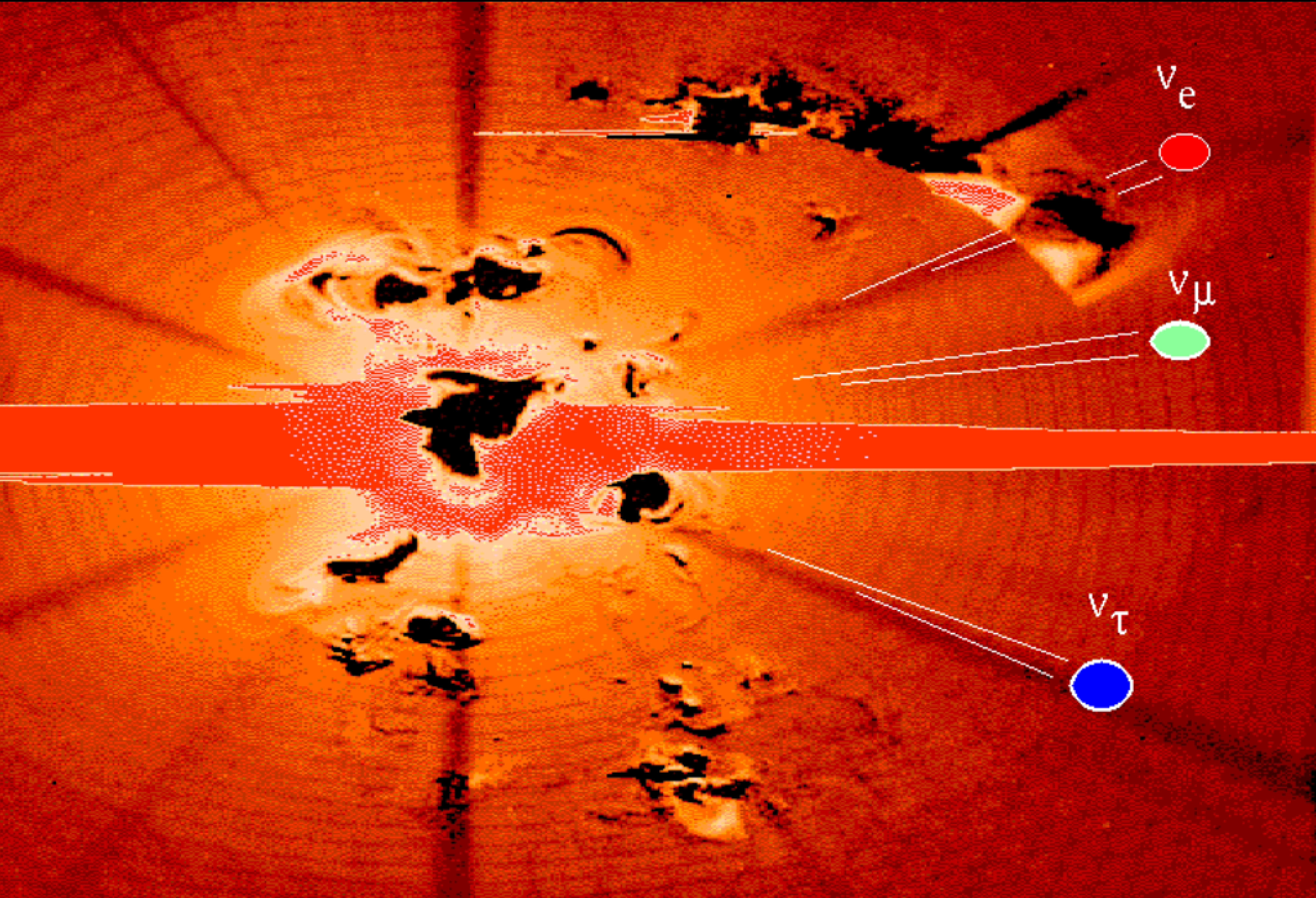
ISAPP 2004, International School on Astro-Particle Physics,
28 June—9 July 2004, Laboratori Nazionali del Gran Sasso, Italy

Neutrinos in Astrophysics and Cosmology
Part IV

Cosmological Neutrinos

Georg G. Raffelt
Max-Planck-Institut für Physik, München, Germany

Creation of the Universe



Cosmological Neutrinos

- The Cosmic Neutrino Sea
- Neutrino Masses and Cosmic Structures
- Big-Bang Nucleosynthesis
- How Many Cosmological Neutrinos?
- Leptogenesis

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Cosmological Neutrinos
Topic I

The Cosmic Neutrino Sea

Georg G. Raffelt
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Neutrino Thermal Equilibrium

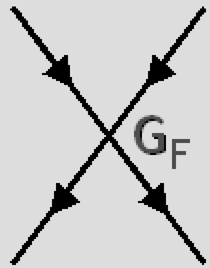
Neutrino reactions

Examples for neutrino processes

$$e^+ + e^- \leftrightarrow \bar{\nu} + \nu$$

$$\bar{\nu} + \nu \leftrightarrow \bar{\nu} + \nu$$

$$\nu + e^\pm \leftrightarrow \nu + e^\pm$$



Dimensional analysis of reaction rate

if $T \ll m_{W,Z}$

$$\Gamma \sim G_F^2 T^5$$

Cosmic expansion rate

Friedmann equation

$$H^2 = \frac{8\pi}{3}\frac{\rho}{m_{Pl}^2}$$

Radiation dominates

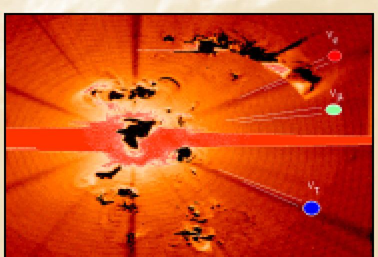
$$\rho \sim T^4$$

Expansion rate

$$H \sim \frac{T^2}{m_{Pl}}$$

Condition for thermal equilibrium: $\Gamma > H$

$$T > (m_{Pl} G_F^2)^{-1/3} \sim [10^{19}\,\mathrm{GeV}(10^{-5}\,\mathrm{GeV}^{-2})^2]^{-1/3} = 1\ \mathrm{MeV}$$



Neutrinos are in thermal equilibrium for $T \gtrsim 1$ MeV corresponding to $t \lesssim 1$ sec

Neutrino Thermal Equilibrium

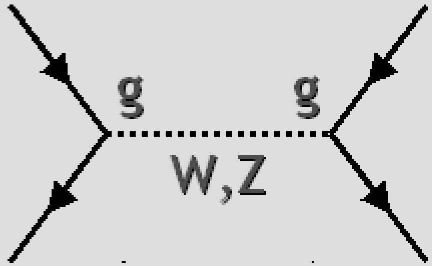
Neutrino reactions

Examples for neutrino processes

$$e^+ + e^- \leftrightarrow \bar\nu + \nu$$

$$\bar\nu + \nu \leftrightarrow \bar\nu + \nu$$

$$\nu + e^\pm \leftrightarrow \nu + e^\pm$$



Dimensional analysis of reaction rate

if $T \gg m_{W,Z}$

$$\Gamma \sim (g^2/4\pi)^2 T$$

Cosmic expansion rate

Friedmann equation

$$H^2 = \frac{8\pi}{3}\frac{\rho}{m_{Pl}^2}$$

Radiation dominates

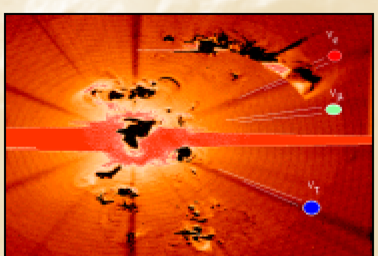
$$\rho \sim T^4$$

Expansion rate

$$H \sim \frac{T^2}{m_{Pl}}$$

Condition for thermal equilibrium: $\Gamma > H$

$$T < (g^2/4\pi)^2\, m_{Pl} \approx \Lambda_{GUT}$$



It depends on very early cosmic history when neutrinos first enter equilibrium, presumably at reheating after inflation

Thermal Radiations

	General	Bosons	Fermions
Energy density ρ	$g\int \frac{d^3\bar{p}}{(2\pi)^3}\frac{E_p}{e^{E_p/T}\pm 1}$	$g_B \frac{\pi^2}{30} T^4$	$\frac{7}{8} g_F \frac{\pi^2}{30} T^4$
Pressure P	$\frac{\rho}{3}$		
Entropy density s	$\frac{\rho+P}{T} = \frac{4}{3}\frac{\rho}{T}$	$g_B \frac{2\pi^2}{45} T^3$	$\frac{7}{8} g_F \frac{2\pi^2}{45} T^3$
Number density n	$g\int \frac{d^3\bar{p}}{(2\pi)^3}\frac{1}{e^{E_p/T}\pm 1}$	$g_B \frac{\zeta_3}{\pi^2} T^3$	$\frac{3}{4} g_F \frac{\zeta_3}{\pi^2} T^3$

Present-Day Neutrino Density

Neutrino decoupling (freeze out)	$H = \Gamma$ $T \approx 2.4$ MeV (electron flavor) $T \approx 3.7$ MeV (other flavors)
Redshift of Fermi-Dirac distribution ("nothing changes at freeze-out")	$\frac{dN_{\nu\bar\nu}}{dE} = \frac{1}{\pi^2}\frac{E^2}{e^{E/T}+1}$ Temperature scales with redshift $T_\nu = T_\gamma \propto (z+1)$
Electron-positron annihilation beginning at $T \approx m_e = 0.511$ MeV	• QED plasma is "strongly" coupled • Stays in thermal equilibrium (adiabatic process) • Entropy of e^+e^- transfered to photons $\underbrace{g_* T_\gamma^3}_{2+\frac{7}{8}4}\Big\vert_{before} = \underbrace{g_* T_\gamma^3}_{2}\Big\vert_{after}$ $\Big\}$ $T_\gamma^3\Big\vert_{after} = \frac{4}{11} T_\gamma^3\Big\vert_{before}$
Redshift of neutrino and photon thermal distributions so that today we have	$n_{\nu\bar\nu}(1\,\text{flavor}) = \frac{4}{11}\times\frac{3}{4}\times n_\gamma = \frac{3}{11} n_\gamma \approx 115\,\text{cm}^{-3}$ $T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \approx 1.95\,\text{K}$ for massless neutrinos

Cosmological Limit on Neutrino Masses

Cosmic neutrino "sea" ~ 115 cm^{-3} neutrinos + anti-neutrinos per flavor

$$\Omega_\nu h^2 = \sum \frac{m_\nu}{92.5\,\mathrm{eV}} < 0.4$$

$m_\nu \lesssim 40$ eV For all stable flavors

REST MASS OF MUONIC NEUTRINO AND COSMOLOGY

S. S. Gershtein and Ya. B. Zel'dovich
Submitted 4 June 1966
ZhETF Pis'ma 4, No. 5, 174-177, 1 September 1966

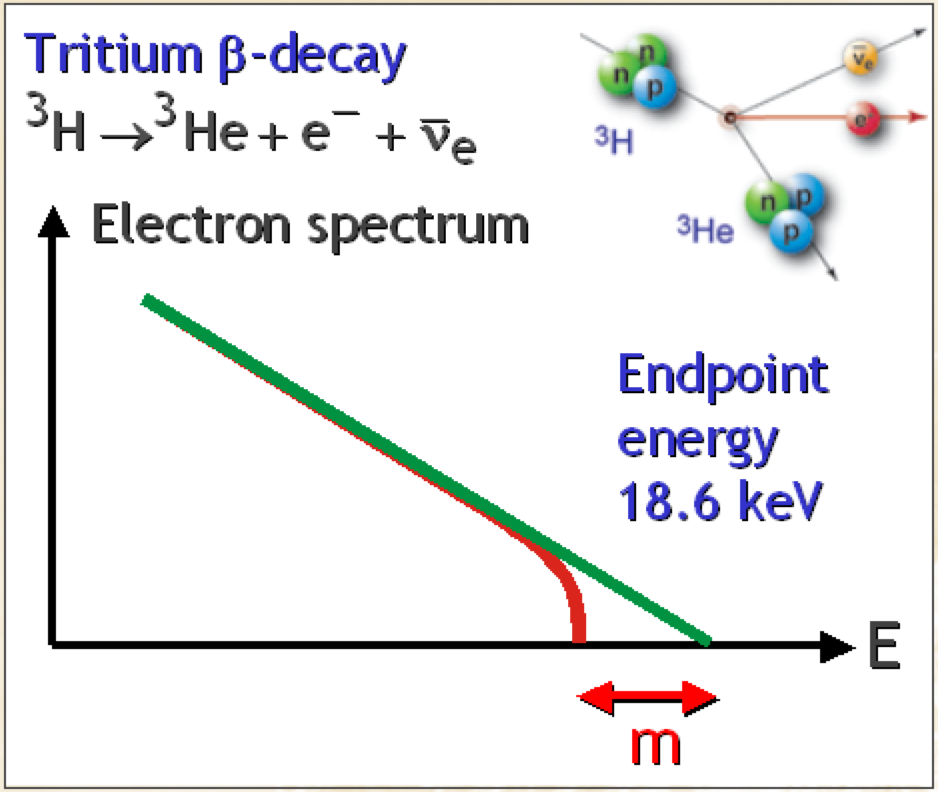
Low-accuracy experimental estimates of the rest mass of the neutrino [1] yield $m(\nu_e) < 200$ eV/c^2 for the electronic neutrino and $m(\nu_\mu) < 2.5 \times 10^6$ eV/c^2 for the muonic neutrino.

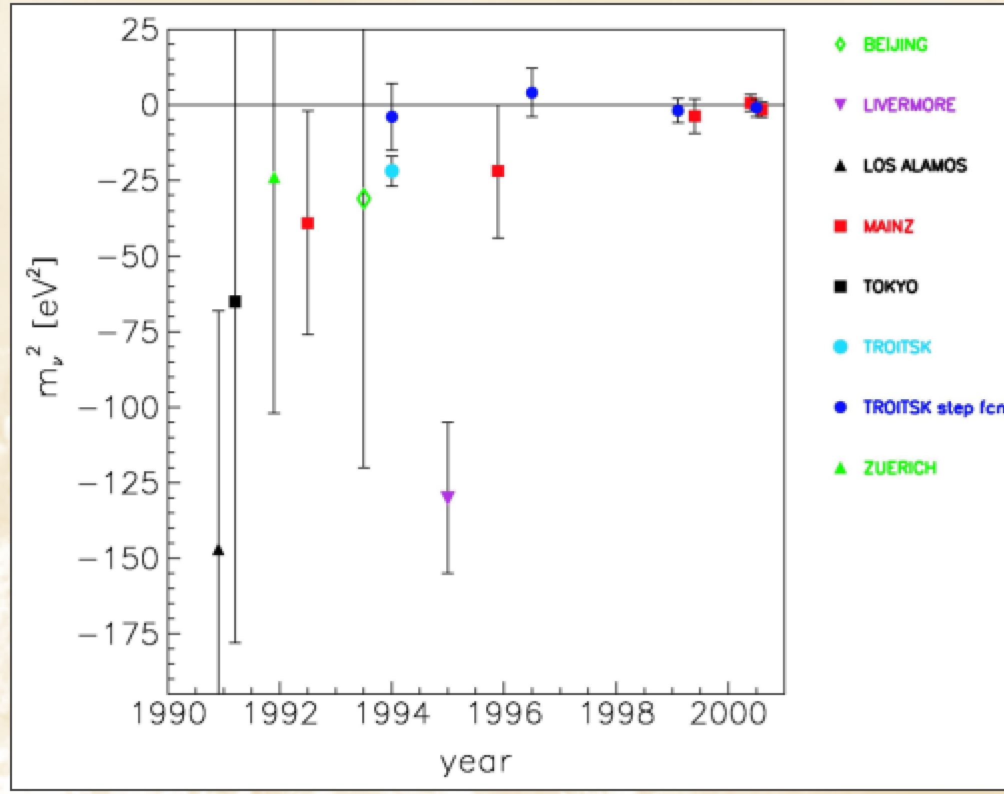
Cosmological considerations connected with the hot model of the Universe [2] make it possible to strengthen greatly the second inequality. Just as in the paper by Ya. B. Zel'dovich and Ya. A. Smorodinskii [3], let us consider the gravitational effect of the neutrinos on the dynamics of the expanding Universe. The age of the known astronomical objects is not smaller than 5×10^9 years, and Hubble's constant H is not smaller than 75 km/sec-Mparsec $= (13 \times 10^9 \text{ years})^{-1}$. It follows therefore that the density of all types of matter in the Universe is at the present time [1)]

$$\rho < 2 \times 10^{-28}\ \mathrm{g/cm^3}.$$

A classic paper:
Gershtein & Zeldovich
JETP Lett. 4 (1966) 120

Tritium Endpoint Spectrum





Currently best limits from Mainz and Troitsk experiments

$m < 2.2$ eV (95% CL)

- Scaled-up spectrometer (KATRIN) should reach 0.2 eV
- Currently under construction
- Measurements to begin 2007

http://ik1au1.fzk.de/~katrin

Three-Flavor Neutrino Parameters

Atmospheric/K2K	CHOOZ	Solar/KamLAND	2σ ranges hep-ph/0405172
$37^\circ < \theta_{23} < 54^\circ$	$\theta_{13} < 11^\circ$	$30^\circ < \theta_{12} < 36^\circ$	

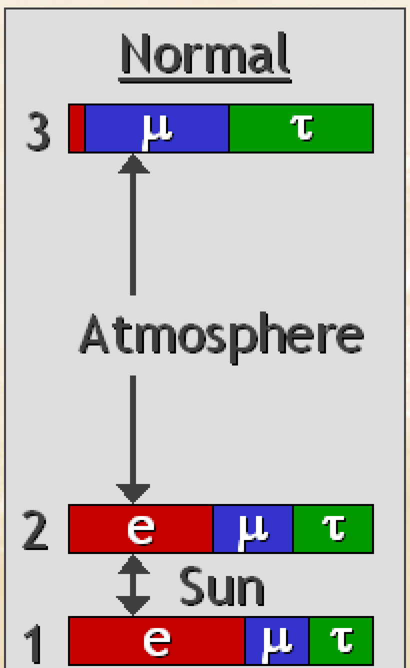
$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}1 & & \\ & c_{23} & s_{23}\\ & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & & e^{-i\delta}s_{13}\\ & 1 & \\ -e^{i\delta}s_{13} & & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

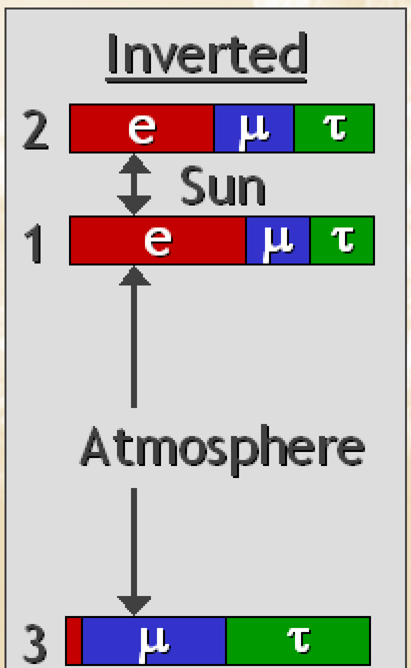
$c_{12} = \cos\theta_{12}$ etc., δ CP-violating phase

Solar 75–92

Atmospheric 1400–3000

$\Delta m^2/\mathrm{meV}^2$





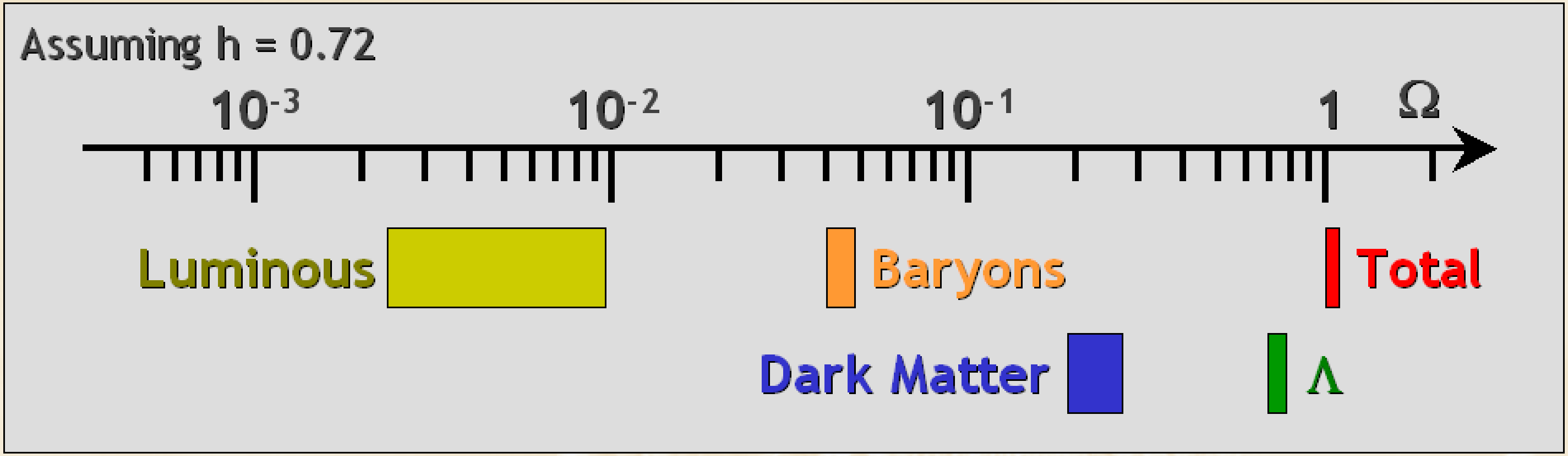
Tasks and Open Questions

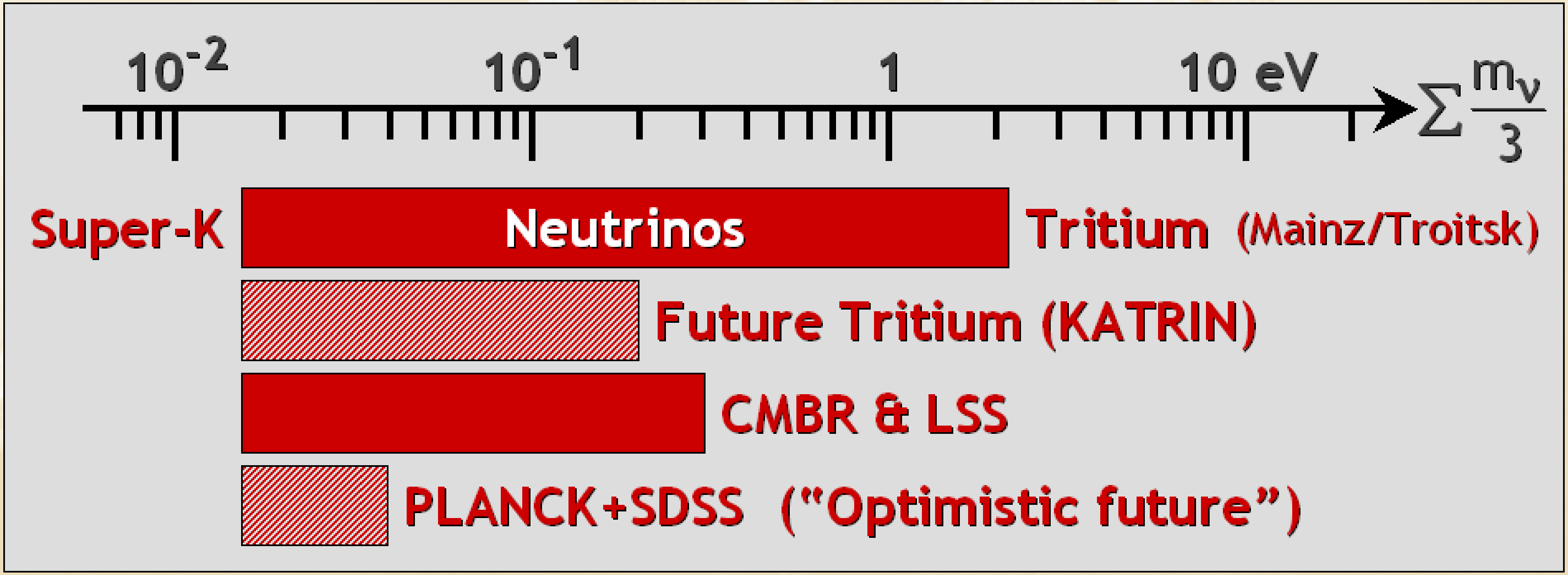
- Precision for θ_{12} and θ_{23}
- How large is θ_{13}?
- CP-violating phase δ?
- Mass ordering? (normal vs inverted)
- Absolute masses? (hierarchical vs degenerate)
- Dirac or Majorana?

Present-Day Neutrino Distribution

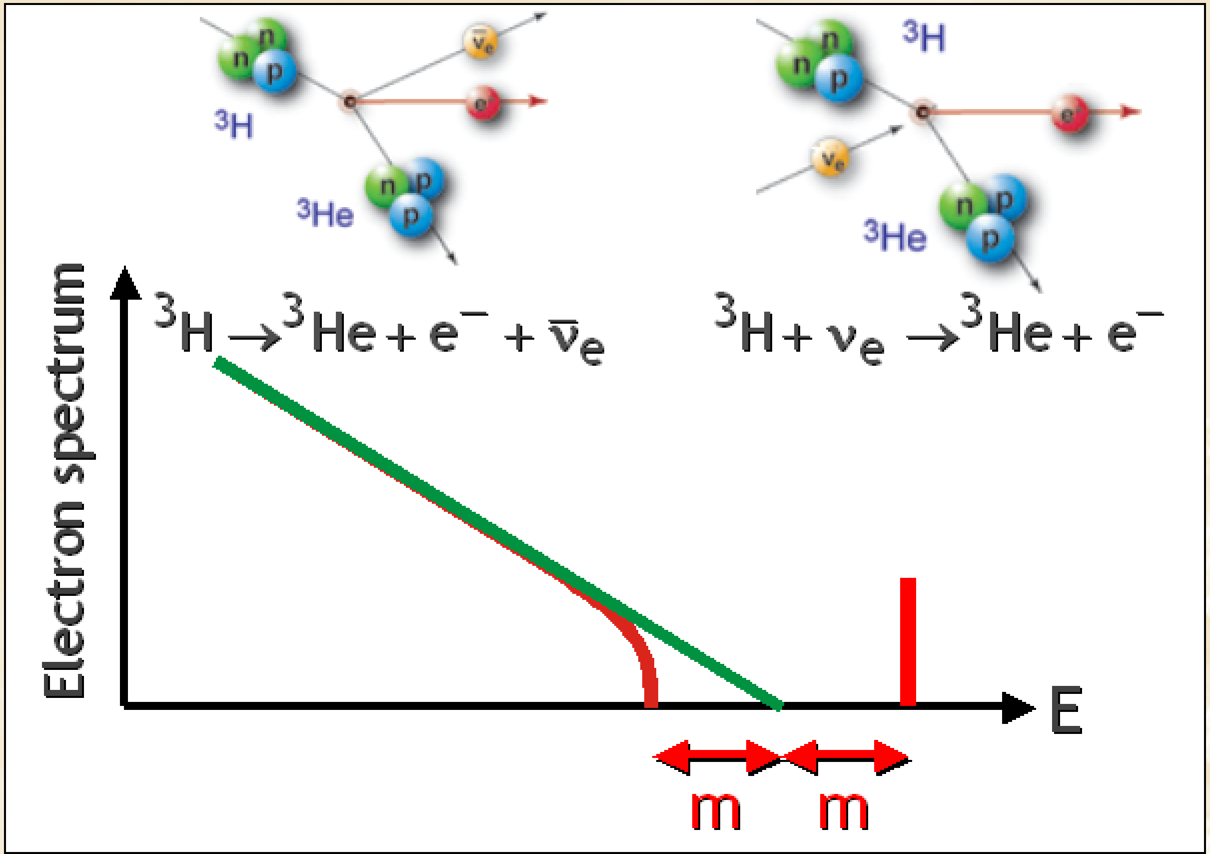
	Normal	Inverted
Minimal neutrino masses from oscillation experiments	$m_3 \gtrsim 50$ meV $m_2 \gtrsim 8$ meV $m_1 \geq 0$	$m_1 \approx m_2 \gtrsim 50$ meV $m_3 \geq 0$
Temperature of massless cosmic background neutrinos	$T = 1.95$ K $= 0.17$ meV	
Cosmic redshift of momenta (not energies)	$\frac{dN_{\nu\bar{\nu}}}{dp} = \frac{1}{\pi^2} \frac{p^2}{e^{p/T}+1}$ Not a thermal distribution unless $T \gg m$	
Average velocity for $m \gg T$	$\langle v \rangle \approx 3\,T/m$	
Normal hierarchy neutrinos	$\langle v_3 \rangle < 1 \times 10^{-2} c$ $\langle v_2 \rangle < 6 \times 10^{-2} c$	
Degenerate case ($m = 0.4$ eV)	$\langle v \rangle < 1 \times 10^{-3} c$ Some clustering possible	

Mass-Energy-Inventory of the Universe





Tritium Endpoint Spectrum Including Cosmic Neutrinos



Event rate above the endpoint relative to the total rate

$$\frac{\Gamma_{E_e>Q}}{\Gamma_{tot}} = \frac{105\,\pi^2}{8}\,\frac{n_{\nu_e}}{(Q-m_e)^3}$$

$$= \frac{105\,\pi^2}{8}\,\frac{57\ \mathrm{cm}^{-3}}{(18.6\ \mathrm{keV})^3}$$

$$= 0.85 \times 10^{-23}$$

Unmeasurably small

Electron spectrum ($E = E_e - Q$, $Q = m_e + 18.6$ keV) in the presence of cosmic neutrinos with occupation numbers $f(E)$ and anti-neutrinos with $\bar{f}(E)$

$$\frac{d\Gamma}{dE} = A\,Q\sqrt{Q^2 - m_e^2}\,\sqrt{E^2(E^2 - m_\nu^2)} \times \begin{cases} [1-\bar{f}(E)] & \text{for } E \le -m_\nu \\ f(E) & \text{for } E \ge +m_\nu \end{cases}$$

See also: Duda et al., Expected signals in relic neutrino detectors, PRD 64 (2001) 122001

Z-Bursts and Highest-Energy Cosmic Rays

Neutrinos
$E_\nu \sim 10^{21} - 10^{22}$ eV
from unknown sources

Resonant
Z-Boson
Production

Cosmic relic
neutrinos
$m_\nu \sim 1$ eV

Neutrino energy
on resonance

$$E_\nu = \frac{M_Z^2}{2m_\nu} = \frac{4.2\times 10^{21}\,\mathrm{eV}}{m_\nu/\mathrm{eV}}$$

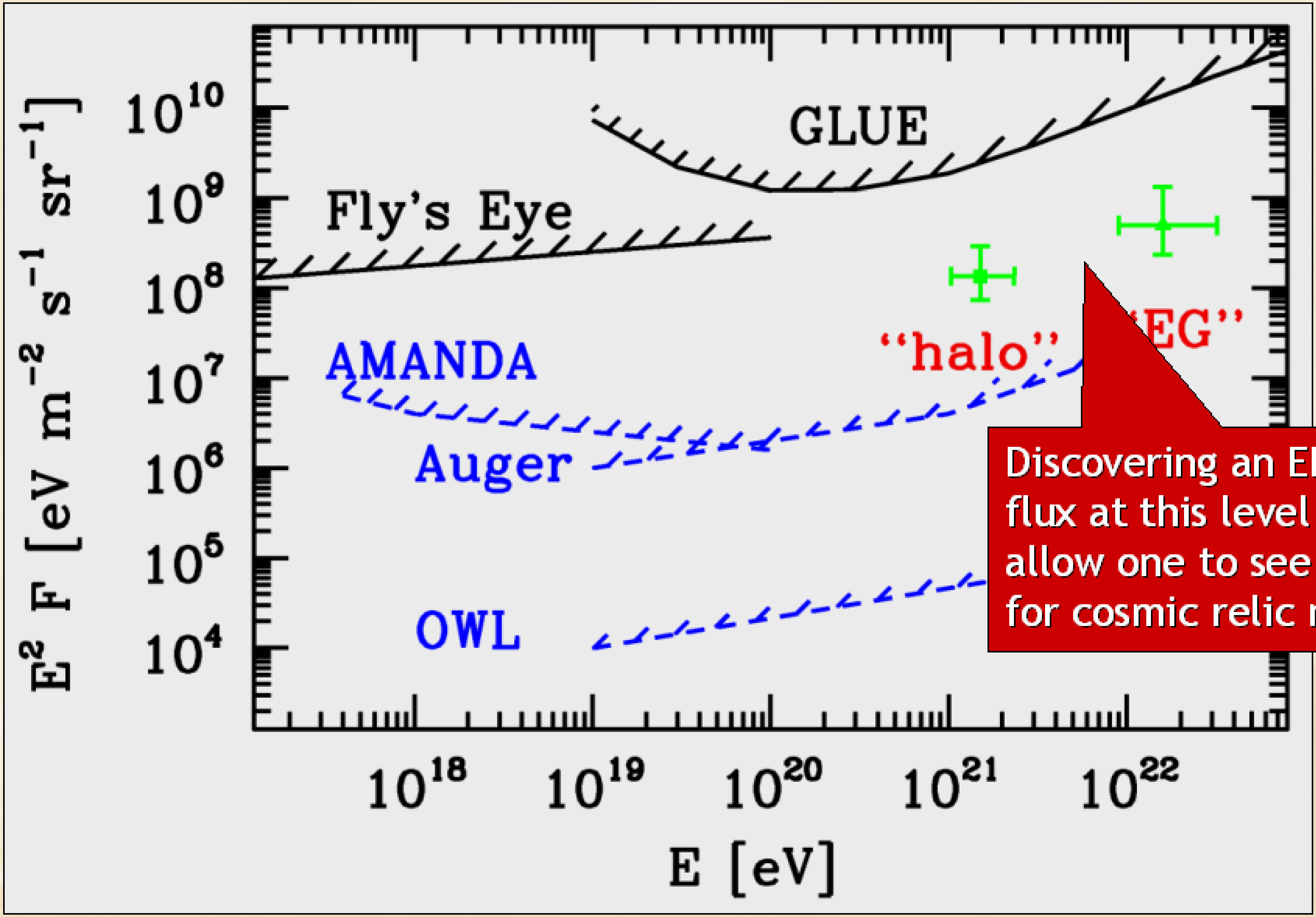
Decay
(Z-Burst)

On average
2 Nucleons
$10\ \pi^0 \to 20\ \gamma$
$17\ \pi^\pm \to e^\pm\,\nu\,\bar\nu$

Measured cosmic rays

For example:
- Weiler
  hep-ph/9710431
- Fargion et al.
  hep-ph/0112014
- Fodor, Katz, Ringwald
  hep-ph/0203198
- Gelmini et al.
  hep-ph/0404272

Discovery Potential for Required Neutrino Fluxes



Discovering an EHE neutrino flux at this level might allow one to see evidence for cosmic relic neutrinos

Fodor, Katz & Ringwald, hep-ph/0203198

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Cosmological Neutrinos
Topic II

Neutrino Masses and Cosmic Structures

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Weakly Interacting Particles as Dark Matter

THE ASTROPHYSICAL JOURNAL, **180**: 7–10, 1973 February 15

GRAVITY OF NEUTRINOS OF NONZERO MASS IN ASTROPHYSICS

R. COWSIK* AND J. MCCLELLAND
Department of Physics, University of California, Berkeley
Received 1972 July 24

ABSTRACT

If neutrinos have a rest mass of a few eV/c^2, then they would dominate the gravitational dynamics of the large clusters of galaxies and of the Universe. A simple model to understand the virial mass discrepancy in the Coma cluster on this basis is outlined.

Subject headings: cosmology — galaxies, clusters of — neutrinos

The possibility of a finite rest mass for the neutrinos has fascinated astrophysicists (Kuchowicz 1969). A recent discussion of such a possibility has been in the context of the solar-neutrino experiments (Bahcall, Cabibbo, and Yahil 1972). Here we wish to point out some interesting consequences of the gravitational interactions of such neutrinos. These considerations become particularly relevant in the framework of big-bang cosmologies which we assume to be valid in our discussion here.

In the early phase of such a Universe when the temperature was ~1 MeV, several processes of neutrino production (Ruderman 1969) would have led to copious production of neutrinos and antineutrinos (Steigman 1972; Cowsik and McClelland 1972). Conditions of thermal equilibrium allow an easy estimate of their number densities (Landau and Lifshitz 1969):

$$n_{\nu i} = \frac{1}{\pi^2 \hbar^3} \int_0^\infty \frac{p^2 dp}{\exp[E/kT(z_{eq})] + 1} . \qquad (1)$$

Here $n_{\nu i}$ = number density of neutrinos of the ith kind (notice that in writing this expression we have assumed that both the helicity states are allowed for the neutrinos because of finite rest mass); $E = c(p^2 + m^2c^2)^{1/2}$; k = Boltzmann's constant; $T(z_{eq}) = T_r(z_{eq}) = T_\nu(z_{eq}) = T_e(z_{eq}) \cdots =$ the common temperature of radiation, neutrinos, electrons, etc., at the latest epoch characterized by redshift z_{eq} when they may be assumed to have been in thermal equilibrium; $kT(z_{eq}) \simeq 1$ MeV.

Since the masses of the neutrinos are expected to be small, $kT(z_{eq}) \gg m_{\nu i}c^2$, in the extreme-relativistic limit equation (1) reduces to

$$n_{\nu i}(z_{eq}) \simeq 0.183[T(z_{eq})/hc]^3 . \qquad (2)$$

As the Universe expands, only the neutrinos (in contrast to all other known particles) survive annihilation because of extremely low cross-sections (deGraff and Tolhock 1966), and their number density decreases with increasing volume of the Universe, simply as $\sim V(z_{eq})/V(z) = [(1 + z)/(1 + z_{eq})]^3$. Noting that $(1 + z_{eq})/(1 + z) = T_r(z_{eq})/T_r(z)$, the number density at the present epoch ($z = 0$) is given by

$$n_{\nu i}(0) = n_{\nu i}(z_{eq})/(1 + z_{eq})^3 \simeq 0.183[T_r(0)/hc]^3 \simeq 300 \text{ cm}^{-3} , \qquad (3)$$

* On leave from the Tata Institute of Fundamental Research, Bombay, India.

More than 30 years ago, beginnings of the idea of weakly interacting particles (neutrinos) as dark matter

Massive neutrinos are no longer a good candidate (hot dark matter)

However, the idea of weakly interacting massive particles as dark matter is now standard

What is wrong with neutrino dark matter?

Galactic Phase Space ("Tremaine-Gunn-Limit")

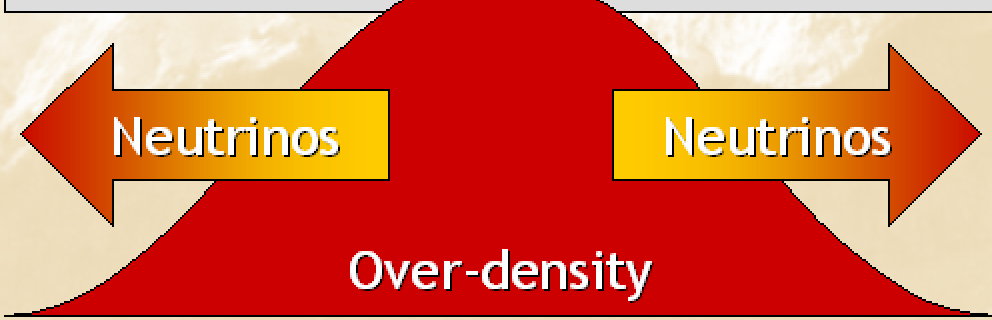
Maximum mass density of a degenerate Fermi gas

$$\rho_{max} = m_\nu \underbrace{\frac{p_{max}^3}{3\pi^2}}_{n_{max}} = \frac{m_\nu (m_\nu v_{escape})^3}{3\pi^2}$$

m_ν > 20 – 40 eV	Spiral galaxies
m_ν > 100 – 200 eV	Dwarf galaxies

Neutrino Free Streaming (Collisionless Phase Mixing)

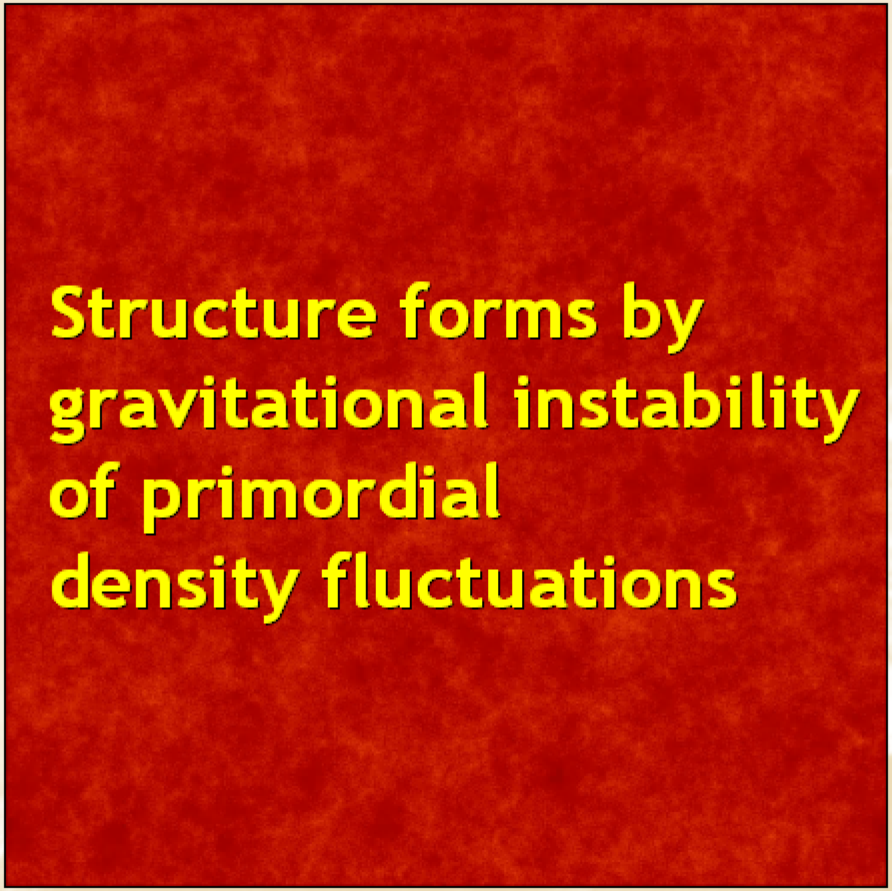
- At T < 1 MeV neutrino scattering in early universe ineffective
- Stream freely until non-relativistic
- Wash out density contrasts on small scales



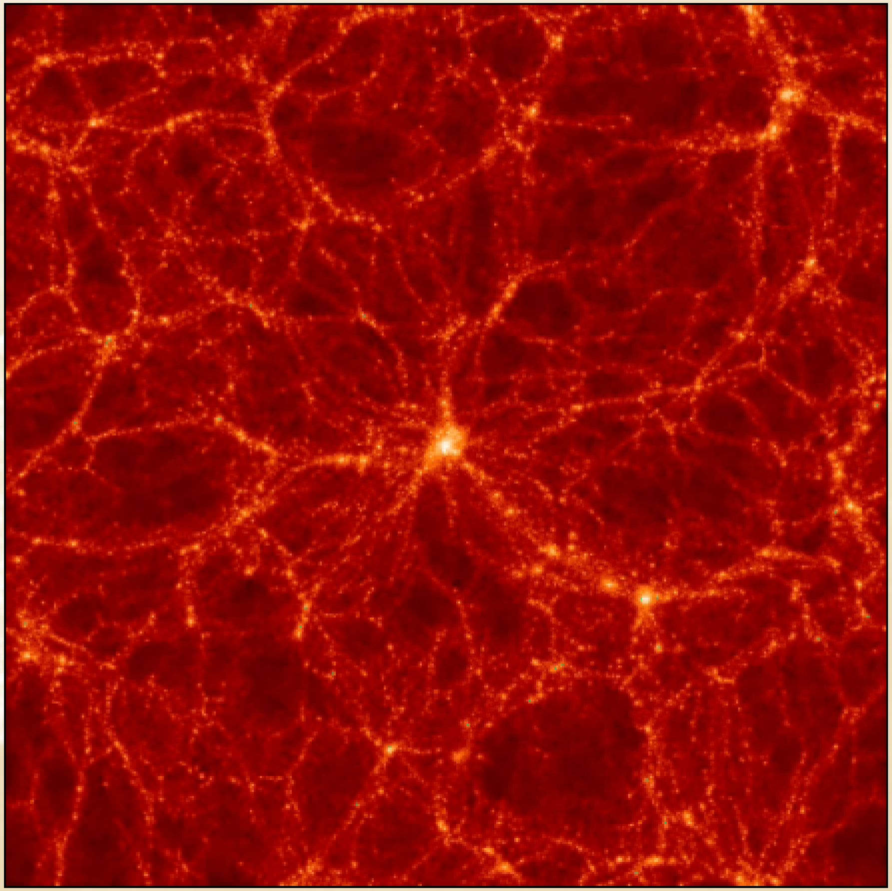
- Nus are "Hot Dark Matter"
- Ruled out
  by structure formation

Formation of Structure

Smooth	→	Structured



Structure forms by gravitational instability of primordial density fluctuations



Processed Power Spectrum in Cold Dark Matter Scenario

Primordial spectrum

Suppressed by stagnation during radiation phase

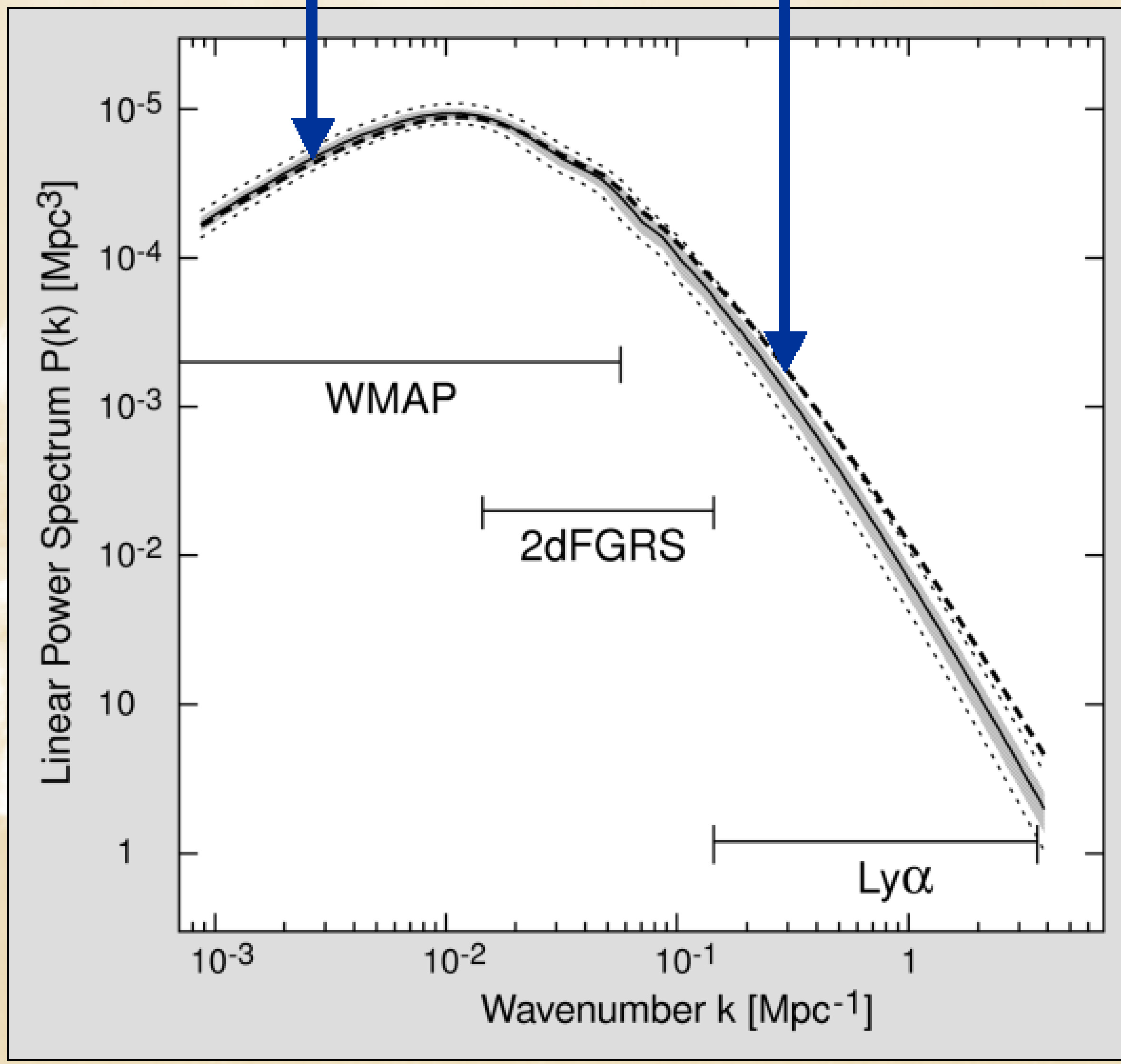
Primordial spectrum usually assumed to be of power-law form

$$P(k) = |\delta_k|^2 \propto k^n$$

Harrison-Zeldovich ("flat") spectrum

$n = 1$

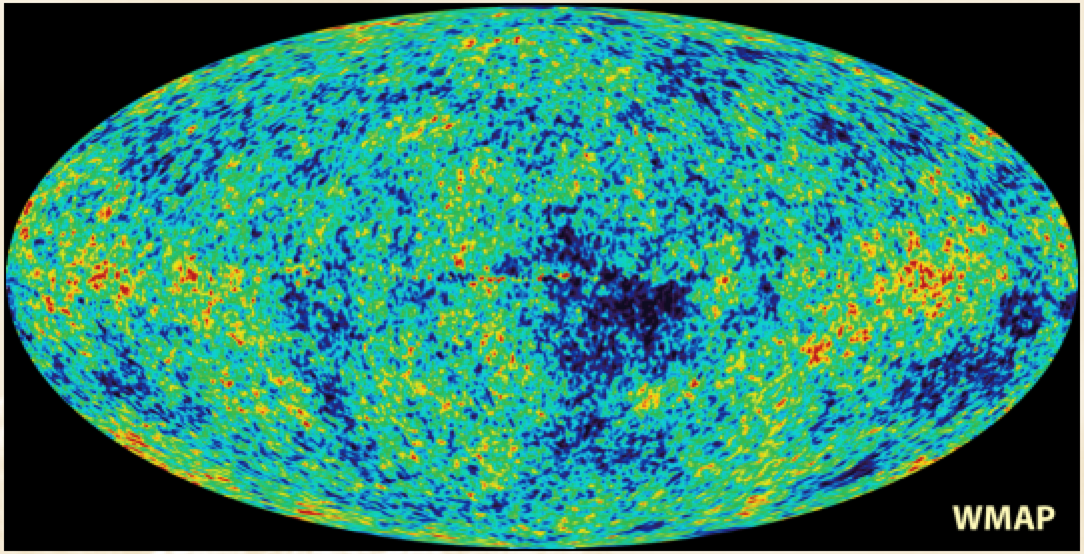
expected from inflation (may be slightly less than 1, depending on details of inflationary phase)



Power Spectrum of CMBR Temperature Fluctuations

Sky map of CMBR temperature fluctuations

$$\Delta(\theta,\varphi) = \frac{T(\theta,\varphi) - \langle T \rangle}{\langle T \rangle}$$

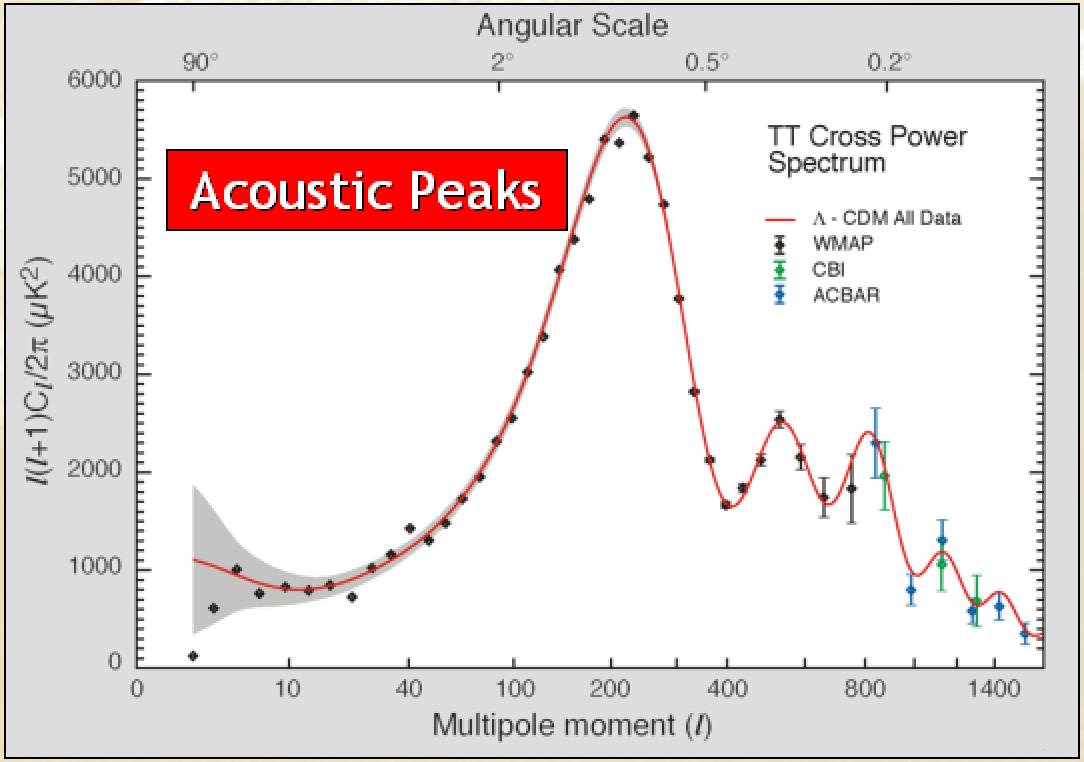


Multipole expansion

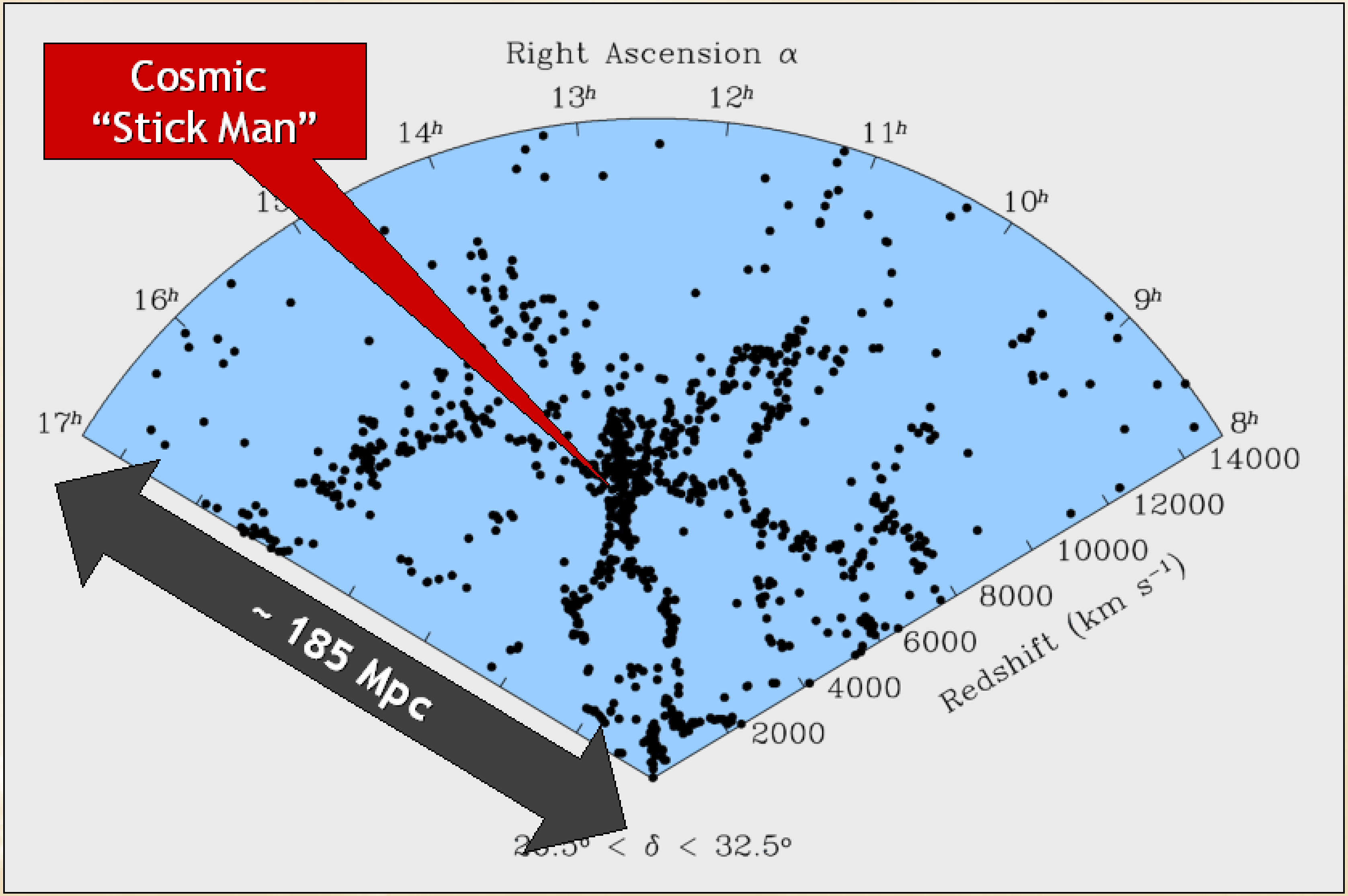
$$\Delta(\theta,\varphi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta,\varphi)$$

Angular power spectrum

$$C_\ell = \langle a_{\ell m}^* a_{\ell m} \rangle = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} a_{\ell m}^* a_{\ell m}$$

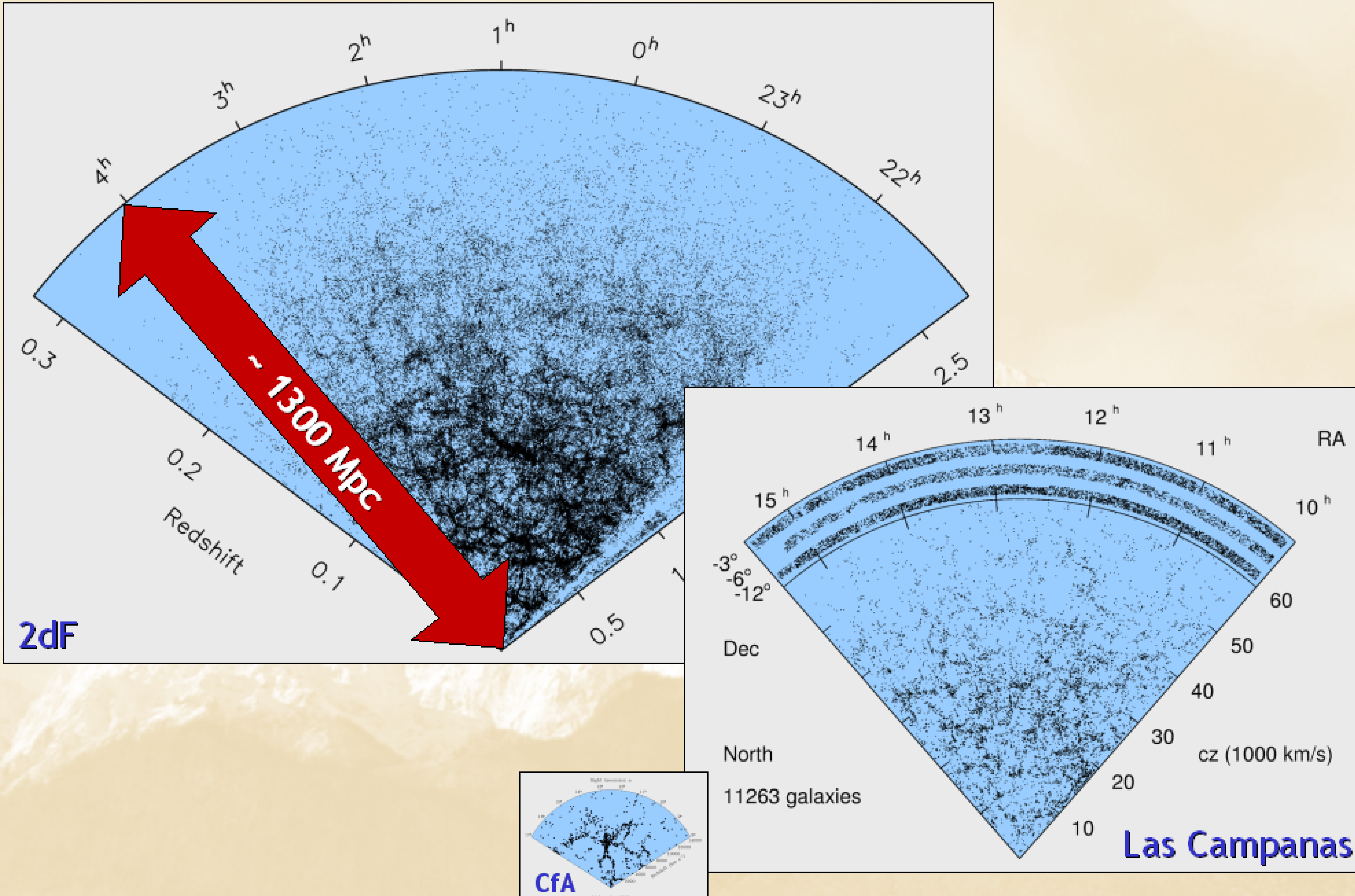


A Slice of the Universe

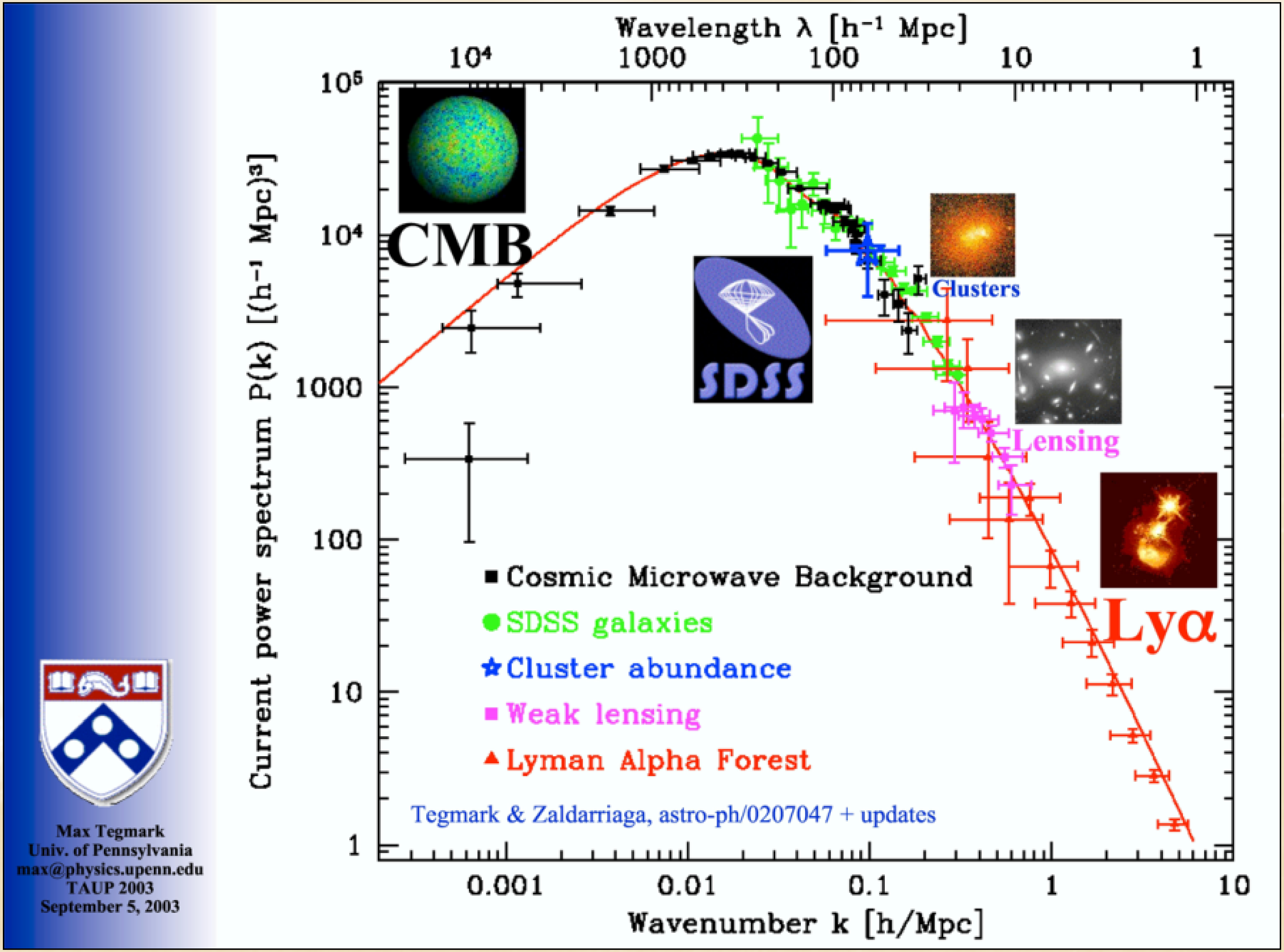


Galaxy distribution from the CfA redshift survey
[ApJ 302 (1986) L1]

2dF Galaxy Redshift Survey (15 May 2002)

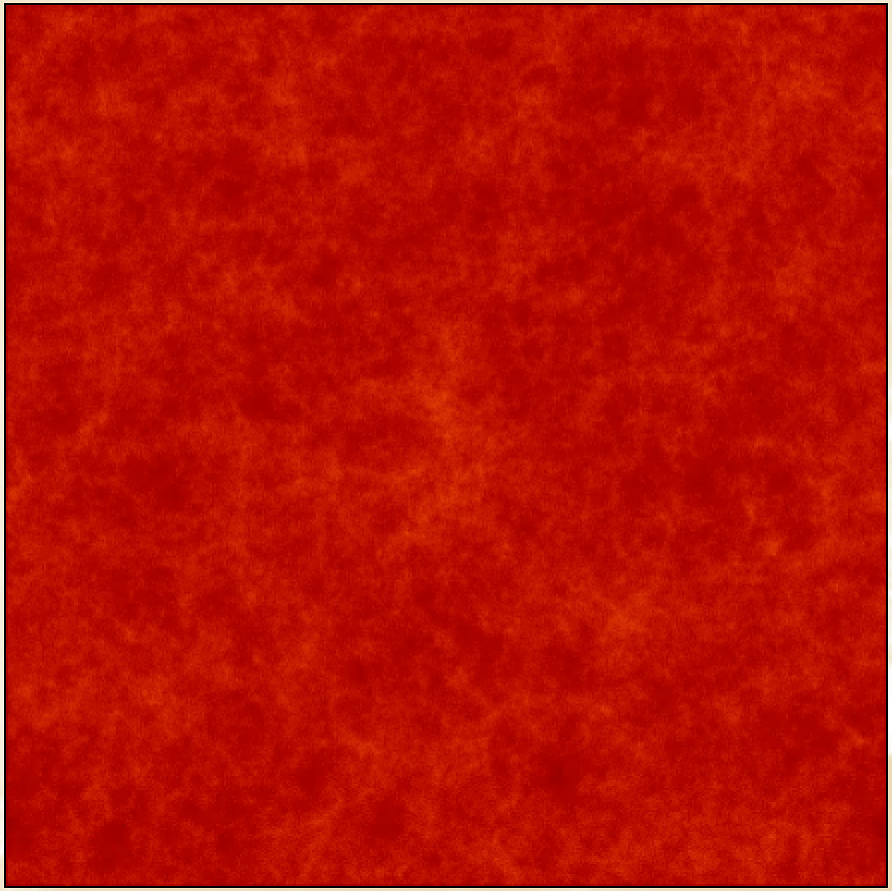


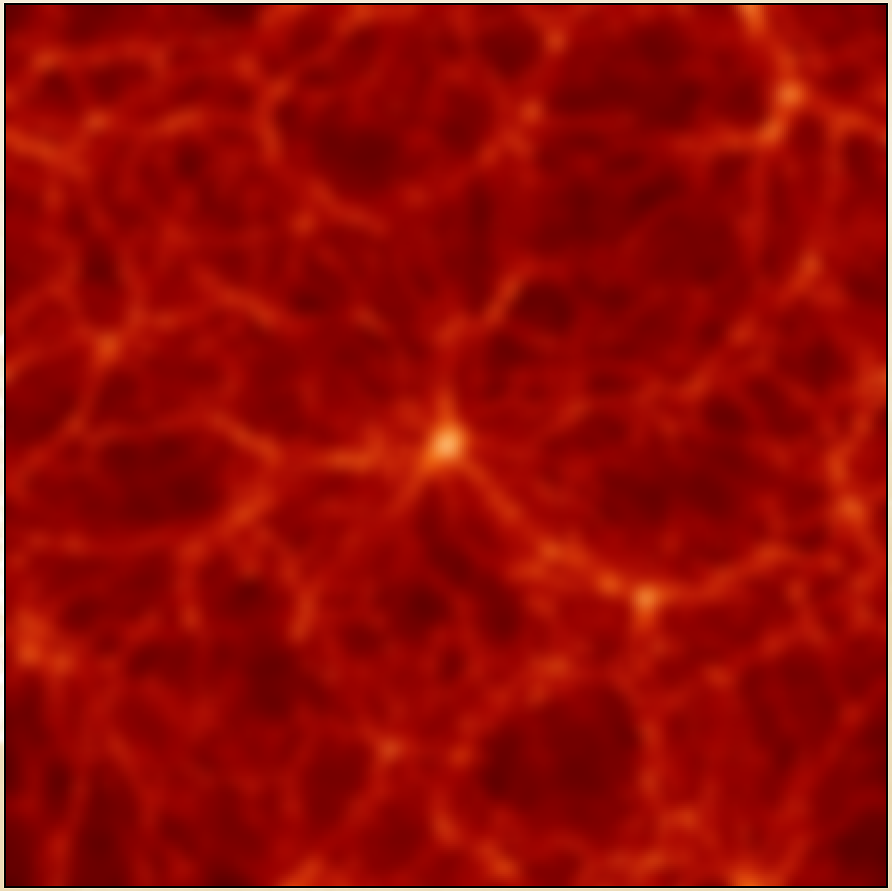
Power Spectrum of Cosmic Density Fluctuations



Formation of Structure

Smooth → Structured

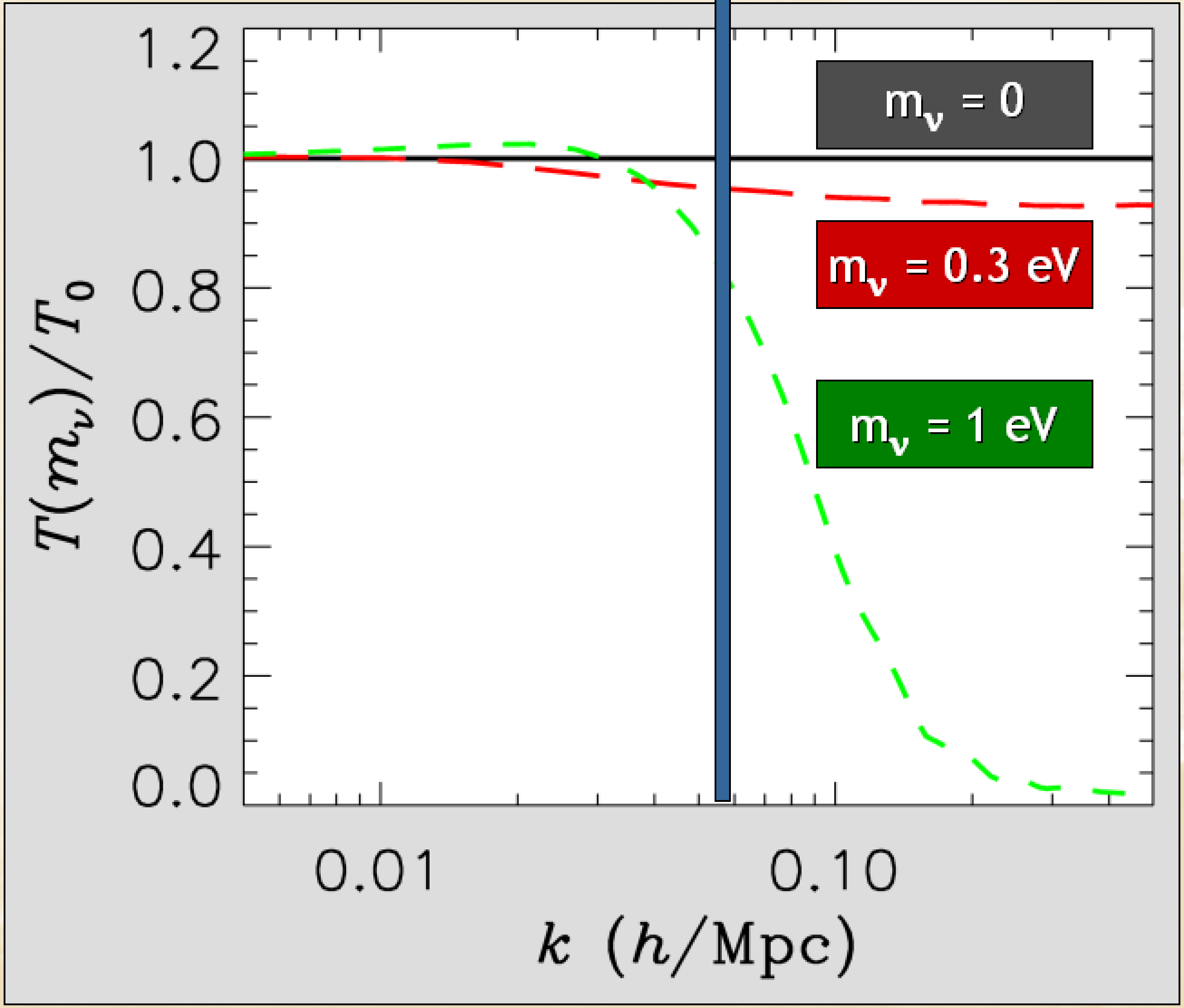




A fraction of hot dark matter suppresses small-scale structure

Neutrino Free Streaming – Transfer Function

Power suppression for $\lambda_{FS} \gtrsim 100$ Mpc/h



Transfer function

$P(k) = T(k)\, P_0(k)$

Effect of neutrino free streaming on small scales

$T(k) = 1 - 8\Omega_\nu/\Omega_M$

valid for

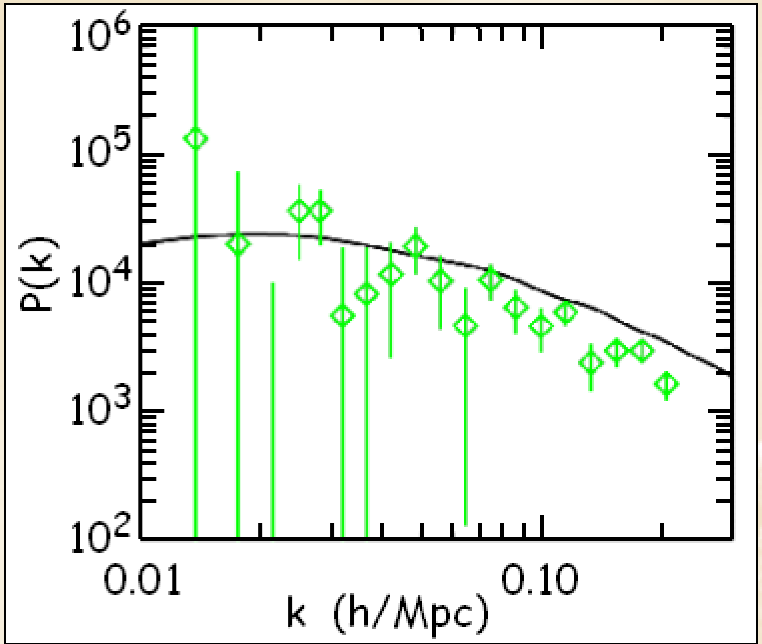
$8\Omega_\nu/\Omega_M \ll 1$

Hannestad, Neutrinos in Cosmology, hep-ph/0404239

Cosmic Structure Modified by Hot Dark Matter

Galaxy Distribution (2dF, PSCz)

Scales 1–200 Mpc



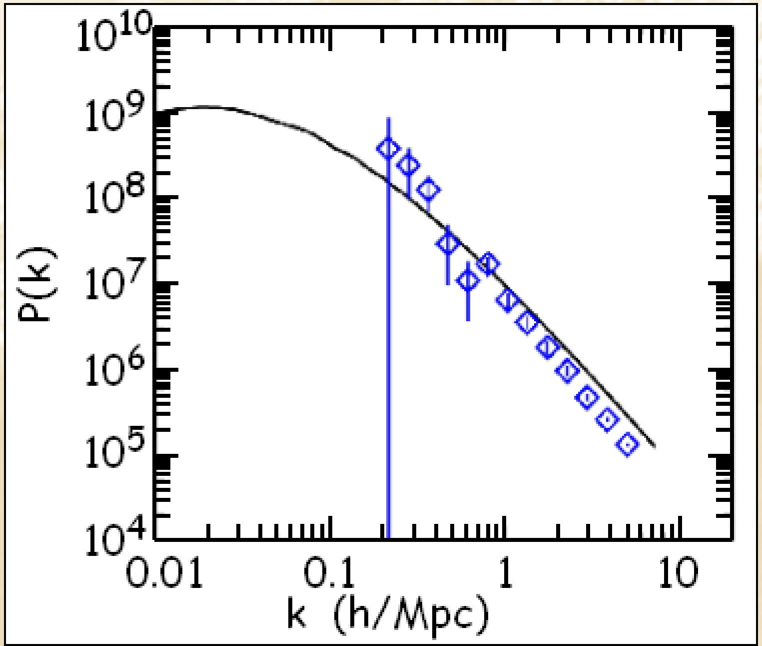
$\Omega_0 = 1$
$\Omega_\Lambda = 0.66$
$\Omega_B = 0.04$
$H_0 = 72$
$n_s = 0.94$

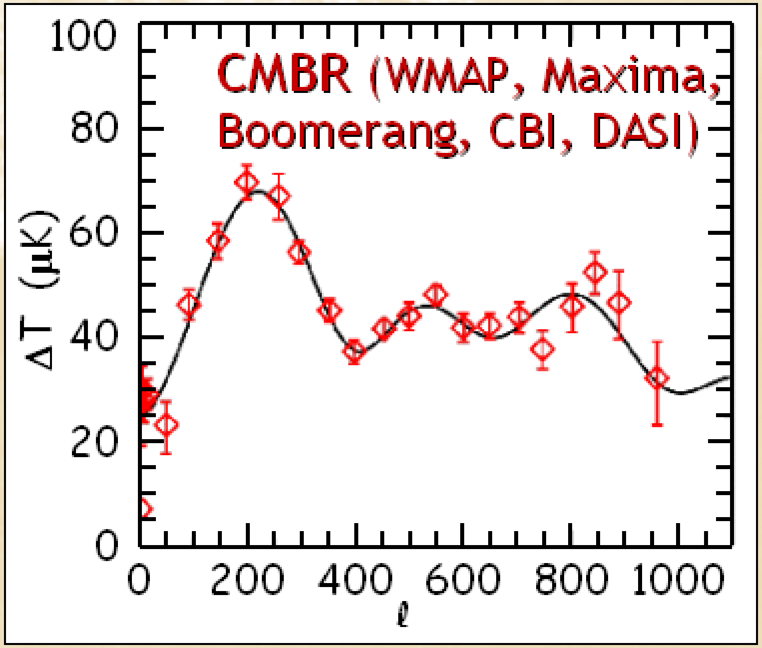
$\Omega_\nu = 0.00$

Adapted from S.Hannestad

Lyman-α forest at large redshift ⟨z⟩ = 2.72

Scales 0.1–10 Mpc

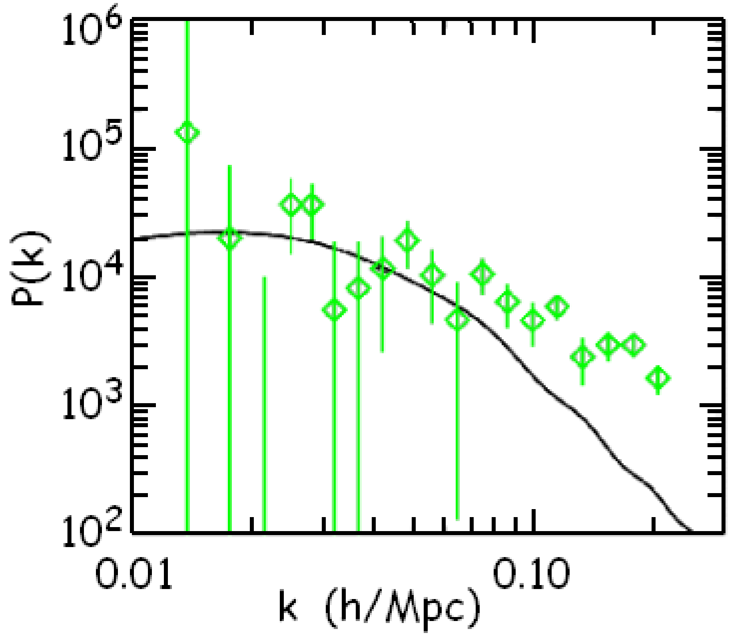




Cosmic Structure Modified by Hot Dark Matter

Galaxy Distribution (2dF, PSCz)

Scales 1–200 Mpc



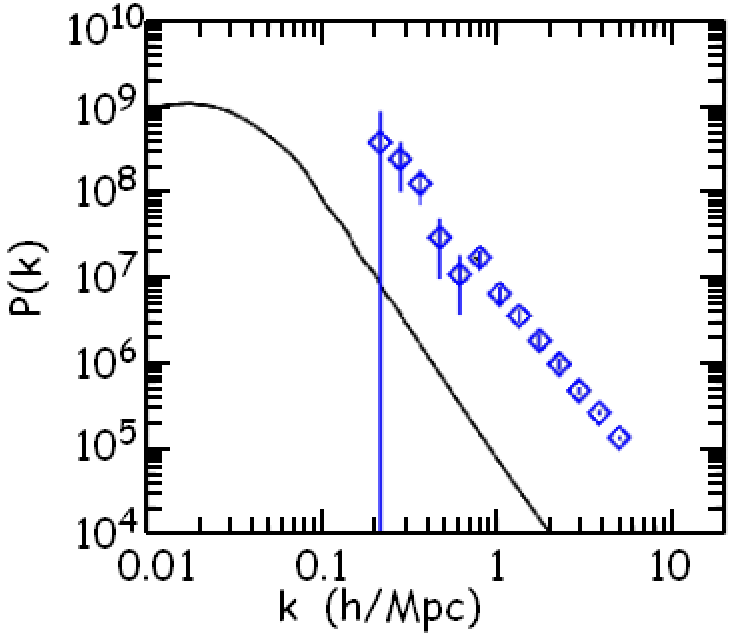
$\Omega_0 = 1$
$\Omega_\Lambda = 0.66$
$\Omega_B = 0.04$
$H_0 = 72$
$n_s = 0.94$

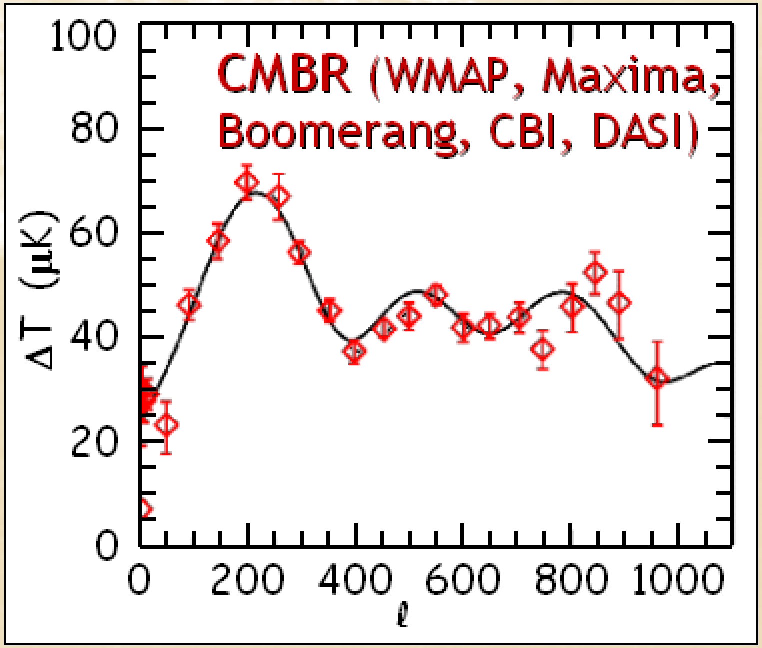
$\Omega_\nu = 0.15$

Adapted from S.Hannestad

Lyman-α forest at large redshift ⟨z⟩ = 2.72

Scales 0.1–10 Mpc





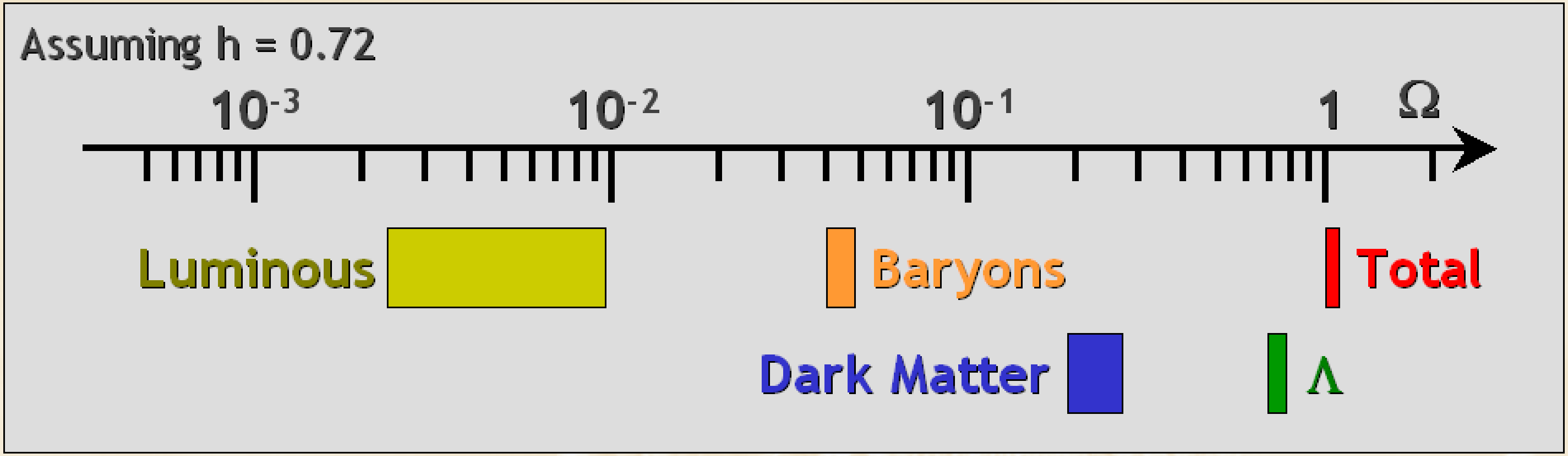
Neutrino Mass Limits from Large-Scale Structure

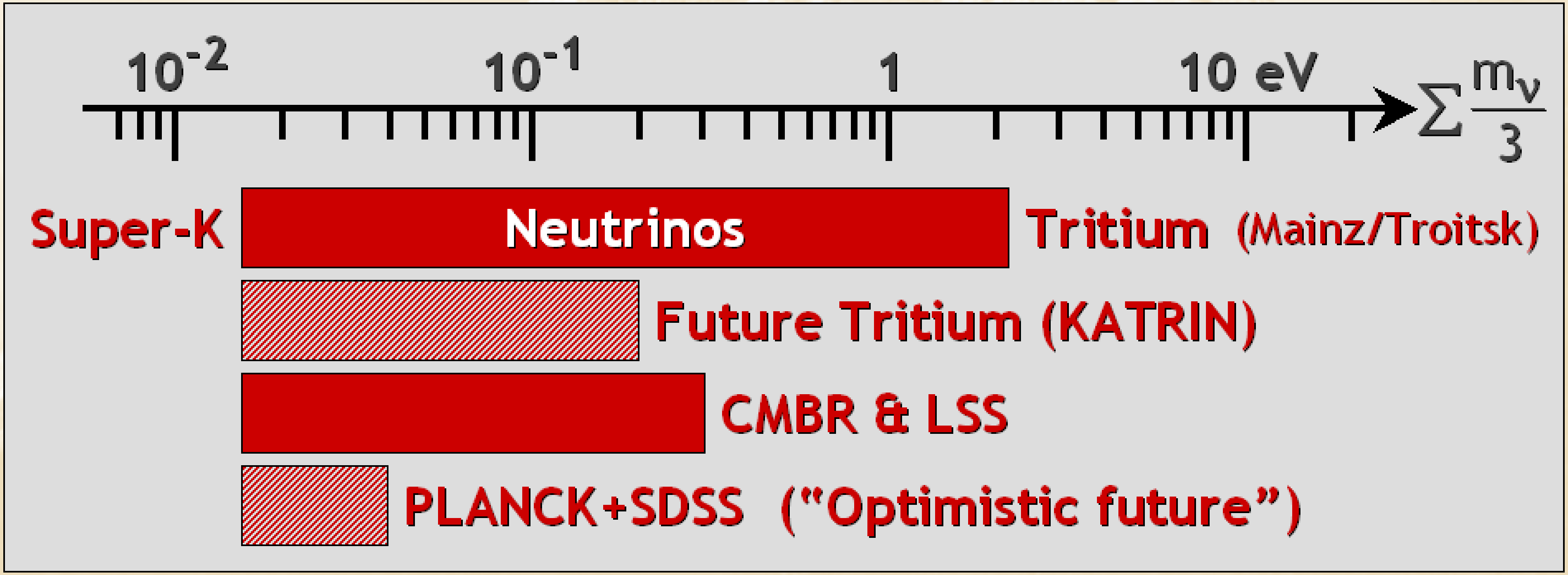
Statistical 95% C.L. limits depend on used data and on priors for other parameters. For detailed analyses see

- Hannestad, astro-ph/0303076
- Elgaroy & Lahav, astro-ph/0303089

$\Sigma m_\nu < 2.1$ eV	2dF (Galaxy-galaxy correlation) + WMAP (Cosmic microwaves)
$\Sigma m_\nu < 1.2$ eV	+ Small-scale CMBR (breaks degeneracy with bias)
$\Sigma m_\nu < 1.0$ eV	+ Priors (1σ) $h = 0.72 \pm 0.08$ $\Omega_M = 0.28 \pm 0.14$

Mass-Energy-Inventory of the Universe

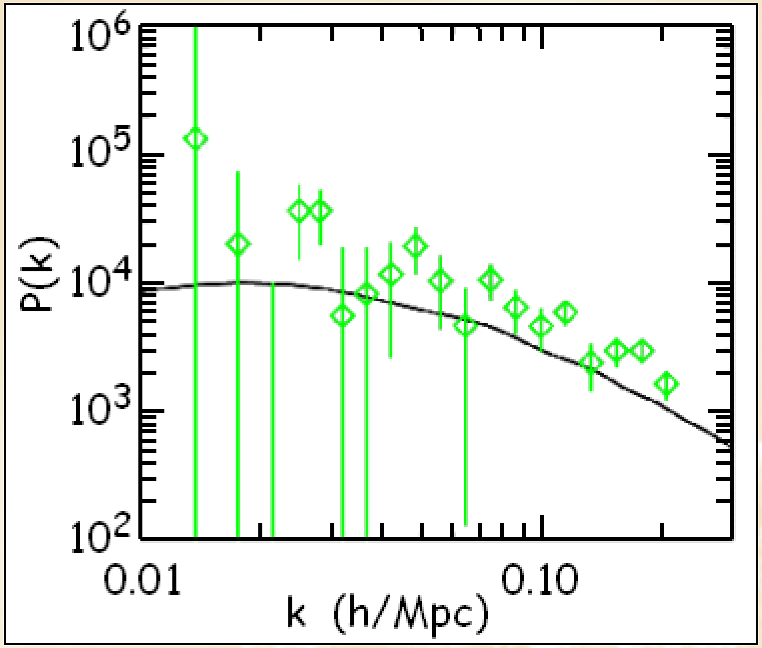




Cosmic Structure Modified by Additional Radiation

Galaxy Distribution (2dF, PSCz)

Scales 1–200 Mpc



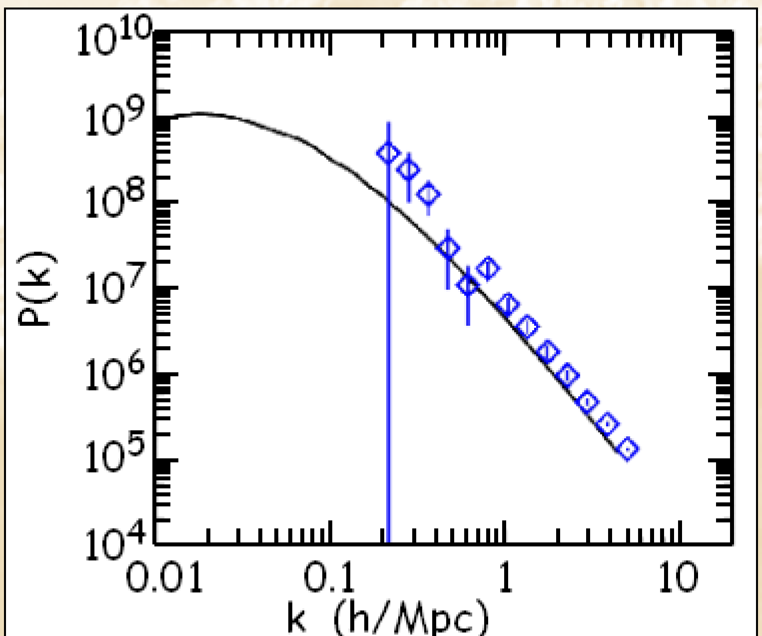
$\Omega_0 = 1$
$\Omega_\Lambda = 0.66$
$\Omega_B = 0.04$
$H_0 = 72$
$n_s = 0.94$

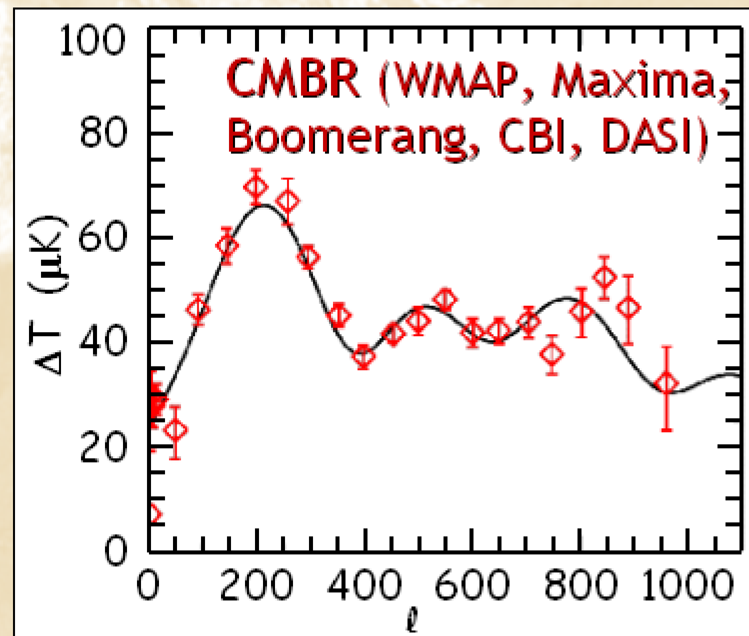
$N_\nu = 3$

Adapted from S.Hannestad

Lyman-α forest at large redshift ⟨z⟩ = 2.72

Scales 0.1–10 Mpc





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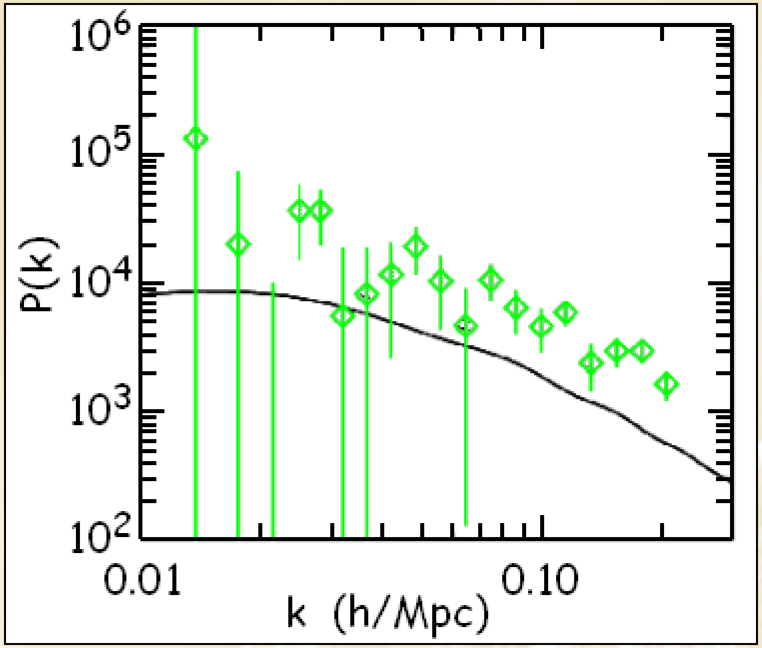

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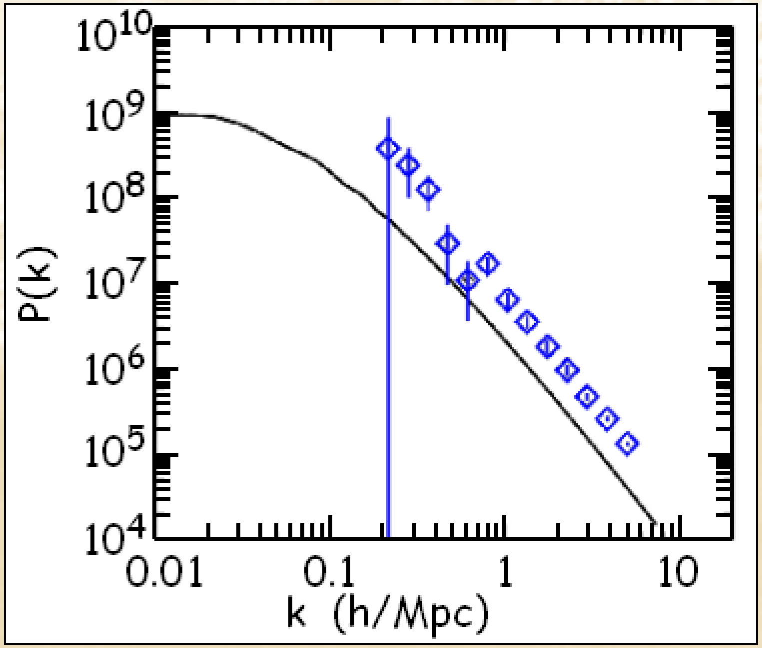
$H_0 = 72$
$n_s = 0.94$

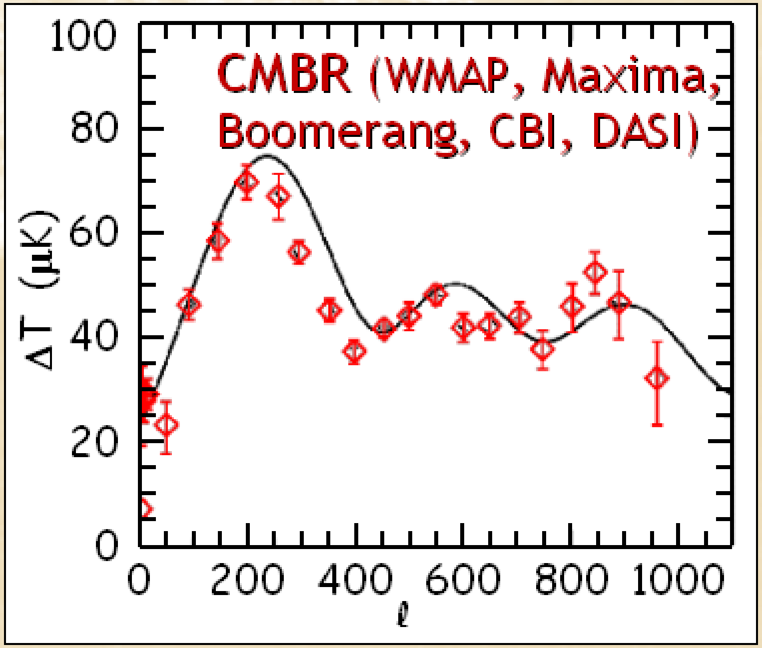
$N_\nu = 9$

Adapted from S.Hannestad

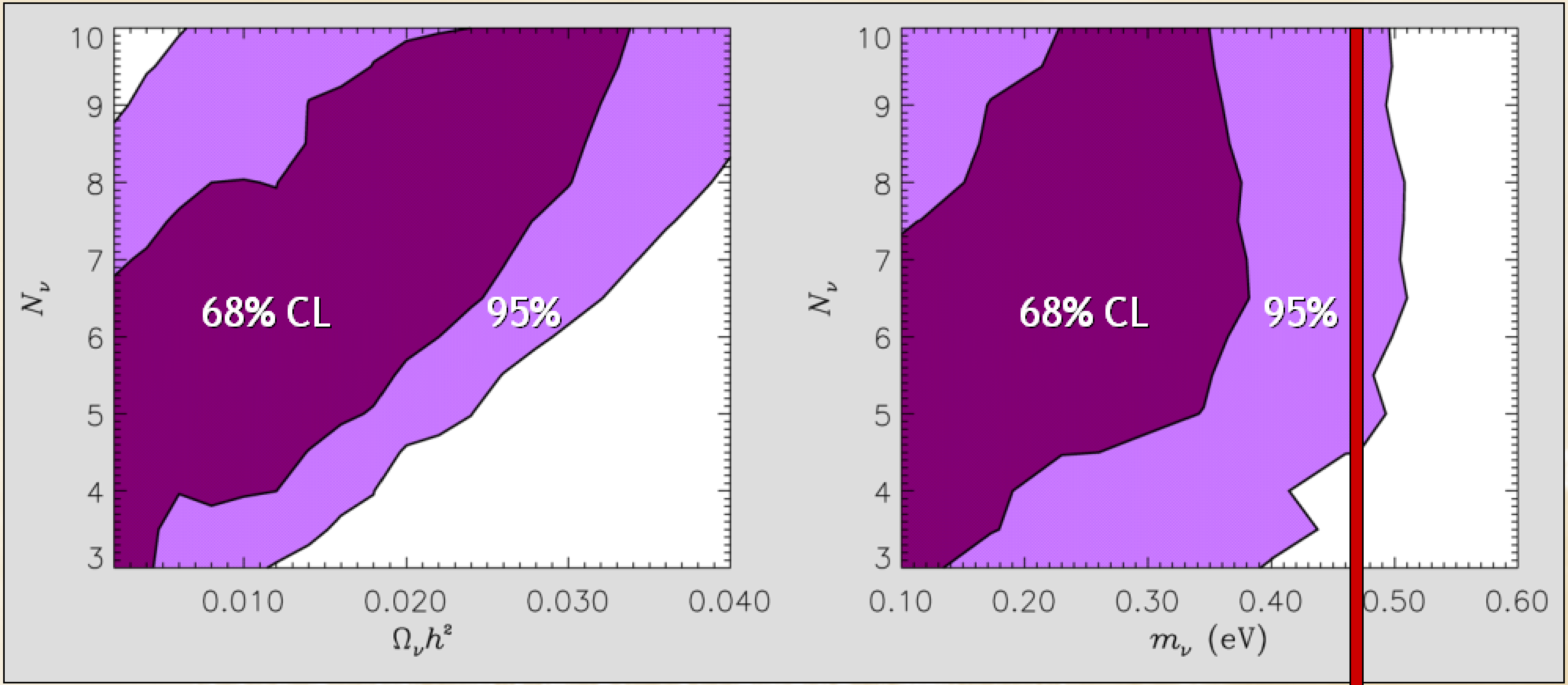
Lyman-α forest at large redshift
⟨z⟩ = 2.72

Scales 0.1–10 Mpc





Neutrino Masses vs. Radiation Density



Neutrino mass limit almost independent of the neutrino number density

Hannestad & Raffelt, Cosmological mass limits on neutrinos, axions, and other light particles, hep/ph0312154

Evidence for Cosmic Neutrinos

CMBR and LSS data, with different priors, provide allowed range (95% CL) on radiation density, expressed in terms of effective neutrino flavors N_{eff}

$1.4 < N_{eff} < 8.5$	Crotty, Lesgourgues, Pastor [hep-ph/0402049]
$0.9 < N_{eff} < 7.0$	Hannestad [astro-ph/0303076]
$1.9 < N_{eff} < 6.6$	Pierpaoli [astro-ph/0302465]
$0.9 < N_{eff} < 8.3$	Barger et al. [hep-ph/0305075]

Presence of cosmic neutrinos indicated with high significance, independently of big-bang nucleosynthesis

ISAPP 2004, International School on Astro-Particle Physics,
28 June—9 July 2004, Laboratori Nazionali del Gran Sasso, Italy

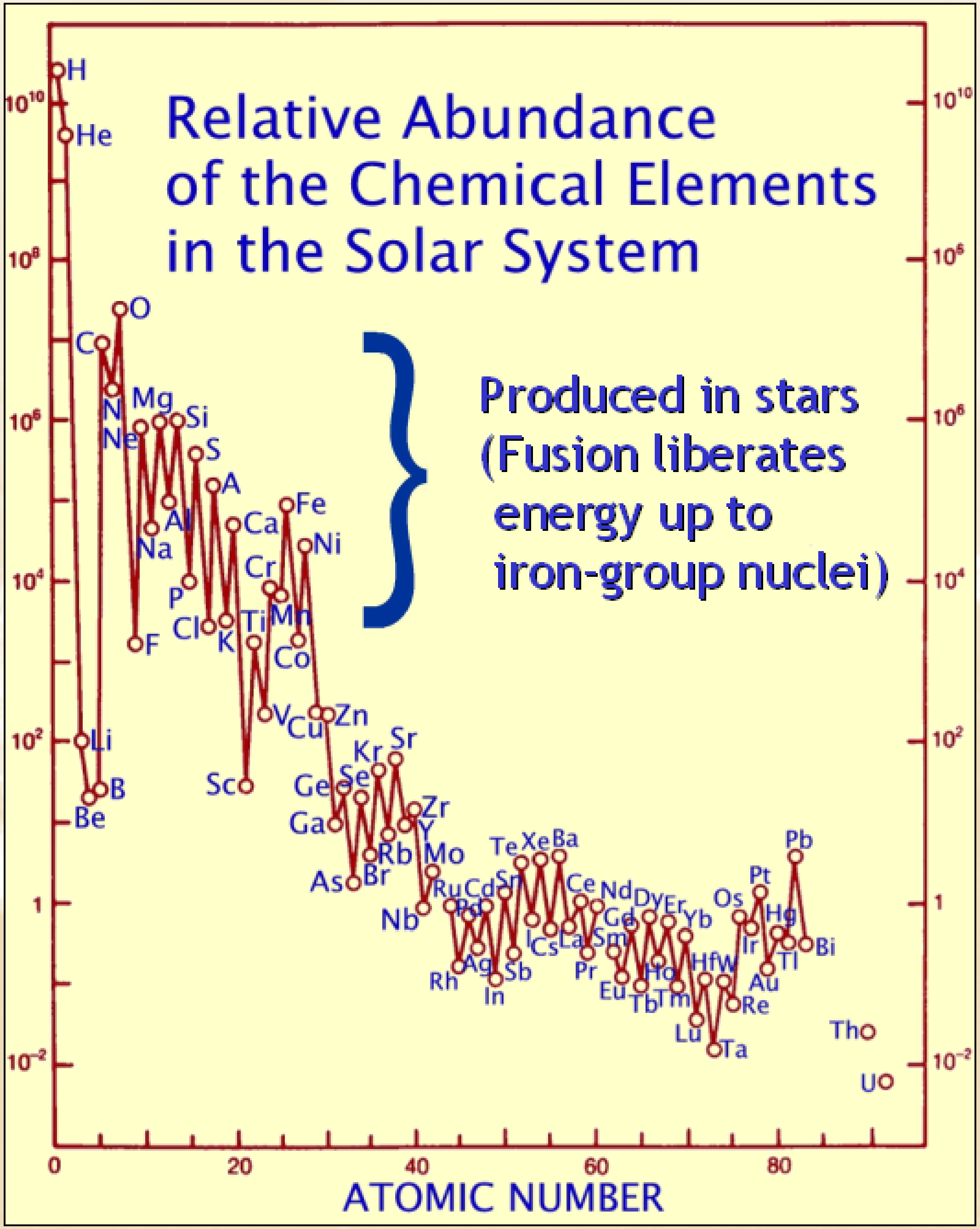
Cosmological Neutrinos
Topic III

Big-Bang Nucleosynthesis

Georg G. Raffelt
Max-Planck-Institut für Physik, München, Germany

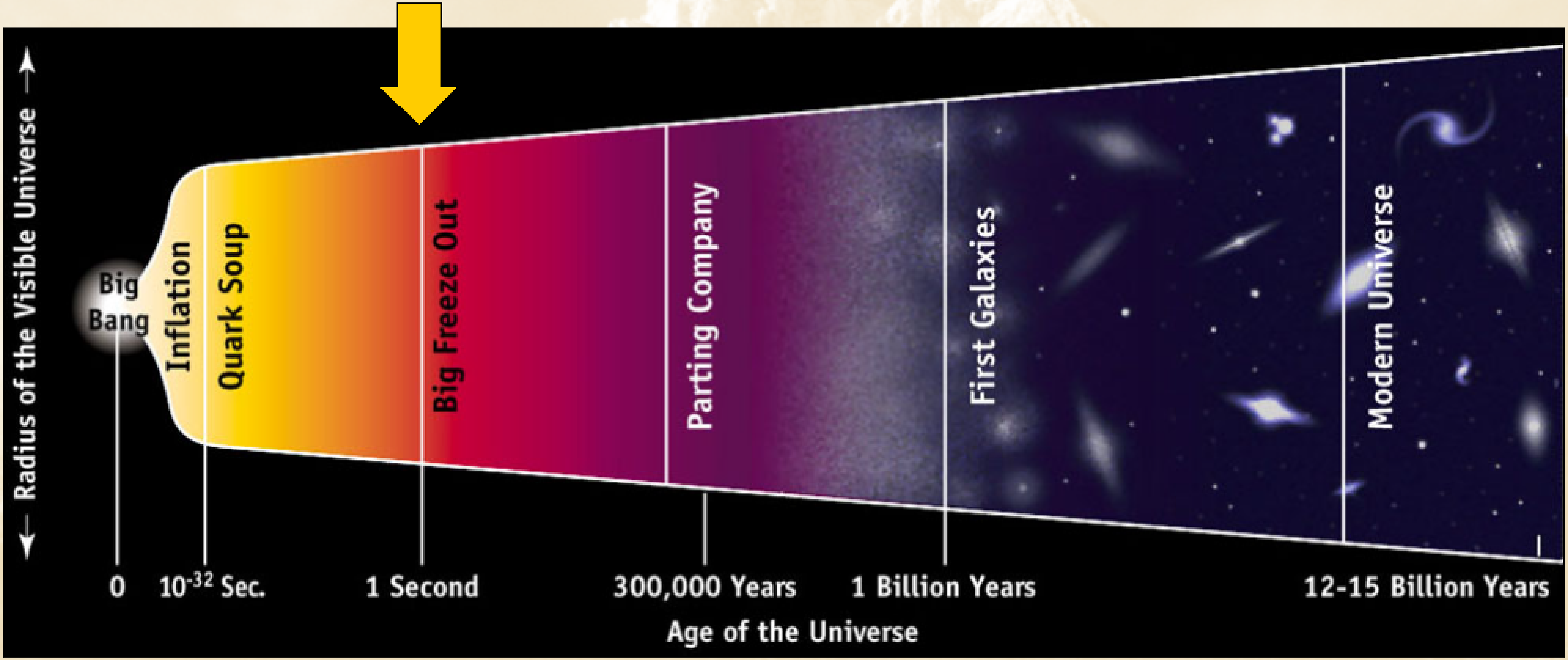
Origin of Elements

- Mass fracion of helium about 25% everywhere in the universe
- Most of it not produced in stars (far too little starlight from liberated energy)
- Big-bang nucleosynthesis a pillar of modern cosmology
- Neutrinos play a crucial role



Where, When and What?

Epoch	From about 1 sec to 3 minutes
Temperature	From about 1 MeV to 30 keV
Constituents	Mostly photons, neutrinos, e^+e^-
Baryons	0.07 g cm^{-3} (at 1 sec)



Helium Synthesis - Three Easy Steps

t (sec): 1 — 100 — 10^4

T (MeV): 1 — 0.1 — 0.01

β equilibrium	$p + e^- \leftrightarrow n + \nu_e$ $p + \bar{\nu}_e \leftrightarrow n + e^+$	$\frac{n}{p} = \exp\left(-\frac{m_n - m_p}{T}\right) \approx 1$ $m_n - m_p = 1.293\ \mathrm{MeV}$
β freeze-out	$\frac{n}{p} \approx \frac{1}{6}$	β rates fall below expansion rate H
• Neutron decay • Nuclear statist. equilibrium	$n_n \propto \exp\left(\frac{-t}{\tau_n}\right)$ $\tau_n = 886\ \mathrm{sec}$	$2p + 2n \leftrightarrow {}^4He + \gamma$ Helium suppressed by large entropy
Neutrons freeze in helium	$\frac{n}{p} \approx \frac{1}{7}$	Helium mass fraction $Y_P \approx 25\%$
Thermonuclear reaction chains	Production and destruction of traces of D, ^3He, ^6Li, ^7Li, ^7Be	
All done		

Simple Estimate of Beta Freeze-Out

β equilibrium

Proton-neutron conversion reactions

$$p + e^- \leftrightarrow n + \nu_e$$

$$p + \bar{\nu}_e \leftrightarrow n + e^+$$

Dimensional analysis of conversion rate

$$\Gamma_{n\leftrightarrow p} \sim G_F^2 T^5$$

Cosmic expansion rate

Friedmann equation

$$H^2 = \frac{8\pi}{3}\frac{\rho}{m_{Pl}^2}$$

Radiation dominates

$$\rho \sim T^4$$

Expansion rate

$$H \sim \frac{T^2}{m_{Pl}}$$

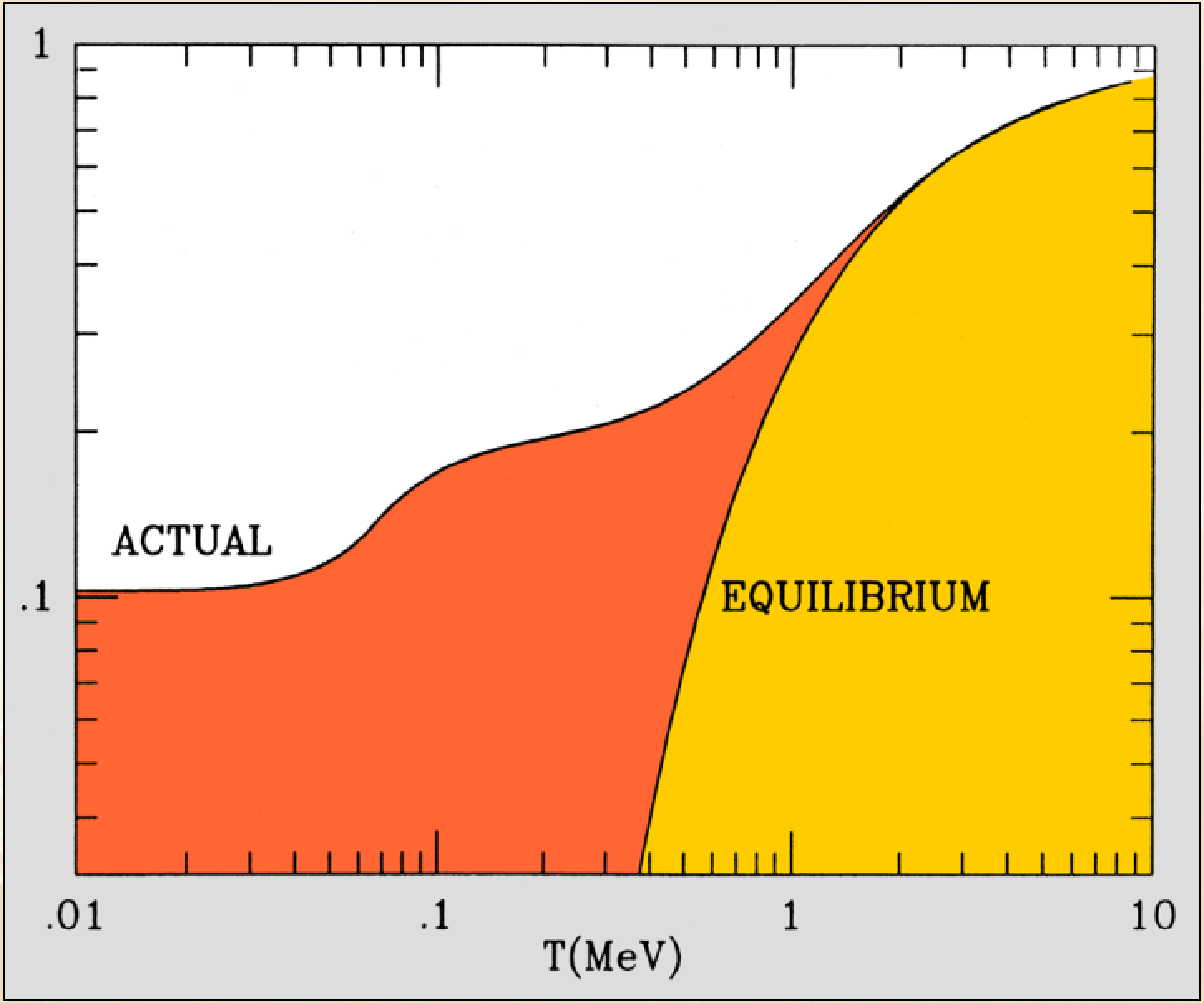
Condition for thermal equilibrium: $\Gamma_{n\leftrightarrow p} > H$

$$T > (m_{Pl} G_F^2)^{-1/3} \sim [10^{19}\,\mathrm{GeV}(10^{-5}\,\mathrm{GeV}^{-2})^2]^{-1/3} = 1\,\mathrm{MeV}$$

Cosmic coincidence that helium and hydrogen survive in comparable amounts

$$m_n - m_p \sim (m_{Pl} G_F^2)^{-1/3} \sim 1\,\mathrm{MeV}$$

Proton-to-Neutron Ratio



Kolb & Turner, The Early Universe

More Exact Estimate of Beta Freeze-Out

β equilibrium in Born approximation

Proton-neutron conversion reactions

$$p + e^- \leftrightarrow n + \nu_e \qquad p + \overline{\nu}_e \leftrightarrow n + e^+$$

Phase-space transformation from rate for neutron decay $n \to p + \overline{\nu}_e + e^-$

$$\Gamma_n = \frac{\displaystyle\int_{m_e}^{\infty} dE \frac{E\sqrt{E^2 - m_e^2}\,(E+Q)^2}{\left(1+e^{E/T}\right)\left(1+e^{-(E+Q)/T}\right)}}{\displaystyle\tau_n \int_{m_e}^{Q} dE\,(Q-E)^2 E\sqrt{E^2 - m_e^2}}$$

For $T \gg Q = m_n - m_p$ and $T \gg m_e$

$$\Gamma_n = 0.15\,\mathrm{s}^{-1}\left(\frac{T}{\mathrm{MeV}}\right)^5$$

Cosmic expansion rate

Friedmann equation

$$H^2 = \frac{8\pi}{3}\frac{\rho}{m_{Pl}^2}$$

Radiation $\gamma, e^{\pm}, \nu_{e,\mu,\tau}, \overline{\nu}_{e,\mu,\tau}$

$$g_* = \sum g_B + \frac{7}{8}\sum g_F = 2 + \frac{7}{8}(4+6) = \frac{43}{4}$$

Expansion rate

$$H = \sqrt{\frac{4\pi^3}{45} g_*}\,\frac{T^2}{m_{Pl}} = \sqrt{\frac{43\pi^3}{45}}\,\frac{T^2}{m_{Pl}}$$

$$= 0.68\,\mathrm{s}^{-1}\left(\frac{T}{\mathrm{MeV}}\right)^2$$

More Exact Estimate of Beta Freeze-Out

β equilibrium in Born approximation

Proton-neutron conversion reactions

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Phase-space transformation from rate for neutron decay $n \to p + \bar{\nu}_e + e^-$

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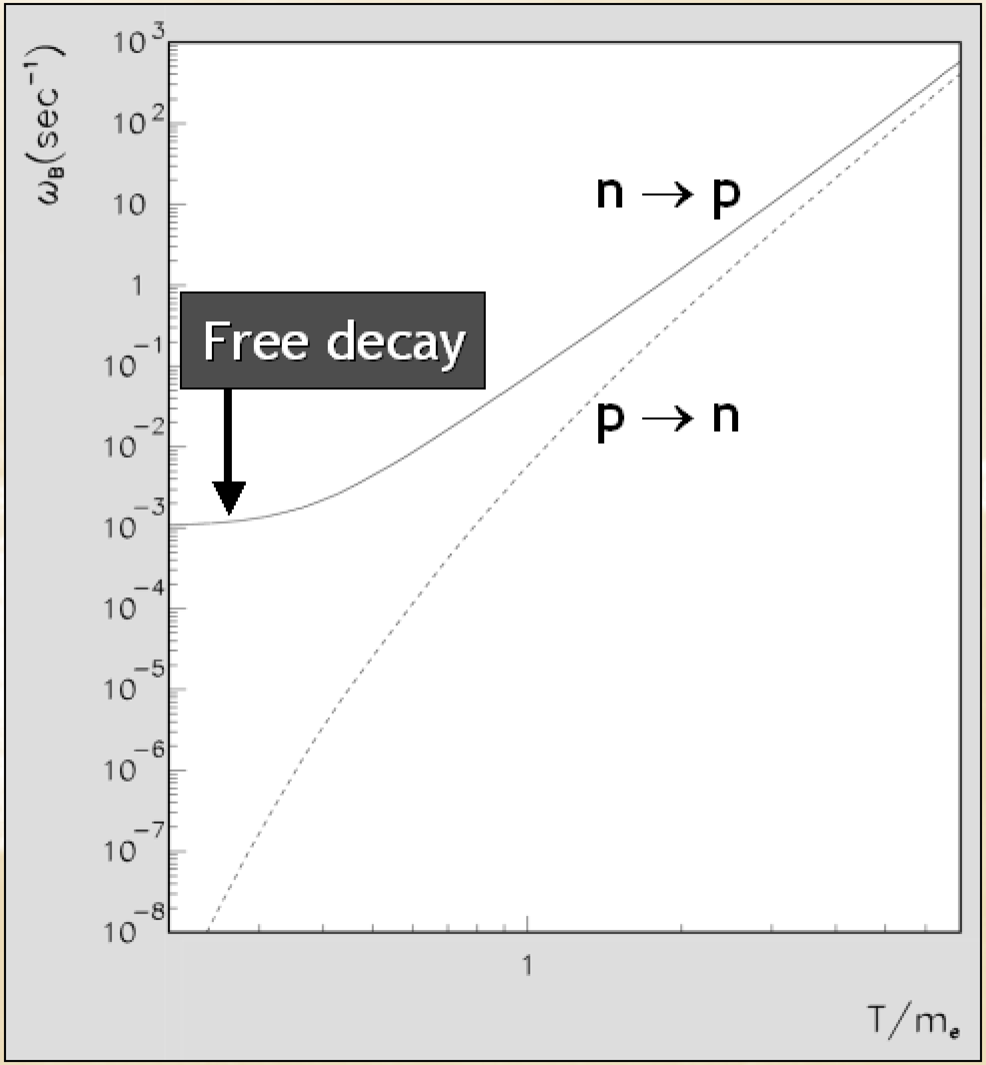
For $T \gg Q = m_n - m_p$ **and** $T \gg m_e$

$$\Gamma_n = 0.15\,s^{-1}\left(\frac{T}{\text{MeV}}\right)^5$$



Esposito et al., astro-ph/9906232

Precision in Weak Interaction Rates

Neutron Lifetime

Measured value

$\tau_n = (885.7 \pm 0.8)$ sec

Calculation in Born approximation

$\tau_n = 961$ sec

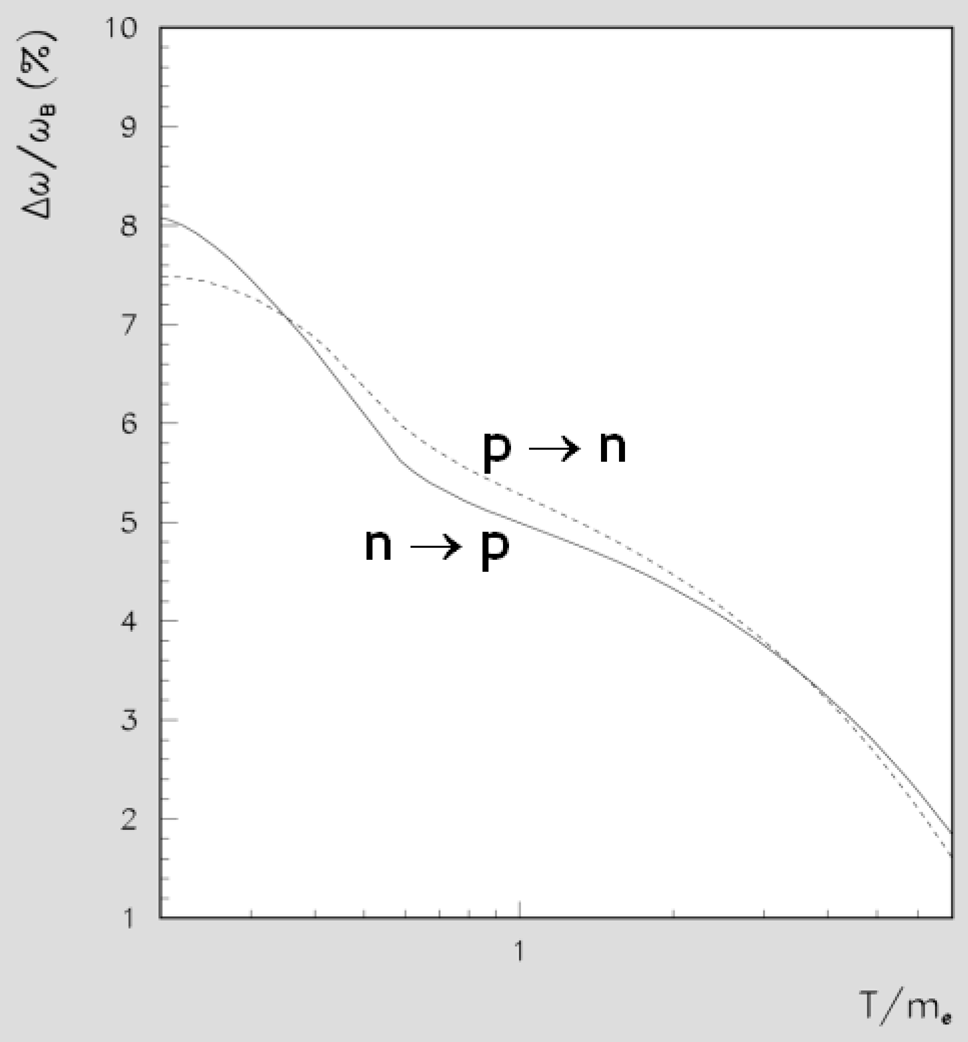
Large deviation of 7.9%
Including QED corrections to first order

$\tau_n = 894$ sec

Another 0.9% remains missing.
Second order effect?

To predict helium abundance with percent precision, need to include corrections

- QED radiative effects $O(\alpha)$
- Finite nucleon mass $O(T/m_n)$
- QED plasma corrections $O(\alpha T/m_e)$



Esposito et al., astro-ph/9906232

Helium Synthesis - Three Easy Steps

t (sec)	T (MeV)
1	1
100	0.1
10^4	0.01

β equilibrium	$p + e^- \leftrightarrow n + \nu_e$ $p + \bar{\nu}_e \leftrightarrow n + e^+$	$\frac{n}{p} = \exp\left(-\frac{m_n - m_p}{T}\right) \approx 1$ $m_n - m_p = 1.293\,\mathrm{MeV}$
β freeze-out	$\frac{n}{p} \approx \frac{1}{6}$	β rates fall below expansion rate H
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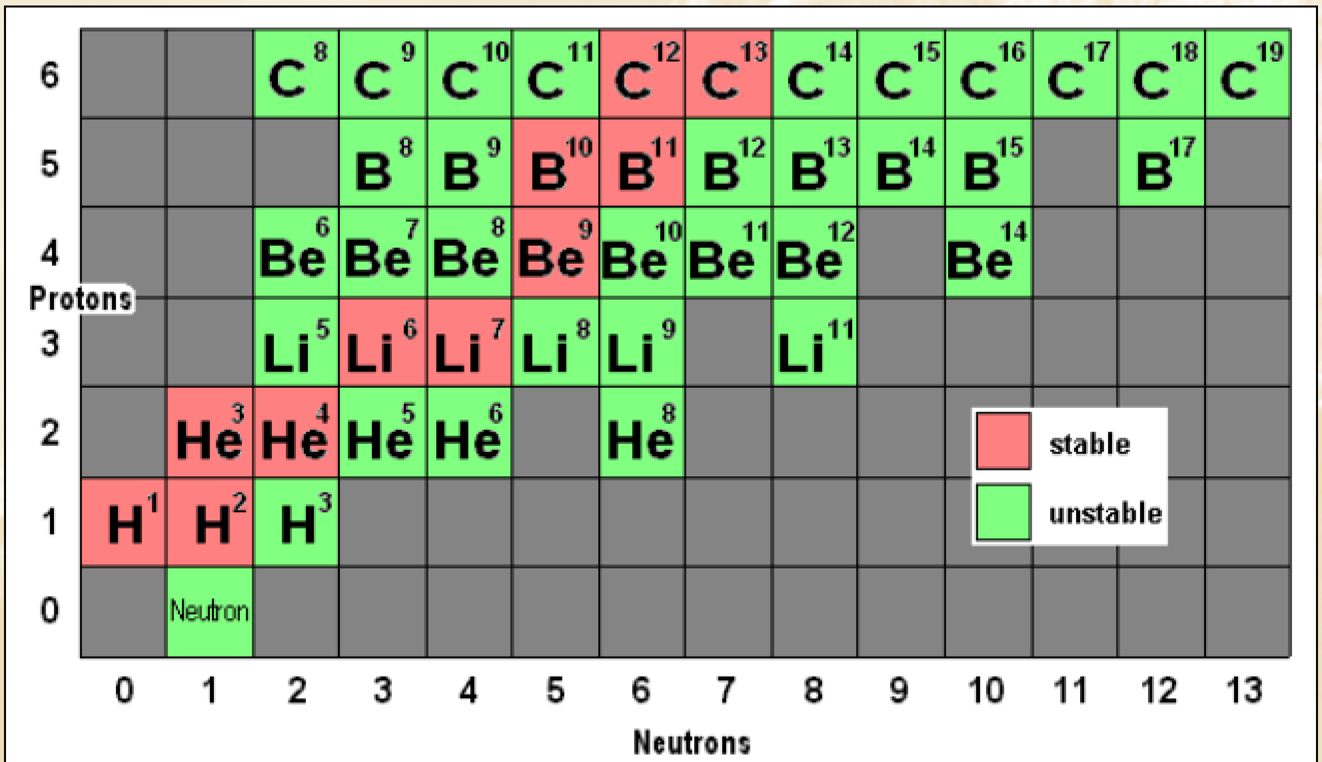
Neutrons freeze in helium	$\frac{n}{p} \approx \frac{1}{7}$	Helium mass fraction $Y_P \approx 25\%$
Thermonuclear reaction chains	Production and destruction of traces of D, ${}^3He$, ${}^6Li$, ${}^7Li$, ${}^7Be$	
All done		

Why do nuclei form so late?

- In thermal equilibrium, all nuclei besides n and p must be present
- Binding energies much larger than MeV, so why are they still dissociated at weak-interaction freeze-out? Why not everything in iron?
- Basic answer: High-entropy environment with ~ 10^9 photons per baryon

$$Nn + Zp \leftrightarrow (\overbrace{A}^{N+Z}, Z) + \text{photons}$$

High-E tail of photon distribution enough to keep nuclei dissociated



	B (MeV)	B/A (MeV)
D	2.23	1.1
^3H	6.92	2.3
^3He	7.72	2.6
^4He	28.30	7.1
^6Li	31.99	5.3
^7Li	39.25	5.6
^7Be	37.60	5.4
^{12}C	92.2	7.7

Nuclear Statistical Equilibrium

In a dilute medium, occupation number of a nucleus ("particle") with chemical potential μ is (Maxwell-Boltzmann)

$$f(E) = e^{-(E-\mu)/T}$$

Number density (nuclei nonrelativistic, g_A spin degrees of freedom)

$$n_A = g_A \left(\frac{m_A T}{2\pi}\right)^{3/2} e^{-(m_A-\mu_A)/T}$$

Effective nuclear reaction

$$Nn + Zp \leftrightarrow (A,Z) + \text{photons}$$

Balance of chemical potentials (photon chemical potential is zero)

$$N\mu_n + Z\mu_p = \mu_A$$

Nucleon mass m_N and binding energy

$$B = m_A - Zm_p - Nm_n$$

Number density of nucleus (A,Z)

$$n_A = g_A \frac{A^{3/2}}{2^A}\left(\frac{2\pi}{m_N T}\right)^{\frac{3}{2}(A-1)} n_p^Z\, n_n^N\, e^{B/T}$$

Baryon density n_B, mass fractions X_A, baryon/photon ratio, photon density

$$n_B = n_n + n_p + \sum A n_A$$

$$X_A = A n_A / n_B \qquad \eta = n_B/n_\gamma$$

$$\sum X_A = 1 \qquad n_\gamma = 2\zeta_3 T^3/\pi^2$$

Mass fraction of nucleus (A,Z)

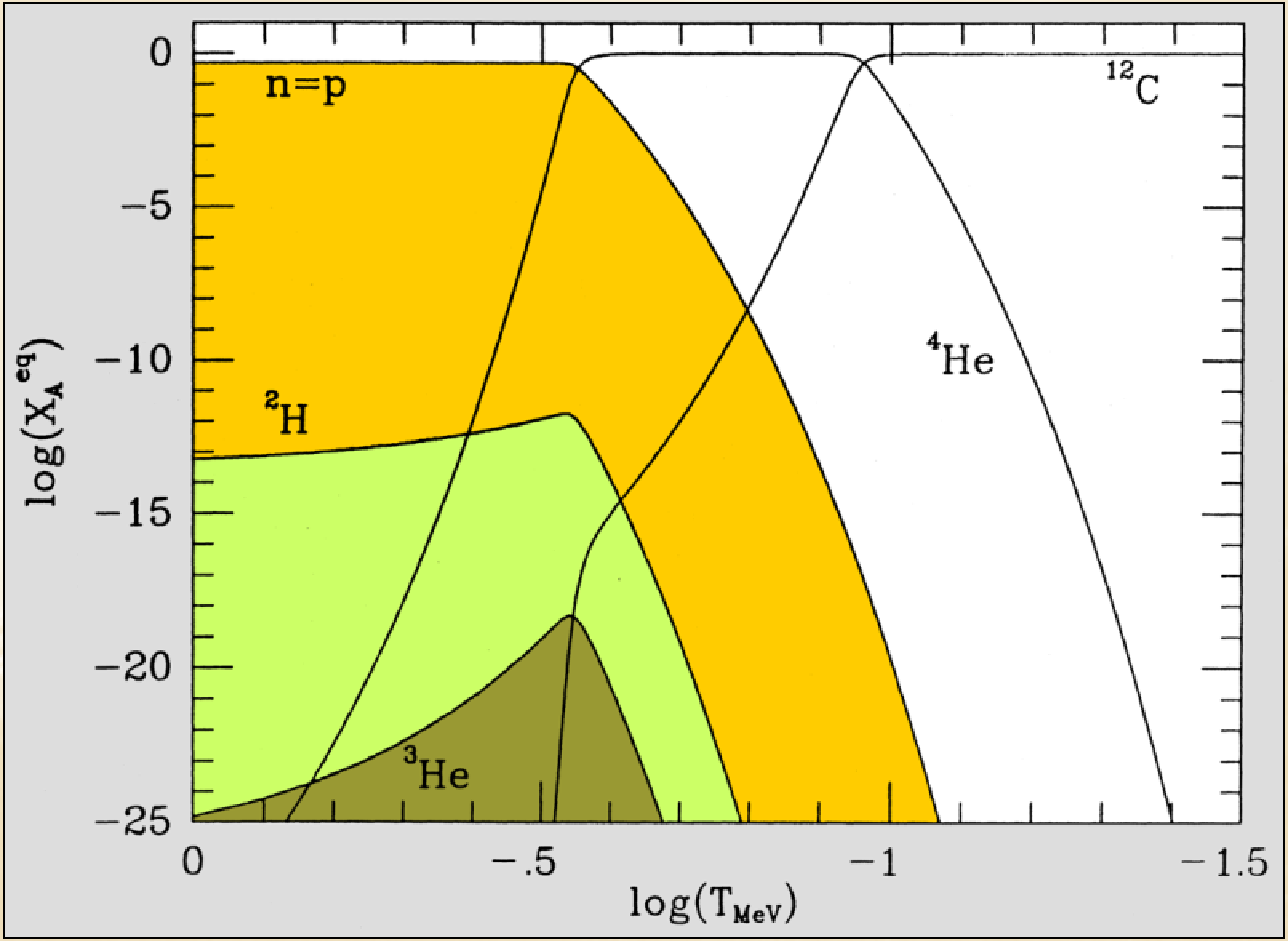
$$X_A = g_A\left(\frac{\zeta_3}{\sqrt{\pi}}\right)^{A-1} 2^{(3A-5)/2} A^{5/2} \times \left[\left(\frac{T}{m_N}\right)^{3/2}\eta\right]^{A-1} X_p^Z\, X_n^N\, e^{B/T}$$

Small number (bracketed term) — Large number ($e^{B/T}$)

Small η prefers dissociated state

Nuclear Statistical Equilibrium – Simplified System

Simplified system of p, n, deuterium, helium and carbon with equal numbers of p and n (Kolb & Turner, The Early Universe)



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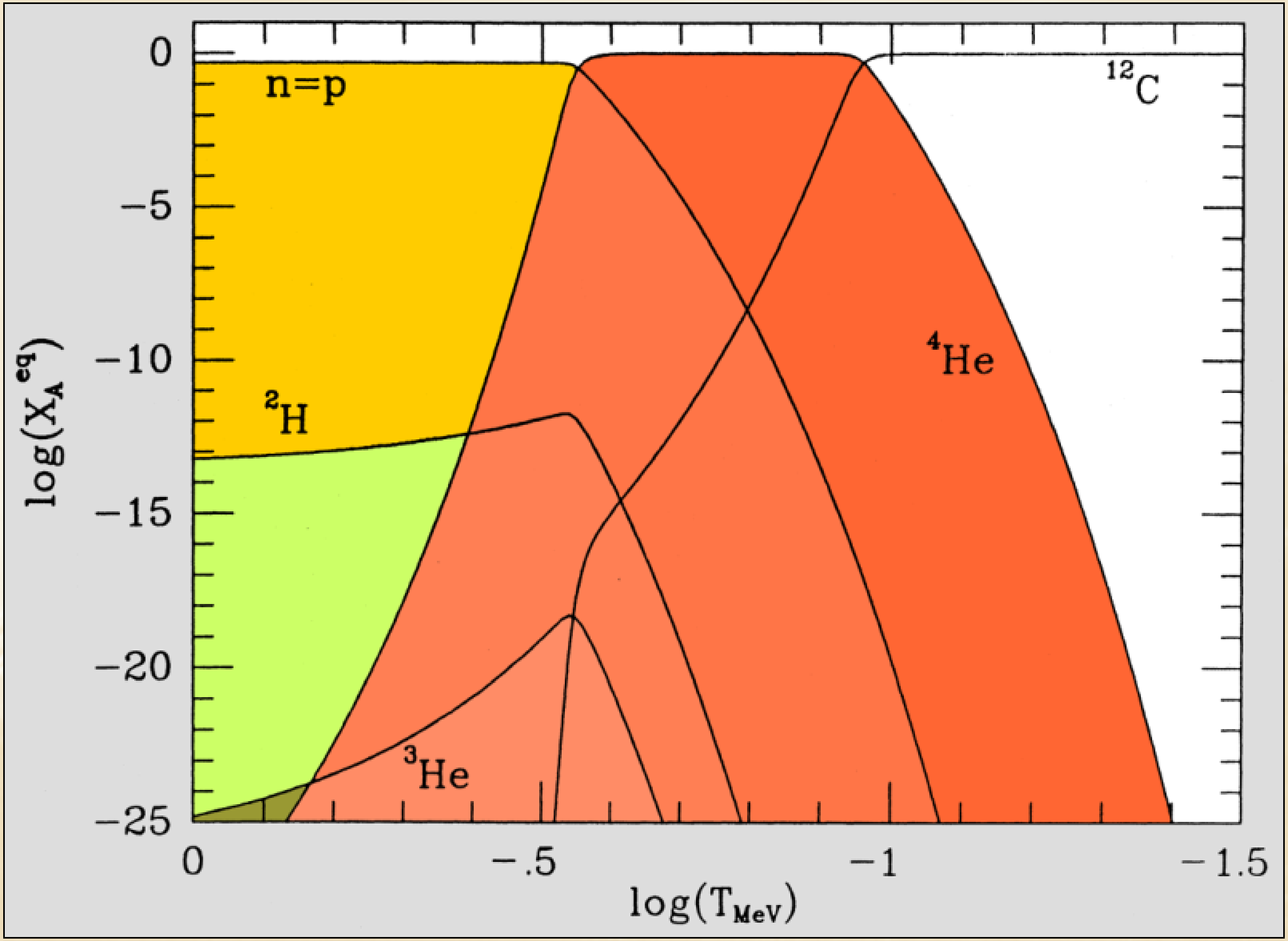

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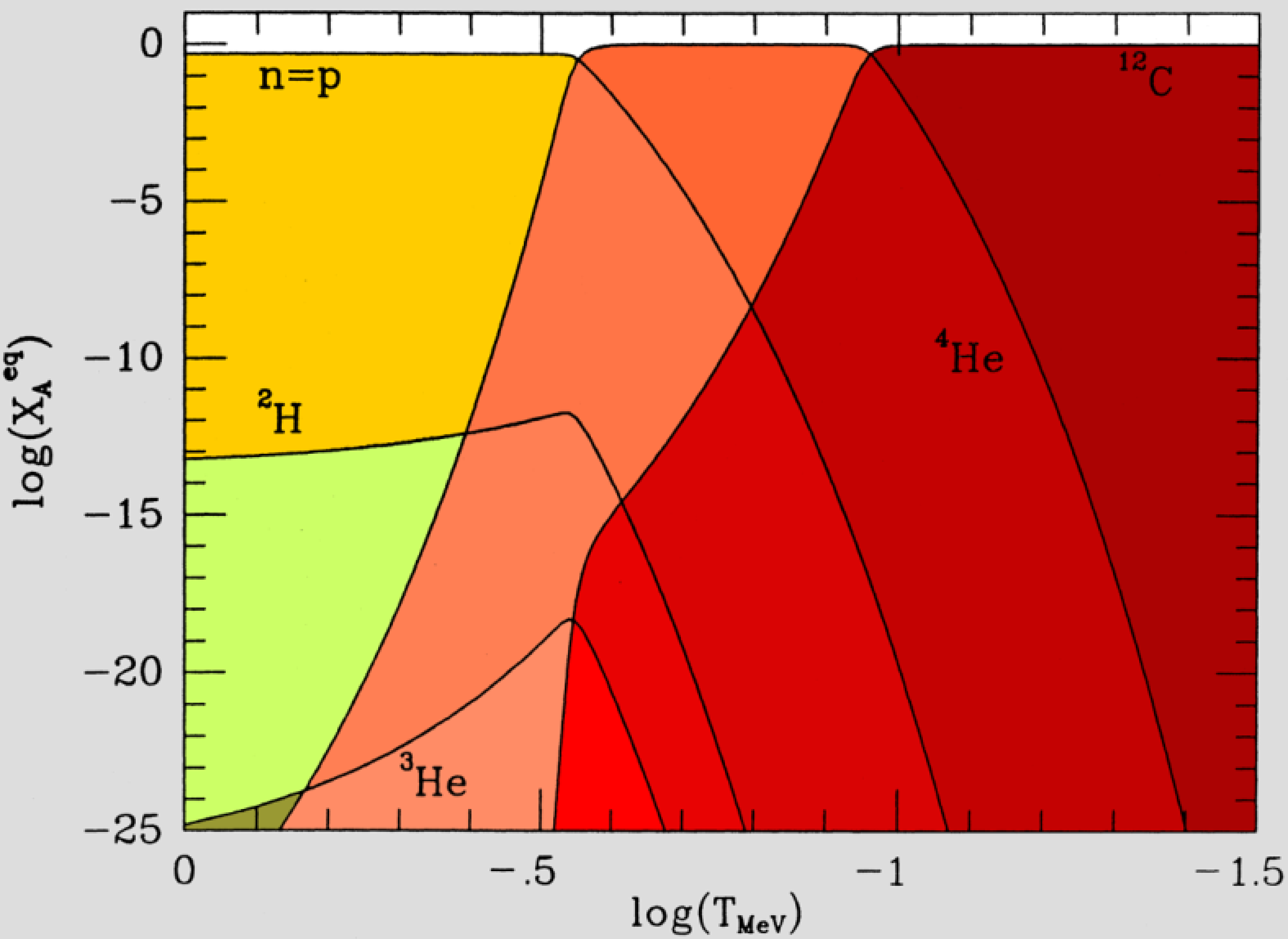

Helium Synthesis - Three Easy Steps

t (sec)	T (MeV)	Step		
		β equilibrium	$p + e^- \leftrightarrow n + \nu_e$ $p + \bar{\nu}_e \leftrightarrow n + e^+$	$\frac{n}{p} = \exp\left(-\frac{m_n - m_p}{T}\right) \approx 1$ $m_n - m_p = 1.293\ \mathrm{MeV}$
1	1	β freeze-out	$\frac{n}{p} \approx \frac{1}{6}$	β rates fall below expansion rate H
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100	0.1	Neutrons freeze in helium	$\frac{n}{p} \approx \frac{1}{7}$	Helium mass fraction $Y_P \approx 25\%$
		Thermonuclear reaction chains	Production and destruction of traces of D, ^3He, ^6Li, ^7Li, ^7Be	
10^4	0.01	All done		

Thermonuclear Reactions and Gamow Peak

Coulomb repulsion prevents nuclear reactions, except for Gamow tunneling

Tunneling probability

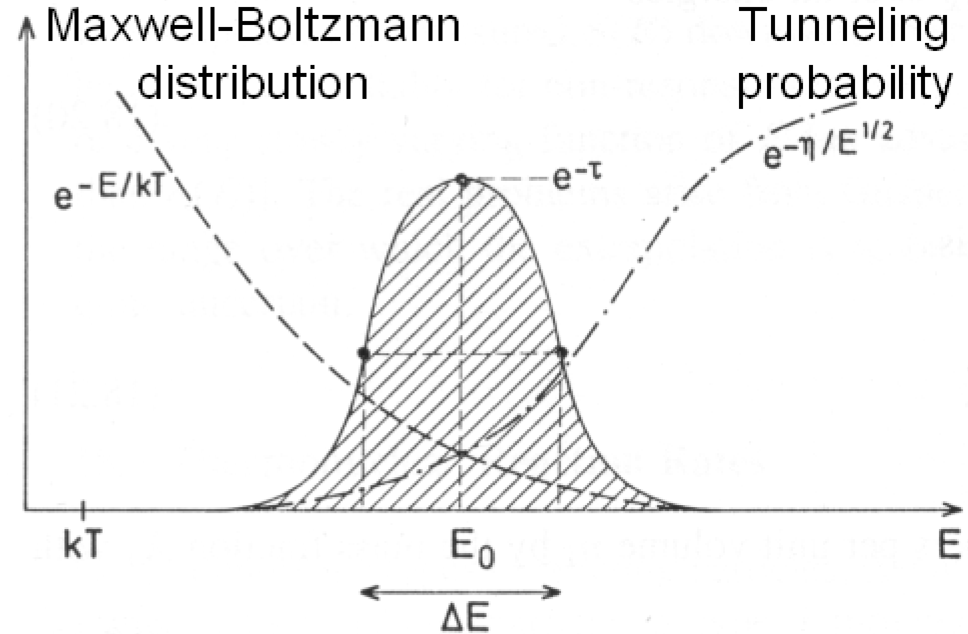
$$p \propto E^{-1/2} e^{-2\pi\eta}$$

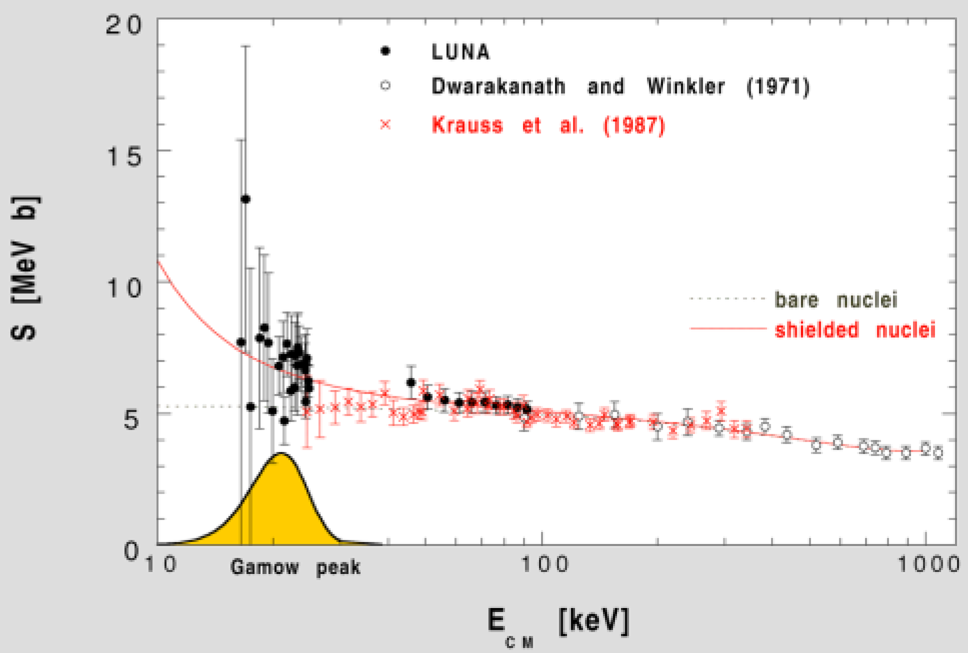
With Sommerfeld parameter

$$\eta = \left(\frac{m}{2E}\right)^{1/2} Z_1 Z_2 e^2$$

Parameterize cross section with astrophysical S-factor

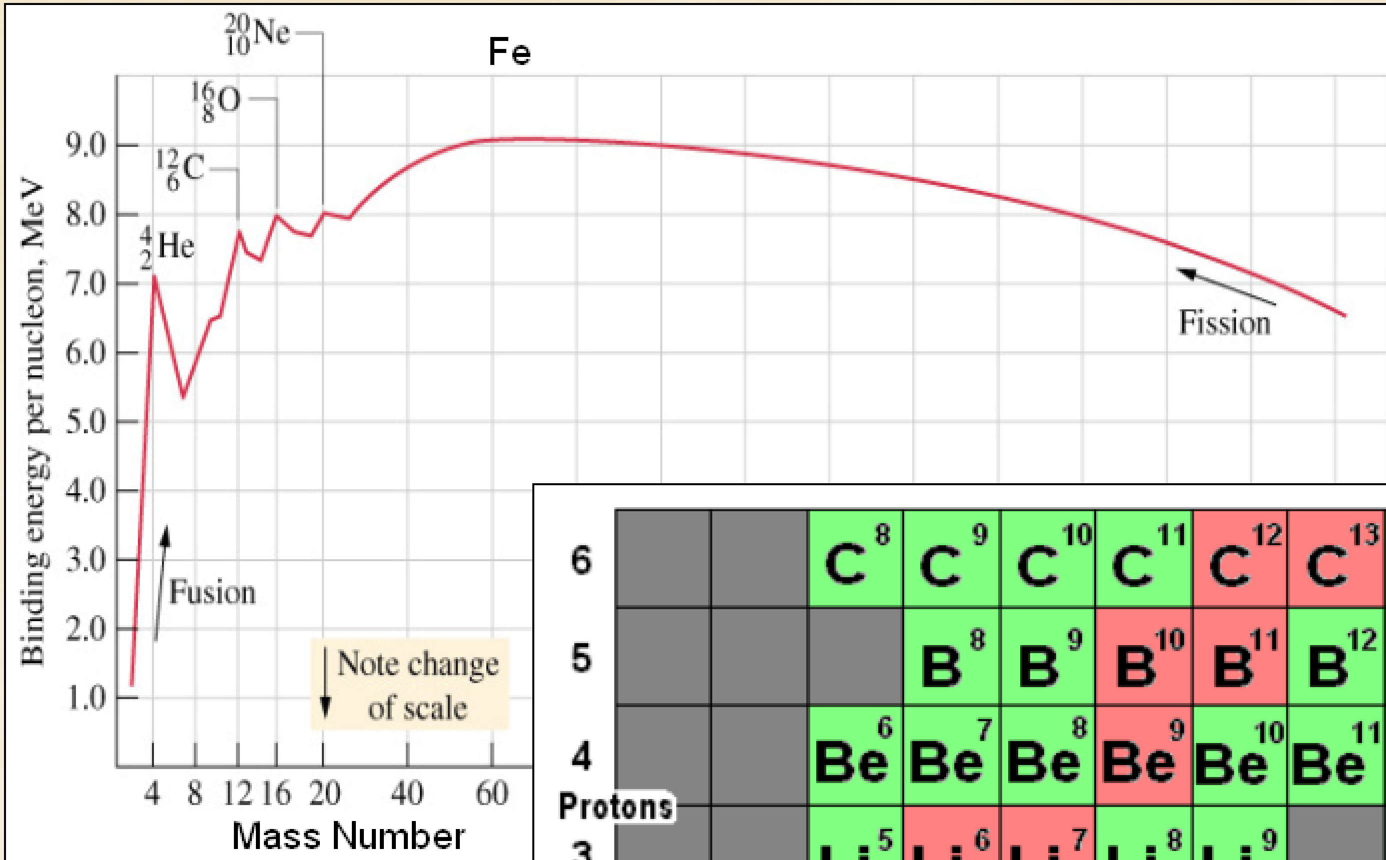
$$S(E) = \sigma(E) E\, e^{2\pi\,\eta(E)}$$

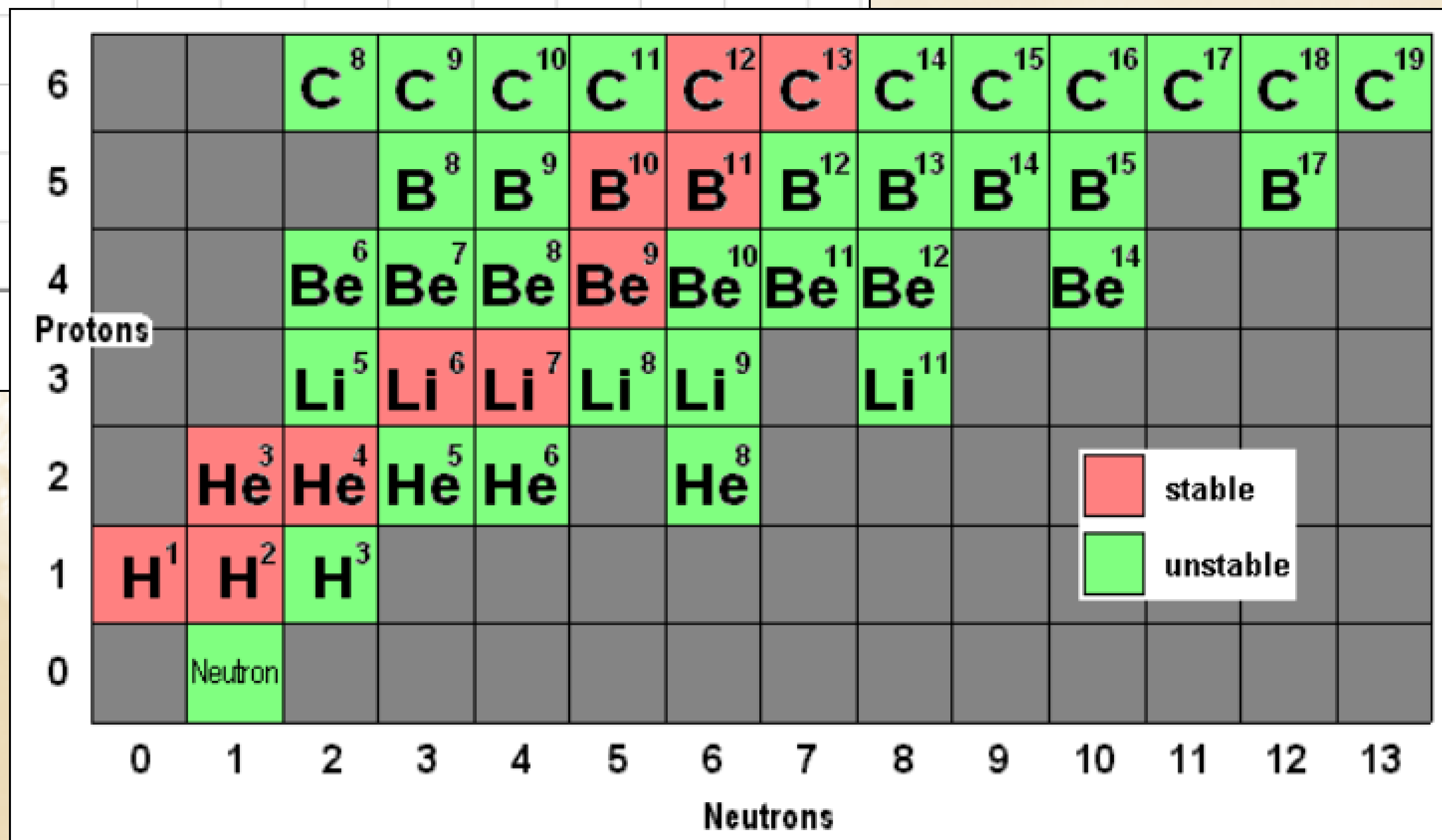




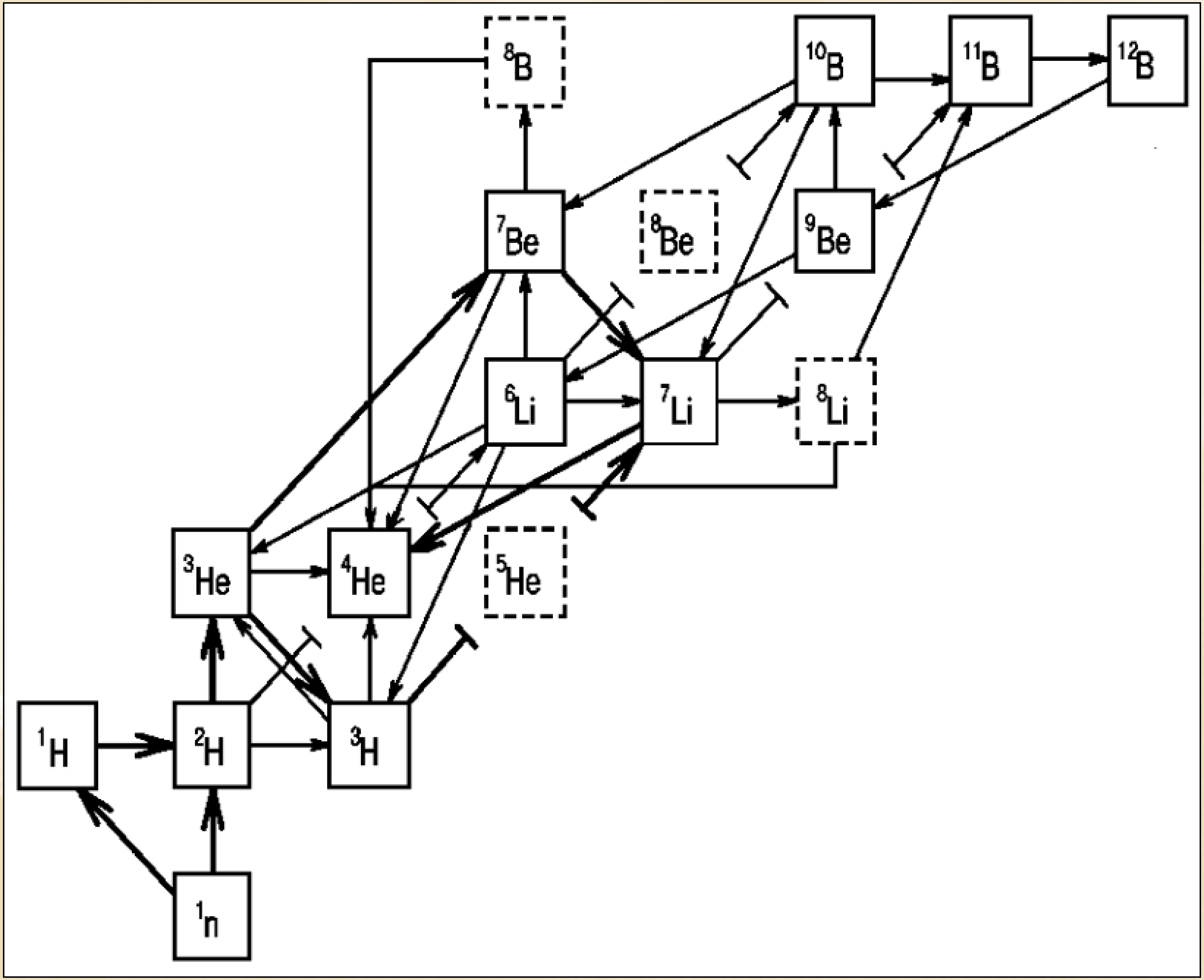
LUNA Collaboration, nucl-ex/9902004

Nuclear Binding Energy

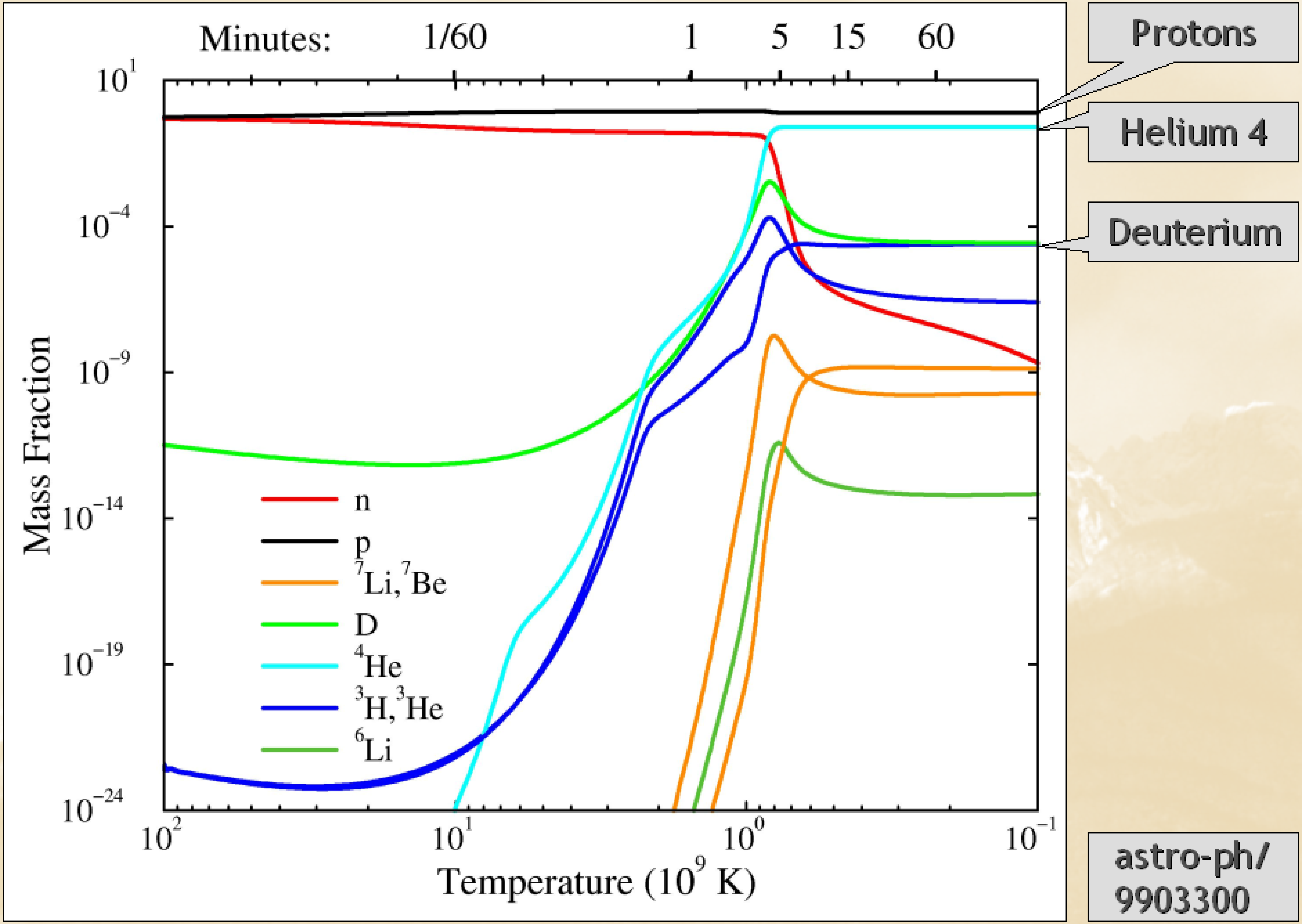




Nuclear Reactions Network



Formation of Light Elements (Big Bang Nucleosynthesis)



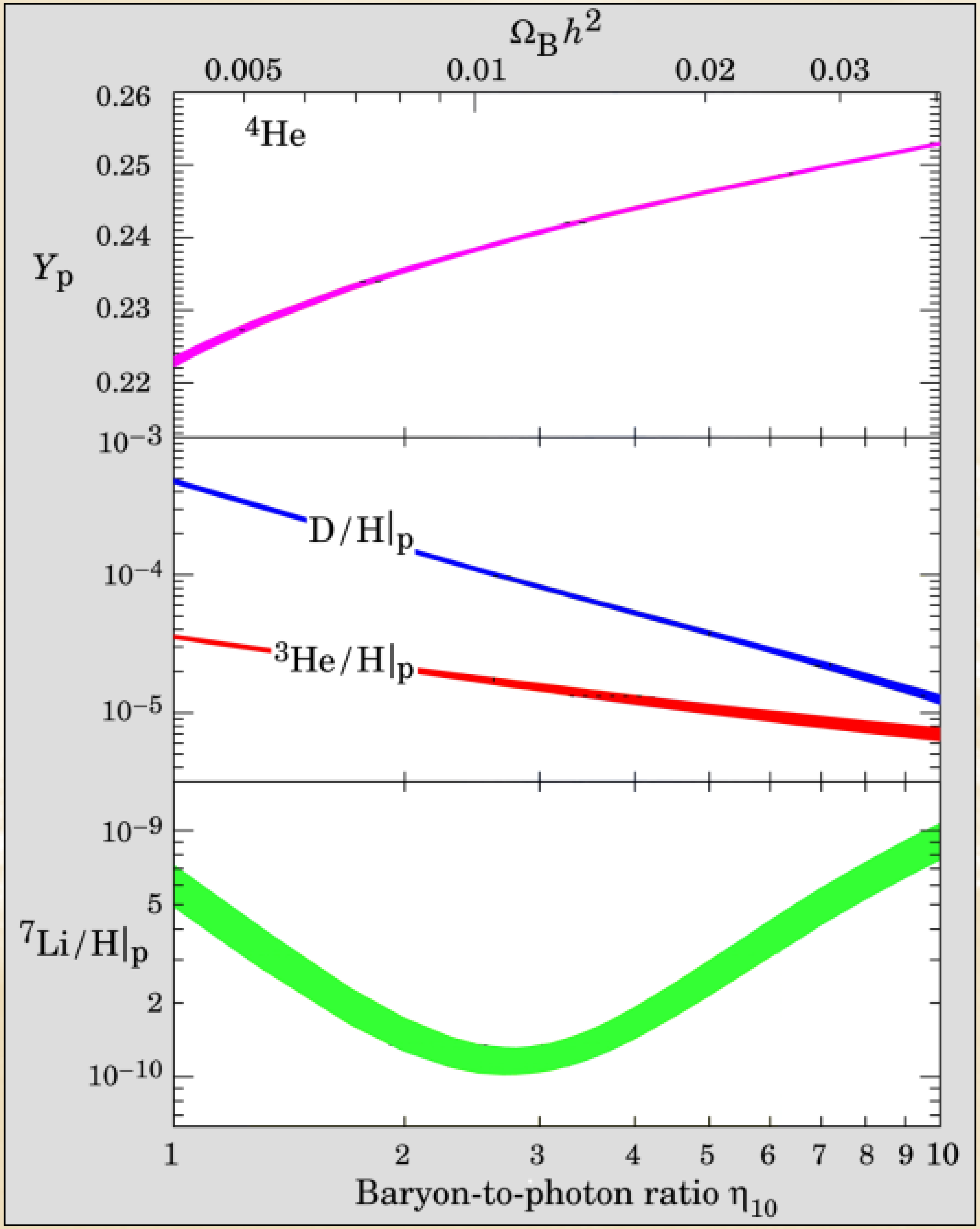
astro-ph/ 9903300

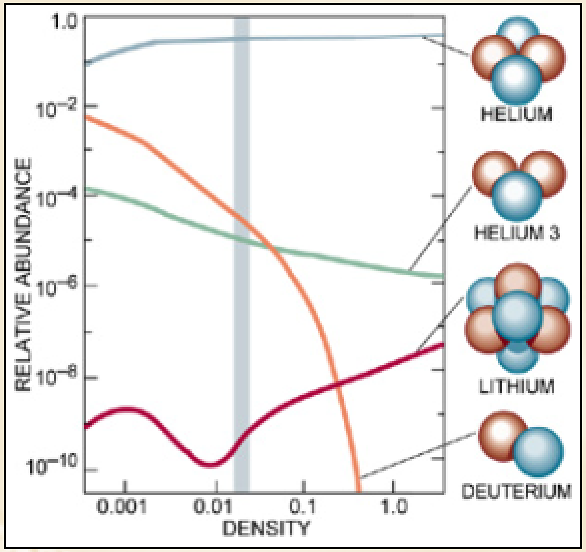
BBN Predictions

Helium

Deuterium

Lithium





Review of Particle Properties

Helium Mass Fraction from HII Regions

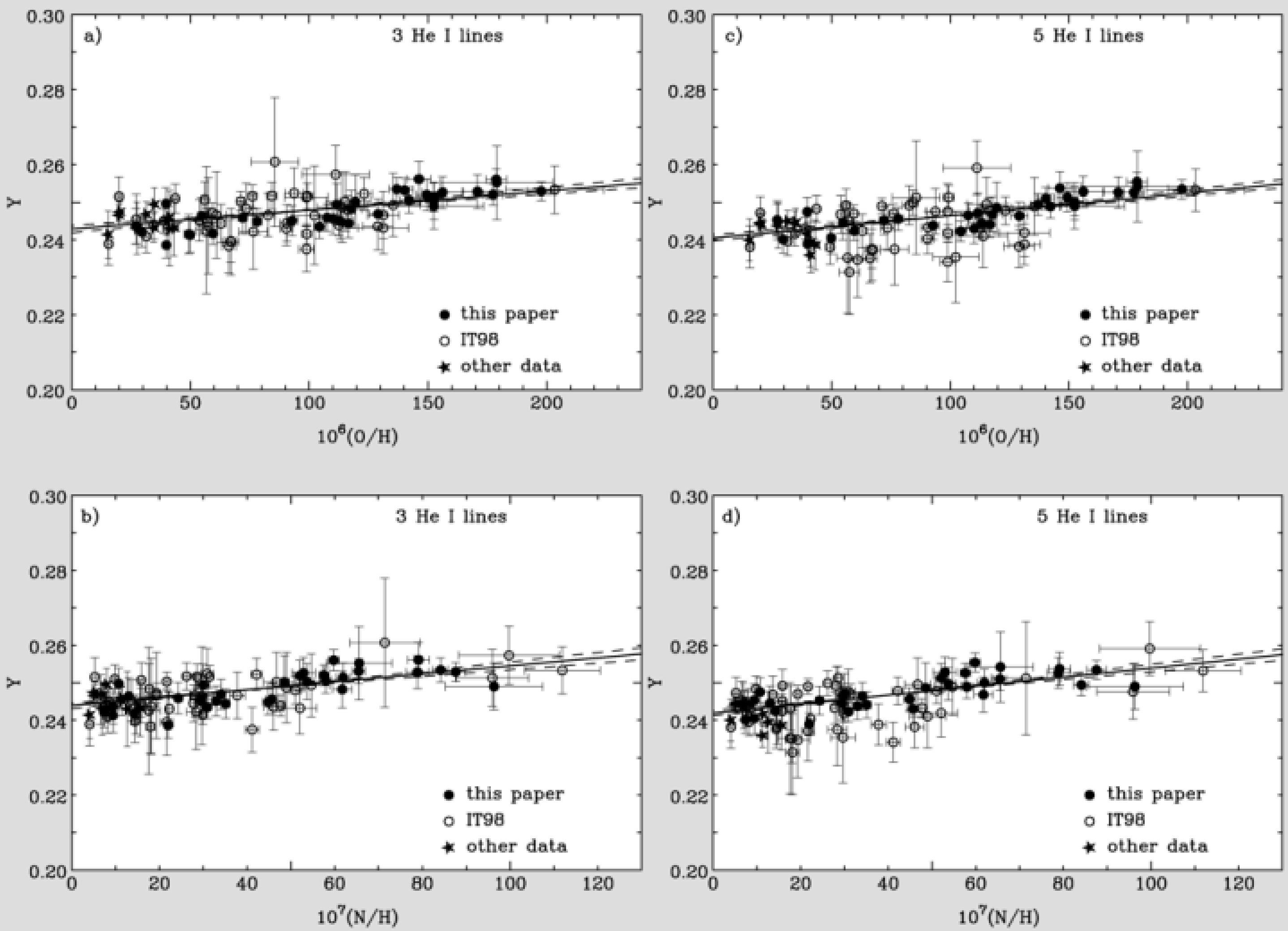


Fig. 2.— Linear regressions of the helium mass fraction Y vs. oxygen and nitrogen abundances for a total of 82 H II regions in 76 blue compact galaxies. In panels a) and b), Y was derived using the 3 λ4471, λ5876 and λ6678 He I lines, and in panels c) and d), Y was derived using the 5 λ3889, λ4471, λ5876, λ6678 and λ7065 He I lines.

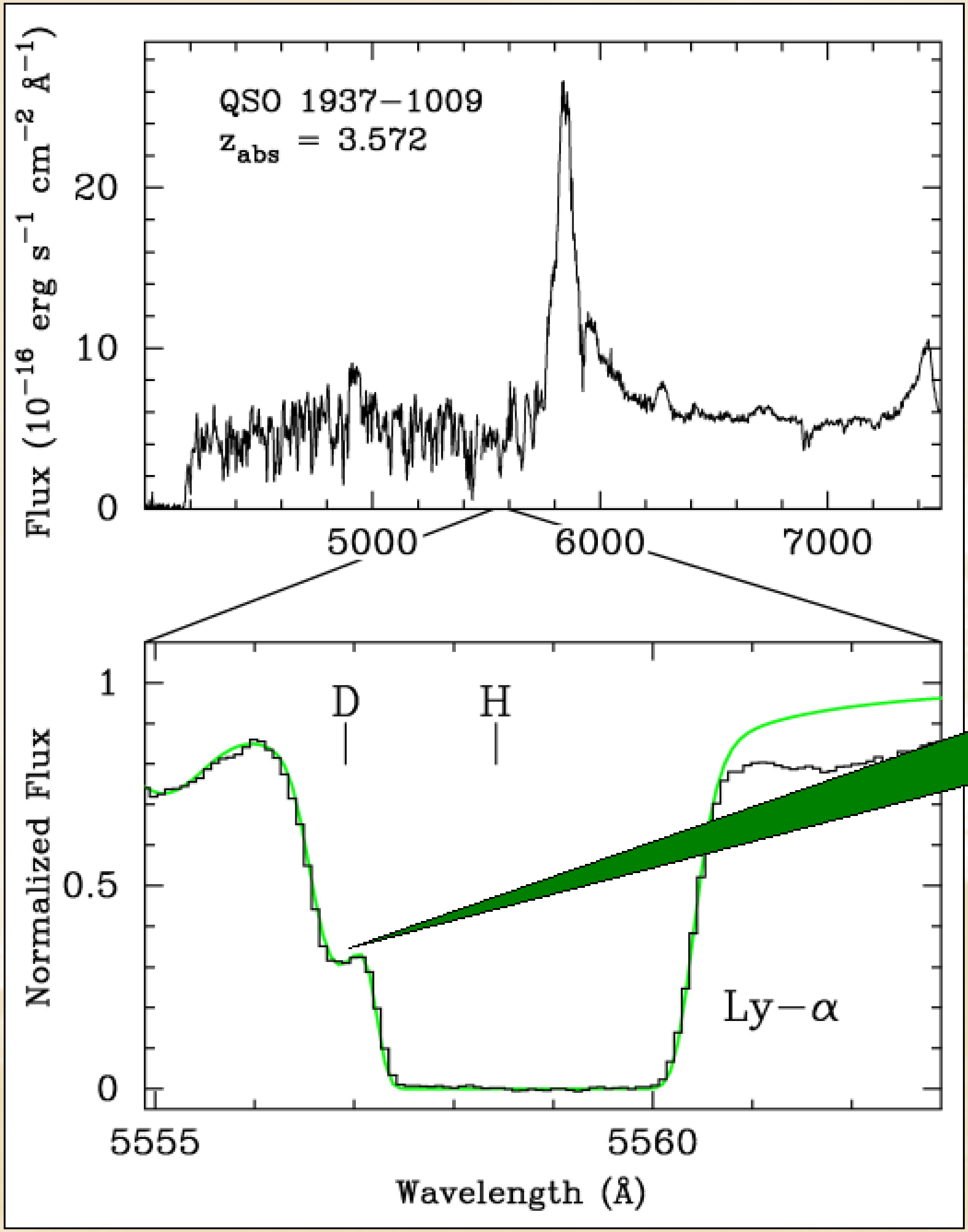
Izotov & Thuan
astro-ph/0310421

Extrapolation to zero oxygen and nitrogen abundance in HII regions in external galaxies

$Y_P = 0.242 \pm 0.002$

Lower values by other authors

Measuring Primordial Deuterium



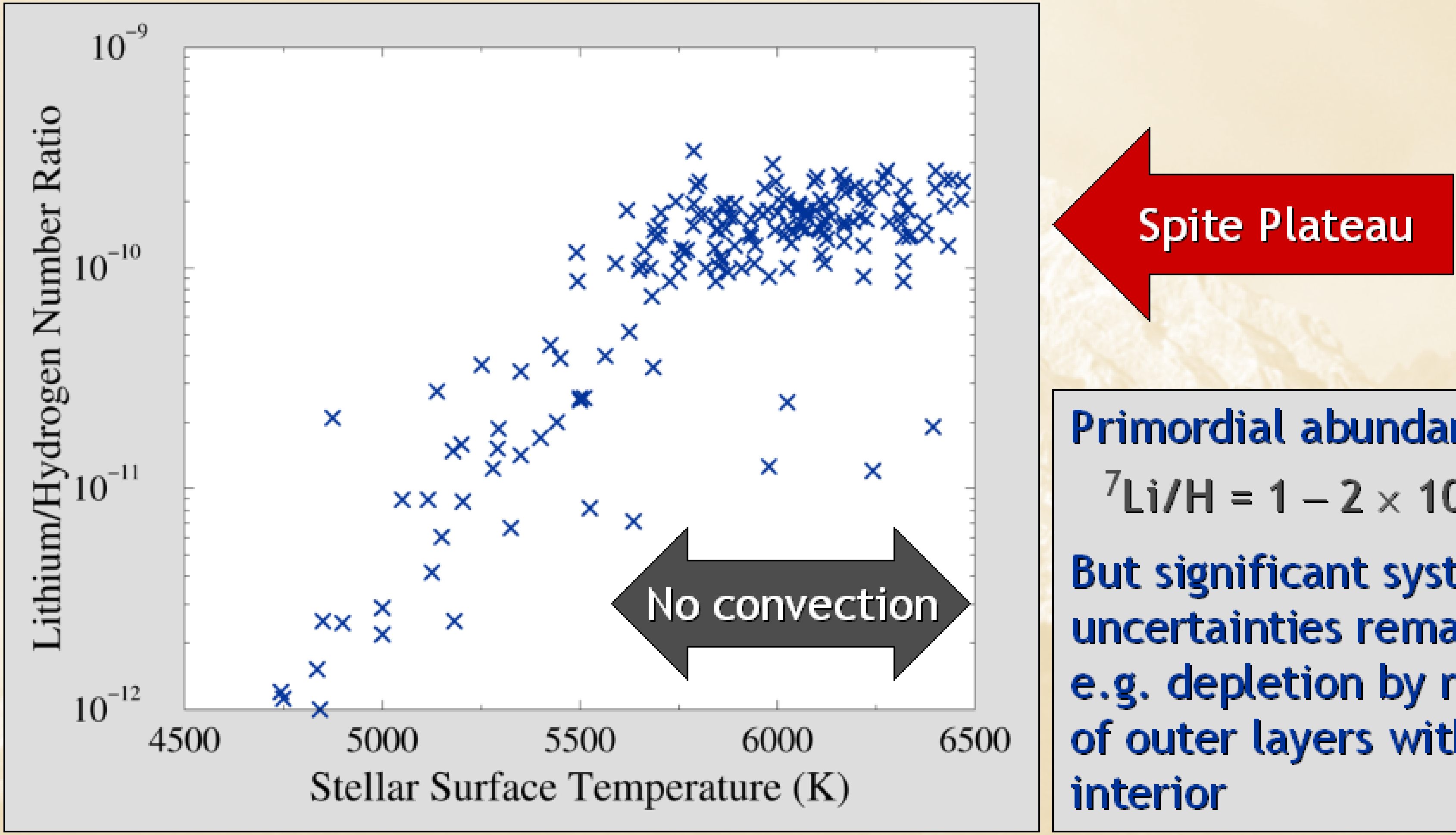
Hydrogen absorption spectrum of a background quasar in high-redshift hydrogen clouds

Deuterium Lyman-α in the flank of the saturated hydrogen Lyman-α line

astro-ph/9903300

Primordial Lithium from Metal-Poor Halo Stars

Lithium very fragile, but primordial abundance may survive in surface layers of most metal-poor oldest stars (Population II)



Primordial abundance

^{7}Li/H = 1 – 2 × 10^{-10}

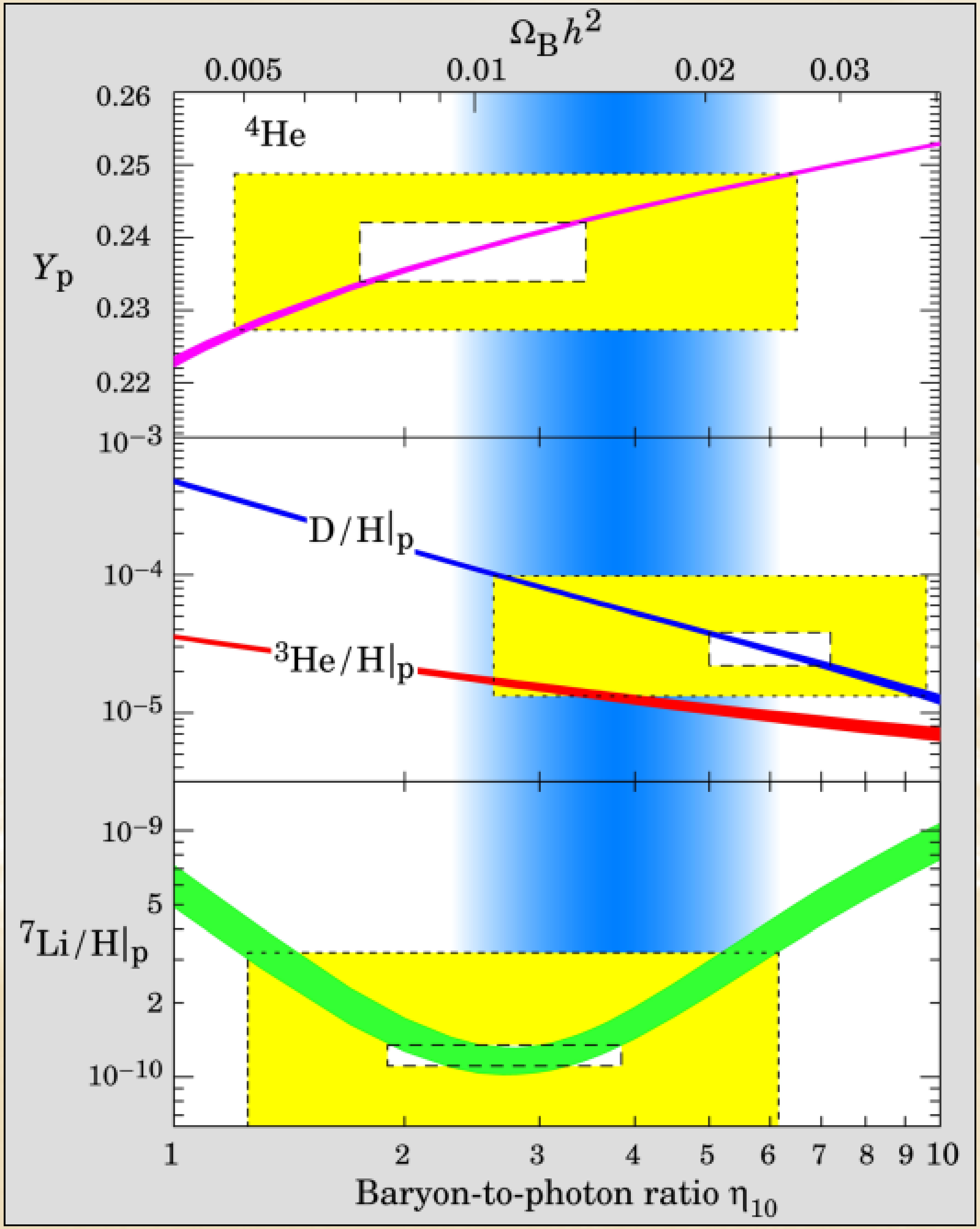
But significant systematic uncertainties remain, e.g. depletion by mixing of outer layers with hotter interior

BBN Concordance

Helium

Deuterium

Lithium

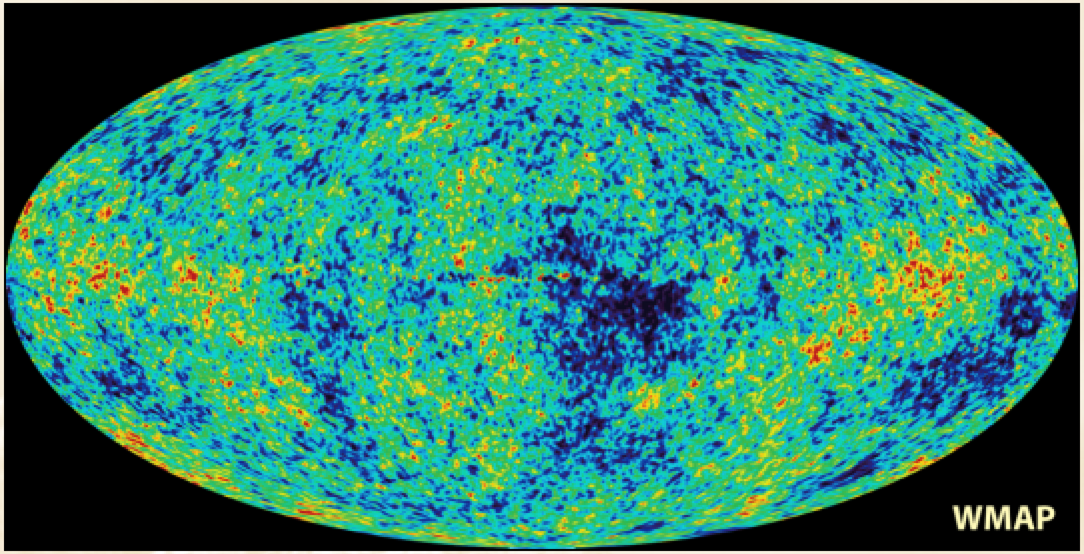


Review of Particle Properties

Power Spectrum of CMBR Temperature Fluctuations

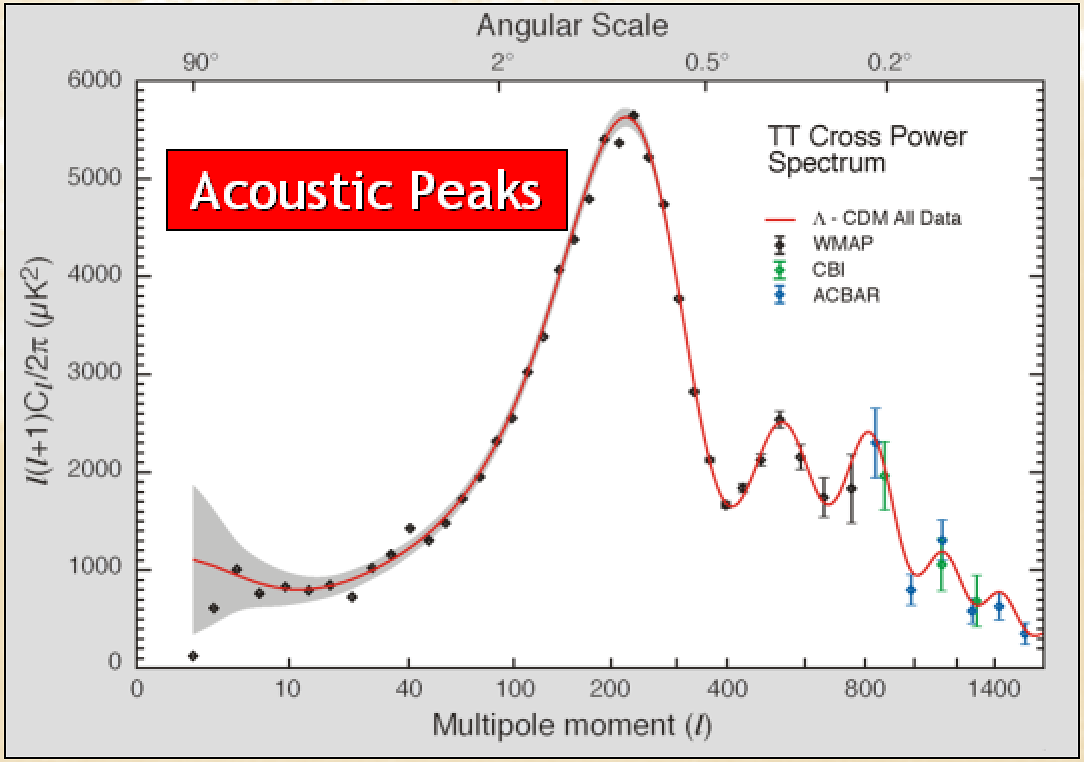
Sky map of CMBR temperature fluctuations

$$\Delta(\theta,\varphi) = \frac{T(\theta,\varphi) - \langle T \rangle}{\langle T \rangle}$$



Multipole expansion

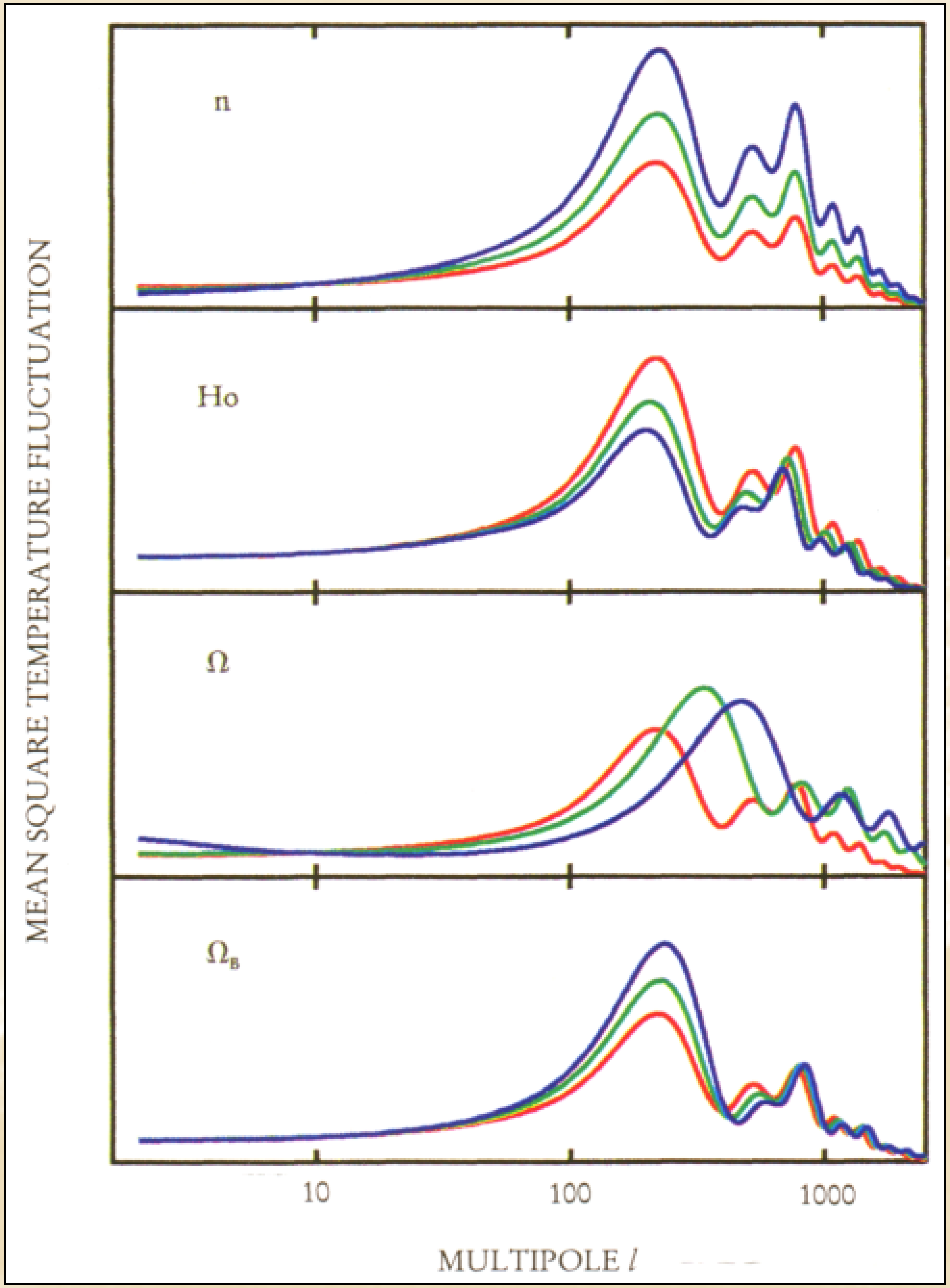
$$\Delta(\theta,\varphi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta,\varphi)$$



Angular power spectrum

$$C_\ell = \left\langle a^*_{\ell m} a_{\ell m} \right\rangle = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} a^*_{\ell m} a_{\ell m}$$

CMBR - The Cosmic Rosetta Stone



Power-law index (tilt)
n = 1.0, 1.1, 1.2

Hubble constant
H_0 = 50, 60, 70

Total density
Ω_{tot} = 1.0, 0.5, 0.3

Baryon density
Ω_B = 5, 7.5, 10×10^{-3}

Physics Today 1997:11, 32

Concordance Model of Cosmology

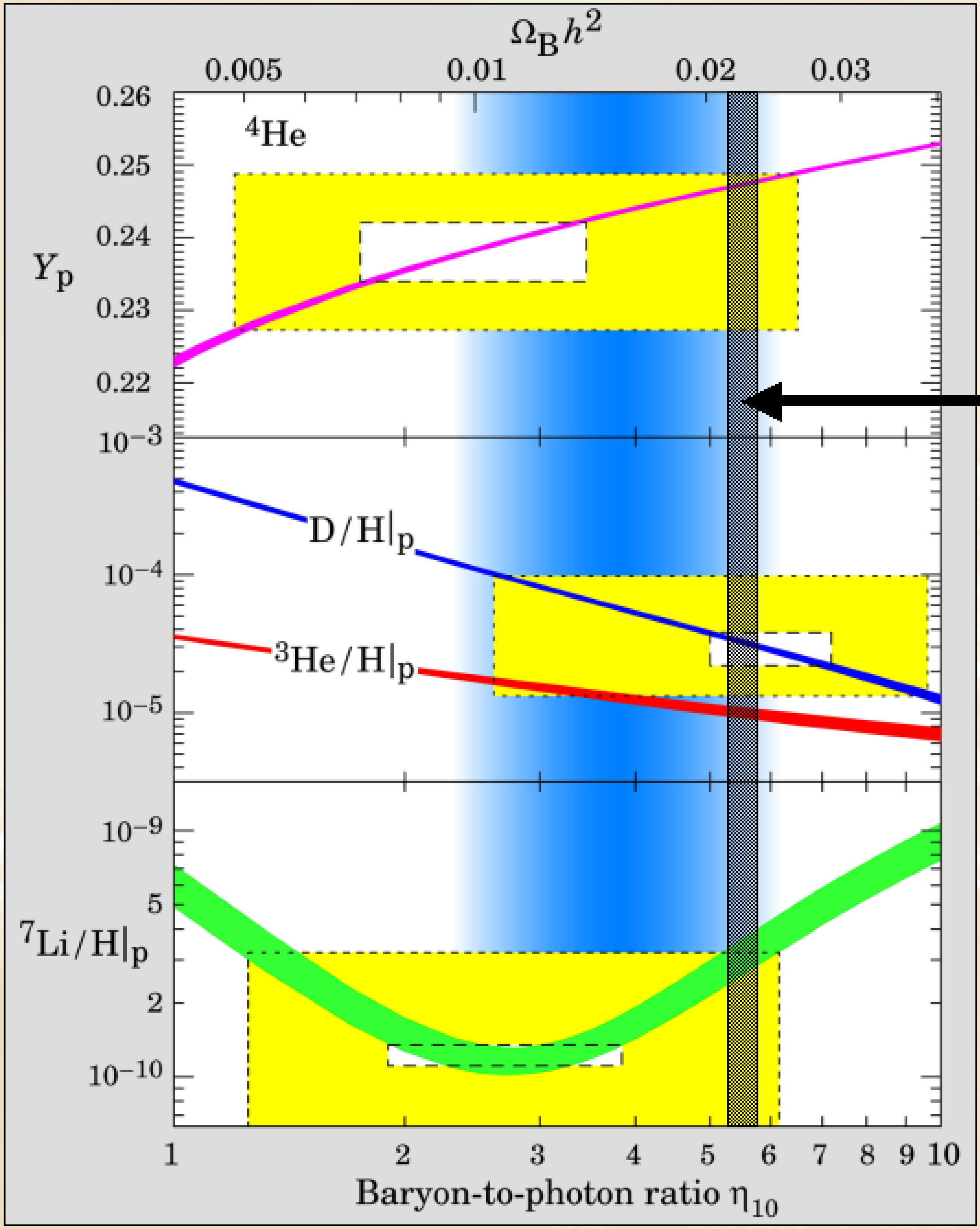
A Friedmann-Lemaître-Robertson-Walker model with the following parameters perfectly describes the global properties of the universe

Expansion rate	$H_0 = (72 \pm 4)\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$	
Spatial curvature	$\lvert R_{curv} \rvert > 5 H_0^{-1}$	$\Omega_{tot} = 1.02 \pm 0.02$
Age	$t_0 = (13.7 \pm 0.2) \times 10^9$ years	
Vacuum energy	$\Omega_\Lambda = 0.73 \pm 0.04$	$\Omega_\Lambda + \Omega_M = 1.02 \pm 0.02$
Matter	$\Omega_M = 0.27 \pm 0.04$	
Baryonic matter	$\Omega_B = 0.044 \pm 0.004$	

The observed large-scale structure and CMBR temperature fluctuations are perfectly accounted for by the gravitational instability mechanism with the above ingredients and a power-law primordial spectrum of adiabatic density fluctuations (curvature fluctuations) $P(k) \propto k^n$

Power-law index	$n = 0.93 \pm 0.03$

BBN Concordance



Helium

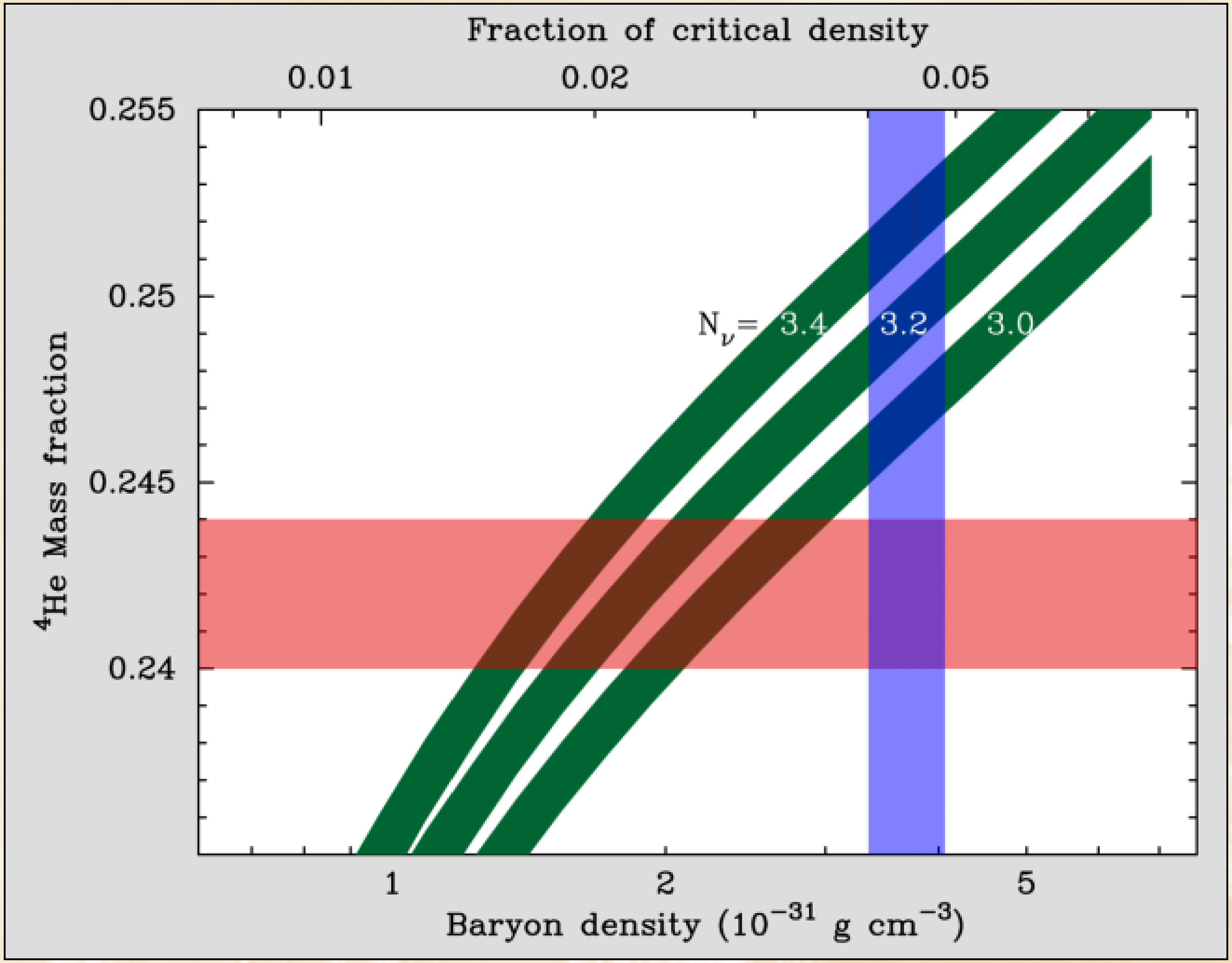
Deuterium

Lithium

Implied by CMBR and large-scale structure

Review of Particle Properties

BBN Limits on Neutrino Flavors



With the baryon density known, the observed helium mass fraction measures the expansion rate at nucleosynthesis

$$H = \sqrt{\frac{4\pi^3}{45} g_*} \frac{T^2}{m_{Pl}}$$

$$g_* = 2 + \frac{7}{8}(4 + 2N_\nu)$$

At BBN one flavor contributes about 16% to cosmic mass-energy so that

$$\Delta Y_P \approx 0.012\, \Delta N_\nu$$

Measured value

$$Y_P = 0.242 \pm 0.002$$

(only statistical error)

- Slight tension between measured and predicted helium abundance
- Certainly no evidence for additional radiation

ISAPP 2004, International School on Astro-Particle Physics,
28 June—9 July 2004, Laboratori Nazionali del Gran Sasso, Italy

Cosmological Neutrinos
Topic IV:

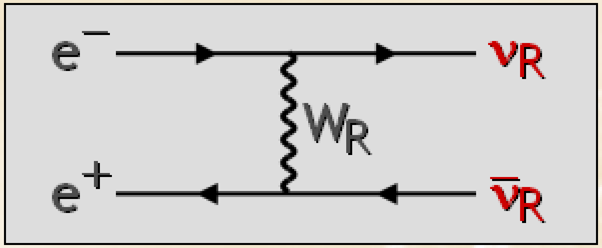
How Many Cosmological Neutrinos?

Georg G. Raffelt
Max-Planck-Institut für Physik, München, Germany

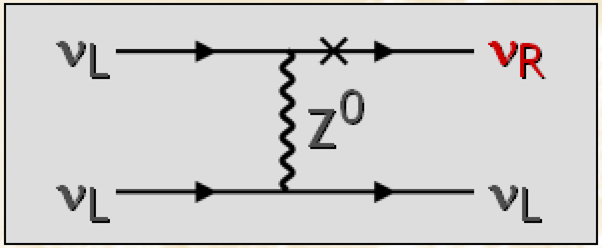
Dirac Neutrinos

- If neutrinos are Dirac particles (not their own antiparticles), right-handed states exist that do not interact by ordinary weak interactions (gauge singlets)
- What is the cosmic population of these right handed (sterile) states?

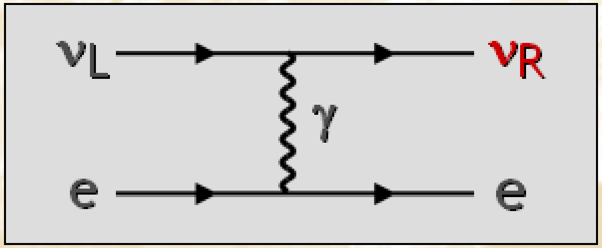
Right-handed currents



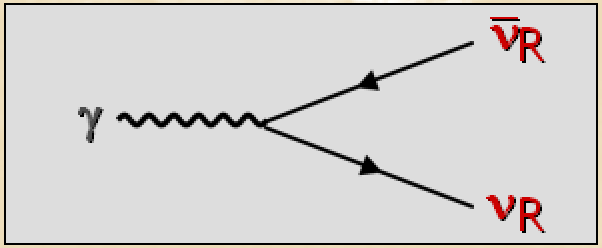
Dirac mass



Dipole moments



Milli charge



- For each case there is a freeze-out temperature $T_{F,R}$ and corresponding number $g_{*F,R}$ of thermally excited degrees of freedom
- Interacting degrees are subsequently heated by entropy transfer
- Dilution of r.h. neutrinos relative to l.h. ones by

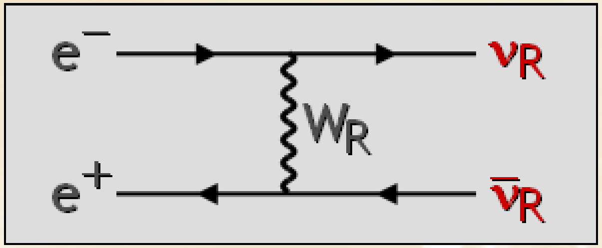
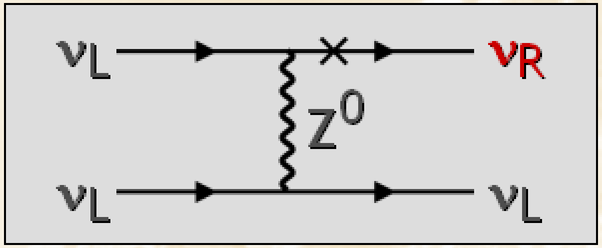
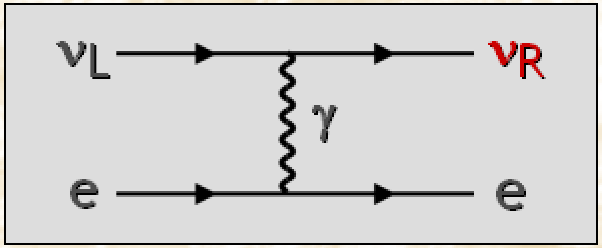
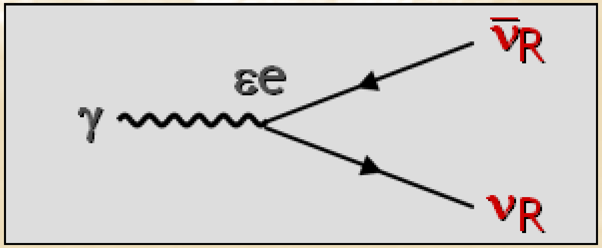
$$\frac{n_L}{n_R} = \frac{g_{*F,R}}{g_{*F,L}} = \frac{g_{*F,R}}{\underbrace{43/4}_{2+\frac{7}{8}(4+6)}}$$

Thermal Degrees of Freedom

Temperature threshold		Particles	g_B	g_F	g_*
	low	γ, 3ν	2	6	(7.25)
m_e	0.5 MeV	$e^\pm$	2	10	10.75
m_μ	105 MeV	$\mu^\pm$	2	14	14.25
m_π	135 MeV	π^0, $\pi^\pm$	5	14	17.25
Λ_{QCD}	100-200 MeV	u, d, s, gluons	18	50	61.75
$m_{c,\tau}$	2 GeV	c, τ	18	66	75.75
m_b	6 GeV	$b^\pm$	18	78	86.25
$m_{W,Z}$	90 GeV	Z^0, $W^\pm$	27	78	92.25
m_t	170 GeV	t	27	90	105.75
m_H	?	Higgs	28	90	106.75
Λ_{SUSY}	~ 1 TeV ?	SUSY particles	118	118	213.50

Thermal Equilibration of Dirac Neutrinos

Condition for thermal equilibrium: $\Gamma > H \sim T^2/m_{Pl}$

Right-handed currents		$\Gamma \sim G_R^2 T^5$ $T_F \sim (m_{Pl} G_F^2)^{-1/3} \sim 1\,\mathrm{MeV}\,(G_F/G_R)^{2/3}$
Dirac mass		$\Gamma \sim G_F^2 T^5 (m_\nu/T)^2 = G_F^2 m_\nu^2 T^3$ $T_F \sim (m_{Pl} G_F^2 m_\nu^2)^{-1} \sim 1\,\mathrm{MeV}\,(1\,\mathrm{MeV}/m_\nu)^2$
Dipole moments		$\Gamma \sim \alpha\mu^2 T^3$ $T_F \sim (m_{Pl}\alpha\mu^2)^{-1}$
Milli charge		$\Gamma \sim \varepsilon^2\alpha^2 T$ $T \sim \varepsilon^2\alpha^2 m_{Pl}$ Recoupling temperature

Thermal Equilibration of Dirac Neutrinos

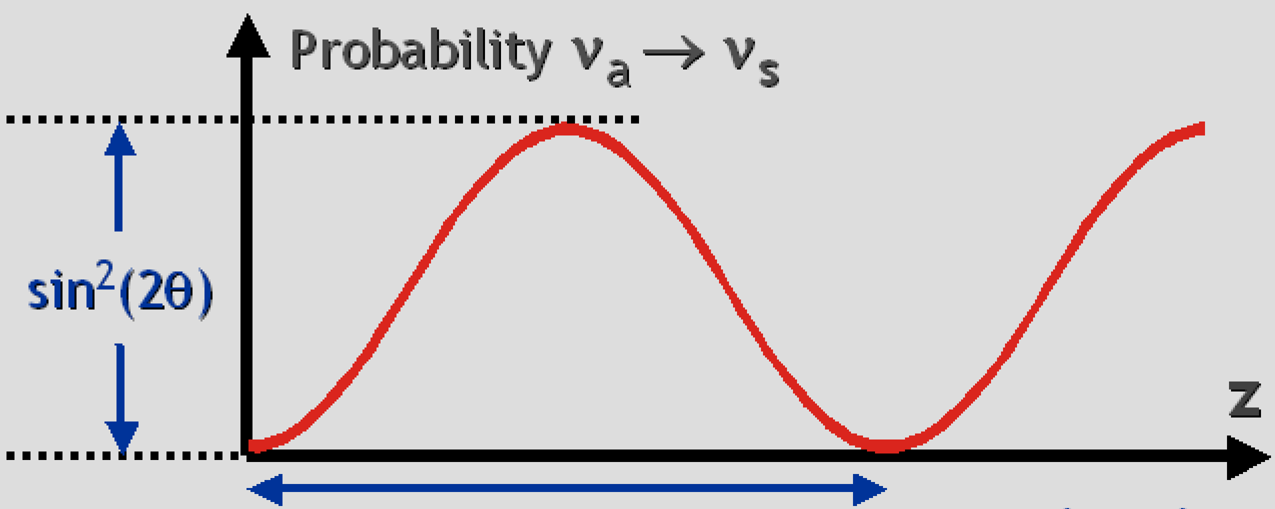
	Equilibrium condition	Limit on coupling	Dilution
Right-handed currents	$\Gamma \sim G_R^2 T^5$ $T_F \sim (m_{Pl} G_F^2)^{-1/3}$	$G_R < 10^{-5} G_F$ Supernova 1987A	$T_F \gtrsim 1$ GeV $g_{*F} \gtrsim 70$ $n_R \lesssim n_L/7$
Dirac mass	$\Gamma \sim G_F^2 m_\nu^2 T^3$ $T_F \sim (m_{Pl} G_F^2 m_\nu^2)^{-1}$	$m_\nu < 1\,\mathrm{eV}$ Structure formation	$T_F \gtrsim 10^9$ GeV $\gg m_W$ (Not self-consistent) $n_R \lesssim n_L/10$
Dipole moments	$\Gamma \sim \alpha \mu_\nu^2 T^3$ $T_F \sim (m_{Pl} \alpha \mu^2)^{-1}$	$\mu_\nu < 10^{-12} \frac{e}{2m_e}$ Supernova 1987A	$T_F \gtrsim 100$ GeV $g_{*F} \gtrsim 80$ $n_R \lesssim n_L/8$
Milli charge	$\Gamma \sim \varepsilon^2 \alpha^2 T$ $T \sim \varepsilon^2 \alpha^2 m_{Pl}$	$\varepsilon < 3 \times 10^{-17}$ Supernova 1987A	$T \lesssim 10^{-9}$ eV Never recouples

Sterile Neutrinos

Sterile (right-handed) neutrinos may exist that are not a Dirac partner to an ordinary neutrino

- Unknown mass m_s
- Unknown mixing angles with ordinary neutrinos Θ_{es}, $\Theta_{\mu s}$, and $\Theta_{\tau s}$

Neutrino flavor oscillations



Oscillation Length $\quad \dfrac{4\pi E}{\delta m^2} = 2.5\,\mathrm{m}\left(\dfrac{E}{\mathrm{MeV}}\right)\left(\dfrac{\mathrm{eV}^2}{\delta m^2}\right)$

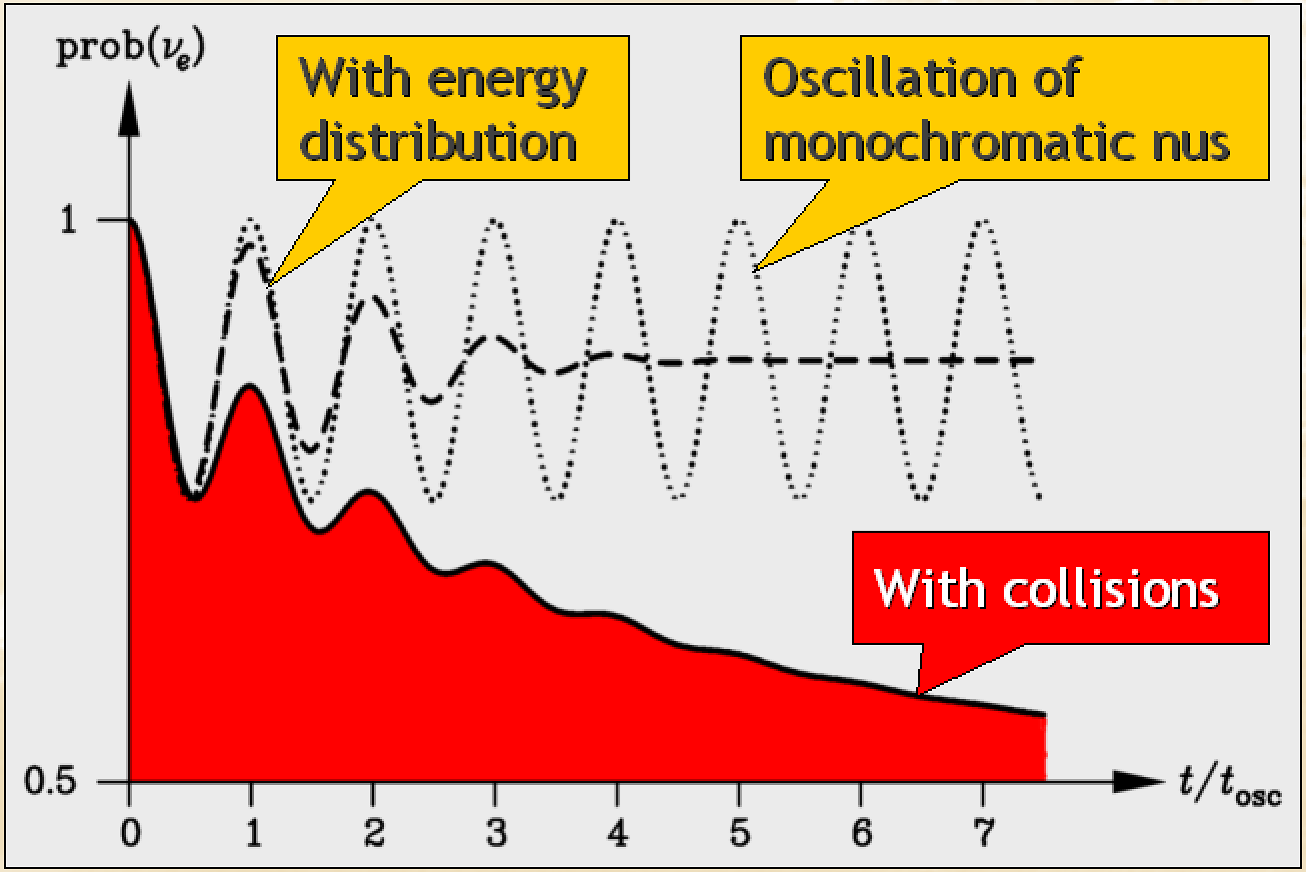
- For small mixing angle, naively expect very small population of sterile neutrinos
- However, collisions of active flavors “decohere” superposition and equilibrate sterile states

Flavor Relaxation in a the Early Universe

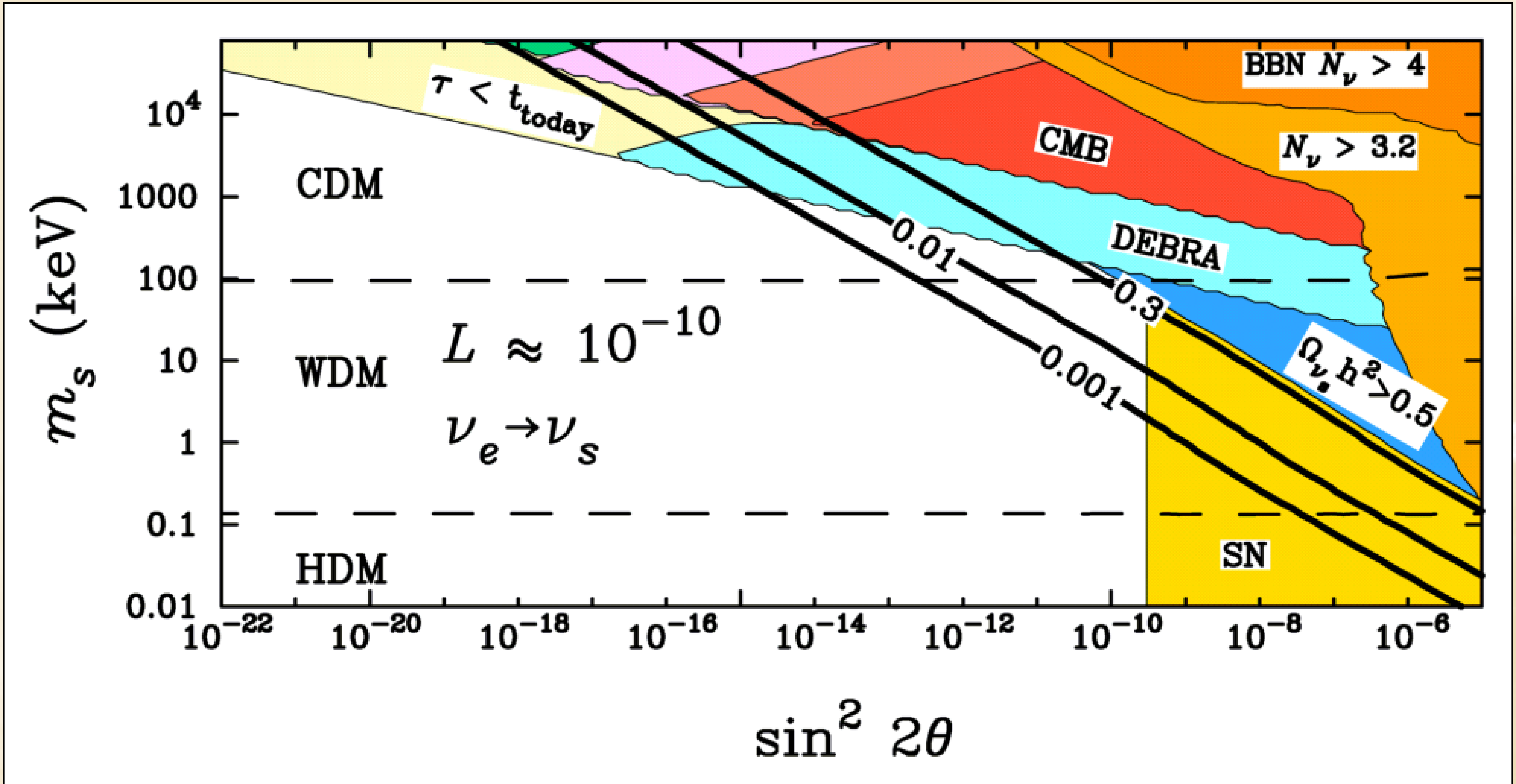
Neutrinos suffer collisions in a medium that can interrupt the coherence of flavor oscillations: The flavor content is "measured" and oscillations start from scratch from the "collapsed state".

Average oscillation probability $\frac{1}{2}\sin^2(2\Theta)$
Collision rate ~ damping rate Γ

→ **Conversion rate** $\frac{1}{2}\sin^2(2\Theta)\,\Gamma$



Sterile Neutrinos as Dark Matter



Abazajian, Fuller & Patel, Sterile neutrino hot, warm, and cold dark matter
astro-ph/0101524

Thermal Neutrino Distribution with Chemical Potential

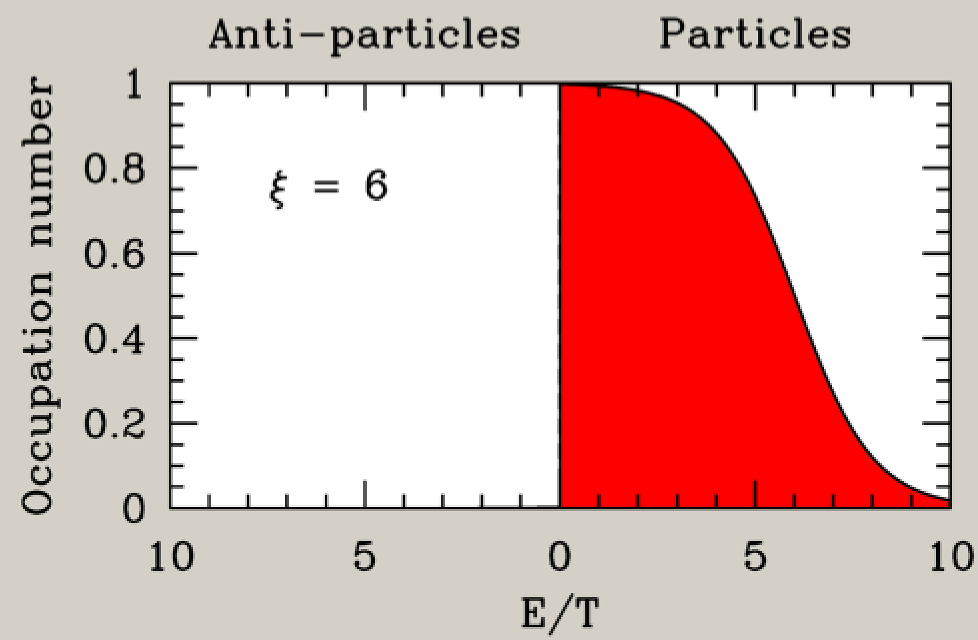
Fermi-Dirac distribution

- Temperature T
- Chemical potential μ

$+\mu$ Particles

$-\mu$ Anti-particles

$$f_p = \frac{1}{\exp\left(\frac{E-\mu}{T}\right)+1}$$



Degeneracy parameter $\xi = \frac{\mu}{T}$ **Invariant under cosmic expansion**

Number density

$$n_{\nu\bar\nu} = \int dE \frac{4\pi}{(2\pi)^3}\left(\frac{E^2}{1+\exp(E/T-\xi)} + \frac{E^2}{1+\exp(E/T+\xi)}\right)$$

$$= \frac{3\zeta_3}{2\pi^2} T_\nu^3 \left[1 + \frac{2\ln(2)}{3\zeta_3}\xi^2 + \frac{1}{72\zeta_3}\xi^4 + \ldots\right]$$

How Many Relic Neutrinos?

Standard thermal population in one flavor $n_{\nu\bar{\nu}} = \frac{3}{11} n_\gamma \approx 115\,\mathrm{cm}^{-3}$

Additional active neutrinos beyond standard population of ν_e, ν_μ, ν_τ	Additional families	Excluded by Z^0 width ($N_\nu = 3$)
	Chemical potentials for ν_e, ν_μ, ν_τ	Possible
Sterile (right-handed) states Populated by $\nu_L \to \nu_R$ transitions	Dirac mass	Not effective in eV range
	Right-handed currents	Constrained by energy loss of SN 1987A
	Electromagnetic dipole moments	Constrained by energy loss of globular cluster stars
	Oscillations/collisions	Hot/warm/cold DM possible

BBN and Neutrino Chemical Potentials

Expansion Rate Effect (all flavors)

Energy density in one neutrino flavor with degeneracy parameter $\xi = \eta/T$

$$\rho_{\nu\bar\nu} = \frac{7\pi^2}{120} T_\nu^4 \left[1 + \underbrace{\frac{30}{7}\left(\frac{\xi}{\pi}\right)^2 + \frac{15}{7}\left(\frac{\xi}{\pi}\right)^4}_{\Delta N_{eff}}\right]$$

Beta equilibrium effect for electron flavor

$$n + \nu_e \leftrightarrow p + e^-$$

Helium abundance essentially fixed by n/p ratio at beta freeze-out

$$\frac{n}{p} = e^{-(m_n - m_p)/T - \xi_{\nu_e}}$$

Effect on helium equivalent to $\Delta N_{eff} \sim -18\,\xi_{\nu_e}$

- ν_e beta effect can compensate expansion-rate effect of $\nu_{\mu,\tau}$
- No significant BBN limit on neutrino number density

Chemical Potentials and Flavor Oscillations

Flavor mixing
(neutrino oscillations)

Flavor lepton numbers
not conserved

Only one common neutrino
chemical potential

Stringent ξ_{ν_e} limit
applies to all flavors
$|\xi_{\nu_{e,\mu,\tau}}| < 0.07$

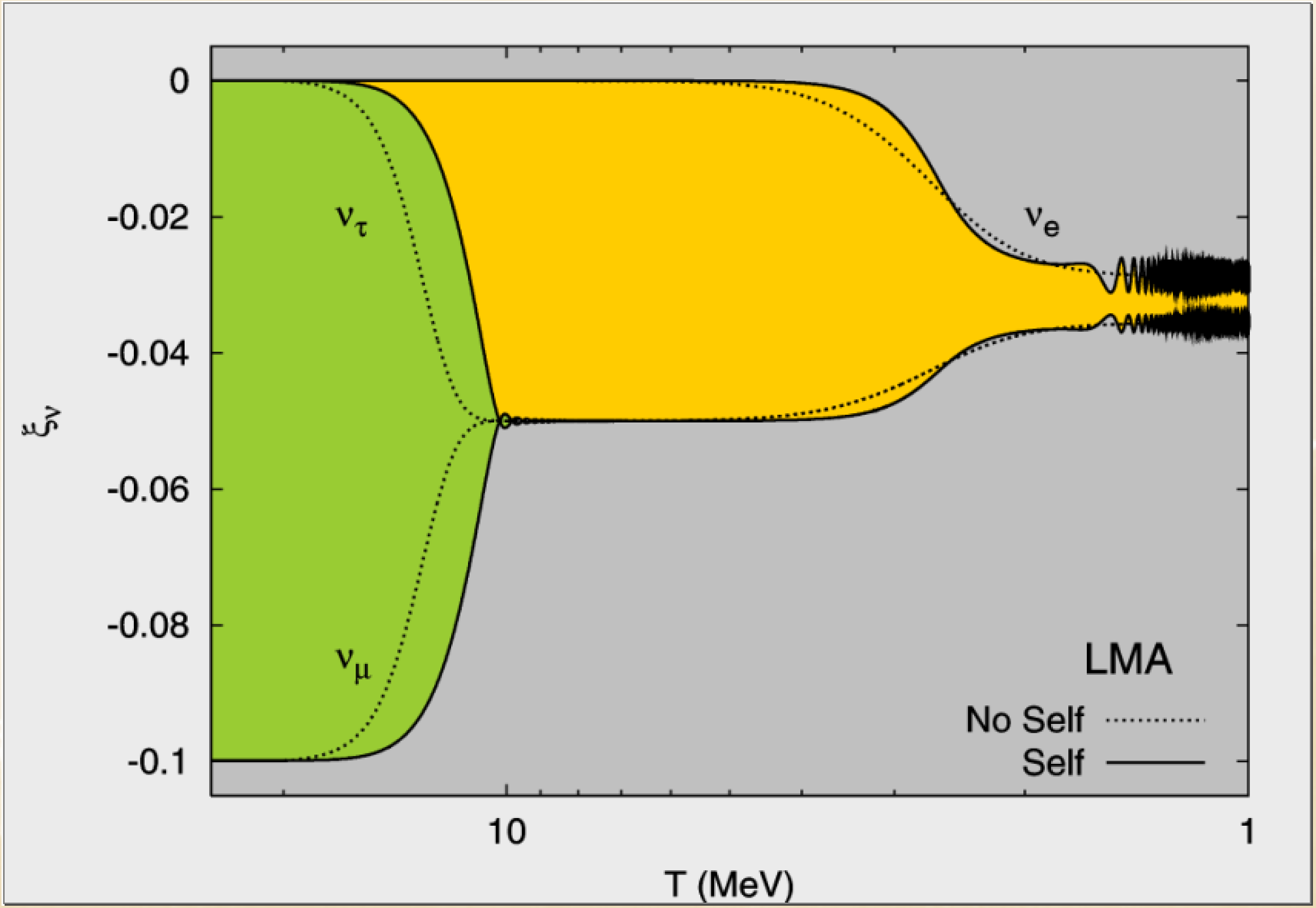
Extra neutrino density
$\Delta N_{eff} < 0.0064$

Cosmic neutrino density
close to standard value

- Flavor equilibrium before n/p freeze out ?
- YES for solar LMA solution
- Our knowledge of the cosmic neutrino density depends on measured oscillation parameters

- Lunardini & Smirnov, hep-ph/0012056
- Dolgov, Hansen, Pastor, Petcov, Raffelt & Semikoz, hep-ph/0201287
- Abazajian, Beacom & Bell, astro-ph/0203442
- Wong, hep-ph/0203180

Flavor Equilibration: LMA Solution



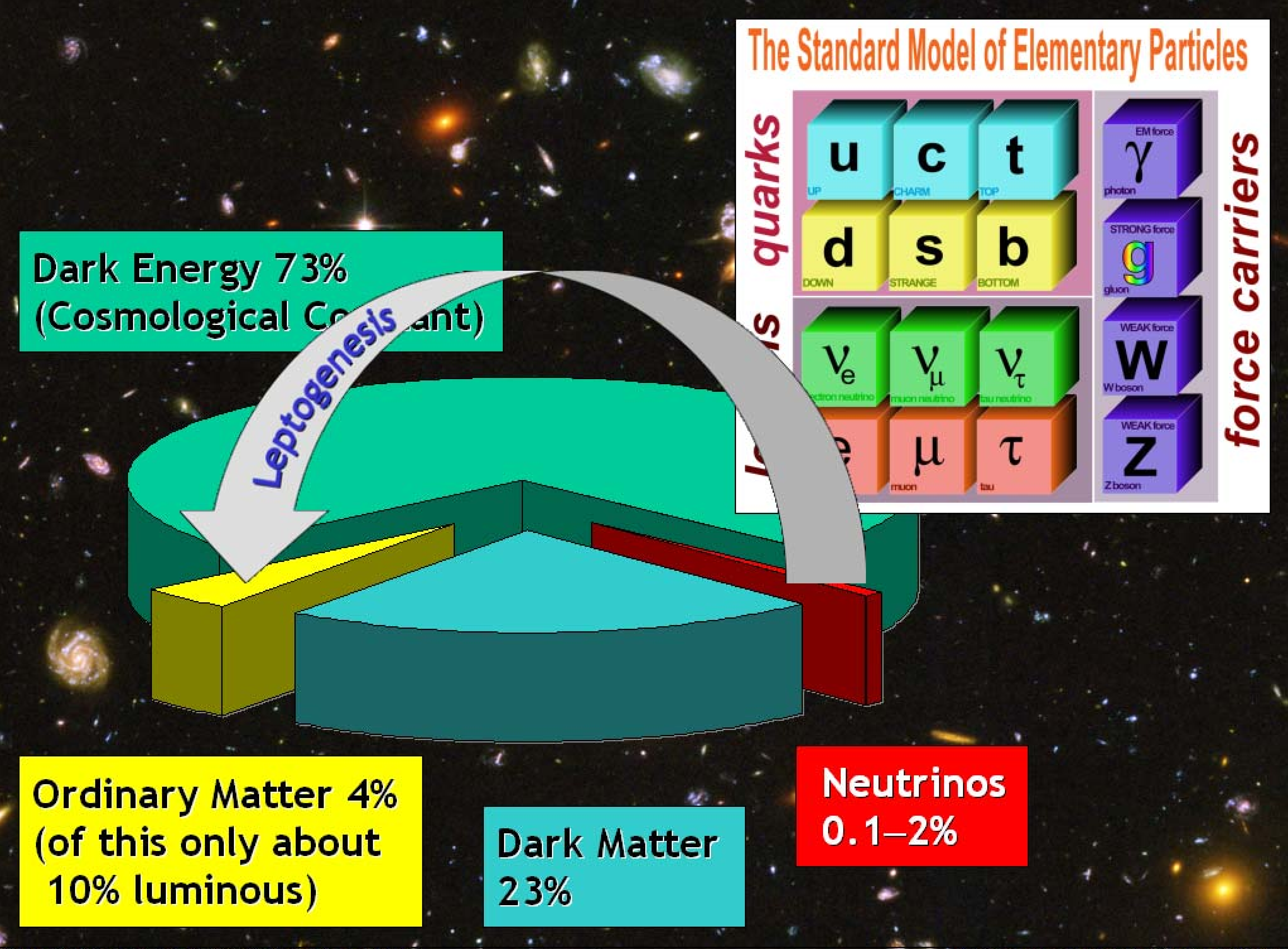
Dolgov, Hansen, Pastor, Petcov, Raffelt & Semikoz, hep-ph/0201287

ISAPP 2004, International School on Astro-Particle Physics,
28 June—9 July 2004, Laboratori Nazionali del Gran Sasso, Italy

Cosmological Neutrinos
Topic V

Leptogenesis

Georg G. Raffelt
Max-Planck-Institut für Physik, München, Germany


Dark Energy 73%
(Cosmological Constant)
Leptogenesis
The Standard Model of Elementary Particles
quarks
u
UP
c
CHARM
t
TOP
d
DOWN
s
STRANGE
b
BOTTOM
νe
νμ
muon neutrino
ντ
tau neutrino
μ
muon
τ
tau
force carriers
EM force
γ
photon
STRONG force
g
gluon
WEAK force
W
W boson
WEAK force
Z
Z boson
Ordinary Matter 4%
(of this only about
10% luminous)
Dark Matter
23%
Neutrinos
0.1–2%

Baryogenesis in the Early Universe

Andrei Sakharov
1921–1989

Sakharov conditions for creating the Baryon Asymmetry of the Universe (BAU)

- C and CP violation
- Baryon number violation
- Deviation from thermal equilibrium

Particle-physics standard model

- Violates C and CP
- Violates B and L by EW instanton effects (B – L conserved)

- However, electroweak baryogenesis not quantitatively possible within particle-physics standard model
- Works in SUSY models for small range of parameters

A.Riotto & M.Trodden: Recent progress in baryogenesis
Ann. Rev. Nucl. Part. Sci. 49 (1999) 35

Leptogenesis by Majorana Neutrino Decays

A classic paper

Volume 174, number 1 PHYSICS LETTERS B 26 June 1986

BARYOGENESIS WITHOUT GRAND UNIFICATION

M. FUKUGITA

Research Institute for Fundamental Physics, Kyoto University, Kyoto 606, Japan

and

T. YANAGIDA

Institute of Physics, College of General Education, Tohoku University, Sendai 980, Japan and Deutsches Elektronen-Synchrotron DESY, D-2000 Hamburg, Fed. Rep. Germany

Received 8 March 1986

A mechanism is pointed out to generate cosmological baryon number excess without resorting to grand unified theories. The lepton number excess originating from Majorana mass terms may transform into the baryon number excess through the unsuppressed baryon number violation of electroweak processes at high temperatures.

See-Saw Model for Neutrino Masses

Dirac masses from coupling to standard Higgs field ϕ

Charged Leptons	
e_L	e_R
μ_L	μ_R
τ_L	τ_R

Neutrinos	
ν_1	N_1
ν_2	N_2
ν_3	N_3

Heavy Majorana masses $M_j > 10^{10}$ GeV

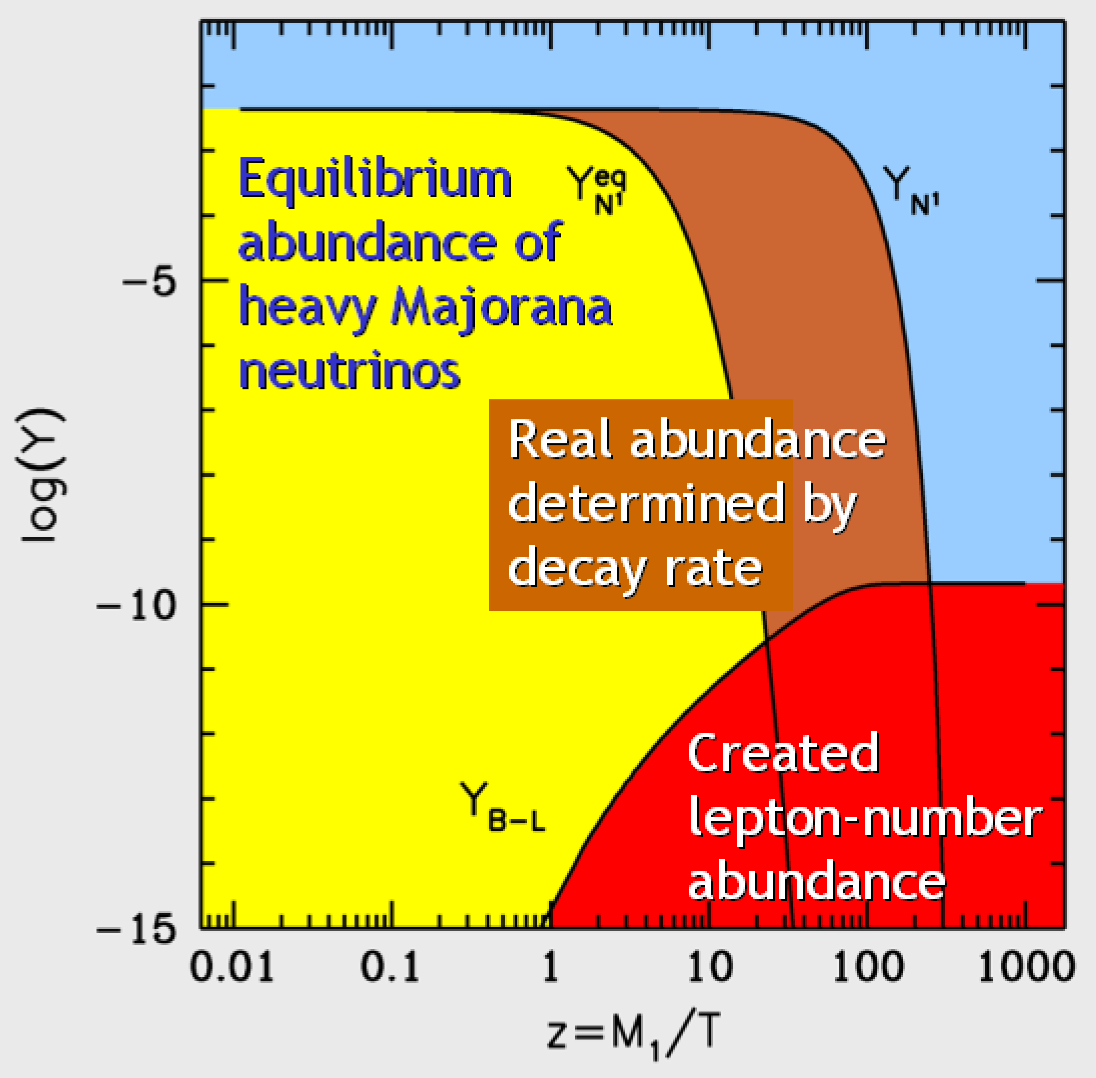
Lagrangian for particle masses

$$L_{mass} = -\bar{\ell}_L \phi g_\ell e_R - \bar{\ell}_L \phi g_\nu N_R - \frac{1}{2}\overline{N_R^c} M N_R + \text{h.c.}$$

Light Majorana mass

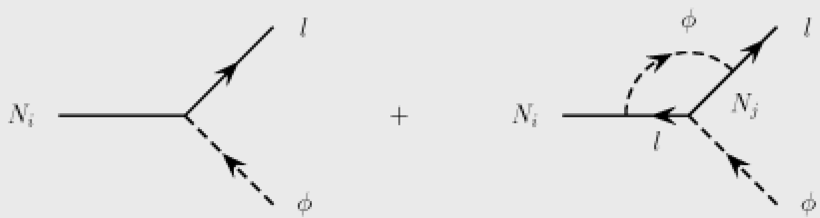
$$\begin{pmatrix}\bar{\nu}_L & \bar{N}_R\end{pmatrix}\begin{pmatrix}0 & g_\nu\langle\phi\rangle \\ g_\nu\langle\phi\rangle & M\end{pmatrix}\begin{pmatrix}\nu_L \\ N_R\end{pmatrix} \xrightarrow{\text{Diagonalize}} \begin{pmatrix}\bar{\nu}_L & \bar{N}_R\end{pmatrix}\begin{pmatrix}\frac{g_\nu^2\langle\phi\rangle^2}{M} & 0 \\ 0 & M\end{pmatrix}\begin{pmatrix}\nu_L \\ N_R\end{pmatrix}$$

Leptogenesis by Out-of-Equilibrium Decay



M. Fukugita & T. Yanagida:
Baryogenesis without Grand Unification
Phys. Lett. B 174 (1986) 45

CP-violating decays by interference of tree-level with one-loop diagram



$$\Gamma_{\rm Decay} = g_\nu^2 \frac{M}{8\pi}$$

W. Buchmüller & M. Plümacher: Neutrino masses and the baryon asymmetry
Int. J. Mod. Phys. A15 (2000) 5047-5086

Connection to Neutrino Mass

Decay rate of heavy Majorana neutrino	$\Gamma_{\rm Decay} = g_\nu^2 \frac{M}{8\pi}$
Cosmic expansion rate	$H \approx \sqrt{g_{\rm eff}}\,\frac{T^2}{m_{\rm Pl}}$
Requirement for strong deviation from equilibrium …	$\Gamma_{\rm Decay} < H\big\vert_{T=M}$
	$g_\nu^2 \frac{M}{8\pi} < \sqrt{g_{\rm eff}}\,\frac{M^2}{m_{\rm Pl}}$
	$\frac{g_\nu^2}{M} < \frac{8\pi\sqrt{g_{\rm eff}}}{m_{\rm Pl}}$
… translates into a limit on the observable neutrino mass	$m_\nu = \frac{g_\nu^2 \langle\phi\rangle^2}{M} < \frac{8\pi\sqrt{g_{\rm eff}}}{m_{\rm Pl}} \langle\phi\rangle^2 \sim 10^{-3}\,{\rm eV}$

Leptogenesis by Majorana Neutrino Decays

In see-saw models for neutrino masses, out-of-equilibrium decays of right-handed heavy Majorana neutrinos provide source for CP- and L-violation

Cosmological evolution

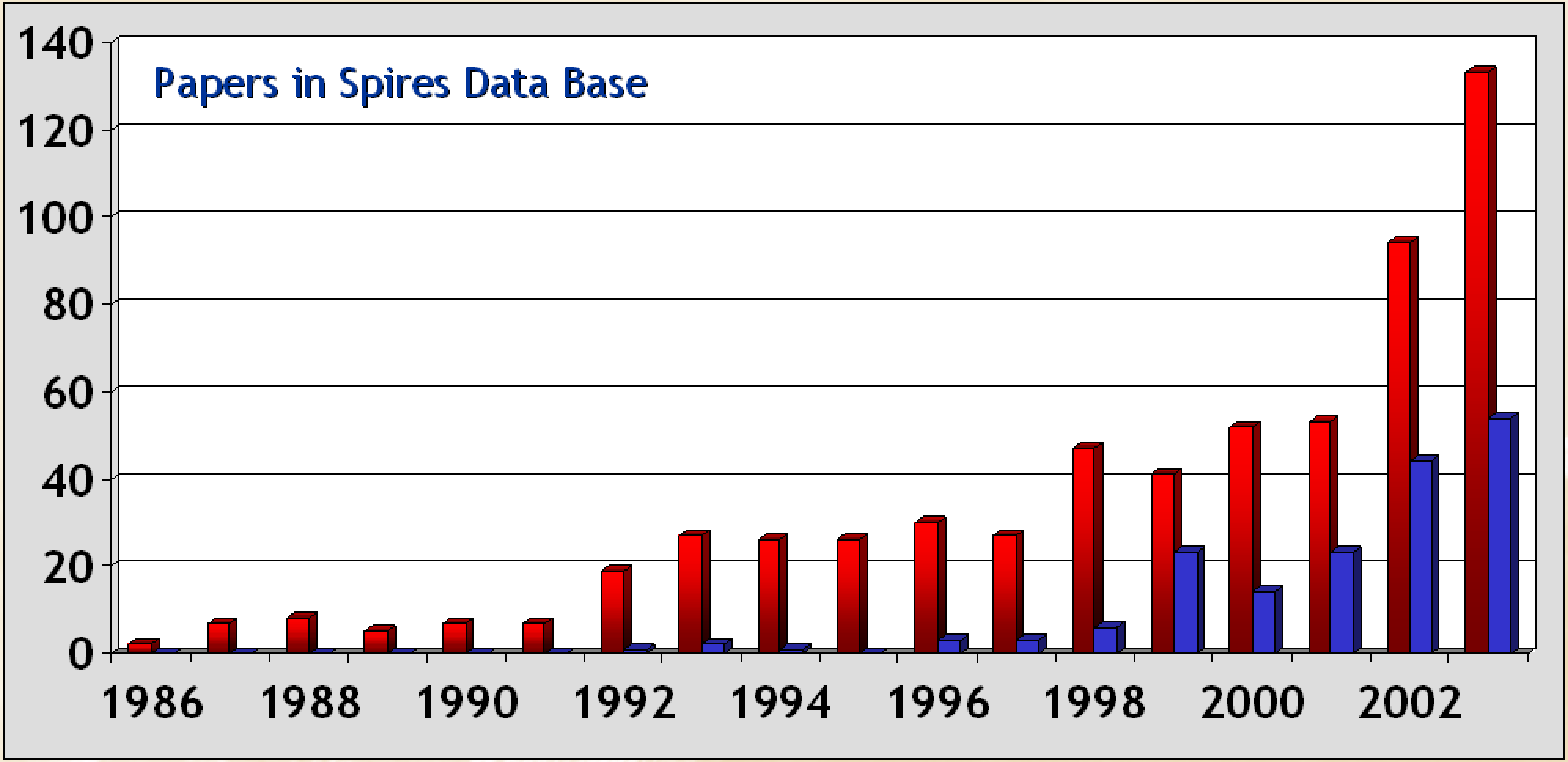
- B = L = 0 early on
- Thermal freeze-out of heavy Majorana neutrinos
- Out-of-equilibrium CP-violating decay creates net L
- Shift L excess into B by sphaleron effects

Sufficient deviation from equilibrium distribution of heavy Majorana neutrinos at freeze-out → Limits on Yukawa couplings → Limits on masses of ordinary neutrinos

Requires Majorana neutrino masses below 0.1 eV

Buchmüller, Di Bari & Plümacher, hep-ph/0209301 & hep-ph/0302092

Leptogenesis as a Research Topic

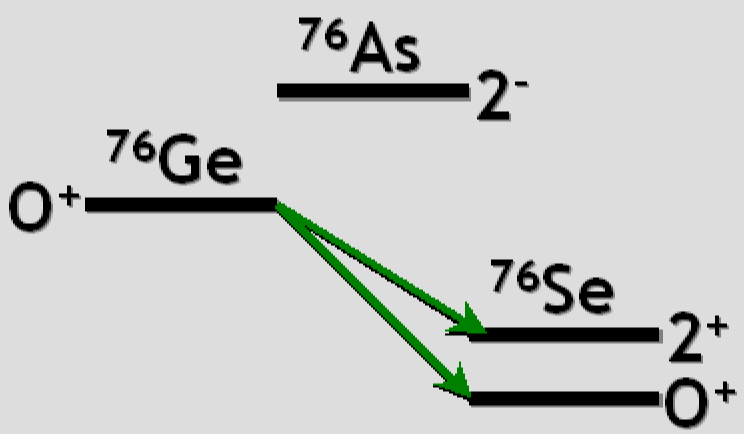


Annual citations of Fukugita & Yanagida, PLB 174 (1986) 45

"Leptogenesis" in title

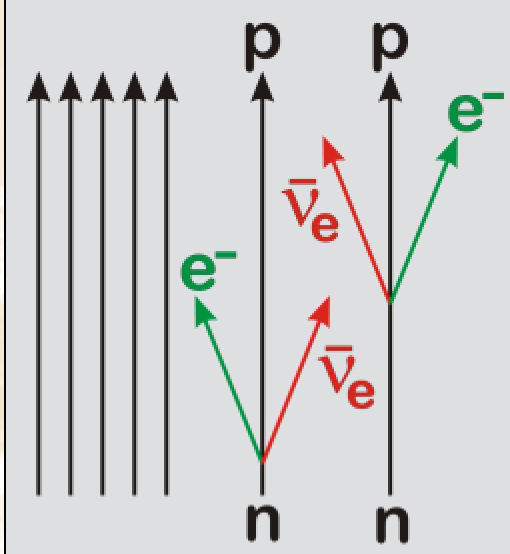
Neutrinoless ββ Decay

Some nuclei decay only by the ββ mode, e.g.

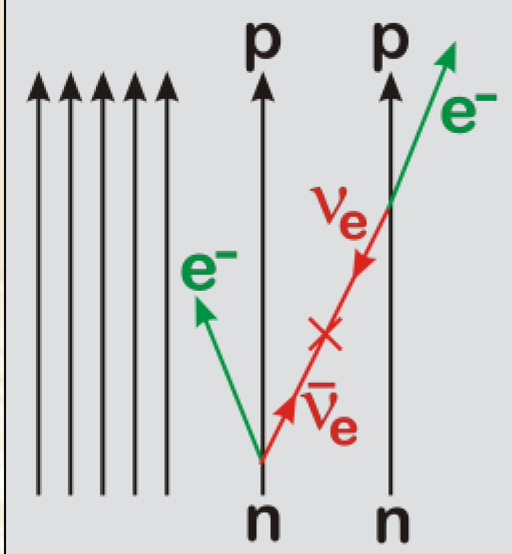


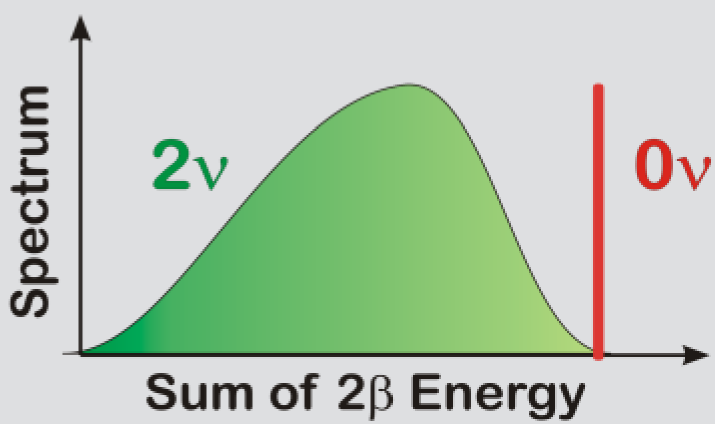
Half life ~ 10^{21} yr

Standard 2ν mode



0ν mode, enabled by Majorana mass





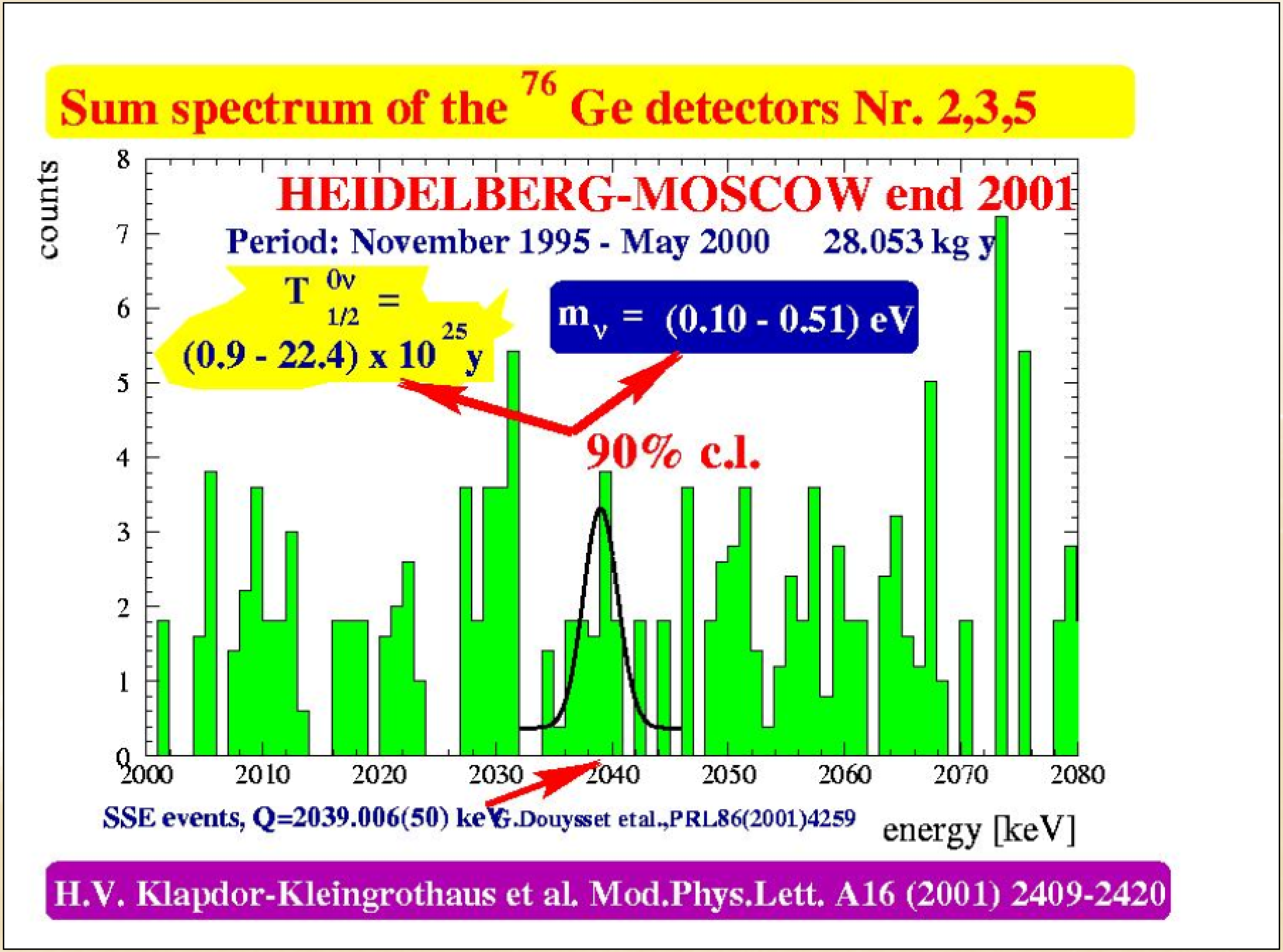
Measured quantity

$$|m_{ee}| = \left| \sum_{i=1}^{N} \lambda_i |U_{ei}|^2 m_i \right|$$

Best limit from ^{76}Ge

$$|m_{ee}| < 0.35\,\text{eV}$$

Evidence for 0ν2β Decay from Heidelberg-Moscow



Improved Evidence for 0ν2β Decay

H.V. Klapdor-Kleingrothaus et al.: Data Acquisition and Analysis of the 76Ge Double Beta Experiment in Gran Sasso 1990-2003, arXiv:hep-ph/0403018

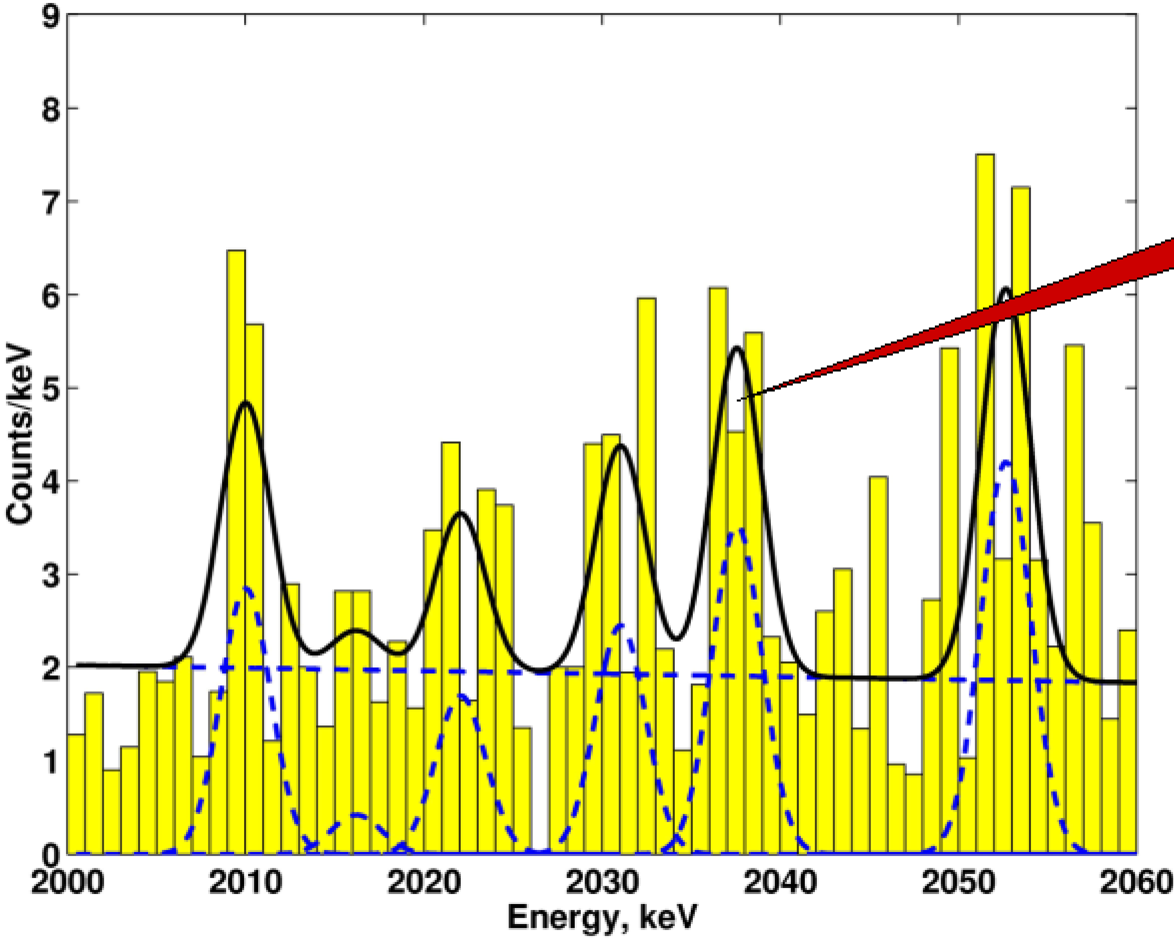
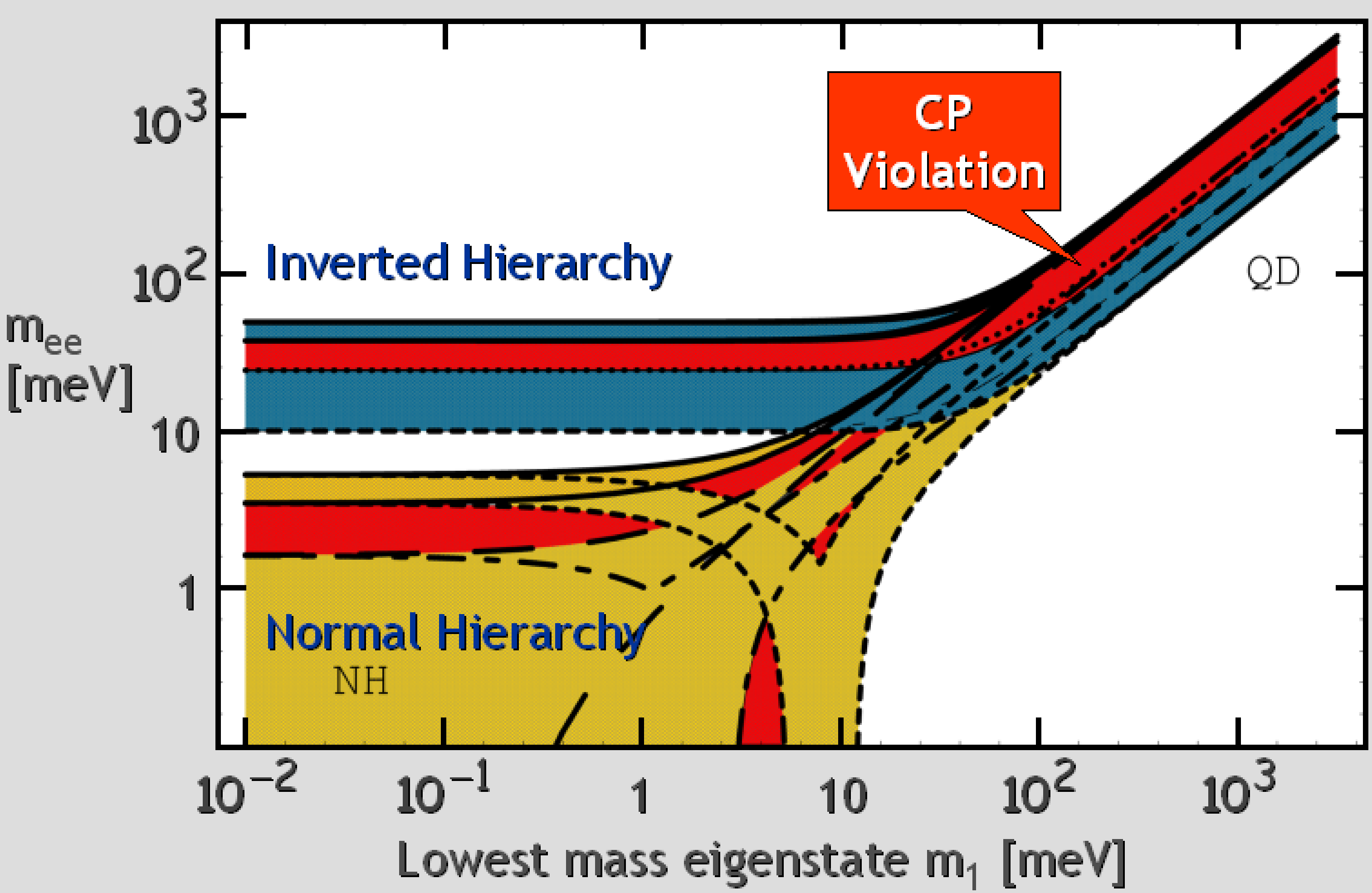
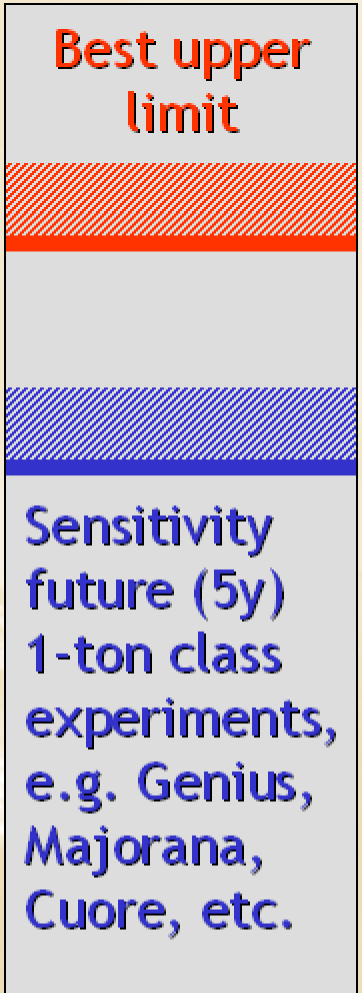


Fig. 31. The single site sum spectrum of the four detectors 2,3,4,5 for the period November 1995 to May 2003 (51.389 kg y), and its fit (see section 3), in the range 2000 - 2060 keV.

Claimed evidence for 0ν2β line now ~ 4σ

Effective Majorana Mass in Plausible Scenarios





Pascoli & Petcov, hep-ph/0310003 & hep-ph/0205022
See also Feruglio, Strumia & Vissani, hep-ph/0201291
Klapdor-Kleingrothaus, Päs & Smirnov, hep-ph/0103076, and others