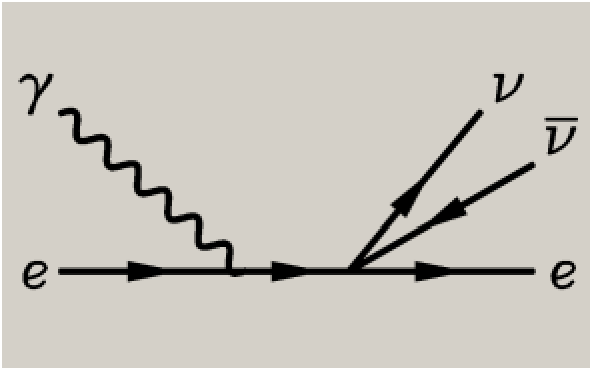
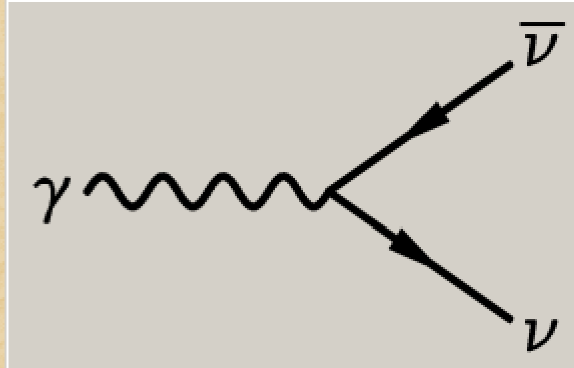
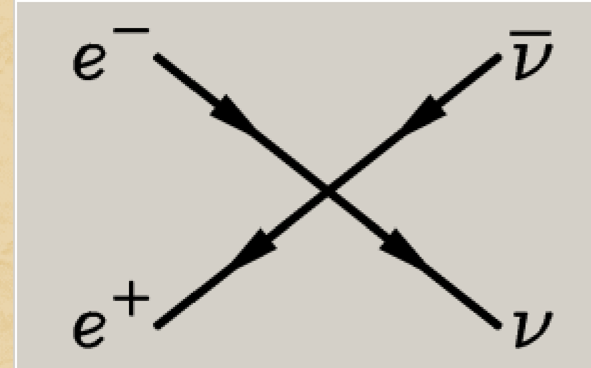


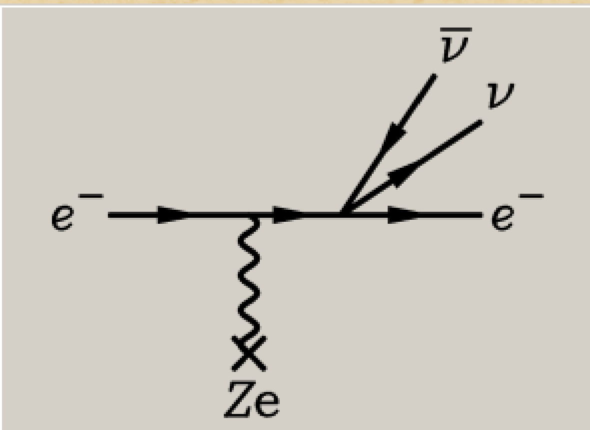
Photo (Compton)



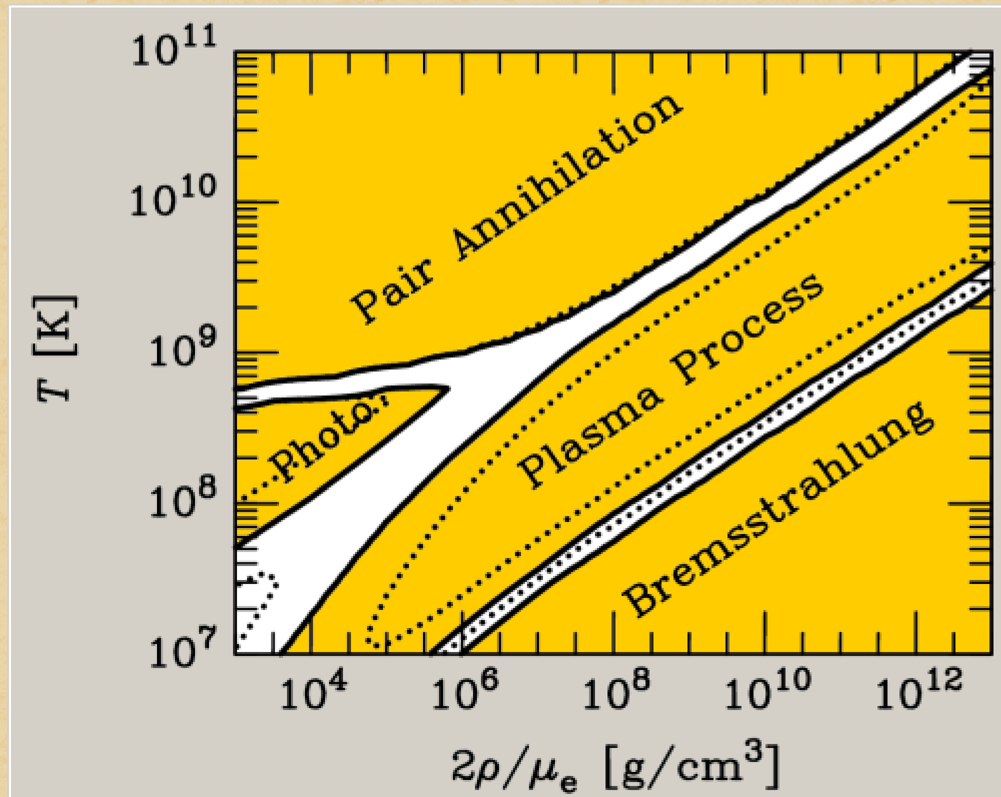
Plasmon decay



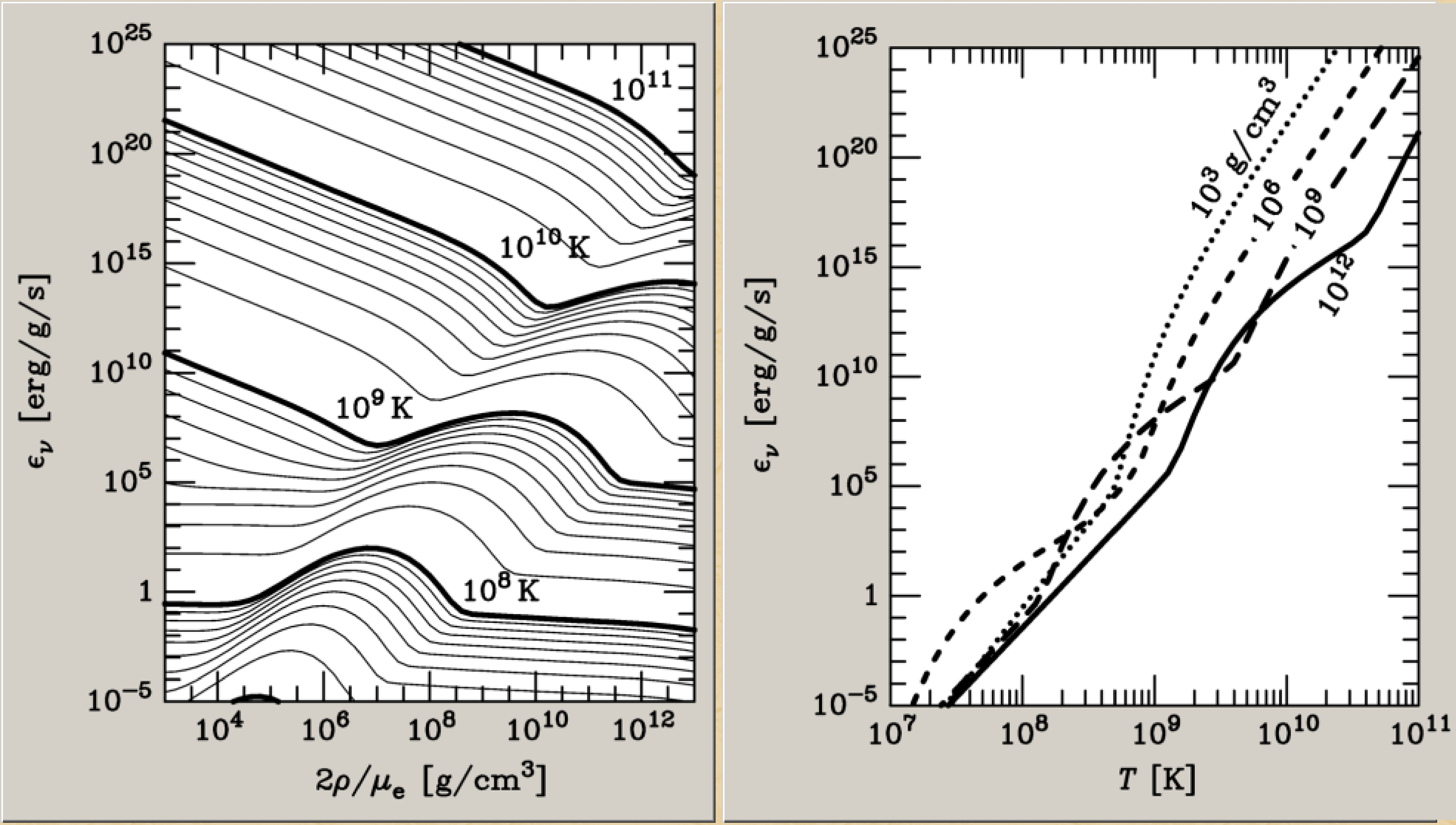
Pair annihilation



Bremsstrahlung



Neutrino Energy Loss Rates



Existence of Direct Neutrino-Electron Coupling

VOLUME 24, NUMBER 10 PHYSICAL REVIEW LETTERS 9 MARCH 1970

ASTROPHYSICAL DETERMINATION OF THE COUPLING CONSTANT FOR THE ELECTRON-NEUTRINO WEAK INTERACTION

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(Received 22 December 1969)

The existence of the $(\bar{e}\nu_e)(\bar{\nu}_e e)$ weak interaction is confirmed by the results of nine astrophysical tests. The value of the coupling constant is equal to, or close to, the coupling constant of beta decay, namely, $g^2 = 10^{0\pm 2} g_\beta{}^2$.

Of all the astrophysical tests applied so far for the inference of a direct electron-neutrino interaction in nature, none has unambiguously provided a useful upper limit on the coupling constant, which in the $V-A$ theory of Feynman and Gell-Mann[1] is taken to be equal to the "universal" weak-interaction coupling constant measured from beta decays (called g_β hereafter). However, it is important to point out that these tests, made by the author and his colleagues during the past eight years, do provide a nonzero lower limit, and therefore establish at least the existence of the $(\bar{e}\nu_e)(\bar{\nu}_e e)$ interaction. It should be emphasized, nonetheless, that all of these tests rely on the validity of various stellar model calculations. These models, while not subject to scrutiny in the same sense as a laboratory ex-

relative theoretical lifetimes, calculated with and without the inclusion of neutrino emission. In this Letter, the unmodified term "luminosity" will mean the photon luminosity L radiated by the star. The "neutrino luminosity" will be designated L_ν. Quantities referring to the sun are subscripted with an encircled dot.

The most accurate available data on white dwarfs are those collected by Eggen[7] for the two clusters Hyades and Pleiades and for the nearby general field. Of chief interest here are the hot white dwarfs, for which the observational data[7,8] have been reduced following the procedure of Van Horn.[9] The resulting luminosities are estimated to have a statistical accuracy of ±0.1 in $\log(L/L_\odot)$, which is adequate here.

Models of cooling white dwarfs have been con-

Self-Regulated Nuclear Burning

Main-Sequence Star

Virial Theorem $\langle E_{kin} \rangle = -\frac{1}{2} \langle E_{grav} \rangle$

Small Contraction
→ Heating
→ Increased nuclear burning
→ Increased pressure
→ Expansion

Additional energy loss ("cooling")
→ Loss of pressure
→ Contraction
→ Heating
→ Increased nuclear burning

Hydrogen burning at a nearly fixed T
→ Gravitational potential nearly fixed:
$G_N M/R \sim$ constant
→ $R \propto M$ (More massive stars bigger)

Degenerate Stars ("White Dwarfs")

Assume T very small
→ No thermal pressure
→ Electron degeneracy for pressure

Pressure ~ Momentum density x Velocity

- Electron density $n_e = p_F^3/(3\pi^2)$
- Momentum p_F (Fermi momentum)
- Velocity $v \sim p_F / m_e$
- Pressure $P \propto p_F^5 \propto \rho^{5/3} \propto M^{5/3} R^{-5}$
- Density $\rho \propto M R^{-3}$

(Stellar mass M and radius R)

Hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{G_N M_r \rho}{r^2}$$

With dP/dr ~ -P/R we have approximately

$$P \sim G_N M \rho R^{-1} \sim G_N M^2 R^{-4}$$

Inverse mass-radius relationship for degenerate stars $R \propto M^{-1/3}$

$$R = 10{,}500\,\text{km} \left(\frac{0.6 M_\odot}{M}\right)^{1/3} (2Y_e)^{5/3}$$

(Y_e electrons per nucleon)

For sufficiently large mass electrons become relativistic

- Velocity = speed of light
- Pressure

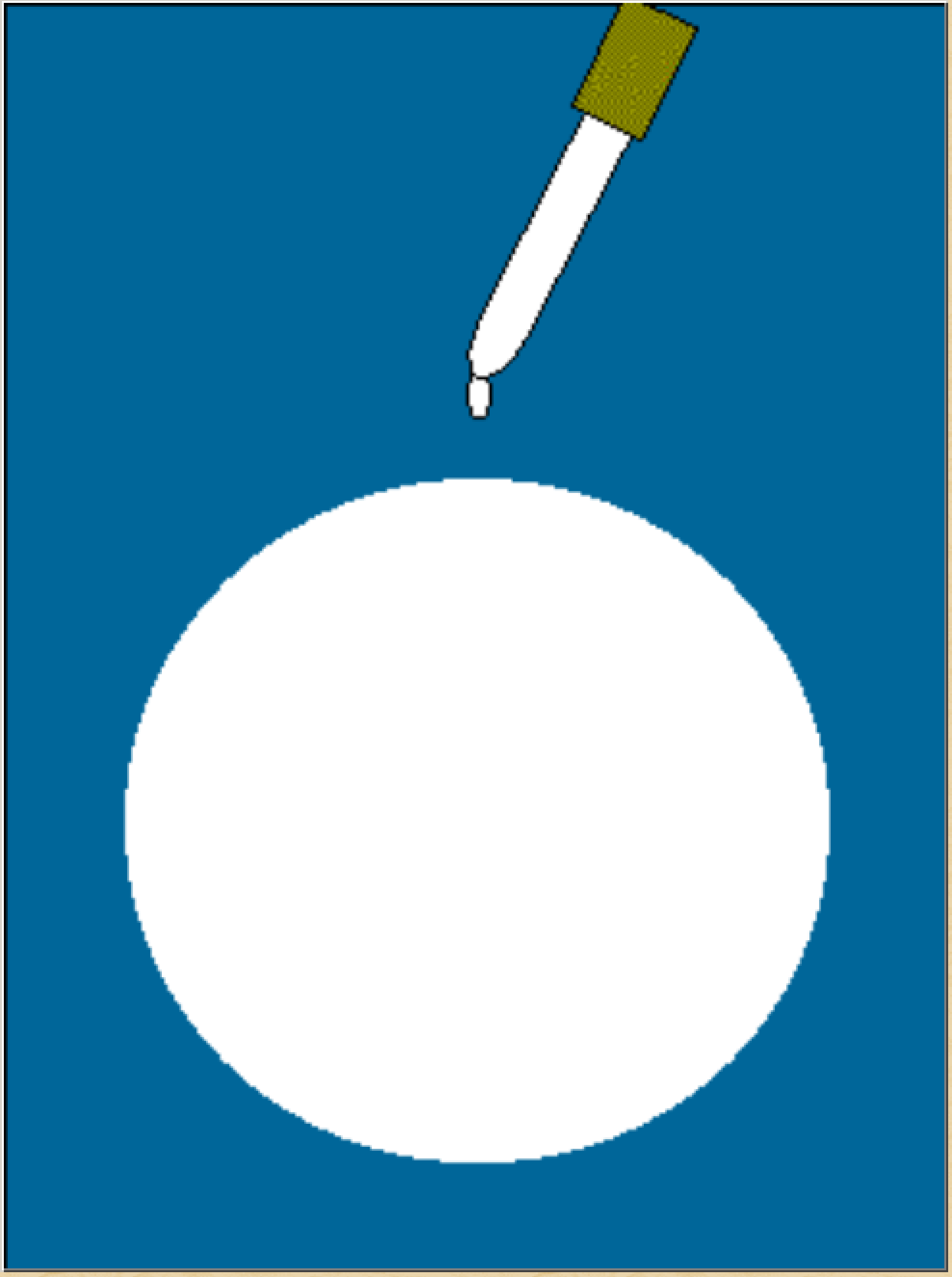
$$P \propto p_F^4 \propto \rho^{4/3} \propto M^{4/3} R^{-4}$$

No stable configuration

Chandrasekhar mass limit

$$M_{Ch} = 1.457 M_\odot (2Y_e)^2$$

Degenerate Stars

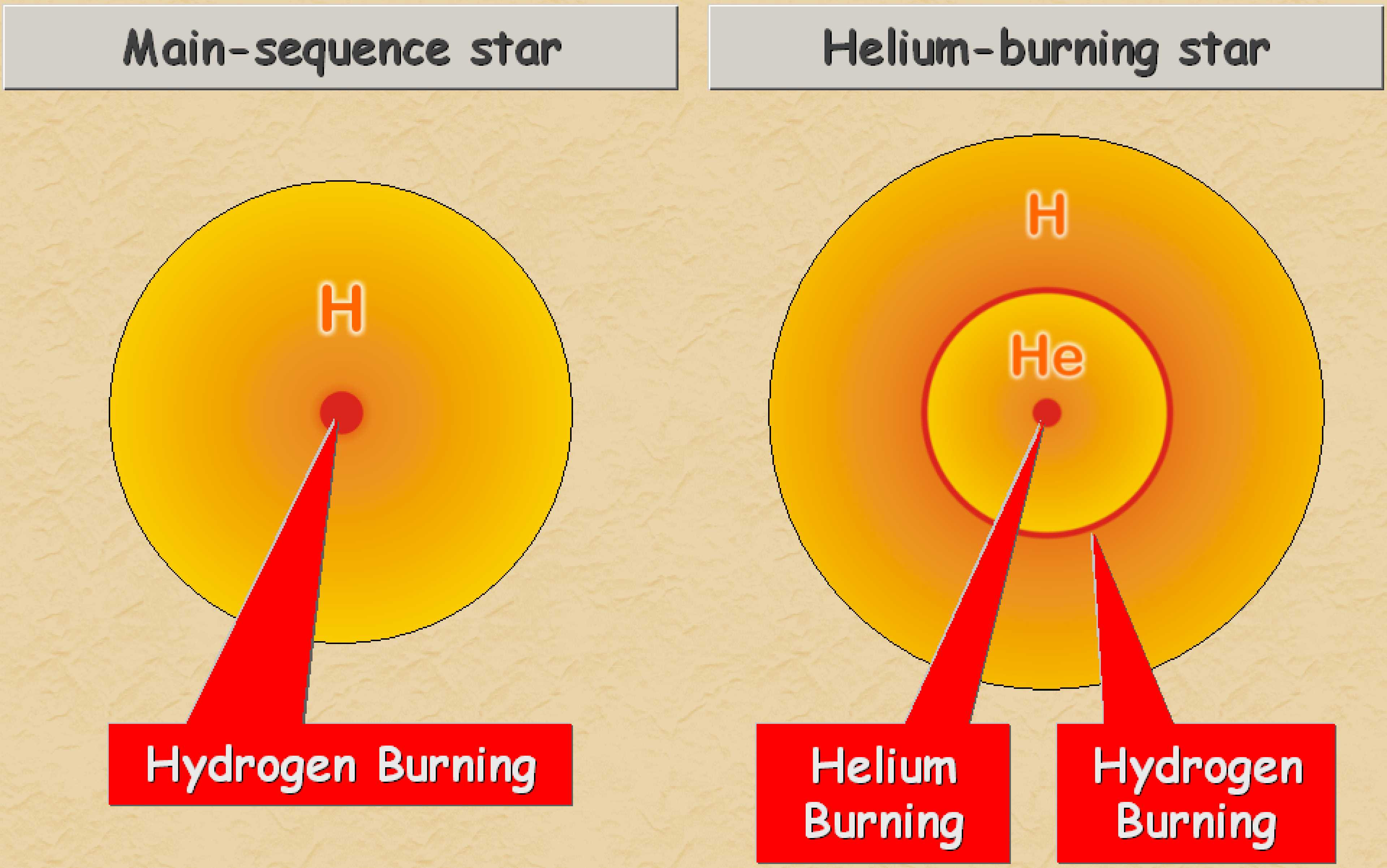


Inverse mass-radius relationship for degenerate stars $R \propto M^{-1/3}$

Chandrasekhar mass limit

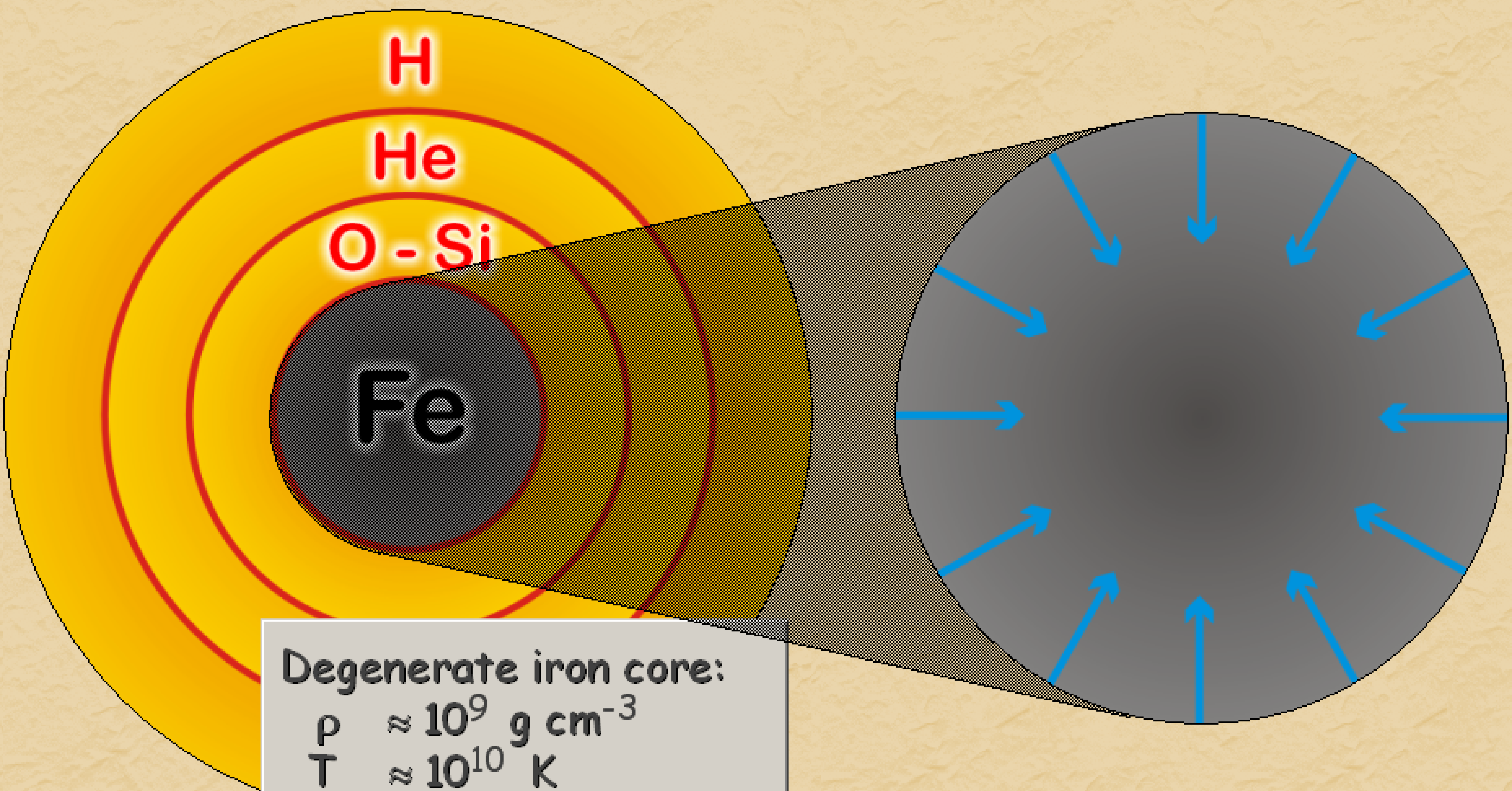
$$M_{Ch} = 1.457 M_{\odot} (2Y_e)^2$$

Stellar Collapse



Stellar Collapse

Onion structure	Collapse (implosion)



Degenerate iron core:

$\rho \approx 10^9$ g cm^{-3}

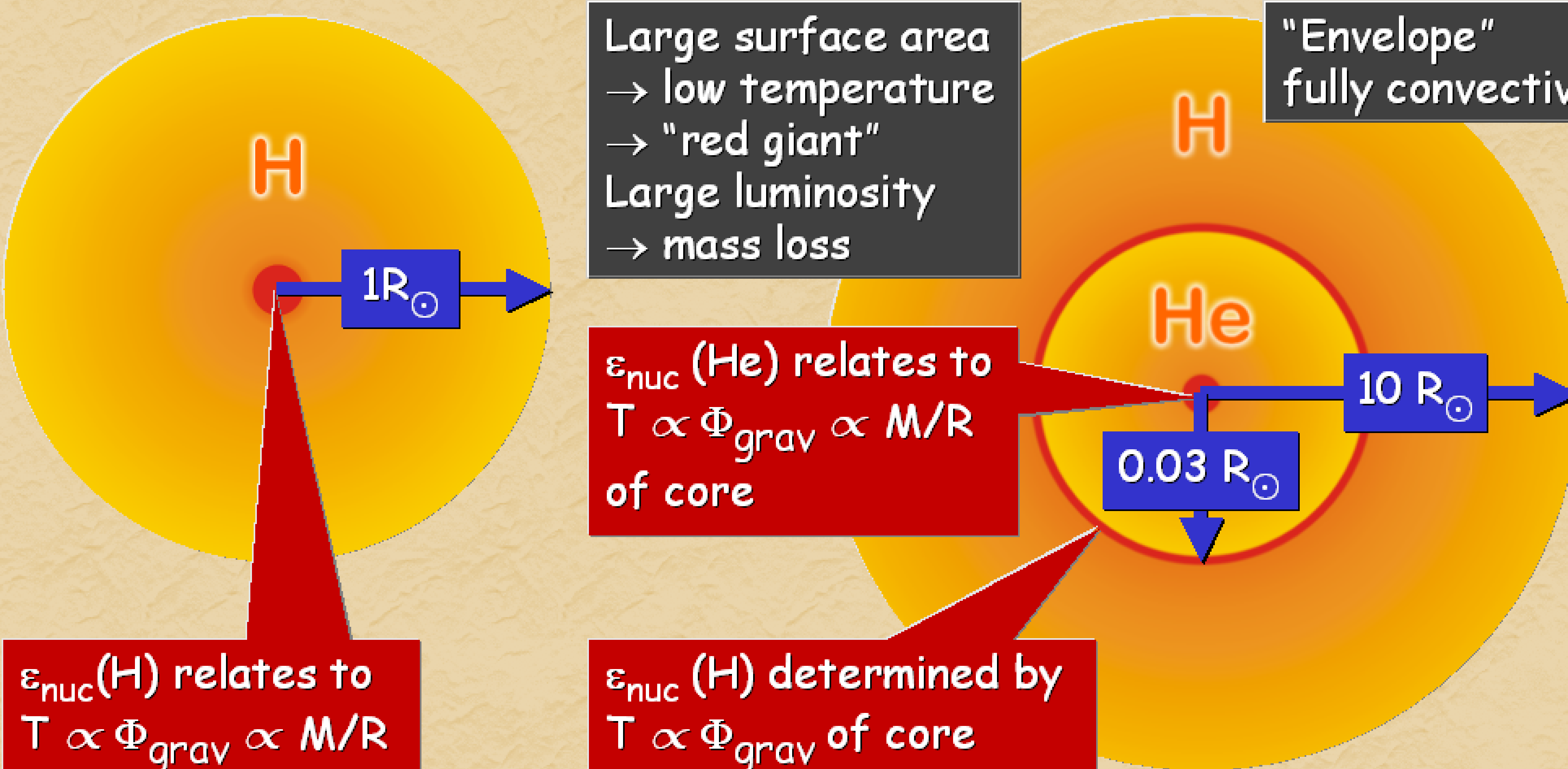
$T \approx 10^{10}$ K

$M_{Fe} \approx 1.5\ M_{sun}$

$R_{Fe} \approx 8000$ km

Giant Stars

Main-sequence star $1M_\odot$ (Hydrogen burning)	Helium-burning star $1M_\odot$



Large surface area
→ low temperature
→ "red giant"
Large luminosity
→ mass loss

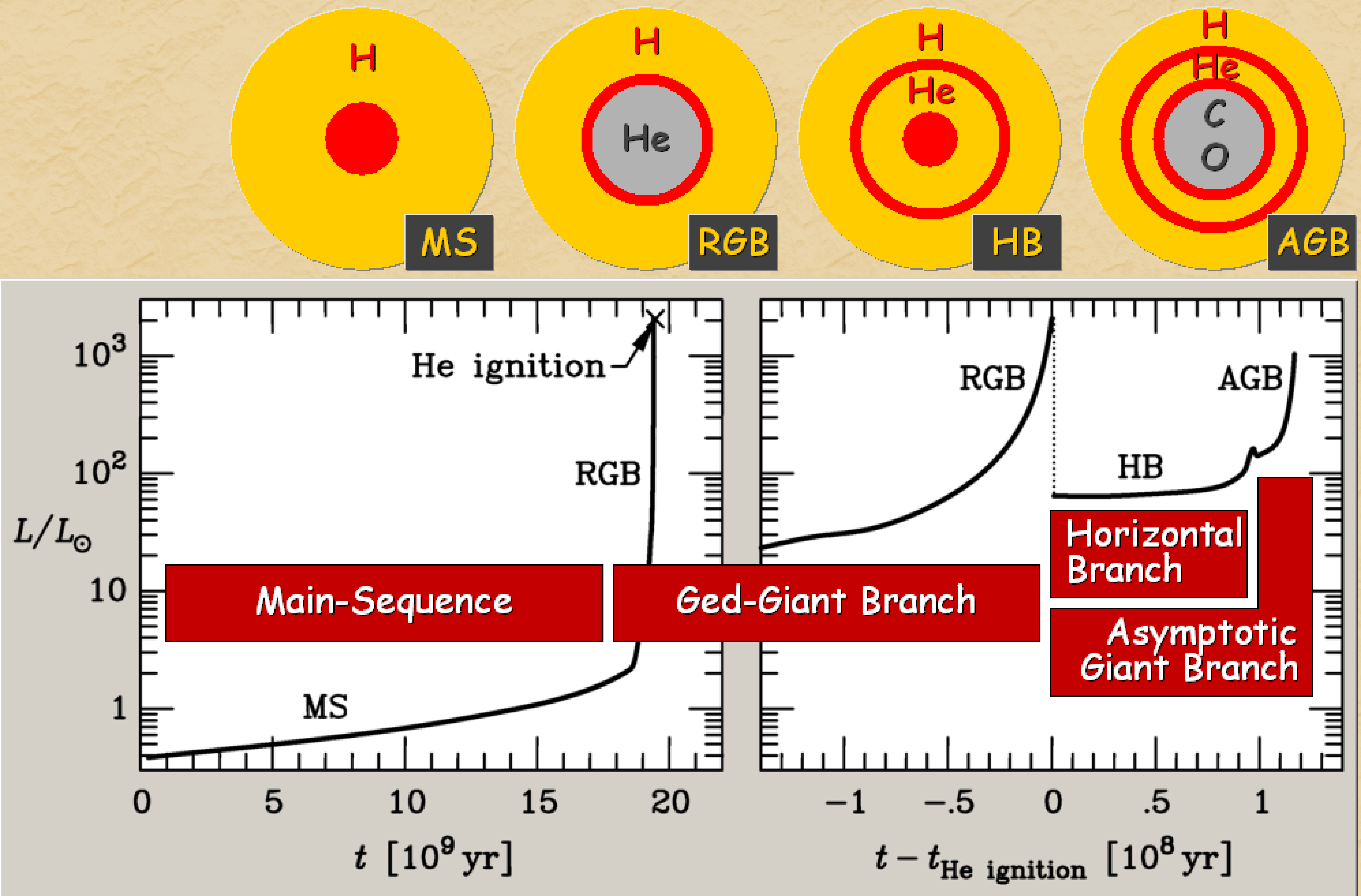
"Envelope" fully convective

ε_{nuc}(H) relates to $T \propto \Phi_{grav} \propto M/R$ of full star

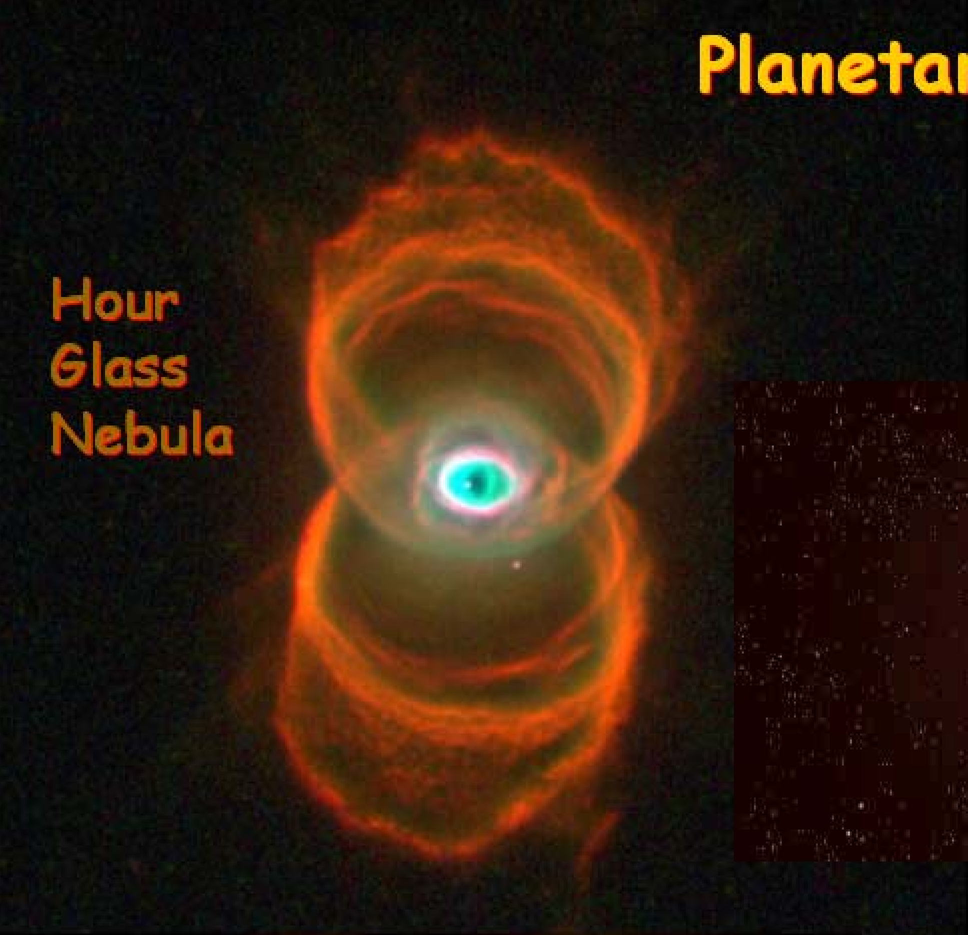
ε_{nuc} (He) relates to $T \propto \Phi_{grav} \propto M/R$ of core

ε_{nuc} (H) determined by $T \propto \Phi_{grav}$ of core → huge L(H)

Evolution of a Low-Mass Star



Planetary Nebulae



Hour Glass Nebula

Planetary Nebula IC 418

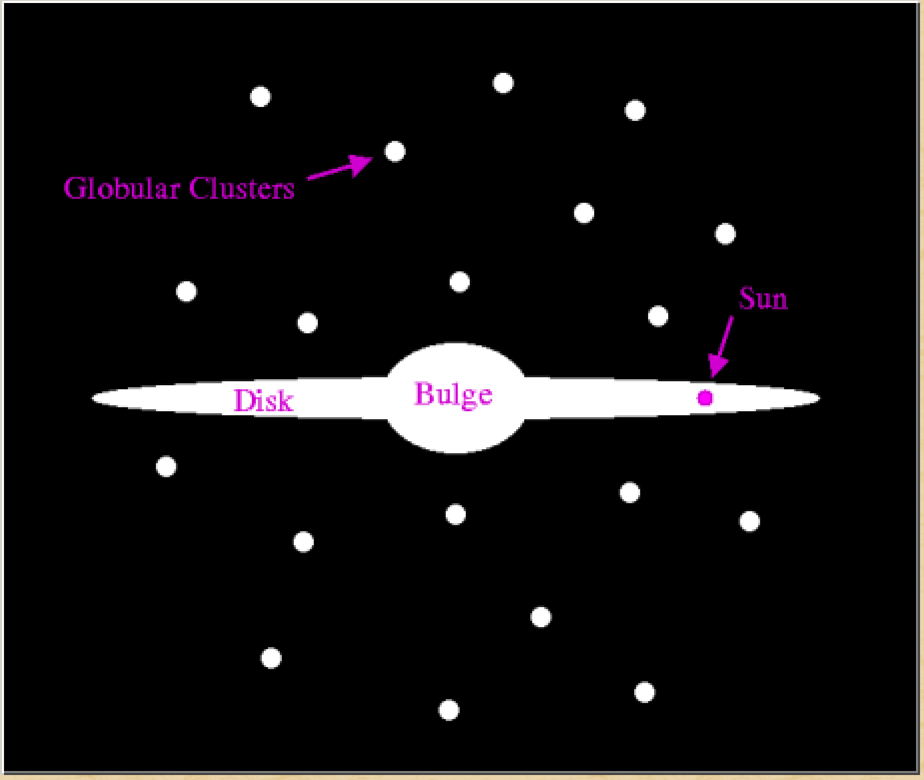
Planetary Nebula NGC 3132

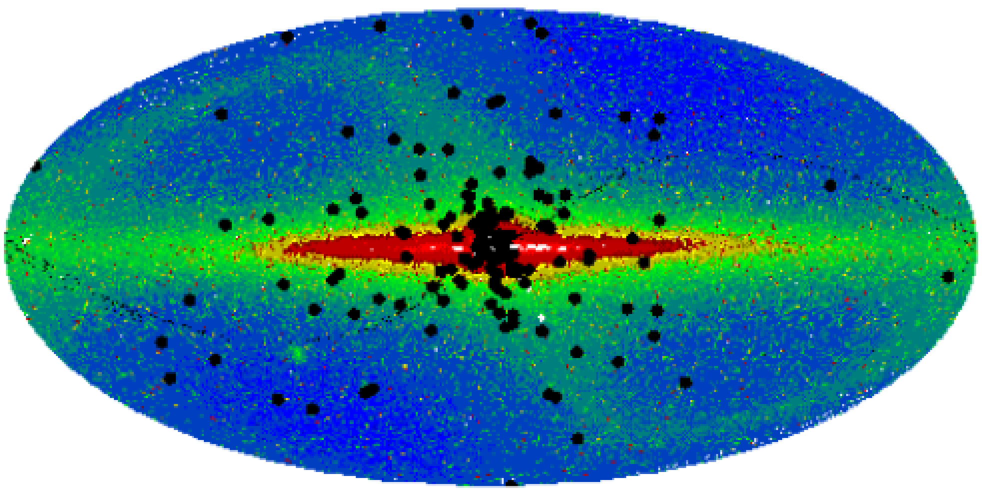
Eskimo Nebula

Evolution of Stars

$M < 0.08\,M_\odot$	Never ignites hydrogen → cools ("hydrogen white dwarf")		Brown dwarf
$0.08 < M \lesssim 0.8\,M_\odot$	Hydrogen burning not completed in Hubble time		Low-mass main-squence star
$0.8 \lesssim M \lesssim 2\,M_\odot$	Degenerate helium core after hydrogen exhaustion		• Carbon-oxygen white dwarf • Planetary nebula
$2\,M_\odot \lesssim M < 5\text{–}8\,M_\odot$	Helium ignition non-degenerate		• Carbon-oxygen white dwarf • Planetary nebula
$5\text{–}8\,M_\odot < M < ???$	All burning cycles → Onion skin structure with degenerate iron core	Core collapse supernova	• Neutron star (often pulsar) • Sometimes black hole? • Supernova remnant (SNR), e.g. crab nebula

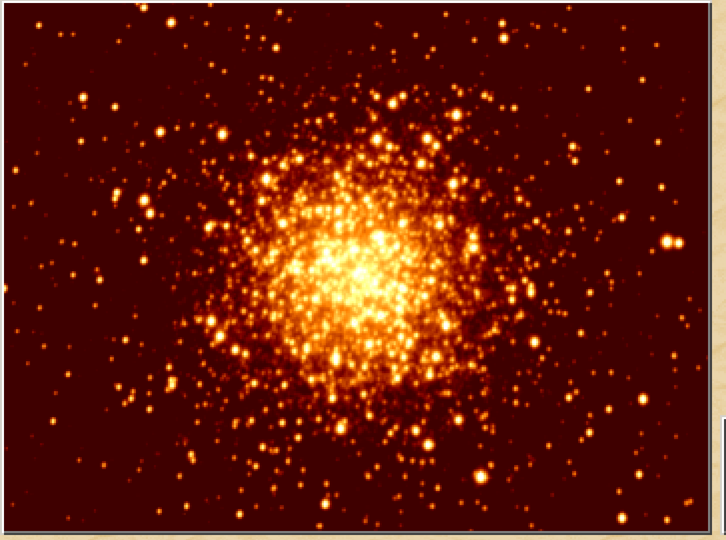
Globular Clusters of the Milky Way





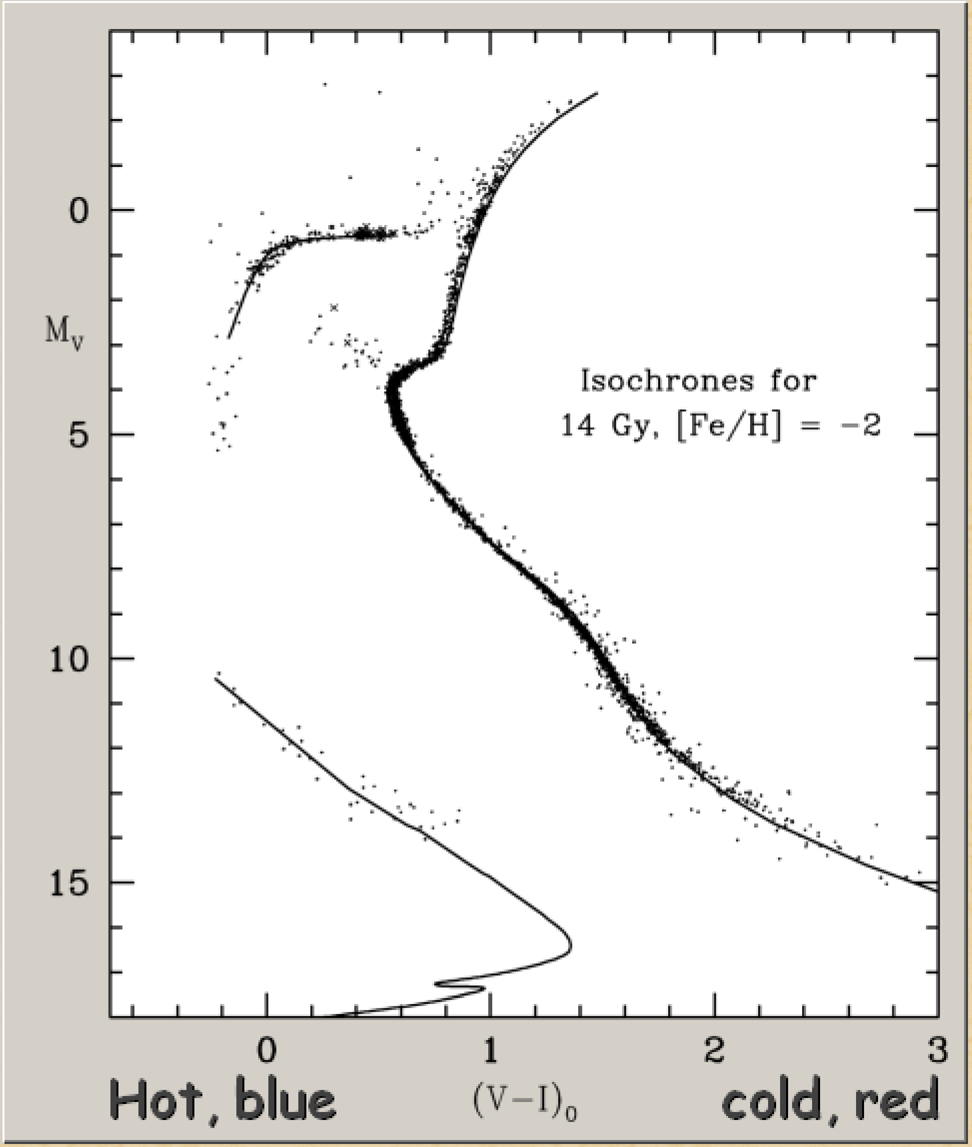
http://www.dartmouth.edu/~chaboyer/mwgc.html

Globular clusters on top of the FIRAS 2.2 micron map of the Galaxy



The galactic globular cluster M3

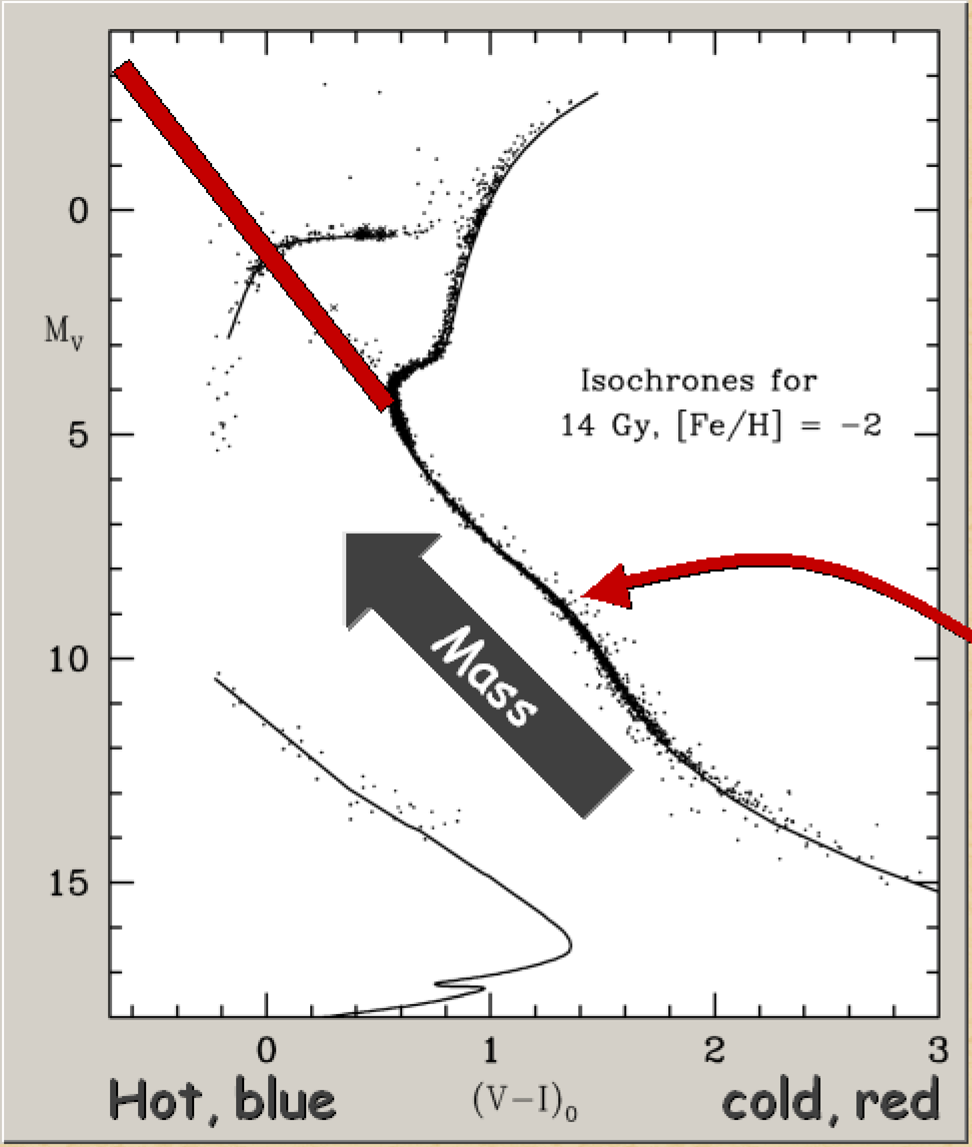
Color-Magnitude Diagram for Globular Clusters

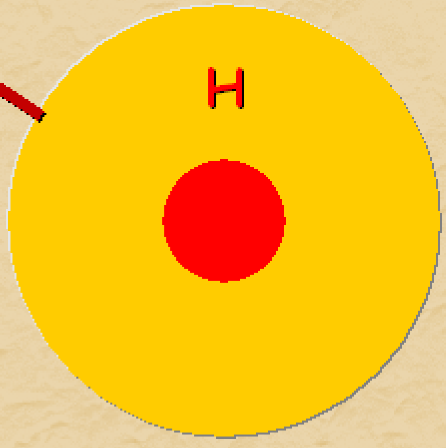


Color-magnitude diagram synthesized from several low-metallicity globular clusters and compared with theoretical isochrones (W.Harris)

Color-Magnitude Diagram for Globular Clusters

- Stars with M so large that they have burnt out in a Hubble time
- No new star formation in globular clusters

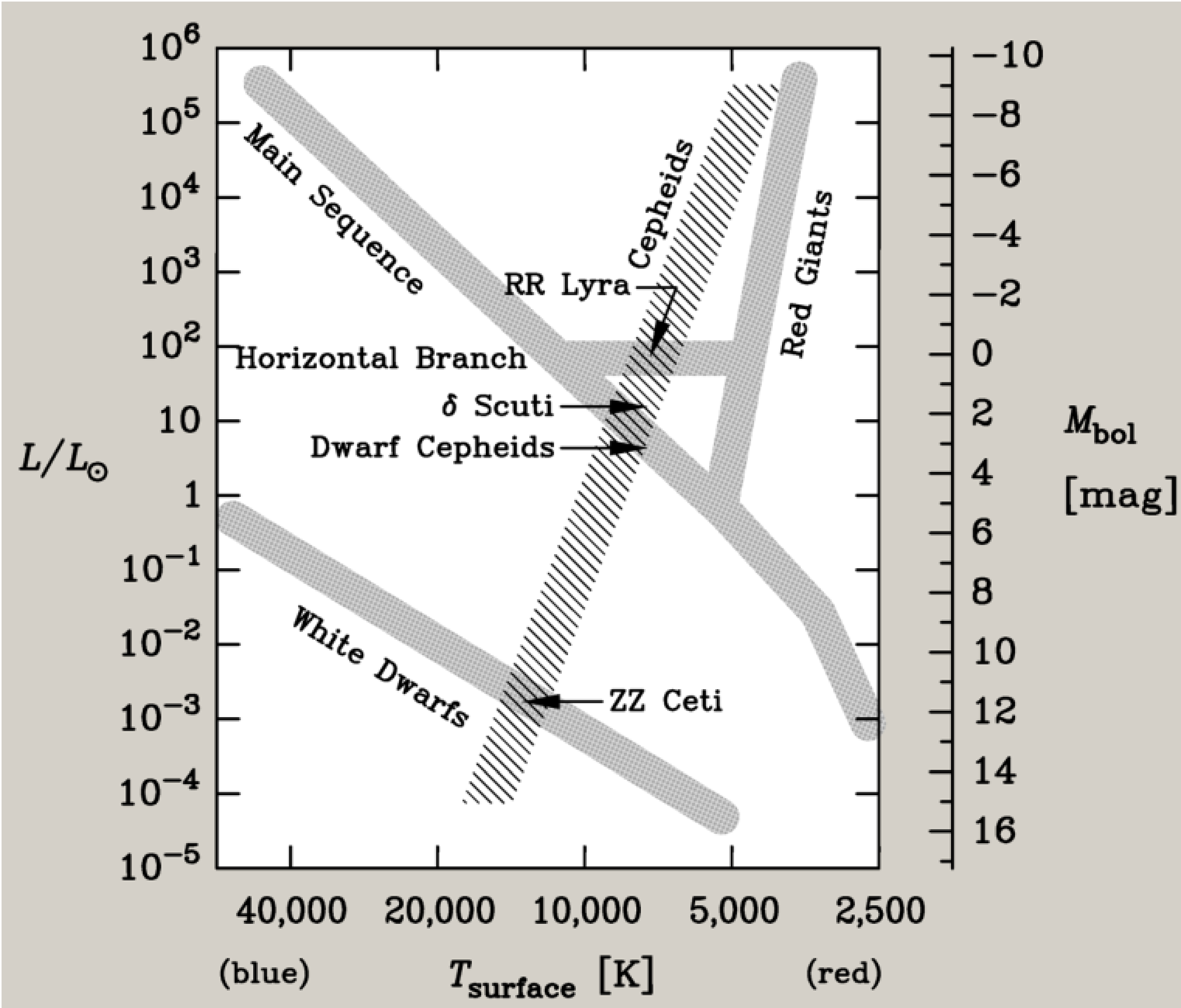




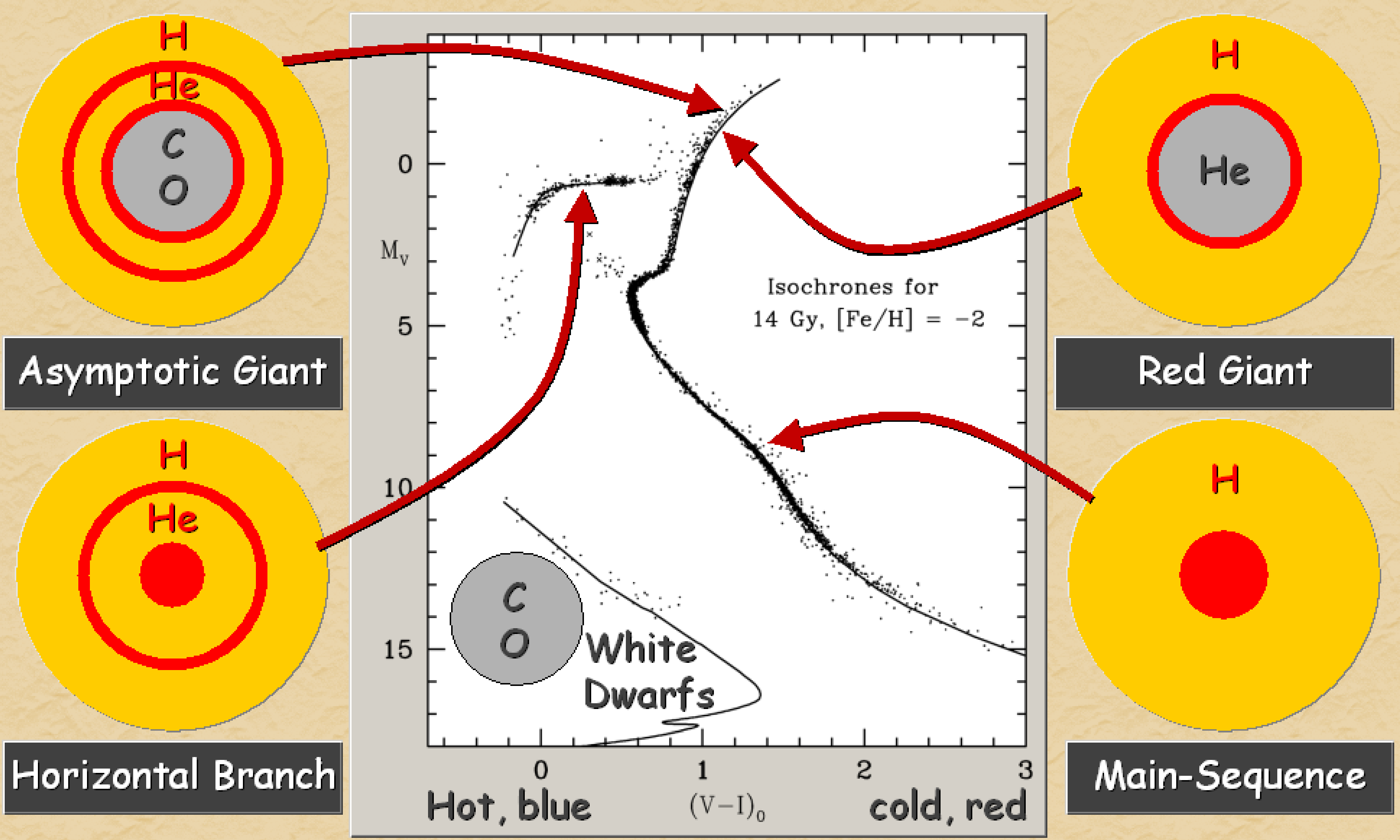
Main-Sequence

Color-magnitude diagram synthesized from several low-metallicity globular clusters and compared with theoretical isochrones (W.Harris)

Schematic Hertzsprung-Russell-Diagram

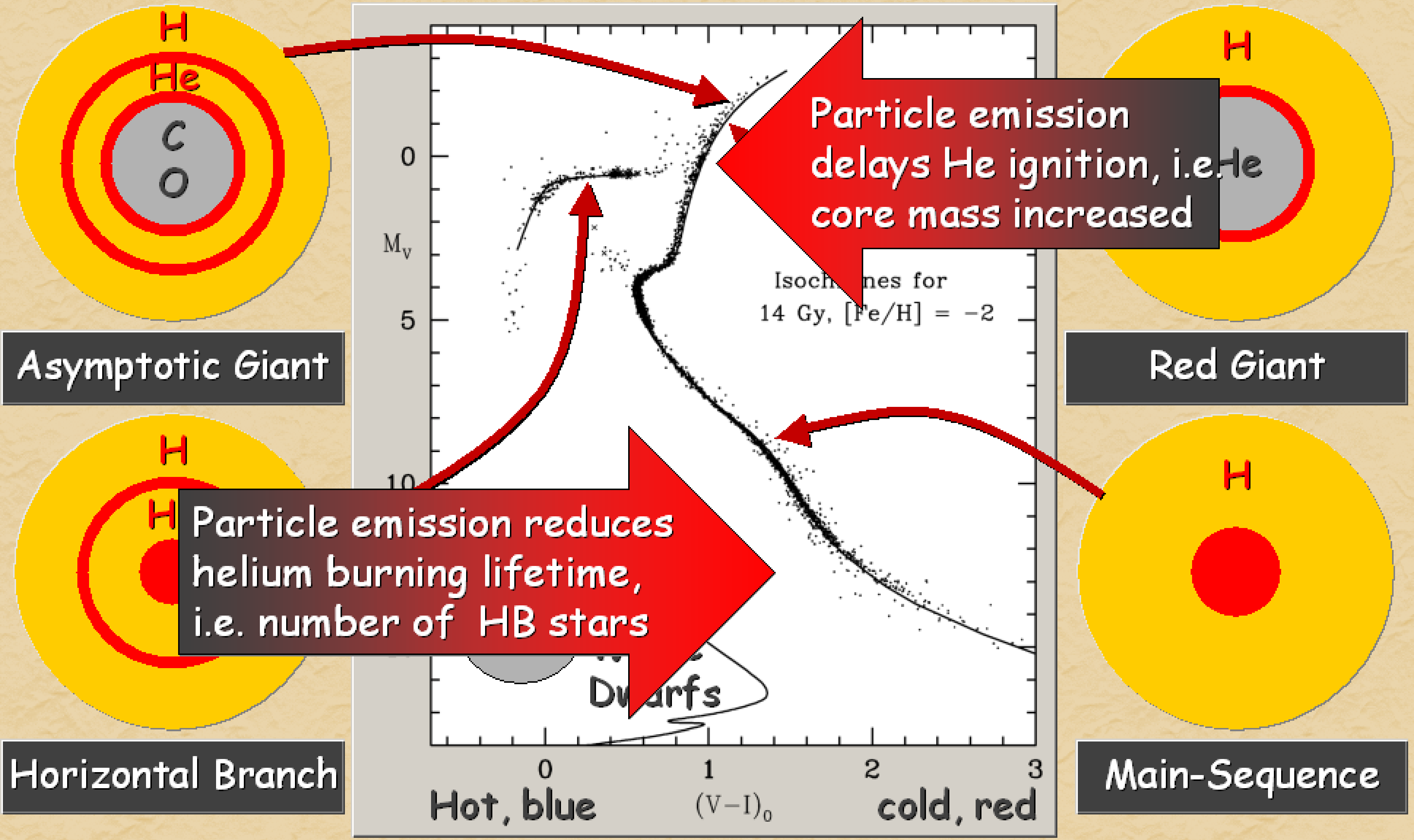


Color-Magnitude Diagram for Globular Clusters



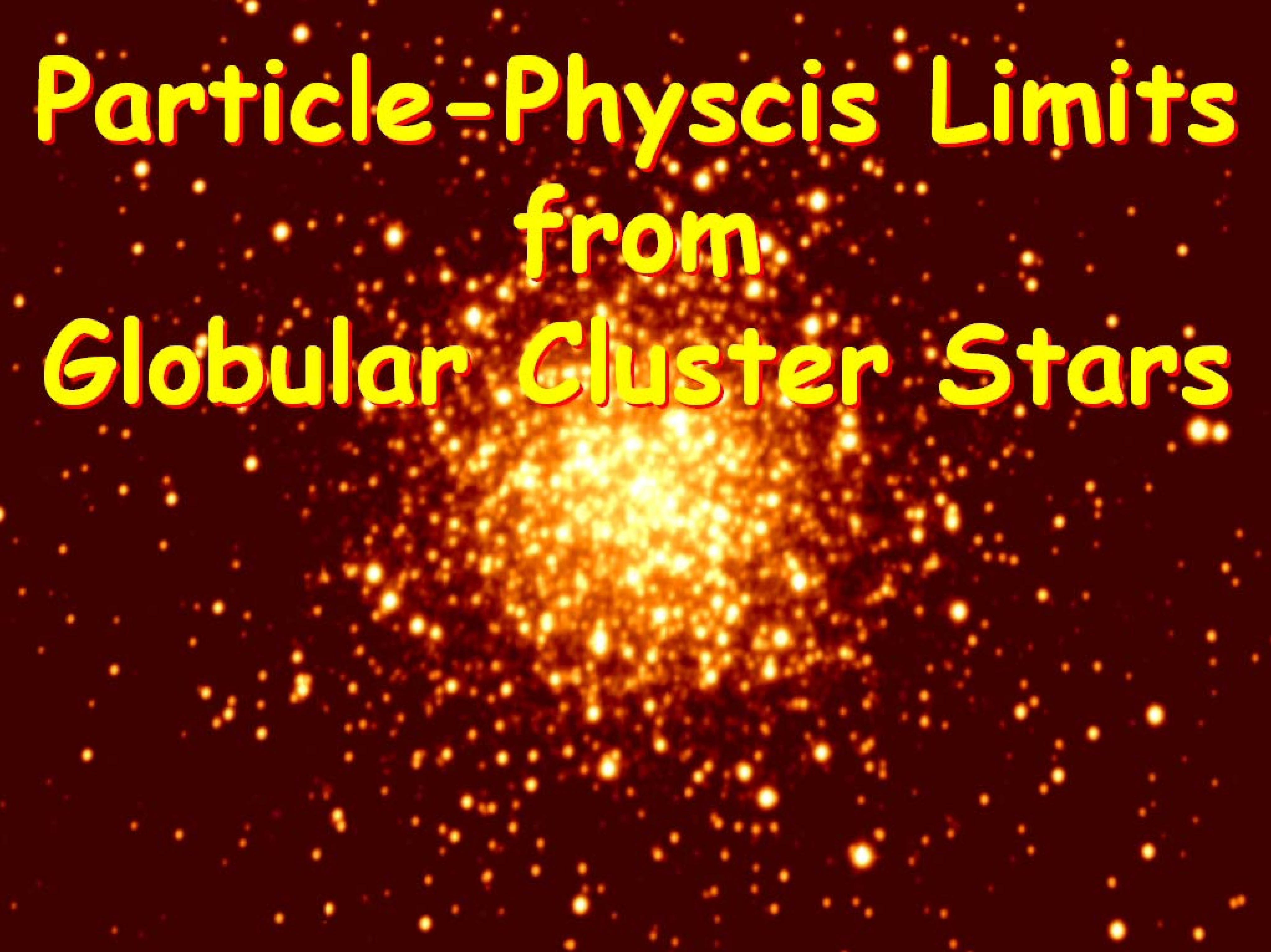
Color-magnitude diagram synthesized from several low-metallicity globular clusters and compared with theoretical isochrones (W.Harris)

Color-Magnitude Diagram for Globular Clusters

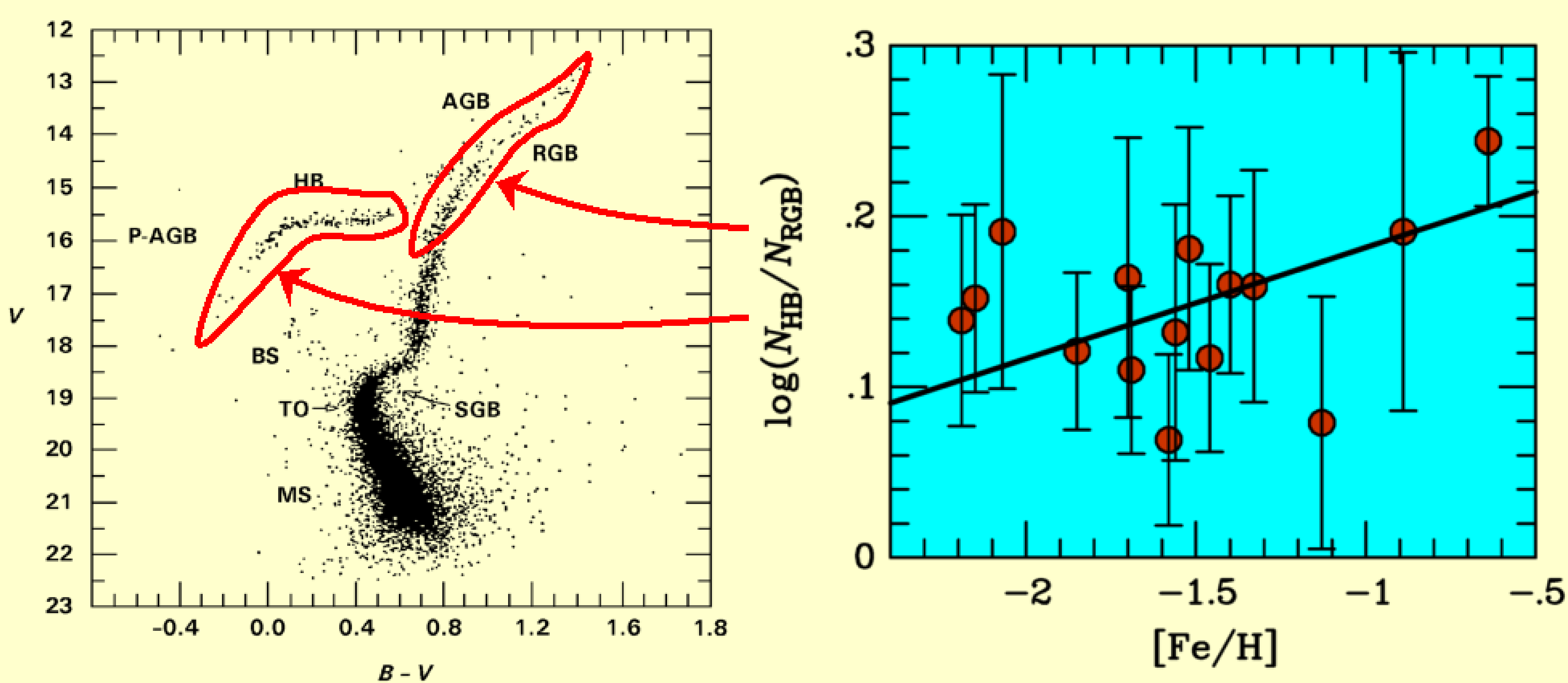


Color-magnitude diagram synthesized from several low-metallicity globular clusters and compared with theoretical isochrones (W.Harris)

Particle-Physcis Limits from Globular Cluster Stars



Helium-Burning Lifetime of Globular Cluster Stars



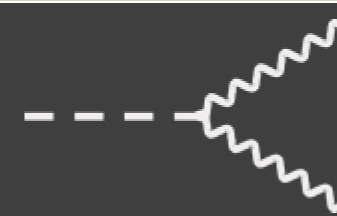
Number ratio of HB-Stars/Red Giants in 15 galactic globular clusters
(Buzzoni et al. 1983)

Helium-burning lifetime established within ±10%

Particles with Two-Photon Coupling

Particles with two-photon vertex:
- Neutral pions (π^0), Gravitons
- Axions (a) and similar hypothetical particles

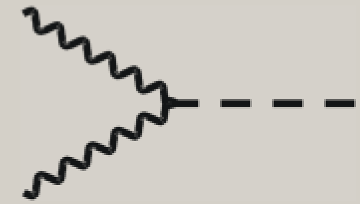
$$L_{a\gamma} = g_{a\gamma} \vec{E} \cdot \vec{B}\, a$$



Two-photon decay

$$\Gamma_{a\gamma} = \frac{g_{a\gamma}^2 m_{a\gamma}^3}{64\,\pi}$$

Photon Coalescence

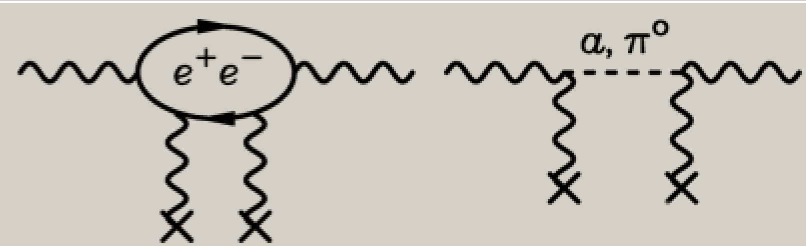


Primakoff Effect

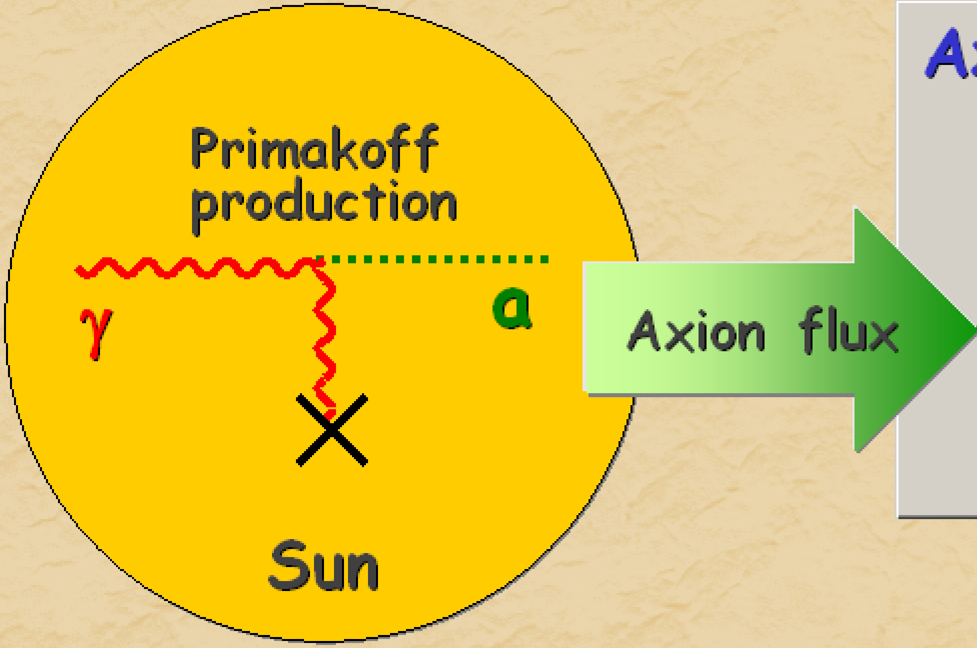
Conversion of photons into pions, gravitons or axions, or the reverse

Magnetically induced vacuum birefringence

In addition to QED Cotton-Mouton-effect

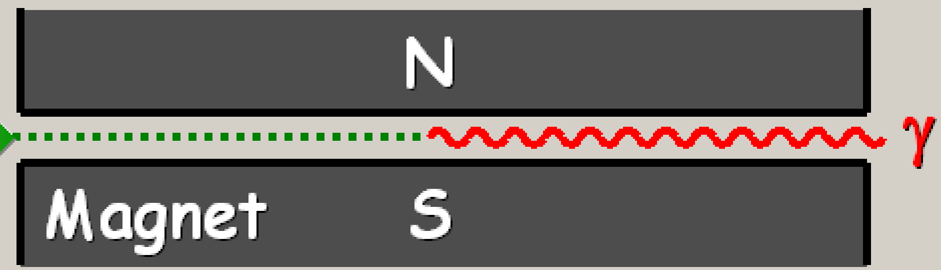


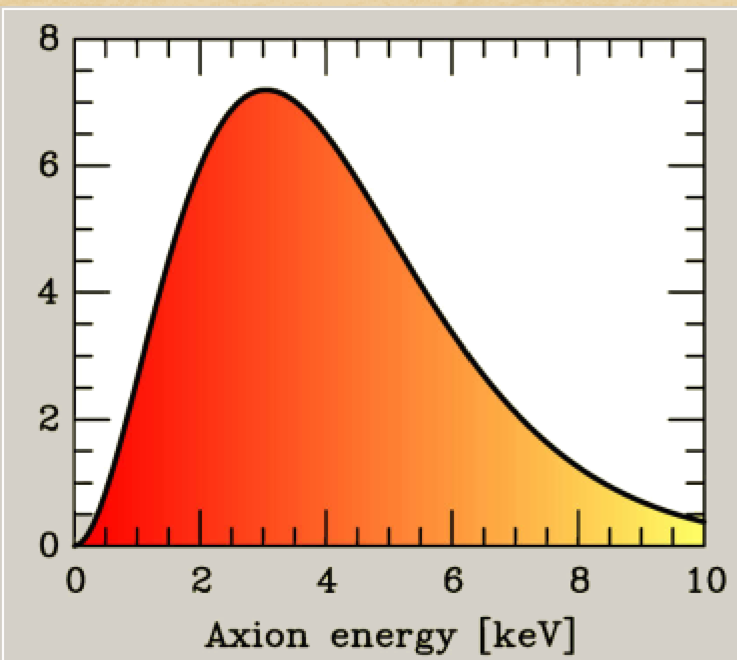
Search for Solar Axions



Axion Helioscope (Sikivie 1983)

Axion-Photon-Oscillation

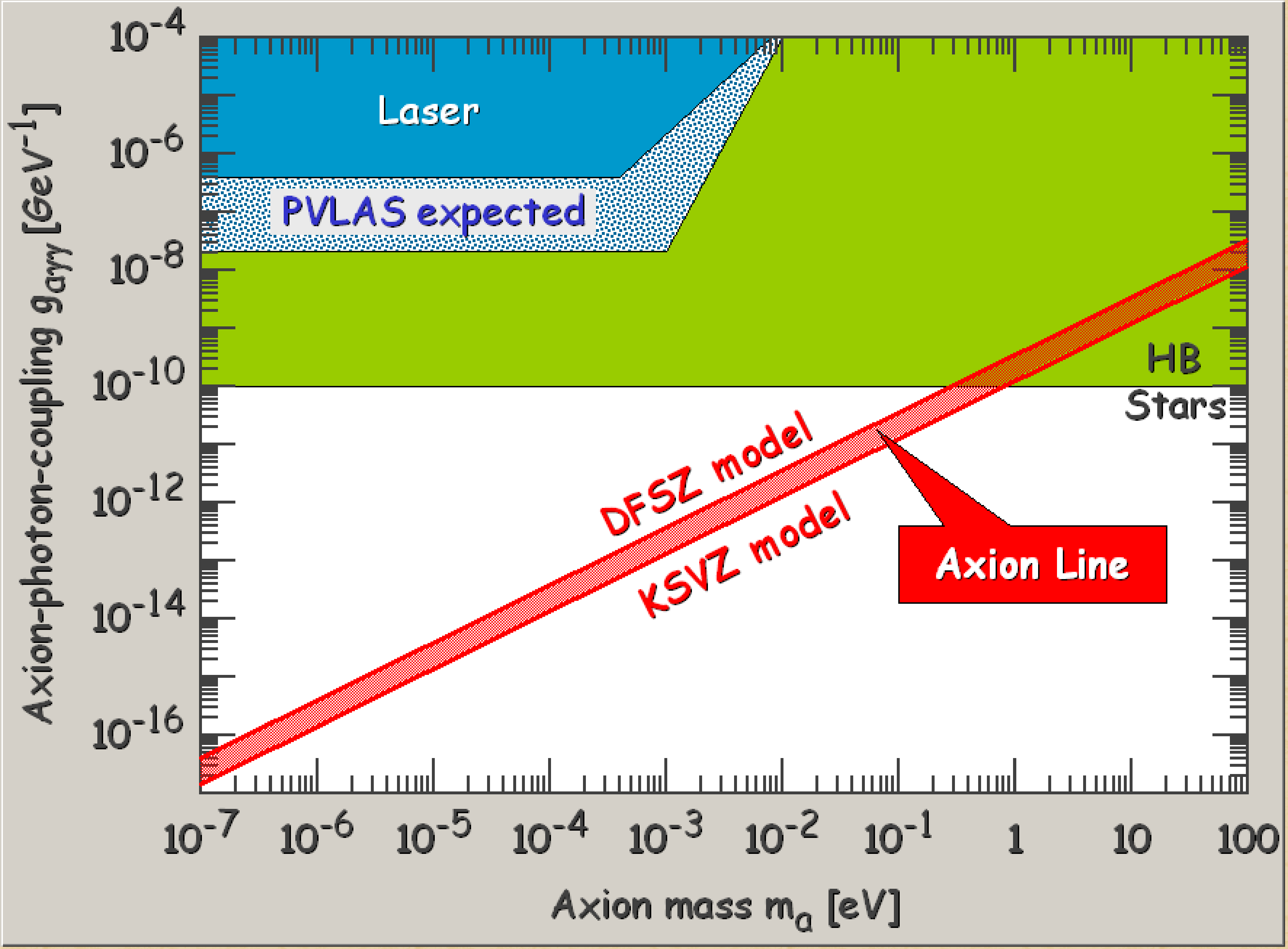




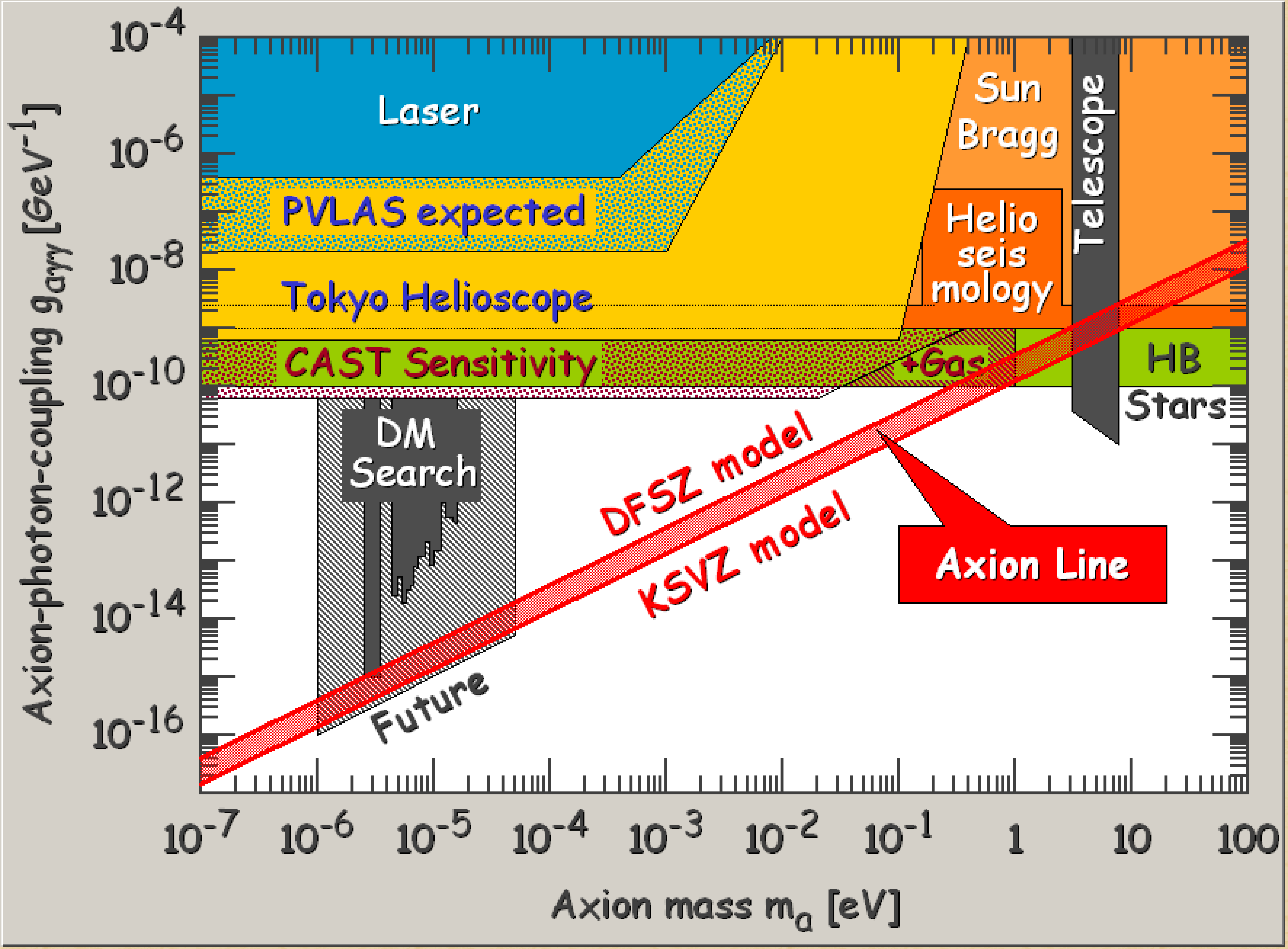
- ➔ Tokyo Axion Helioscope (Results since 1998)
- ➔ CERN Axion Solar Telescope (CAST) (in preparation)

Alternative Technique:
Bragg conversion in crystal
Experimental limits on solar axion flux from dark-matter experiments (SOLAX, COSME, DAMA, ...)

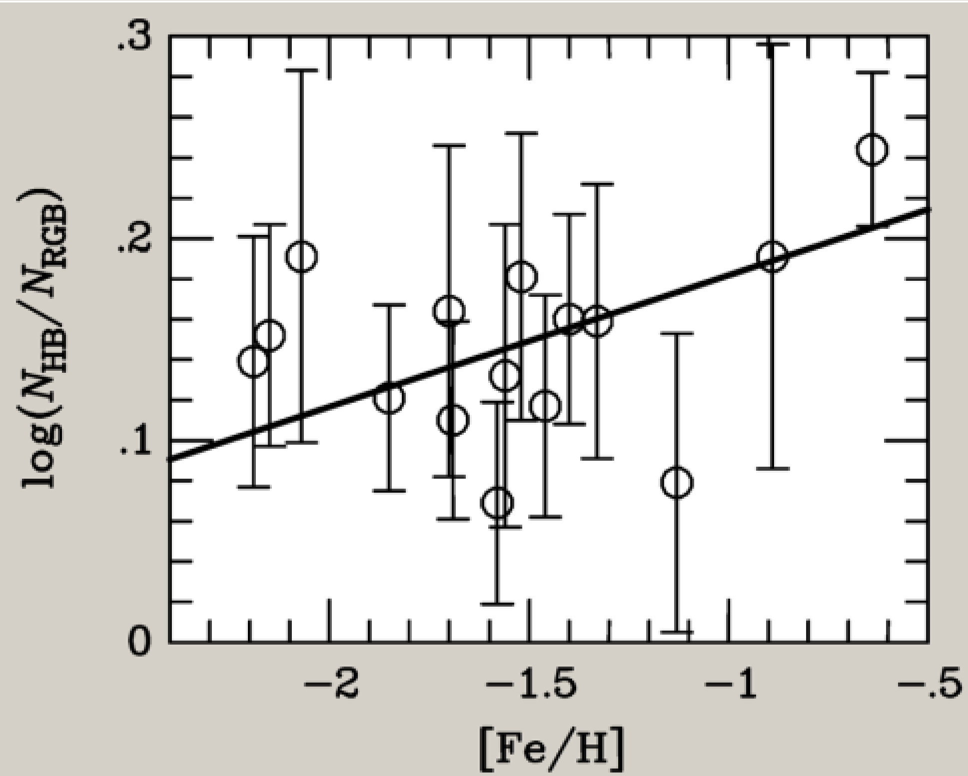
Limits on Axion-Photon-Coupling

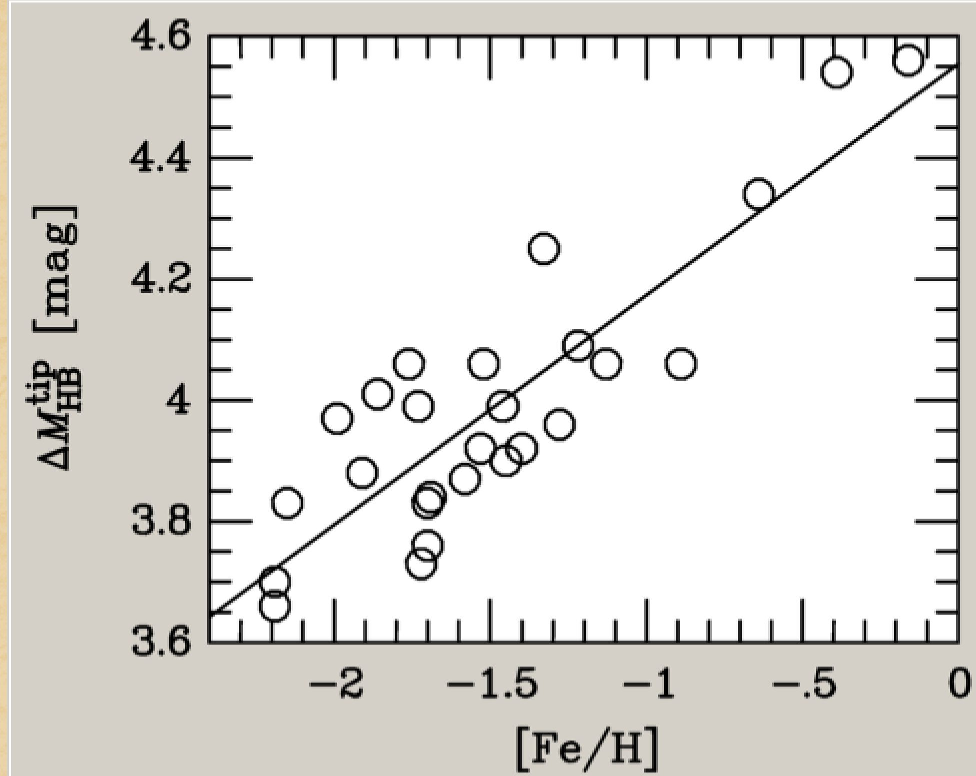


Limits on Axion-Photon-Coupling



Measurements of Globular Cluster Observables

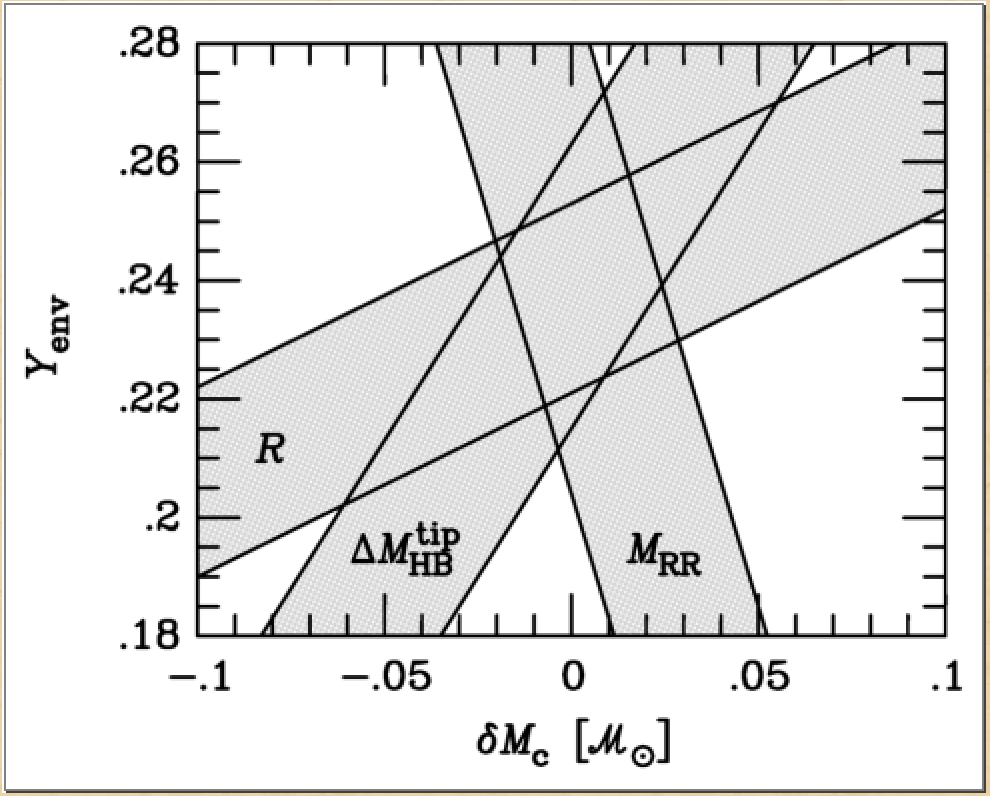




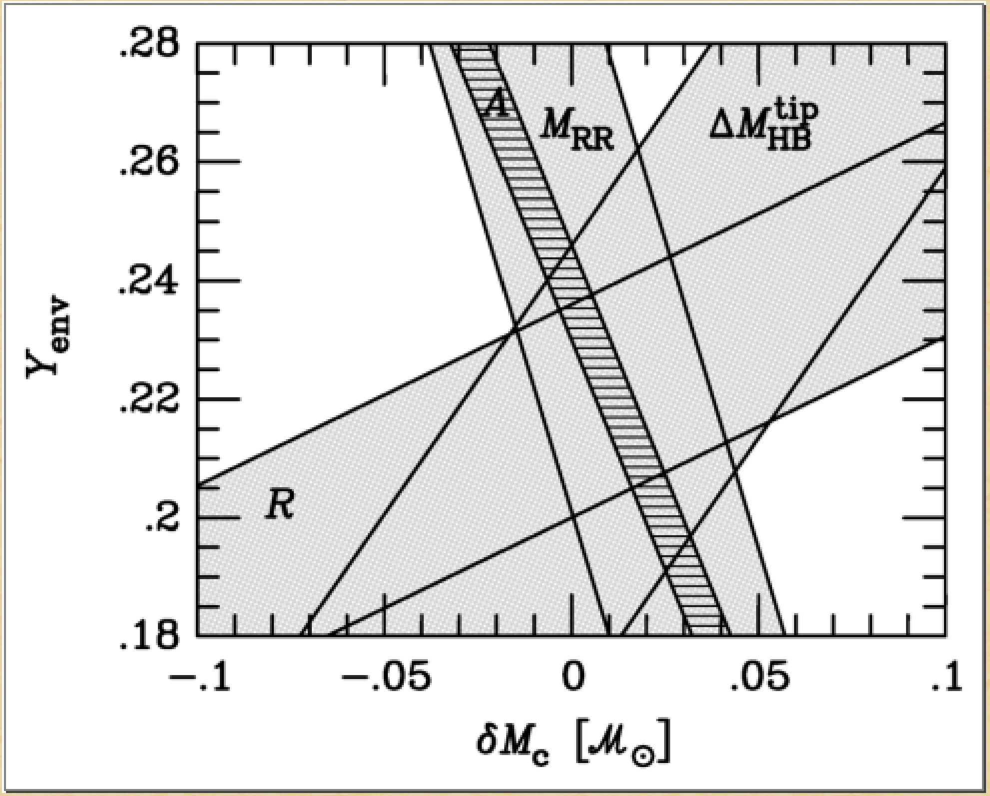
Number ratio of HB vs. RGB stars in 15 globular clusters

Brightness difference between HB (RR Lyrae stars) and brightest red giant in 26 globular clusters

Core-Mass at Helium Ignition

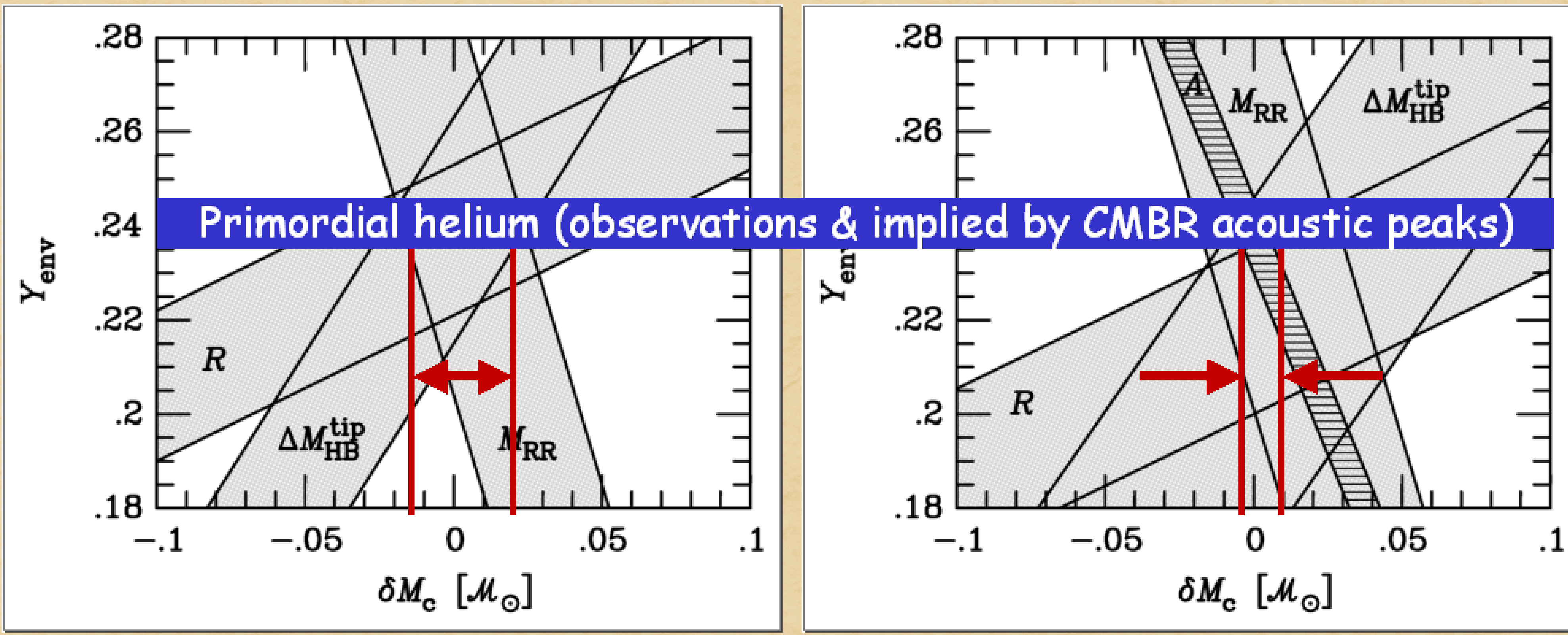


G.Raffelt, Stars as Laboratories for Fundamental Physics (1996)



Catalan et al., astro-ph/9509062

Core-Mass at Helium Ignition



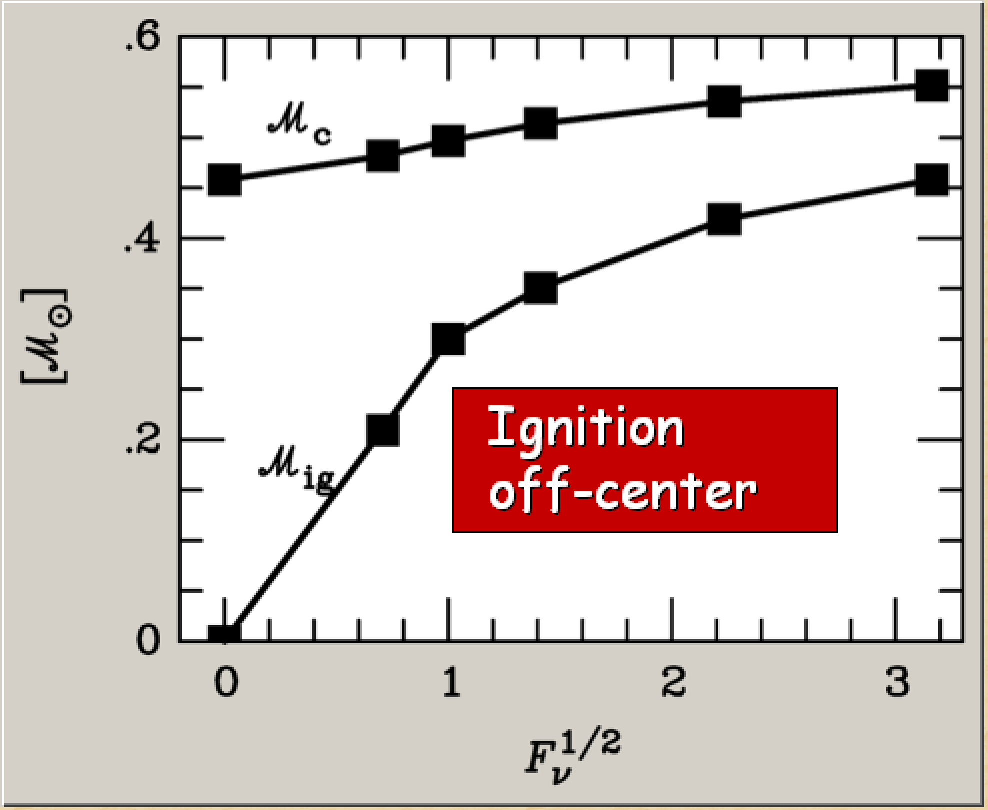
G.Raffelt, Stars as Laboratories for Fundamental Physics (1996)

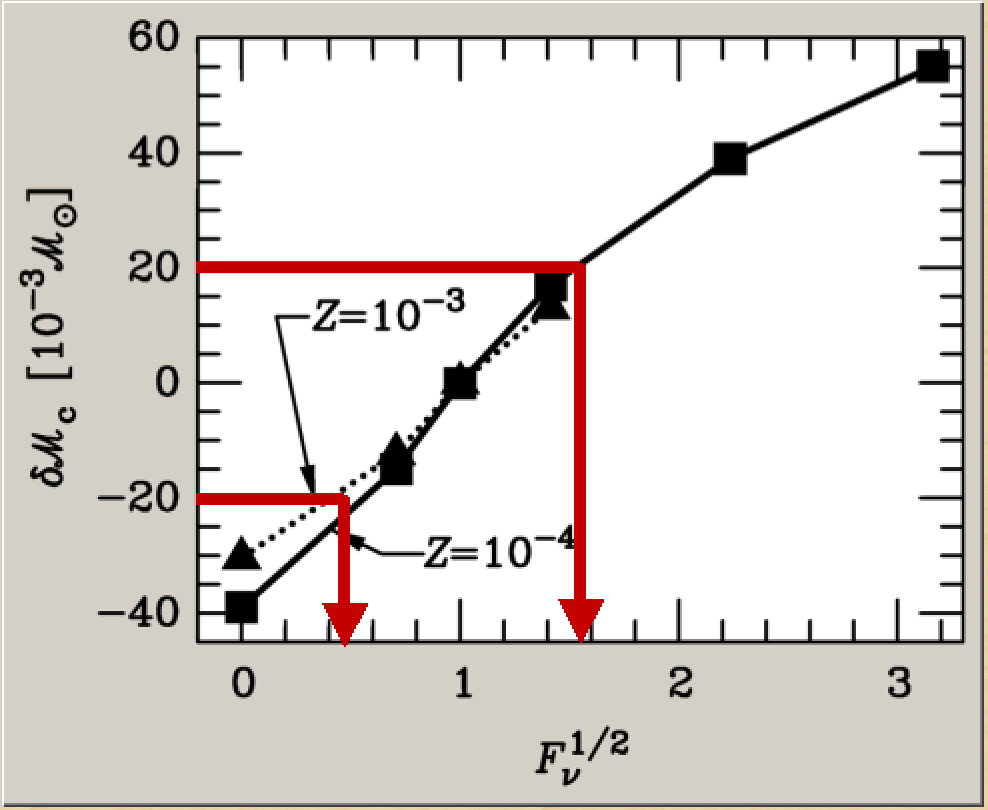
Catalan et al., astro-ph/9509062

Core mass at helium ignition established to ± 0.02 M_{sun} or ± 4%

Core-Mass Dependence on Neutrino Cooling

Multiply standard neutrino loss rates with a factor F_ν





- Core mass (upper curve)
- Radial coordinate of helium ignition (lower curve)

Change of core mass

Dominant Neutrino Process in Degenerate Helium Core

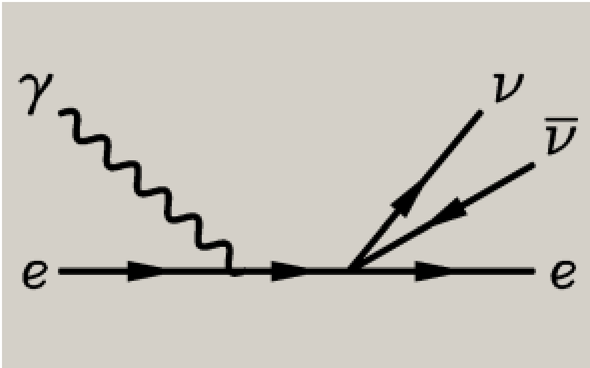
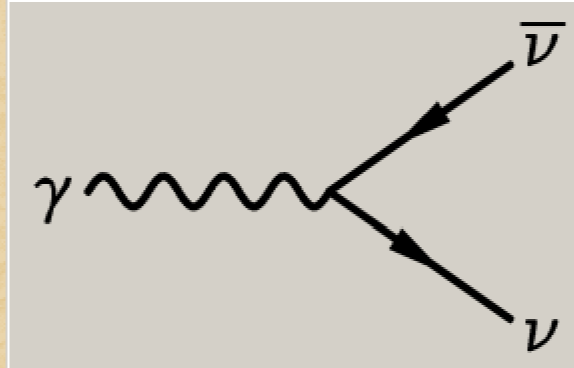
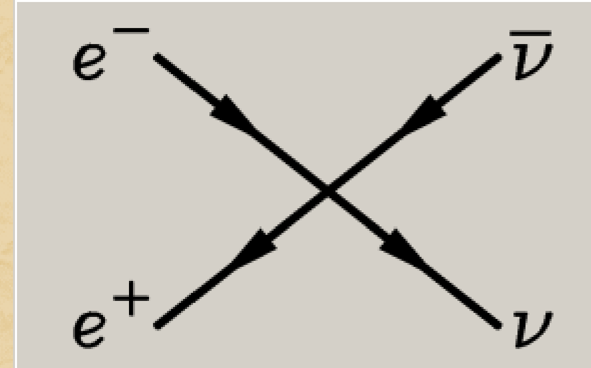


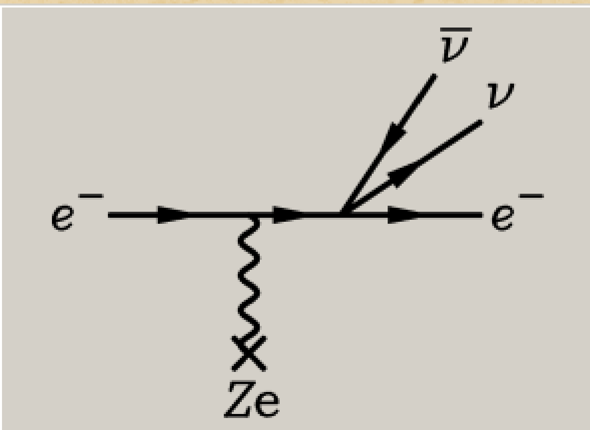
Photo (Compton)



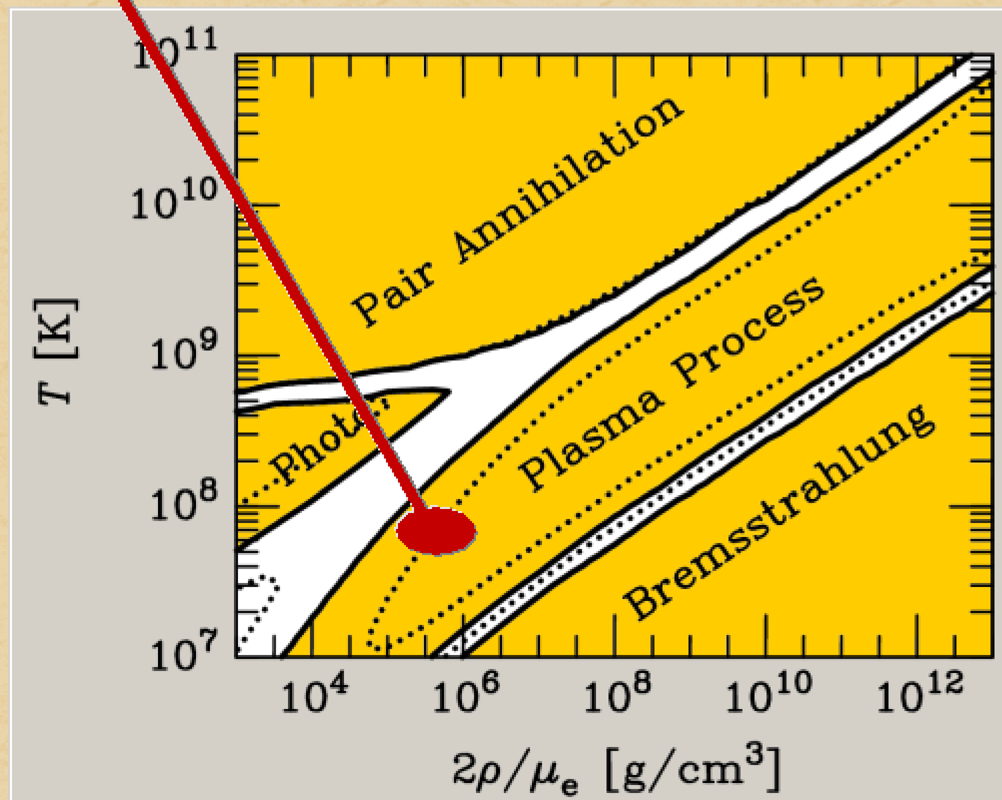
Plasmon decay



Pair annihilation

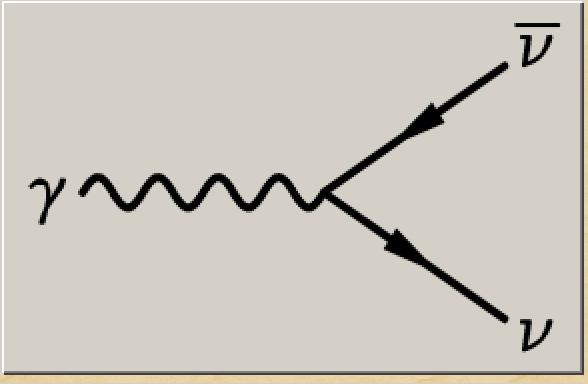


Bremsstrahlung



Plasmon Decay in Neutrinos

Vacuum:
- Photon massless
- Can not decay into other particles, even if they themselves are massless



Plasmon decay

Vacuum:
- Massless neutrinos do not couple to photons
- May have dipole moments or even "millicharges"

Propagation in a medium:
- Photon acquires a "refractive index"
- In a non-relativistic plasma (e.g. Sun, white dwarfs, core of red giant before helium ignition, ...) behaves like massive particle:

$$\omega^2 - k^2 = \omega_{pl}^2$$

Plasma frequency (electron density n_e)

$$\omega_{pl}^2 = \frac{4\pi\alpha n_e}{m_e}$$

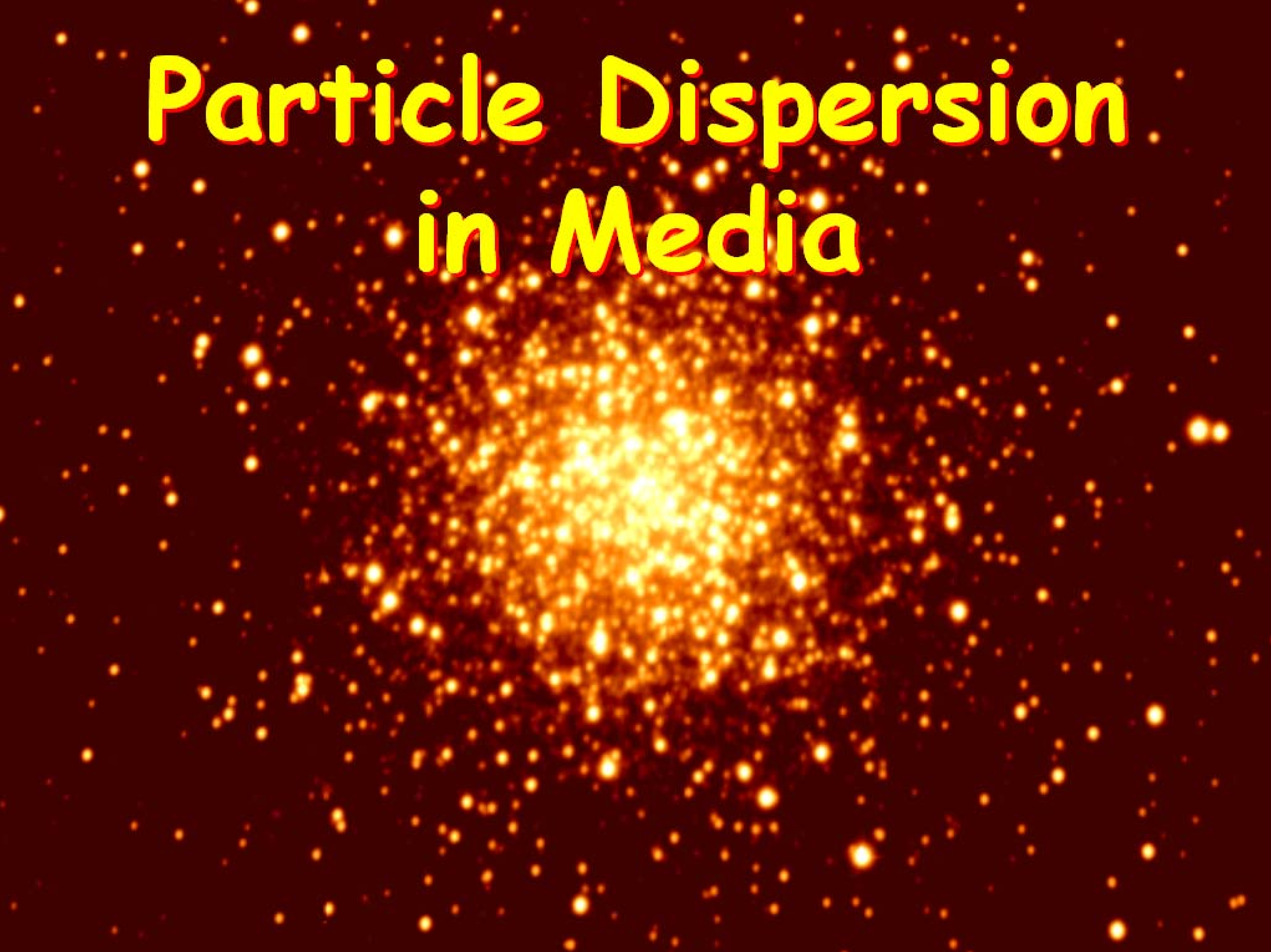
- Degenerate helium core $(\rho = 10^6\,\mathrm{g\,cm^{-3}}, T = 8.6\ \mathrm{keV})$ $\quad \omega_{pl} \approx 18\,\mathrm{keV}$

In a medium:
- Neutrinos interact coherently with the charged particles which themselves couple to photons
- Induces an "effective charge"
- In a degenerate plasma (electron Fermi energy E_F and Fermi momentum p_F)

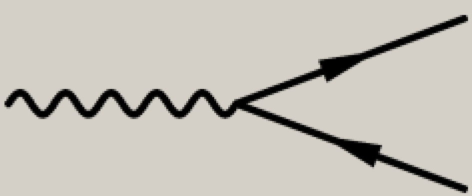
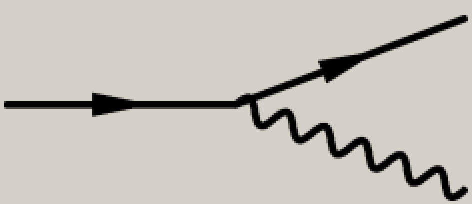
$$\frac{e_\nu}{e} = 16\sqrt{2}\,C_V G_F E_F p_F$$

- Degenerate helium core (and $C_V = 1$)

$$e_\nu = 6 \times 10^{-11}\,e$$


Particle Dispersion
in Media

Plasmon Decay vs. Cherenkov Effect

Photon dispersion in a medium can be	"Time-like" $\omega^2 - k^2 > 0$	"Space-like" $\omega^2 - k^2 < 0$
Refractive index n ($k = n\,\omega$)	$n < 1$	$n > 1$
Example	• Ionized plasma • Normal matter for large photon energies	Water ($n \approx 1.3$), air, glass for visible frequencies
Allowed process that is forbidden in vacuum	Plasmon decay to neutrinos $\gamma \to \nu\bar{\nu}$ 	Cherenkov effect $e \to e + \gamma$ 

Particle Dispersion in Media

Vacuum

Most general Lorentz-invariant dispersion relation

$$\omega^2 - k^2 = m^2$$

ω = frequency, k = wave number, m = mass

Gauge invariance implies m = 0 for photons and gravitons

Medium

Particle interaction with medium breaks Lorentz invariance so that

$$\omega^2 - k^2 = \pi(\omega, k)$$

Implies a relationship between ω and k (dispersion relation) that is often written in the form

- Refractive index n

  $$k = n\,\omega$$

- Effective mass m_{eff} (note that m_{eff}^2 can be negative)

  $$\omega^2 - k^2 = m_{eff}^2$$

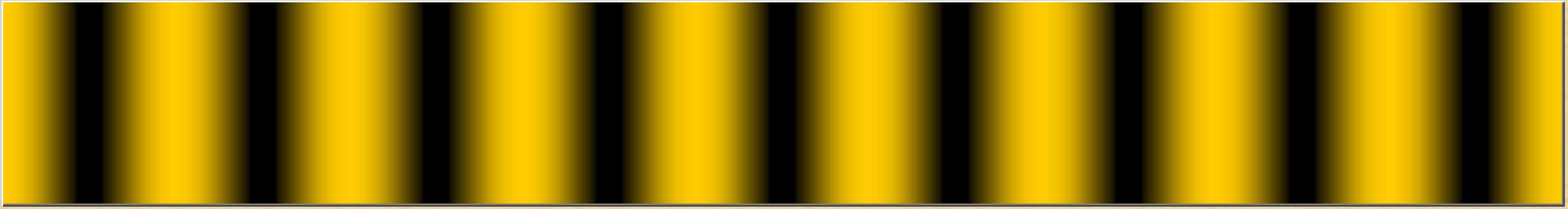
- Effective potential V (natural for neutrinos in a medium)

  $$(\omega - V)^2 - k^2 = m_{vac}^2$$

Refraction and Forward Scattering

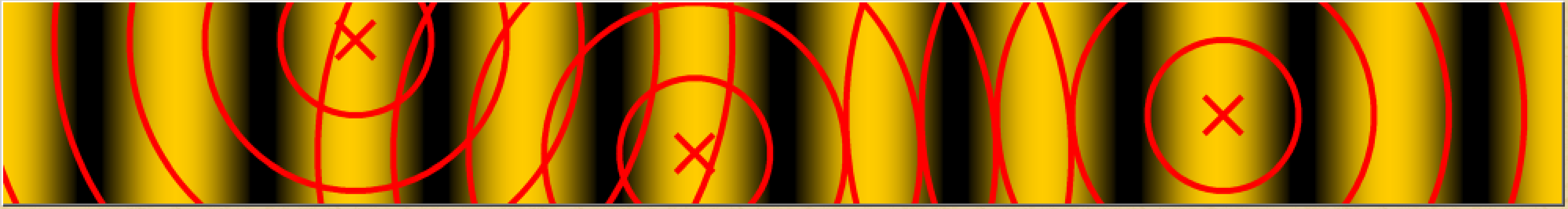
Plane wave in vacuum

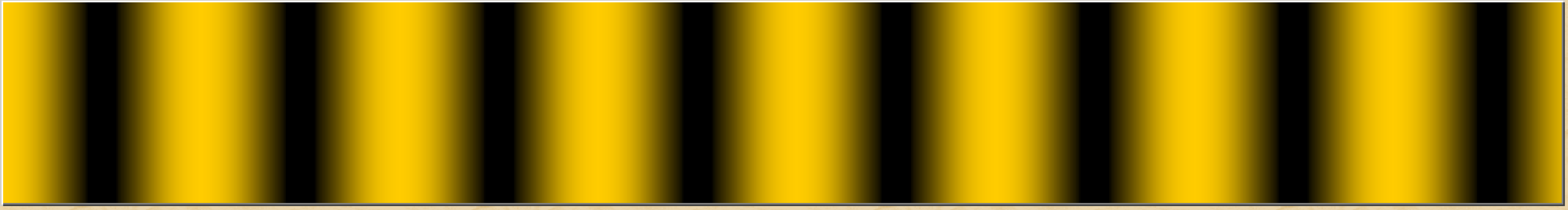
$$\Phi(\vec{r},t) \propto \exp\left(-i\omega t + i\vec{k}\cdot\vec{r}\right)$$



Refraction and Forward Scattering

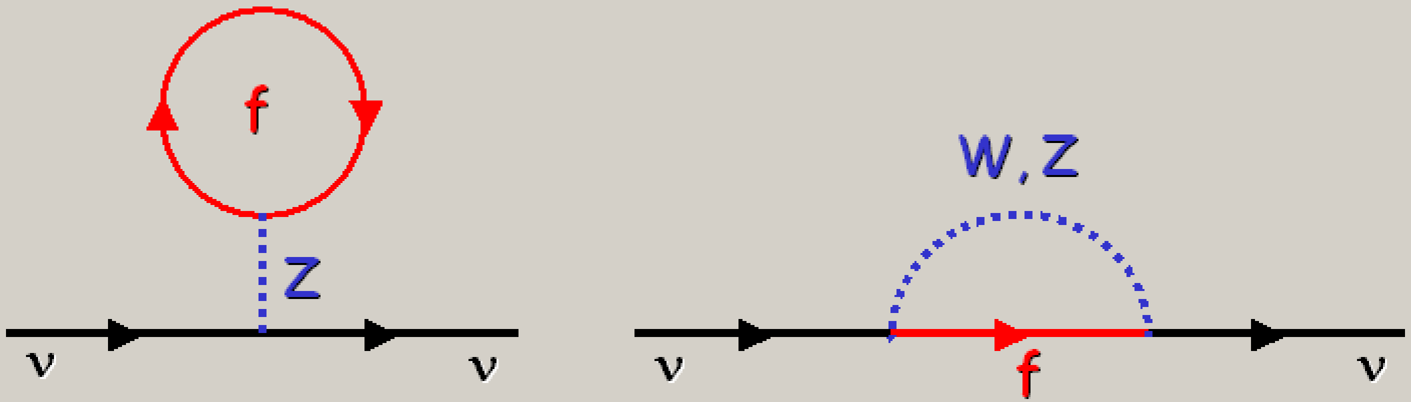
Plane wave in vacuum	$\Phi(\vec{r},t) \propto \exp\left(-i\omega t + i\vec{k}\cdot\vec{r}\right)$
With scattering centers	$\Phi(\vec{r},t) \propto \exp(-i\omega t)\left[\exp\left(i\vec{k}\cdot\vec{r}\right) + f(\omega,\theta)\frac{\exp(ikr)}{r}\right]$
In forward direction, adds coherently to a plane wave with modified wave number	$k = n_{refr}\omega$ $n_{refr} = 1 + \frac{2\pi}{\omega^2} n\, f(\omega,0)$ n density of scattering centers $f(\omega,0)$ forward scattering amplitude





Neutrino Refraction in Media

Neutrinos propagating in a medium suffer refraction (Wolfenstein 1978)



Effect is usually different for different flavors

For small neutrino energies and large matter-antimatter asymmetry of the medium (not true in early universe)

$$(E-V)^2 - p^2 = m_{vac}^2$$

Normal medium, consisting of protons, neutrons (or nuclei), and electrons

$$V = \pm\sqrt{2}G_F n_B \times \begin{cases} -\frac{1}{2}Y_n + Y_e & \text{for } \nu_e \\ -\frac{1}{2}Y_n & \text{for } \nu_\mu, \nu_\tau \end{cases}$$

$$\sqrt{2}G_F n_B = 0.762 \times 10^{-13}\,\text{eV}\,\frac{\rho}{\text{g cm}^{-3}}$$

Electromagnetic Polarization Tensor

Klein-Gordon-Equation in Fourier space	$\left(-K^2 g^{\mu\nu} + K^\mu K^\nu + \Pi^{\mu\nu}\right) A_\nu = 0$
Polarization tensor (self-energy of photon)	
Gauge invariance and current conservation	$\Pi^{\mu\nu} K_\mu = \Pi^{\mu\nu} K_\nu = 0$

Vacuum: $\Pi^{\mu\nu} = a\, g^{\mu\nu} + b\, K^\mu K^\nu$

$\rightarrow a = b = 0$ (photon massless)

Medium: Four-velocity U available to construct Π

QED plasma

$$\Pi^{\mu\nu}(K) = 16\pi\alpha \int \frac{d^3\vec{p}}{2E(2\pi)^3}\, f_e(\vec{p})\, \frac{(PK)^2 g^{\mu\nu} + K^2 P^\mu P^\nu - (PK)(P^\mu K^\nu + K^\mu P^\nu)}{(PK)^2 - \frac{1}{4}(K^2)^2}$$

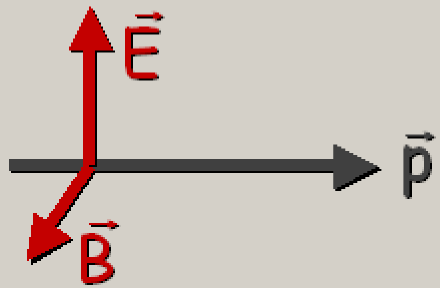
Photon: $K = (\omega, \vec{k})$ Electron or positron: $P = (E, \vec{p})$ $E = \sqrt{\vec{p}^2 + m_e^2}$

Electron plus positron phase-space distribution with chemical potential μ_e

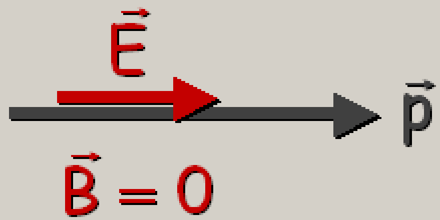
$$f_e(\vec{p}) = \left[\exp\left(\frac{E - \mu_e}{T}\right) + 1\right]^{-1} + \left[\exp\left(\frac{E + \mu_e}{T}\right) + 1\right]^{-1}$$

Transverse and Longitudinal "Plasmons"

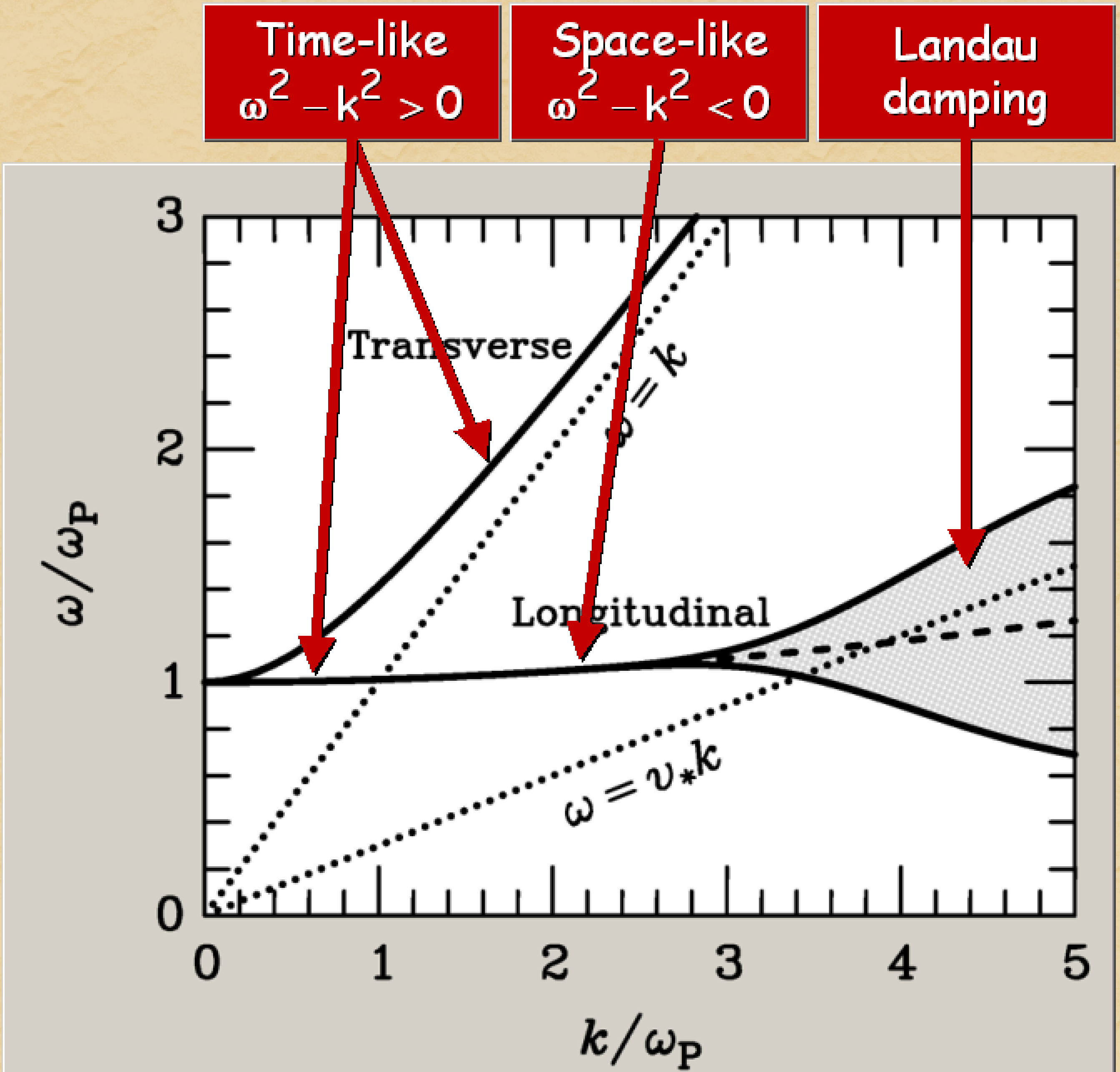
Transverse excitations



Longitudinal excitations (collective oscillations of electrons against positive charges)



Dispersion relation in a non-relativistic, non-degenerate plasma



Electron (Positron) Dispersion Relation

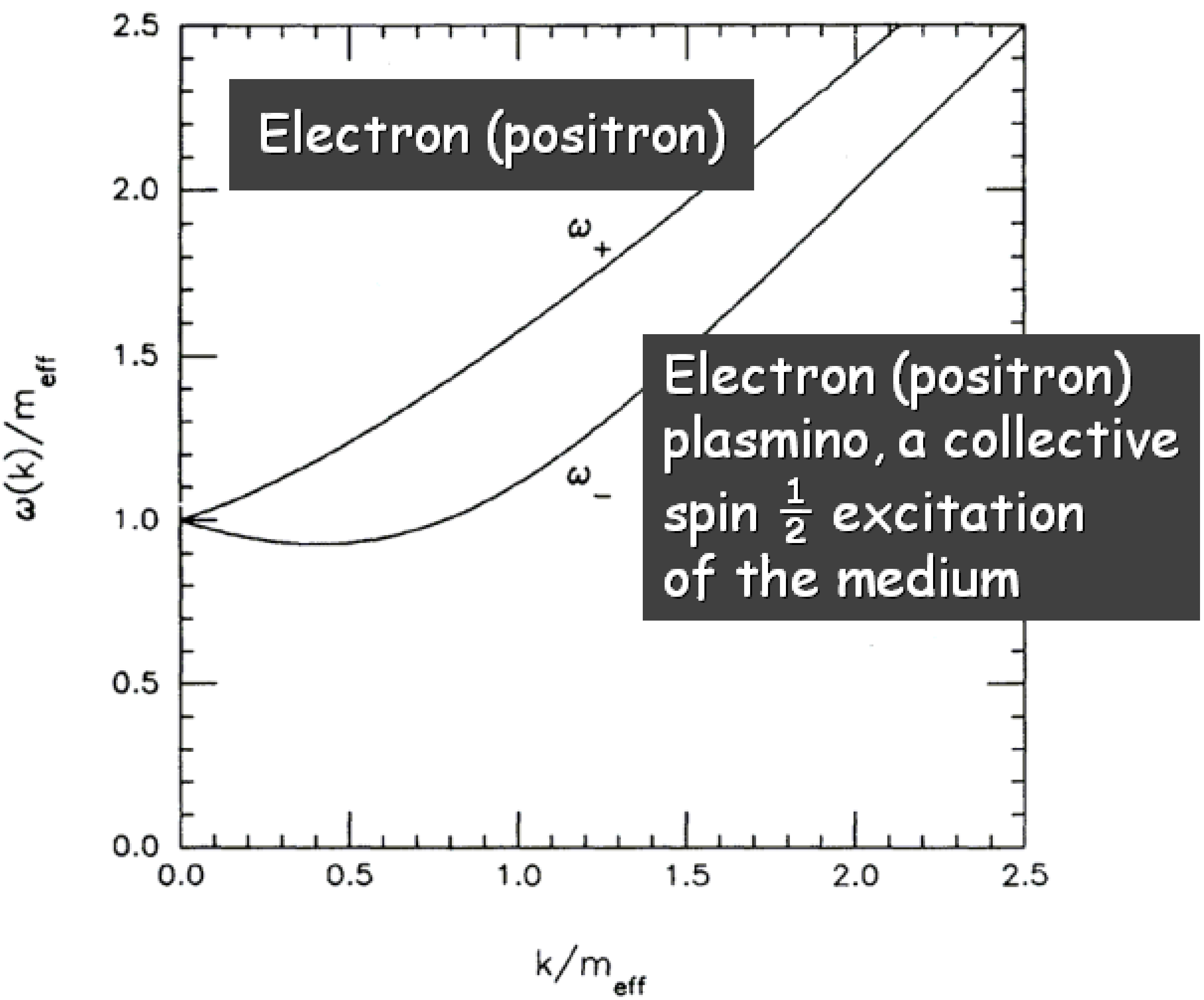
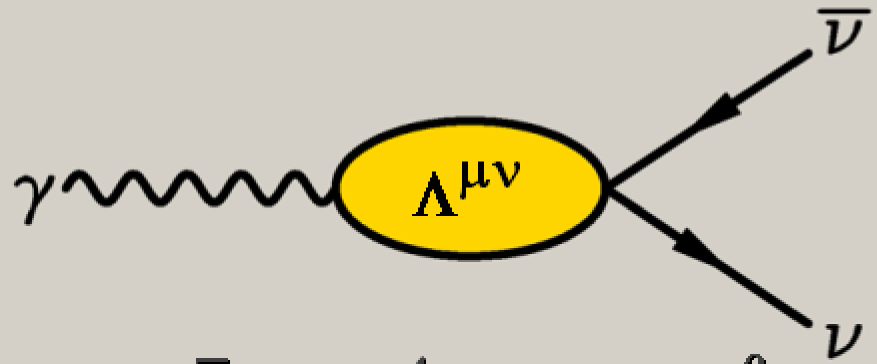


FIG. 1.—Ultrarelativistic dispersion relations for the electron or positron $[\omega_+(k)]$ and for the electron plasmino or positron plasmino $[\omega_-(k)]$.

E. Braaten, Neutrino emissivity of an ultrarelativistic plasma from positron and plasmino annihilation, Astrophys. J. 392 (1992) 70

Neutrino-Photon-Coupling in a Plasma

Neutrino effective in-medium coupling



$$L_{eff} = -\sqrt{2} G_F \bar{\Psi} \gamma_\alpha \frac{1}{2}(1-\gamma_5)\Psi \Lambda^{\alpha\beta} A_\beta$$

For vector-current analogous to photon polarization tensor

$$\Lambda_V^{\mu\nu}(K) = 4eC_V \int \frac{d^3\vec{p}}{2E(2\pi)^3}\left[f_{e^-}(\vec{p}) + f_{e^+}(\vec{p})\right] \frac{(PK)^2 g^{\mu\nu} + K^2 P^\mu P^\nu - (PK)(P^\mu K^\nu + K^\mu P^\nu)}{(PK)^2 - \frac{1}{4}(K^2)^2}$$

$$= \frac{C_V}{e} \Pi_V^{\mu\nu}(K)$$

$$\Lambda_A^{\mu\nu}(K) = 2ieC_A \varepsilon^{\mu\nu\alpha\beta} \int \frac{d^3\vec{p}}{2E(2\pi)^3}\left[f_{e^-}(\vec{p}) - f_{e^+}(\vec{p})\right] \frac{K^2 P_\alpha K_\beta}{(PK)^2 - \frac{1}{4}(K^2)^2}$$

Usually negligible

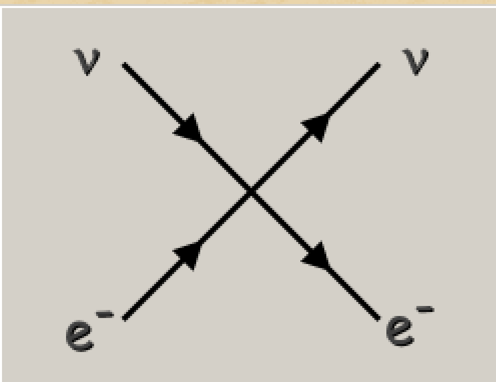
Neutral-Current Couplings and Plasmon Decay

Standard-model plasmon decay process $\propto C_V^2$

$\sin^2\Theta_W \approx \frac{1}{4}$

Standard-model plasmon decay produces almost exclusively $\nu_e\bar{\nu}_e$

Neutral-current process, yet was never useful for "neutrino counting" unlike big-bang nucleosynthesis (of course today Z^0-decay width fixes $N_\nu = 3$)



$G_F =$

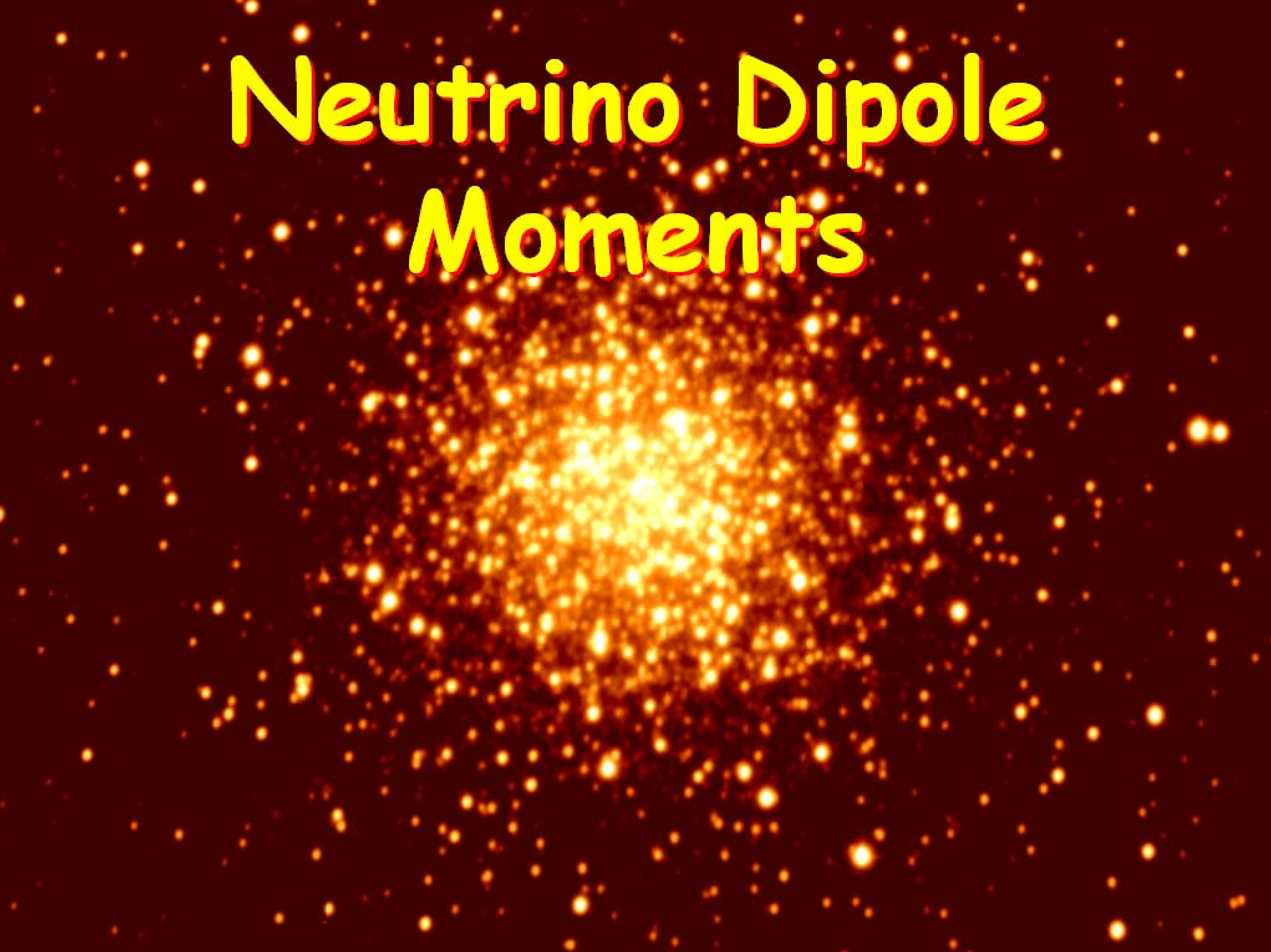
$1.166\times10^{-5}\,\mathrm{GeV}^{-2}$

$\sin^2\Theta_W = 0.231$

$$H_{int} = \frac{G_F}{\sqrt{2}}\,\Psi_f\gamma_\mu\,(C_V - C_A\gamma_5)\,\Psi_f\;\Psi_\nu\gamma^\mu\,(1-\gamma_5)\,\Psi_\nu$$

Neutrino	Fermion	C_V	C_A
ν_e	Electron	$+\frac{1}{2}+2\sin^2\Theta_W \approx 1$	$+\frac{1}{2}$
ν_μ,ν_τ		$-\frac{1}{2}+2\sin^2\Theta_W \approx 0$	$-\frac{1}{2}$
ν_e,ν_μ,ν_τ	Proton	$+\frac{1}{2}-2\sin^2\Theta_W \approx 0$	$+\frac{1.26}{2}$
	Neutron	$-\frac{1}{2}$	$-\frac{1.26}{2}$

Neutrino Dipole Moments



Neutrino Dipole Moments

Effective coupling of electromagnetic field to a neutral fermion

$$L_{eff} = -F_1 \bar{\Psi}\gamma_\mu \Psi A^\mu - G_1 \bar{\Psi}\gamma_\mu \gamma_5 \Psi \partial_\nu F^{\mu\nu} - \frac{1}{2} F_2 \bar{\Psi}\sigma_{\mu\nu}\Psi F^{\mu\nu} - \frac{1}{2} G_2 \bar{\Psi}\sigma_{\mu\nu}\gamma_5 \Psi F^{\mu\nu}$$

Charge $e_\nu = F_1(0) = 0$

Anapole moment $G_1(0)$

Magnetic dipole moment $\mu = F_2(0)$

Electric dipole moment $\varepsilon = G_2(0)$

- Charge form factor $F_1(q^2)$ and anapole $G_1(q^2)$ are short-range interactions if charge $F_1(0) = 0$
- Connect states of equal chirality
- In standard model they represent radiative corrections to weak interaction

- Dipole moments connect states of opposite chirality
- Violation of individual flavor lepton numbers (neutrino mixing)
  → Magnetic or electric dipole moments can connect different flavors or different mass eigenstates ("Transition moments")
- Usually measured in "Bohr magnetons" $\mu_B = e/(2m_e)$

Plasmon Decay And Stellar Energy Loss Rates

Assume photon dispersion relation like a massive particle (nonrelativistic plasma)

$$E_\gamma^2 - p_\gamma^2 = \omega_{pl}^2 = \frac{4\pi\alpha n_e}{m_e}$$

Decay rate of photon (transverse plasmon) with energy E_γ

$$\Gamma(\gamma \to \nu\bar{\nu}) = \frac{4\pi}{3}\frac{1}{E_\gamma} \times \begin{cases} \alpha_\nu \left(\omega_{pl}^2/4\pi\right) & \text{Millicharge} \\ \frac{\mu_\nu^2}{2}\left(\omega_{pl}^2/4\pi\right)^2 & \text{Dipole moment} \\ \frac{C_V^2 G_F^2}{\alpha}\left(\omega_{pl}^2/4\pi\right)^3 & \text{Standard model} \end{cases}$$

Energy-loss rate of stellar plasma (temperature T and plasma frequency ω_{pl})

$$Q(\gamma \to \nu\bar{\nu}) = \int \frac{2d^3\vec{p}}{(2\pi)^3}\frac{E_\gamma \Gamma}{e^{E_\gamma/T}-1} = \frac{8\zeta_3}{3\pi}T^3 \times \begin{cases} \alpha_\nu \left(\omega_{pl}^2/4\pi\right) \\ \frac{\mu_\nu^2}{2}\left(\omega_{pl}^2/4\pi\right)^2 \\ \frac{C_V^2 G_F^2}{\alpha}\left(\omega_{pl}^2/4\pi\right)^3 \end{cases}$$

Globular Cluster Limits on Neutrino Dipole Moments

Compare magnetic-dipole plasma emission with standard case

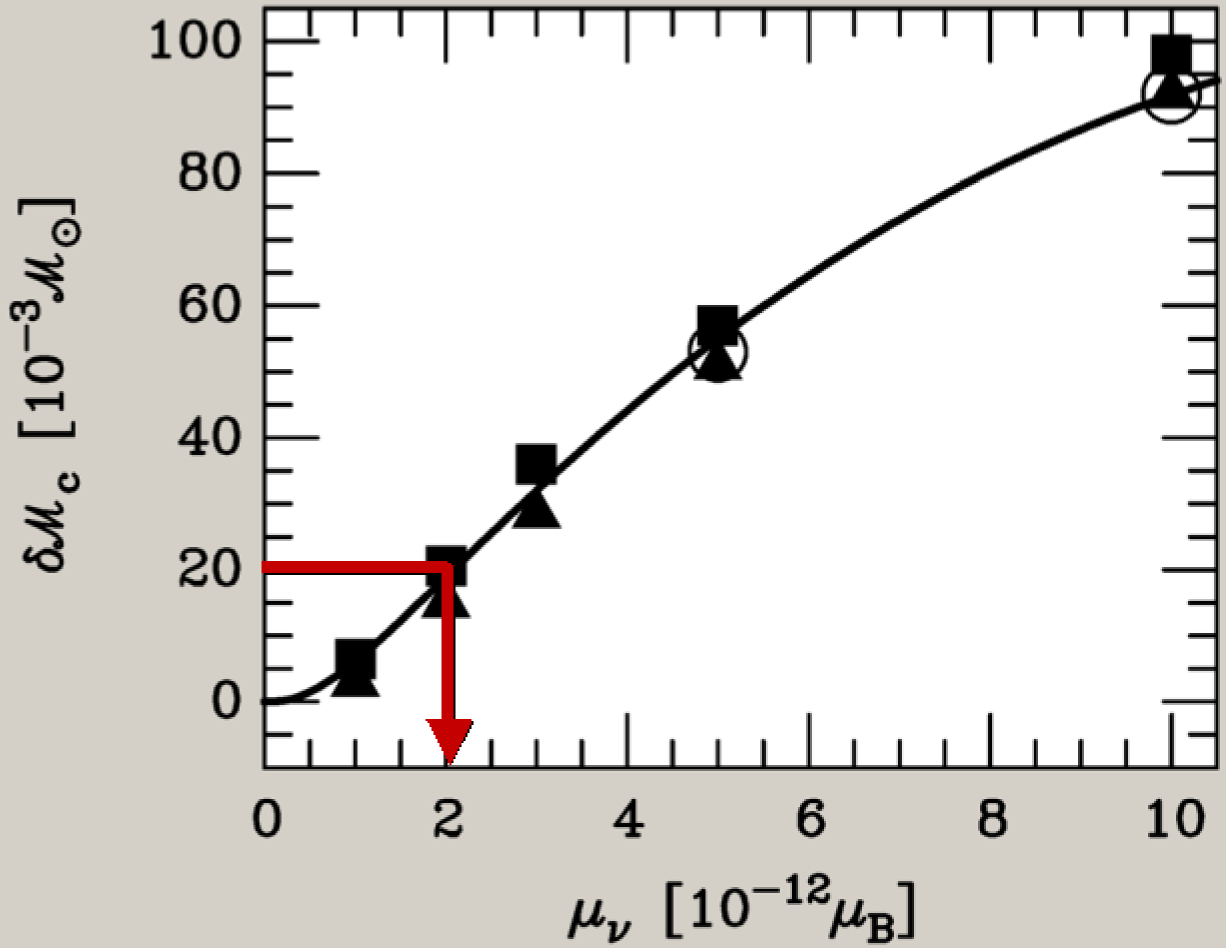
$$\frac{Q_\mu}{Q_{SM}} = \frac{2\pi\alpha\mu_\nu^2}{C_V^2 G_F^2 \omega_{pl}^2}$$

For red-giant core before helium ignition $\omega_{pl} = 18$ keV

$$\frac{Q_\mu}{Q_{SM}} = 9\times10^{22}\left(\frac{\mu_\nu}{\mu_B}\right)^2$$

Require this to be < 1

$$\mu_\nu < 3\times10^{-12}\mu_B$$

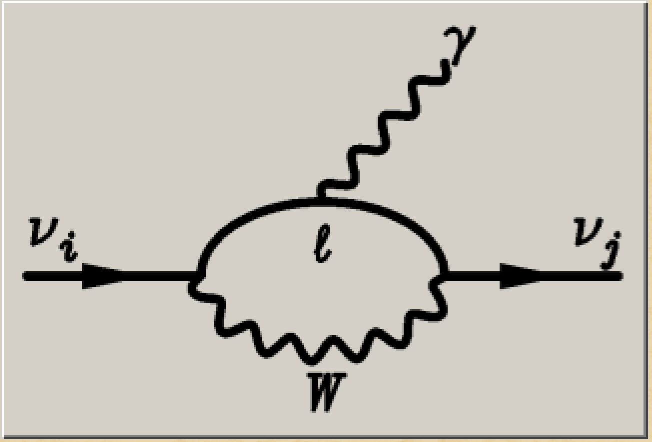


Globular-cluster limit on neutrino dipole moment

$$\mu_\nu < 2\times10^{-12}\mu_B$$

Standard Dipole Moments for Massive Neutrinos

In standard electroweak model, neutrino dipole and transition moments are induced at higher order



Massive neutrinos ν_i $(i = 1,2,3)$, mixed to form weak eigenstates

$$\nu_\ell = \sum_{i=1}^{3} U_{\ell i} \nu_i$$

Explicit evaluation for Dirac neutrinos
(Magnetic moments μ_{ij}
electric moments ε_{ij})

$$\mu_{ij} = \frac{e\sqrt{2}G_F}{(4\pi)^2}(m_i + m_j) \sum_{\ell = e,\mu,\tau} U_{\ell j} U_{\ell i}^* f\left(\frac{m_\ell}{m_W}\right)$$

$$\varepsilon_{ij} = \ldots (m_i - m_j) \ldots$$

$$f\left(\frac{m_\ell}{m_W}\right) = -\frac{3}{2} + \frac{3}{4}\left(\frac{m_\ell}{m_W}\right)^2 + O\left(\left(\frac{m_\ell}{m_W}\right)^4\right)$$

Standard Dipole Moments for Massive Neutrinos

Diagonal case
(Magnetic moments of Dirac neutrinos)

$$\mu_{ii} = \frac{3e\sqrt{2}G_F}{(4\pi)^2} m_i = 3.20 \times 10^{-19} \mu_B \frac{m_i}{\text{eV}} \qquad \mu_B = \frac{e}{2m_e}$$

$$\varepsilon_{ii} = 0$$

Off-diagonal case
(Transition moments)

First term in $f(m_\ell/m_W)$ does not contribute ("GIM cancellation")

$$\mu_{ij} = \frac{3e\sqrt{2}G_F}{4(4\pi)^2} (m_i + m_j) \left(\frac{m_\tau}{m_W}\right)^2 \sum_{\ell=e,\mu,\tau} U_{\ell j} U^*_{\ell i} \left(\frac{m_\ell}{m_\tau}\right)^2$$

$$= 3.96 \times 10^{-23} \mu_B \frac{m_i + m_j}{\text{eV}} \sum_{\ell=e,\mu,\tau} U_{\ell j} U^*_{\ell i} \left(\frac{m_\ell}{m_\tau}\right)^2$$

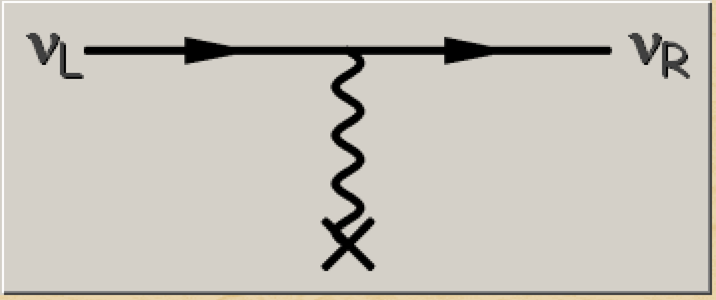
$$\varepsilon_{ij} = \ldots (m_i - m_j) \ldots$$

Largest neutrino mass eigenstate $0.05\ \text{eV} < m < 0.7\ \text{eV}$
For Dirac neutrino expect

$$1.6 \times 10^{-20} \mu_B < \mu_\nu < 2.2 \times 10^{-19} \mu_B$$

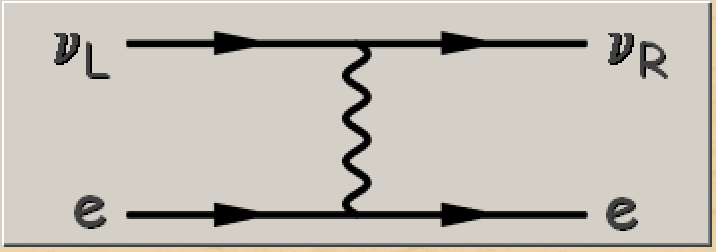
Consequences of Neutrino Dipole Moments

Spin precession in external E or B fields



$$i\frac{\partial}{\partial t}\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix} = \begin{pmatrix}0 & \mu_\nu B_T\\ \mu_\nu B_T & 0\end{pmatrix}\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix}$$

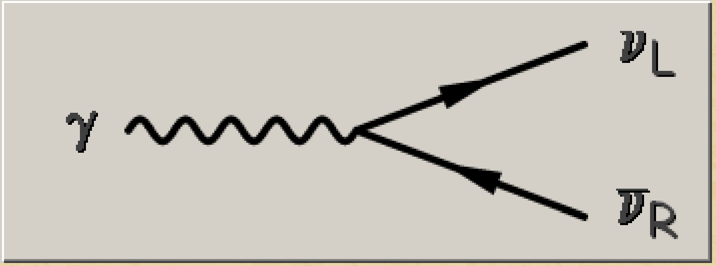
Scattering



$$\frac{d\sigma}{dT} = \frac{G_F^2 m_e}{2\pi}\left[(C_V + C_A)^2 + (C_V - C_A)^2\left(1 - \frac{T}{E_\nu}\right)^2 + (C_V^2 - C_A^2)\frac{m_e T}{E_\nu^2}\right] + \alpha\mu_\nu^2\left[\frac{1}{T} - \frac{1}{E_\nu}\right]$$

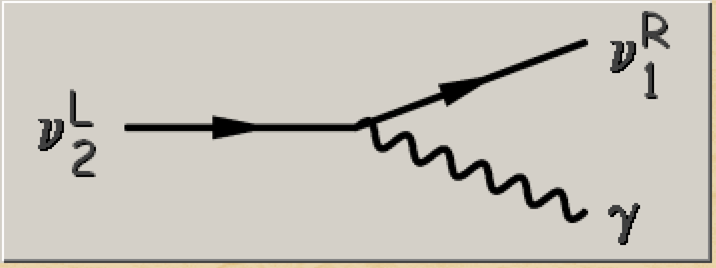
T electron recoil energy

Plasmon decay in stars



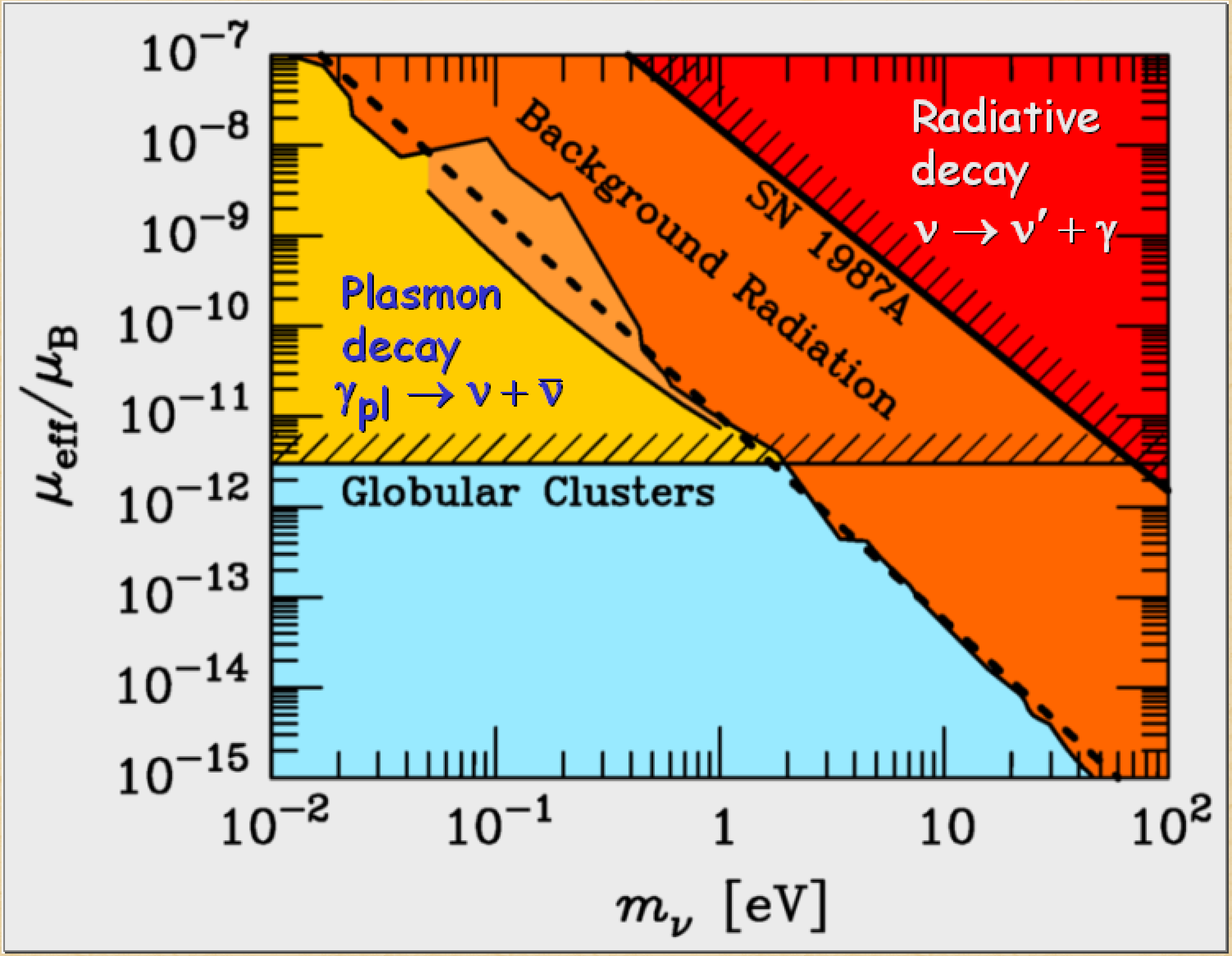
$$\Gamma = \frac{\mu_\nu^2}{24\pi}\omega_{pl}^3$$

Decay or Cherenkov effect



$$\Gamma = \frac{\mu_\nu^2}{8\pi}\left(\frac{m_2^2 - m_1^2}{m_2}\right)^3$$

Neutrino Radiative Lifetime Limits



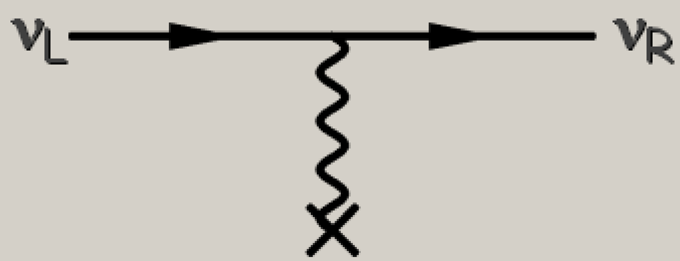
$$\Gamma_{\nu\to\nu'\gamma} = \frac{\mu_{eff}^2}{8\pi} m_\nu^3$$

$$\Gamma_{\gamma\to\nu\bar\nu} = \frac{\mu_{eff}^2}{24\pi} \omega_{pl}^3$$

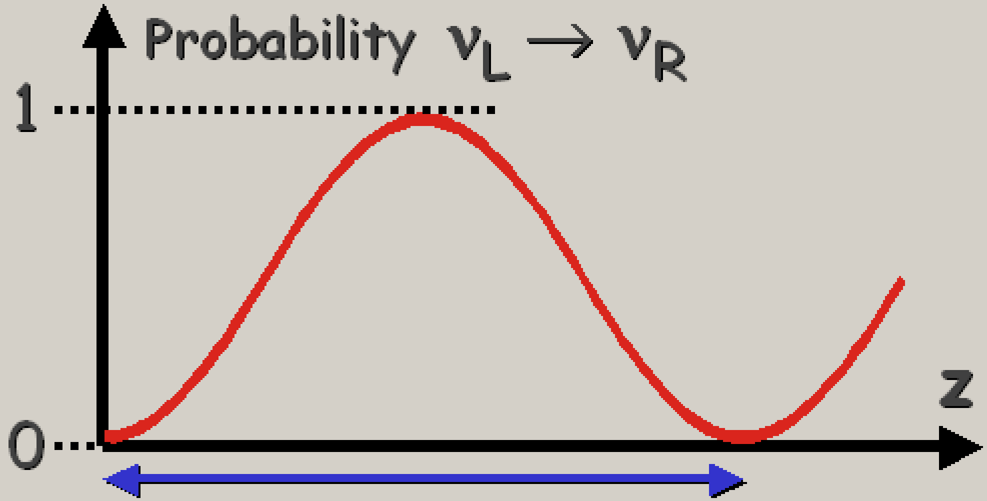
For low-mass neutrinos, plasmon decay in globular cluster stars yields most restrictive limits

Neutrino Spin Oscillations

Spin precession in external E or B fields



$$i\frac{\partial}{\partial t}\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix} = \begin{pmatrix}0 & \mu_\nu B_T\\ \mu_\nu B_T & 0\end{pmatrix}\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix}$$



Oscillation Length $\dfrac{\pi}{\mu_\nu B_T}$

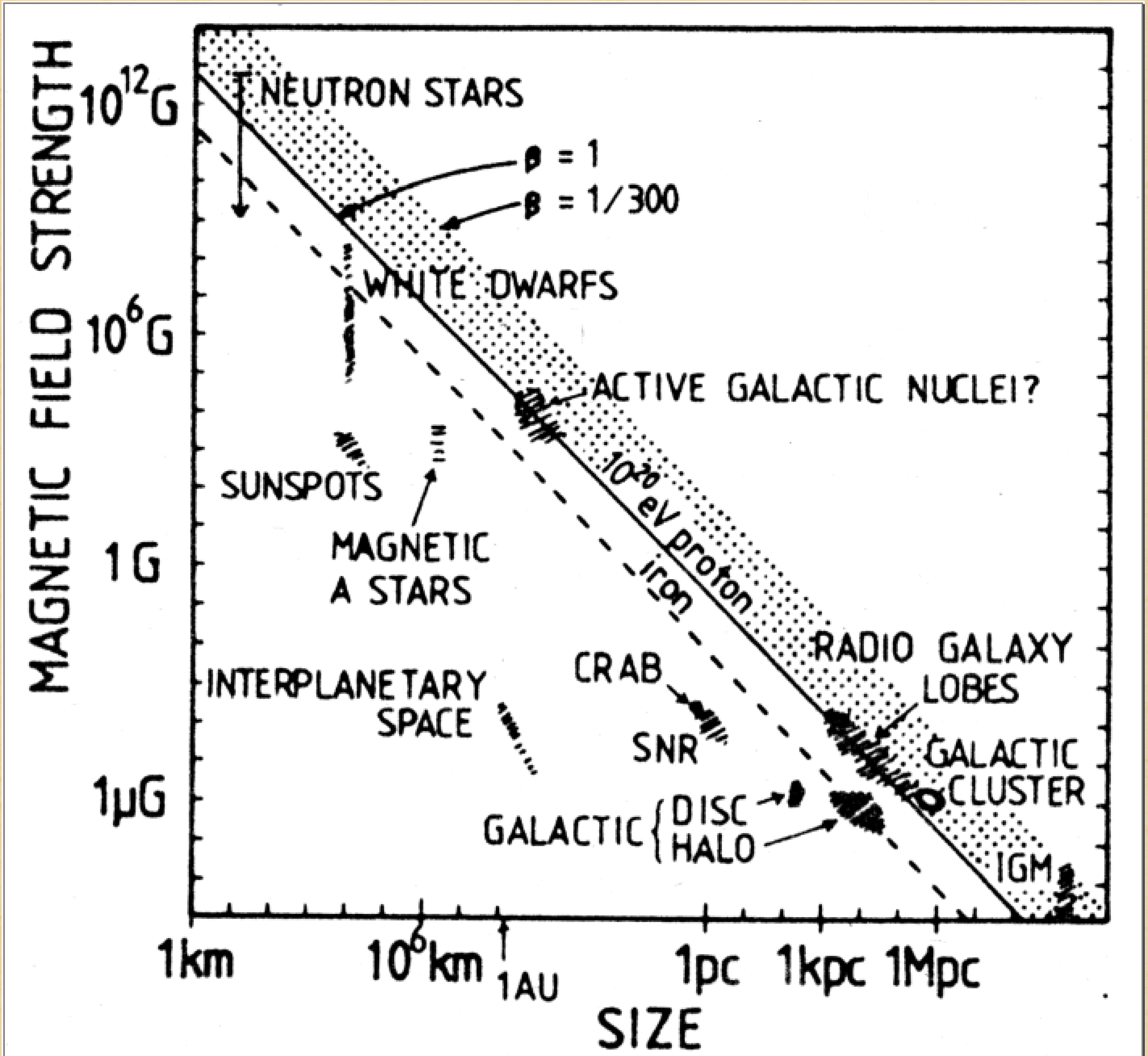
For relativistic neutrinos, the oscillation equation

- is independent of energy
- involves only the transverse field

Distance for helicity reversal

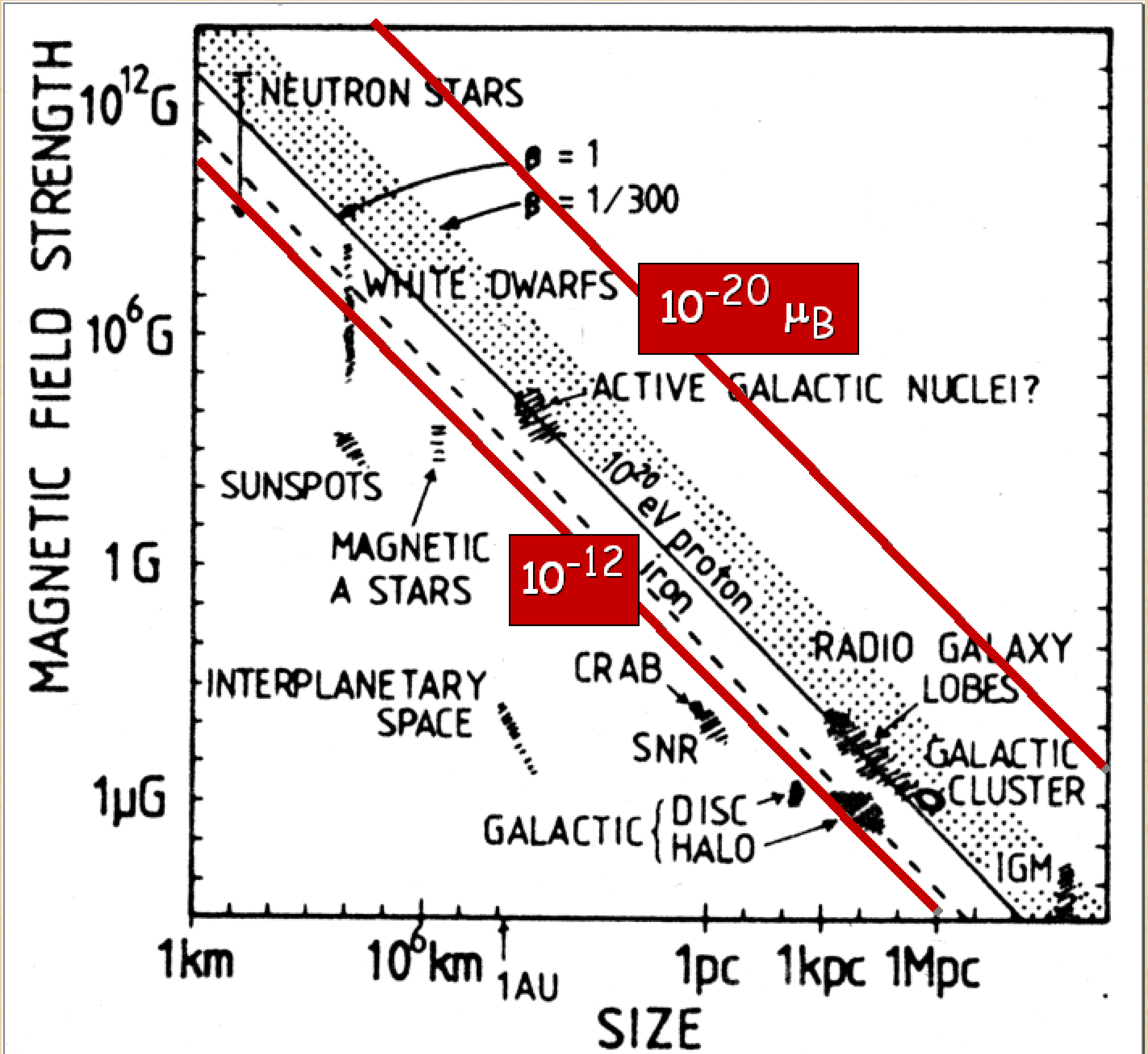
$$\frac{\pi}{2\mu_\nu B_T} = 5.36\times10^{13}\,\text{cm}\;\frac{10^{-10}\mu_B}{\mu_\nu}\;\frac{1\,\text{G}}{B_T}$$

Astrophysical Magnetic Fields



"Hillas Plot" [ARAA 22, 425 (1984)]

Astrophysical Magnetic Fields



Field strength and length scale where neutrinos with specified dipole moment would suffer complete depolarization

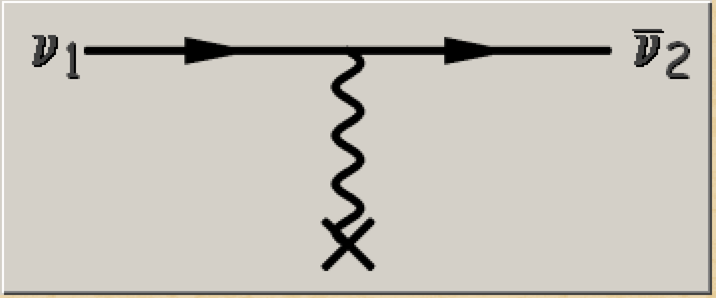
"Hillas Plot" [ARAA 22, 425 (1984)]

Neutrino Spin-Flavor Oscillations

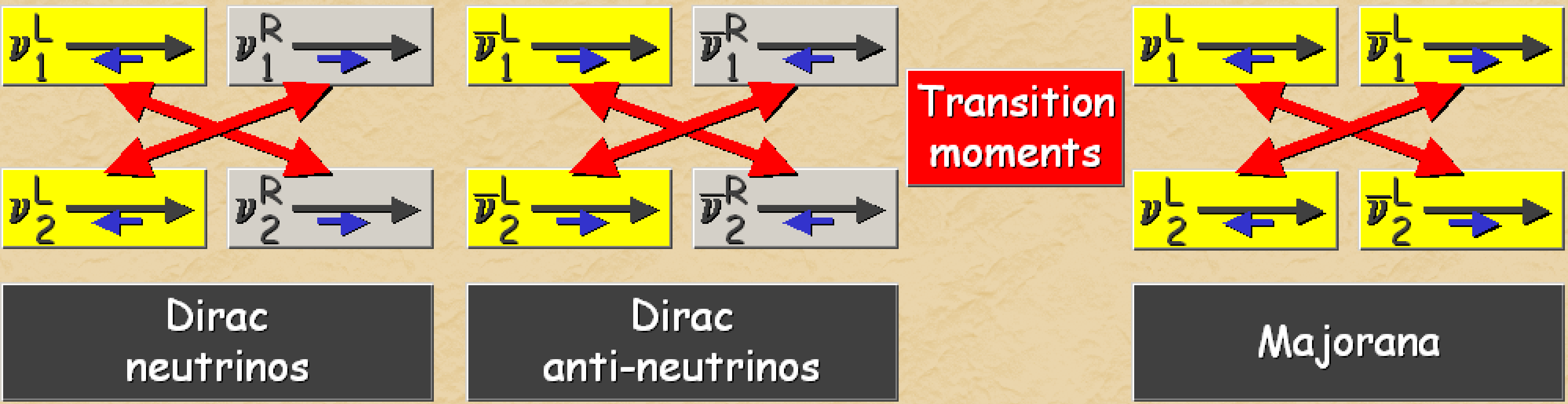
Majorana neutrinos
- Diagonal dipole moments vanish
- Transition moments inevitably exist
- Standard-model calculation ~ Dirac case

Transition moments couple neutrinos with anti-neutrinos

Spin-flavor precession in external E or B fields



$$i\frac{\partial}{\partial t}\begin{pmatrix}\nu_1\\ \bar\nu_2\end{pmatrix} = \begin{pmatrix}0 & \mu_\nu B_T\\ \mu_\nu B_T & 0\end{pmatrix}\begin{pmatrix}\nu_1\\ \bar\nu_2\end{pmatrix}$$



Neutrino Spin-Flavor Oscillations in a Medium

Consider general case of two-flavor oscillations for Majorana neutrinos with a transition magnetic moment μ and ordinary flavor mixing in a medium (the most general case for two-flavor oscillations in the Sun)

$$i\frac{\partial}{\partial r}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \bar\nu_e\\ \bar\nu_\mu\end{pmatrix}=\begin{pmatrix}c\Delta+a_e & s\Delta & 0 & \mu B\\ s\Delta & -c\Delta+a_\mu & \mu B & 0\\ 0 & \mu B & c\Delta-a_e & s\Delta\\ \mu B & 0 & s\Delta & -c\Delta-a_\mu\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \bar\nu_e\\ \bar\nu_\mu\end{pmatrix}$$

$$c=\cos(2\theta)$$

$$s=\sin(2\theta)$$

$$\Delta=\frac{m_2^2-m_1^2}{4E}$$

$$a_e=\sqrt{2}G_F\left(n_e-\tfrac{1}{2}n_n\right)$$

$$a_\mu=\sqrt{2}G_F\left(-\tfrac{1}{2}n_n\right)$$

Resonant spin-flavor precession (RSFP) obtains an excellent fit to solar neutrino data

- But requires very large B-fields in view of globular-cluster bound on μ
- Requires large non-standard transition moment of order $(10^{-12}-10^{-10})\,\mu_B$
- After KamLAND not the dominant effect
- May still produce some solar anti-neutrinos
- May play some role for supernova neutrinos