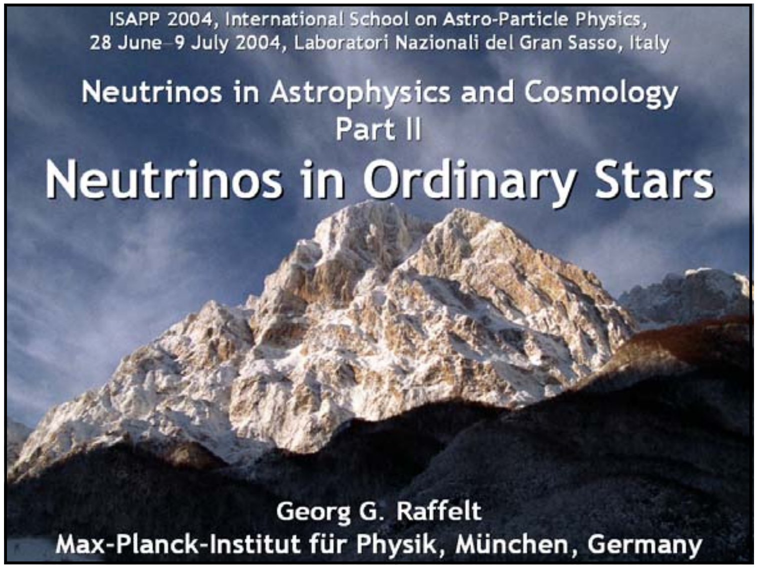

ISAPP 2004, International School on Astro-Particle Physics,
28 June–9 July 2004, Laboratori Nazionali del Gran Sasso, Italy

Neutrinos in Astrophysics and Cosmology
Part II

Neutrinos in Ordinary Stars

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Equations of Stellar Structure

Assume spherical symmetry and static structure (neglect kinetic energy)
Excludes: Rotation, convection, magnetic fields, supernova-dynamics, …

Hydrostatic equilibrium	$\frac{dP}{dr} = -\frac{G_N M_r \rho}{r^2}$
Energy conservation	$\frac{dL_r}{dr} = 4\pi r^2 \epsilon \rho$
Energy transfer	$L_r = \frac{4\pi r^2}{3\kappa\rho} \frac{d(aT^4)}{dr}$

- r Radius from center
- P Pressure
- G_N Newton's constant
- ρ Mass density
- M_r Integrated mass up to r
- L_r Luminosity (energy flux)
- ϵ Local rate of energy generation [erg/g/s]
  $\epsilon = \epsilon_{nuc} + \epsilon_{grav} - \epsilon_\nu$
- κ Opacity
  $\kappa^{-1} = \kappa_\gamma^{-1} + \kappa_c^{-1}$
- κ_γ Radiative opacity
  $\kappa_\gamma \rho = \langle \lambda_\gamma \rangle_{\rm Rosseland}^{-1}$
- κ_c Electron conduction

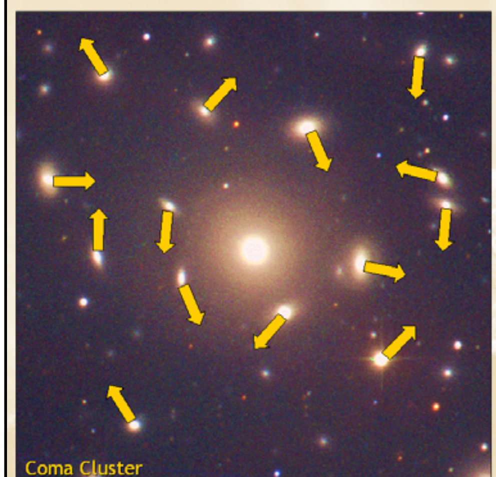
Literature
- Clayton: Principles of stellar evolution and nucleosynthesis (Univ. Chicago Press 1968)
- Kippenhahn & Weigert: Stellar structure and evolution (Springer 1990)

Virial Theorem and Hydrostatic Equilibrium

Hydrostatic equilibrium	$\frac{dP}{dr} = -\frac{G_N M_r \rho}{r^2}$
Integrate both sides	$\int_0^R dr\, 4\pi r^3 P' = -\int_0^R dr\, 4\pi r^3 \frac{G_N M_r \rho}{r^2}$
L.h.s. partial integration with P = 0 at surface R	$-3\int_0^R dr\, 4\pi r^2 P = E_{grav}^{tot}$
Classical monatomic gas: $P = \frac{2}{3}U$ (U density of internal energy)	$U^{tot} = -\frac{1}{2}E_{grav}^{tot}$
Average energy of single "atoms" of the gas	$\langle E_{kin}\rangle = -\frac{1}{2}\langle E_{grav}\rangle$ — Virial Theorem

Most important tool to understand self-gravitating systems

Dark Matter in Galaxy Clusters



A gravitationally bound system of many particles obeys the virial theorem

$$2\langle E_{kin}\rangle = -\langle E_{grav}\rangle$$

$$2\left\langle \frac{mv^2}{2} \right\rangle = \left\langle \frac{G_N M_r m}{r} \right\rangle$$

$$\langle v^2 \rangle \approx G_N M_r \langle r^{-1} \rangle$$

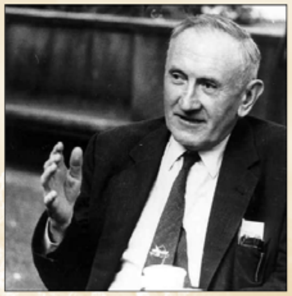
Velocity dispersion from Doppler shifts and geometric size

↓

Total Mass

Dark Matter in Galaxy Clusters

Fritz Zwicky:
Die Rotverschiebung von Extragalaktischen Nebeln
(The redshift of extragalactic nebulae)
Helv. Phys. Acta 6 (1933) 110



In order to obtain the observed average Doppler effect of 1000 km/s or more, the average density of the Coma cluster would have to be at least 400 times larger than what is found from observations of the luminous matter.
Should this be confirmed one would find the surprising result that dark matter is far more abundant than luminous matter.

Virial Theorem Applied to the Sun

$\langle E_{kin} \rangle = -\frac{1}{2} \langle E_{grav} \rangle$ Virial Theorem

Approximate Sun as a homogeneous sphere with

Mass $M_{sun} = 1.99 \times 10^{33}$ g
Radius $R_{sun} = 6.96 \times 10^{10}$ cm

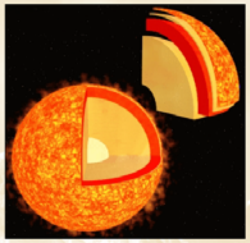
Gravitational potential energy of a proton near center of the sphere

$$\langle E_{grav} \rangle = -\frac{3}{2} \frac{G_N M_{sun} m_p}{R_{sun}} = -3.2 \text{ keV}$$

Thermal velocity distribution

$$\langle E_{kin} \rangle = \frac{3}{2} k_B T = -\frac{1}{2} \langle E_{grav} \rangle$$

Estimated temperature
T = 1.1 keV



Central temperature from standard solar models
$T_c = 1.56 \times 10^7$ K
$= 1.34$ keV

Hydrogen burning: Proton-Proton Chains

$p + p \to {}^2H + e^+ + \nu_e$ < 0.420 MeV (100%)
$p + e^- + p \to {}^2H + \nu_e$ 1.442 MeV (0.24%)

$^2H + p \to {}^3He + \gamma$

85% → PPI: $^3He + {}^3He \to {}^4He + 2p$
15% → $^3He + {}^4He \to {}^7Be + \gamma$
hep: $^3He + p \to {}^4He + e^+ + \nu_e$ < 18.8 MeV

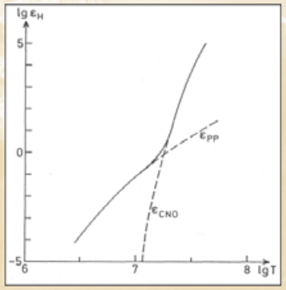
90%: $^7Be + e^- \to {}^7Li + \nu_e$ 0.862 MeV
10%: $^7Be + e^- \to {}^7Li^* + \nu_e$ 0.384 MeV
0.02%: $^7Be + p \to {}^8B + \gamma$; $^8B \to {}^8Be^* + e^+ + \nu_e$ < 15 MeV

PPII: $^7Li + p \to {}^4He + {}^4He$
PPIII: $^8Be^* \to {}^4He + {}^4He$

Hydrogen Burning: CNO Cycle

$^{12}_{6}C$ (p,γ) $^{13}_{7}N$ → e^+ ν_e → $^{13}_{6}C$ (p,γ) $^{14}_{7}N$ (p,γ) $^{15}_{8}O$ → e^+ ν_e → $^{15}_{7}N$ (p,α) → $^{12}_{6}C$

$^{15}_{7}N$ (p,γ) $^{16}_{8}O$ (p,γ) $^{17}_{9}F$ → e^+ ν_e → $^{17}_{8}O$ (p,α) → $^{14}_{7}N$



Thermonuclear Reactions and Gamow Peak

Coulomb repulsion prevents nuclear reactions, except for Gamow tunneling

Tunneling probability

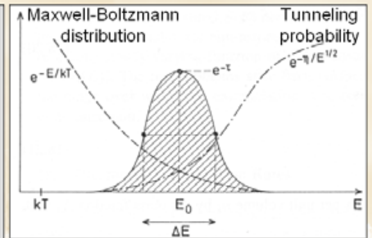
$$p \propto E^{-1/2} e^{-2\pi\eta}$$

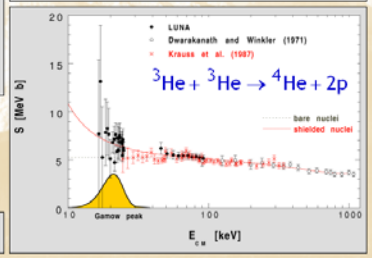
With Sommerfeld parameter

$$\eta = \left(\frac{m}{2E}\right)^{1/2} Z_1 Z_2 e^2$$

Parameterize cross section with astrophysical S-factor

$$S(E) = \sigma(E)\, E\, e^{2\pi\,\eta(E)}$$





LUNA Collaboration, nucl-ex/9902004

Main Nuclear Burnings

Hydrogen burning $4p + 2e^- \rightarrow {}^4He + 2\nu_e$

- Proceeds by pp chains and CNO cycle
- No higher elements are formed because no stable isotope with mass number 8
- Neutrinos from $p \rightarrow n$ conversion
- Typical temperatures: 10^7 K (~1 keV)

- Each type of burning occurs at a very different T but a broad range of densities
- Never co-exist in same location

Helium burning

$${}^4He + {}^4He + {}^4He \leftrightarrow {}^8Be + {}^4He \rightarrow {}^{12}C$$

"Triple alpha reaction" because ^8Be unstable, builds up with concentration ~ 10^{-9}

$${}^{12}C + {}^4He \rightarrow {}^{16}O$$

$${}^{16}O + {}^4He \rightarrow {}^{20}Ne$$

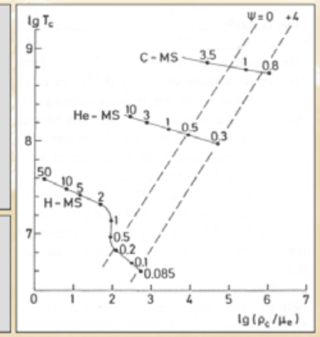
Typical temperatures: 10^8 K (~10 keV)

Carbon burning

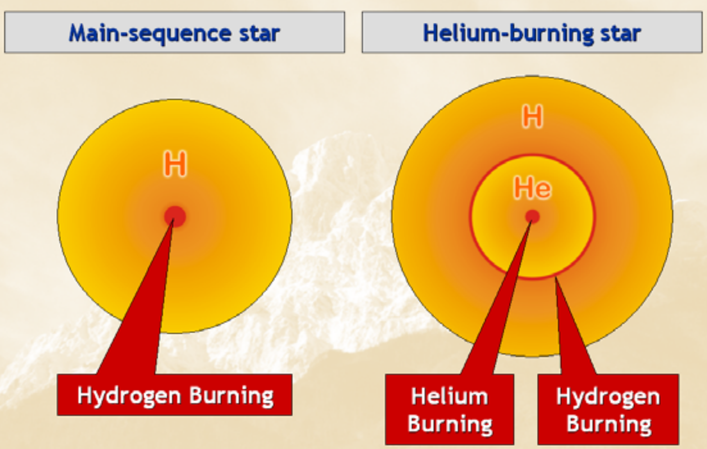
Many reactions, for example

$${}^{12}C + {}^{12}C \rightarrow {}^{23}Na + p \quad \text{or} \quad {}^{20}Ne + {}^4He \quad \text{etc}$$

Typical temperatures: 10^9 K (~100 keV)



Hydrogen Exhaustion



Burning Phases of a 15 Solar-Mass Star

Burning Phase	Dominant Process	T_c [keV]	ρ_c [g/cm³]	L_γ [10^4 L_{sun}]	L_ν/L_γ	Duration [years]
Hydrogen	$H \rightarrow He$	3	5.9	2.1	–	1.2×10^7
Helium	$He \rightarrow C, O$	14	1.3×10^3	6.0	1.7×10^{-5}	1.3×10^6
Carbon	$C \rightarrow Ne, Mg$	53	1.7×10^5	8.6	1.0	6.3×10^3
Neon	$Ne \rightarrow O, Mg$	110	1.6×10^7	9.6	1.8×10^3	7.0
Oxygen	$O \rightarrow Si$	160	9.7×10^7	9.6	2.1×10^4	1.7
Silicon	$Si \rightarrow Fe, Ni$	270	2.3×10^8	9.6	9.2×10^5	6 days

Neutrinos from Thermal Plasma Processes

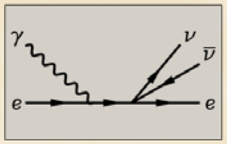
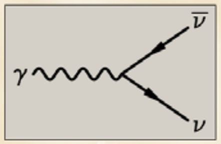
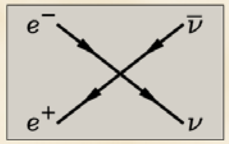


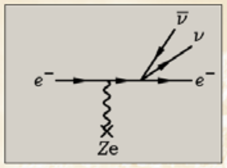
Photo (Compton)



Plasmon decay

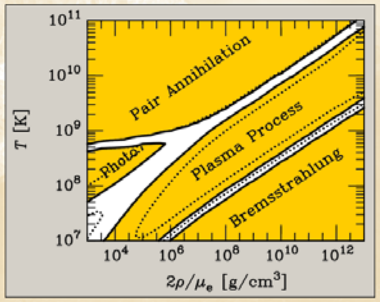


Pair annihilation



Bremsstrahlung

These processes first discussed in 1961-63 after V–A theory



Existence of Direct Neutrino-Electron Coupling

VOLUME 24, NUMBER 10 PHYSICAL REVIEW LETTERS 9 MARCH 1970

ASTROPHYSICAL DETERMINATION OF THE COUPLING CONSTANT FOR THE ELECTRON-NEUTRINO WEAK INTERACTION

Richard B. Stothers*
Goddard Institute for Space Studies, National Aeronautics and Space Administration, New York, New York 10025
(Received 22 December 1969)

The existence of the $(\bar{e}\nu_e)(\bar{\nu}_e e)$ weak interaction is confirmed by the results of nine astrophysical tests. The value of the coupling constant is equal to, or close to, the coupling constant of beta decay, namely, $g^2 = 10^{0\pm2} g_\beta^2$.

Of all the astrophysical tests applied so far for the inference of a direct electron-neutrino interaction in nature, none has unambiguously provided a useful upper limit on the coupling constant, which in the $V-A$ theory of Feynman and Gell-Mann[1] is taken to be equal to the "universal" weak-interaction coupling constant measured from beta decays (called g_β hereafter). However, it is important to point out that these tests, made by the author and his colleagues during the past eight years, do provide a nonzero lower limit, and therefore establish at least the existence of the $(\bar{e}\nu_e)(\bar{\nu}_e e)$ interaction. It should be emphasized, nonetheless, that all of these tests rely on the validity of various stellar model calculations. These models, while not subject to scrutiny in the same sense as a laboratory ex- relative theoretical lifetimes, calculated with and without the inclusion of neutrino emission. In this Letter, the unmodified term "luminosity" will mean the photon luminosity L radiated by the star. The "neutrino luminosity" will be designated L_ν. Quantities referring to the sun are subscripted with an encircled dot.

The most accurate available data on white dwarfs are those collected by Eggen[7] for the two clusters Hyades and Pleiades and for the nearby general field. Of chief interest here are the hot white dwarfs, for which the observational data[7,8] have been reduced following the procedure of Van Horn.[9] The resulting luminosities are estimated to have a statistical accuracy of ±0.1 in $\log(L/L_\odot)$, which is adequate here.

Models of cooling white dwarfs have been con-

Georg Raffelt, Max-Planck-Institut für Physik, München, Germany
ISAPP, 28 June-9 July 2004, LNGS, Gran Sasso, Italy

Self-Regulated Nuclear Burning

Virial Theorem $\langle E_{kin}\rangle = -\frac{1}{2}\langle E_{grav}\rangle$

Main-Sequence Star

Small Contraction
→ Heating
→ Increased nuclear burning
→ Increased pressure
→ Expansion

Additional energy loss ("cooling")
→ Loss of pressure
→ Contraction
→ Heating
→ Increased nuclear burning

Hydrogen burning at a nearly fixed T
→ Gravitational potential nearly fixed: $G_N M/R$ ~ constant
→ $R \propto M$ (More massive stars bigger)

Georg Raffelt, Max-Planck-Institut für Physik, München, Germany
ISAPP, 28 June-9 July 2004, LNGS, Gran Sasso, Italy

Degenerate Stars ("White Dwarfs")

Assume T very small
→ No thermal pressure
→ Electron degeneracy is pressure source

Pressure ~ Momentum density x Velocity
- Electron density $n_e = p_F^3/(3\pi^2)$
- Momentum p_F (Fermi momentum)
- Velocity $v \propto p_F/m_e$
- Pressure $P \propto p_F^5 \propto \rho^{5/3} \propto M^{5/3}R^{-5}$
- Density $\rho \propto MR^{-3}$
  (Stellar mass M and radius R)

Hydrostatic equilibrium
$$\frac{dP}{dr} = -\frac{G_N M_r \rho}{r^2}$$
With dP/dr ~ −P/R we have approximately
$$P \propto G_N M \rho R^{-1} \propto G_N M^2 R^{-4}$$

Inverse mass-radius relationship for degenerate stars: $R \propto M^{-1/3}$

$$R = 10{,}500\ \text{km}\left(\frac{0.6\,M_{sun}}{M}\right)^{1/3}(2Y_e)^{5/3}$$
(Y_e electrons per nucleon)

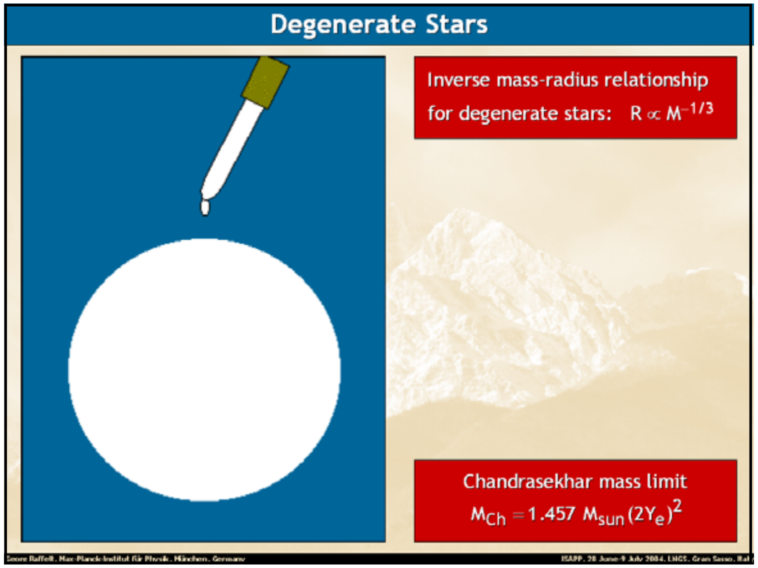
For sufficiently large mass, electrons become relativistic
- Velocity = speed of light
- Pressure

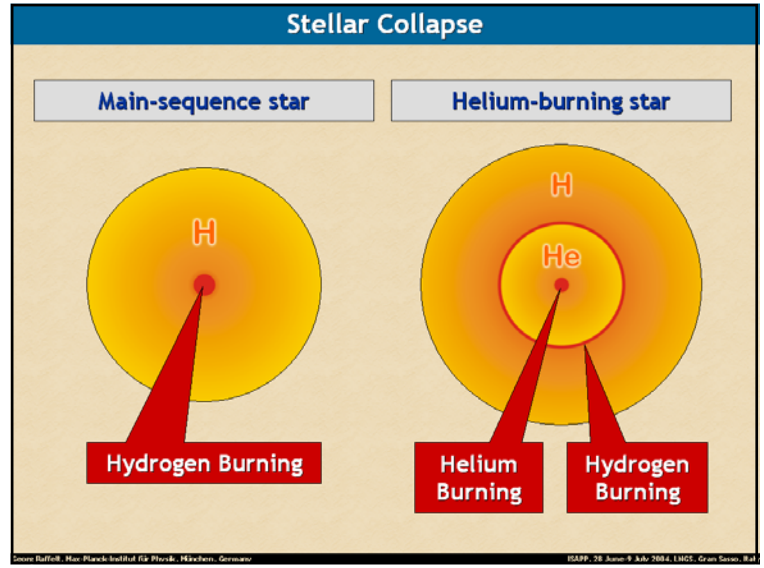
$P \propto p_F^4 \propto \rho^{4/3} \propto M^{4/3}R^{-4}$

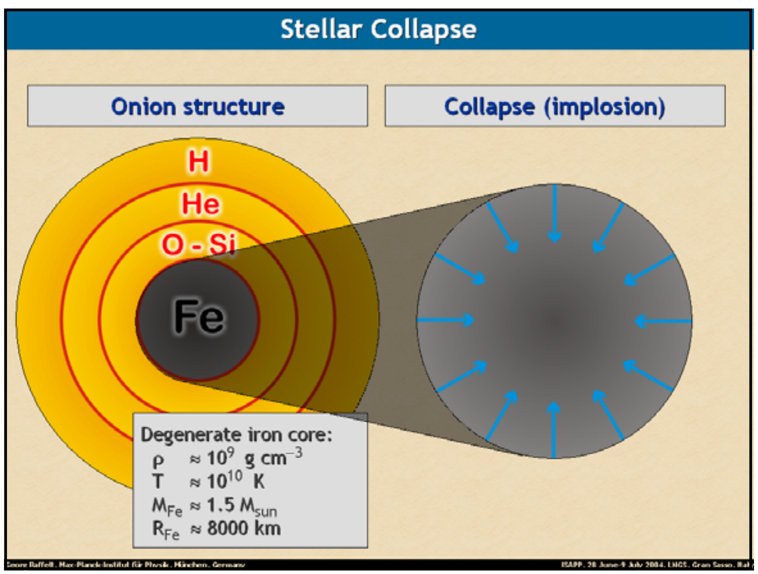
No stable configuration

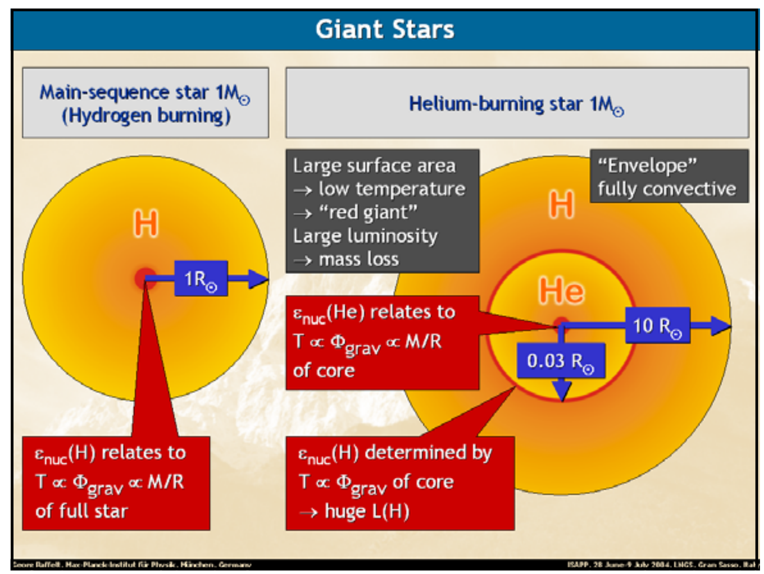
Chandrasekhar mass limit
$M_{Ch} = 1.457\ M_{sun}(2Y_e)^2$

Georg Raffelt, Max-Planck-Institut für Physik, München, Germany
ISAPP, 28 June-9 July 2004, LNGS, Gran Sasso, Italy

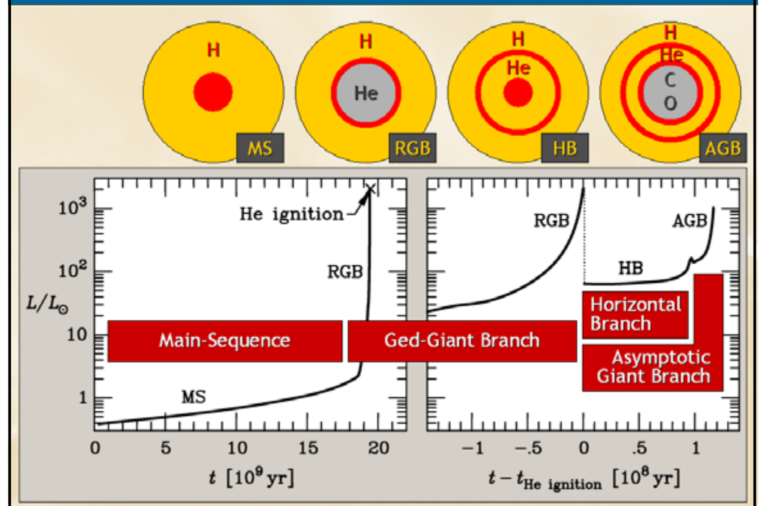

Degenerate Stars
Inverse mass-radius relationship
for degenerate stars: $R \propto M^{-1/3}$
Chandrasekhar mass limit
$M_{Ch} = 1.457\ M_{sun}\,(2Y_e)^2$

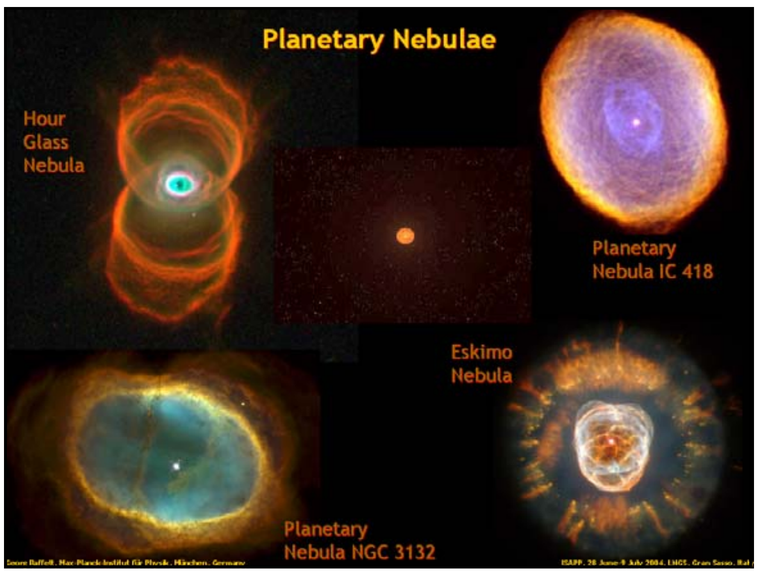

Stellar Collapse
Main-sequence star
Helium-burning star
H
H
He
Hydrogen Burning
Helium Burning
Hydrogen Burning


Stellar Collapse
Onion structure
Collapse (implosion)
H
He
O - Si
Fe
Degenerate iron core:
$\rho \approx 10^9$ g cm^{-3}
$T \approx 10^{10}$ K
$M_{Fe} \approx 1.5\ M_{sun}$
$R_{Fe} \approx 8000$ km


Giant Stars
Main-sequence star $1M_\odot$ (Hydrogen burning)
Helium-burning star $1M_\odot$
Large surface area → low temperature → "red giant"
Large luminosity → mass loss
"Envelope" fully convective
H
H
He
$1R_\odot$
$10\ R_\odot$
$0.03\ R_\odot$
ε_{nuc}(He) relates to $T \propto \Phi_{grav} \propto M/R$ of core
ε_{nuc}(H) relates to $T \propto \Phi_{grav} \propto M/R$ of full star
ε_{nuc}(H) determined by $T \propto \Phi_{grav}$ of core → huge L(H)

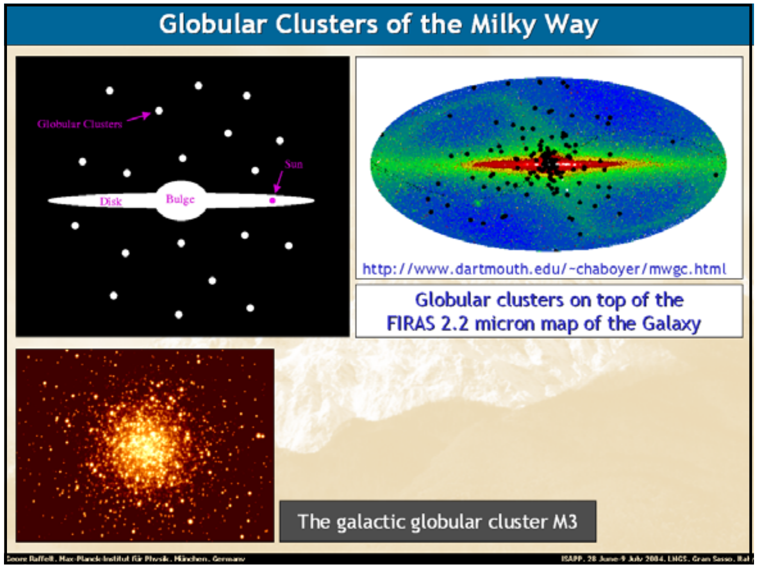
Evolution of a Low-Mass Star

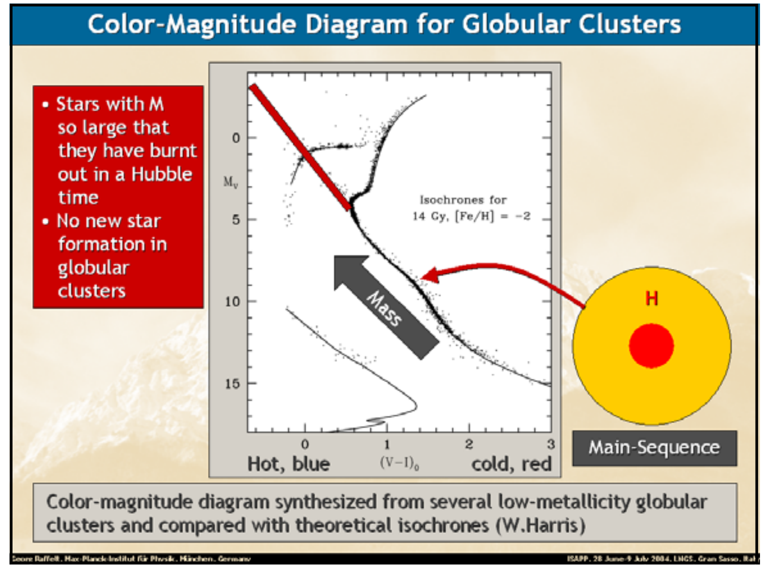


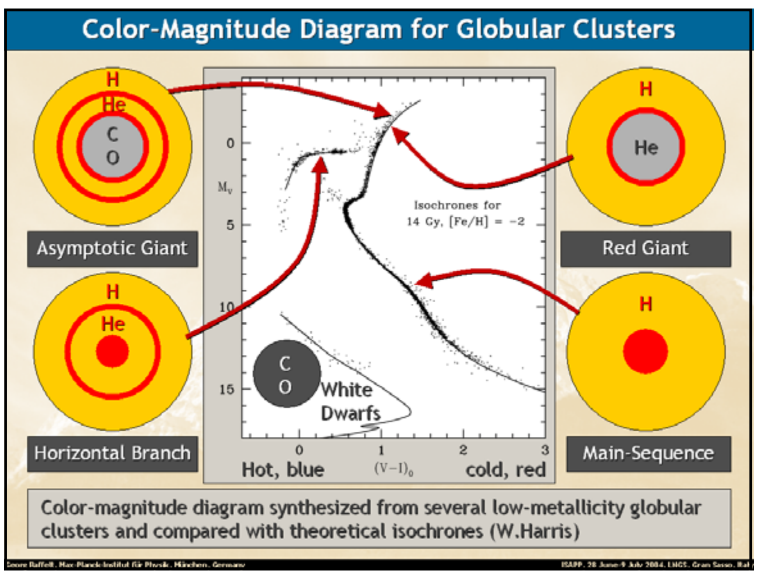


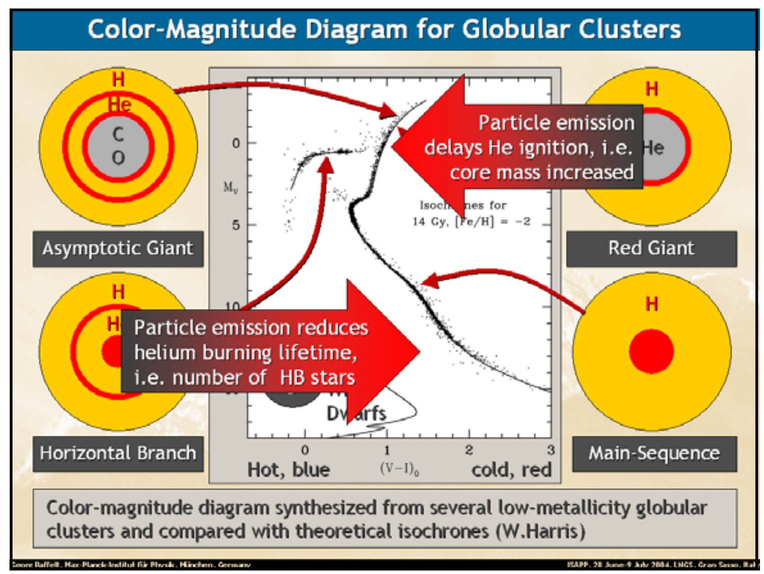
Evolution of Stars

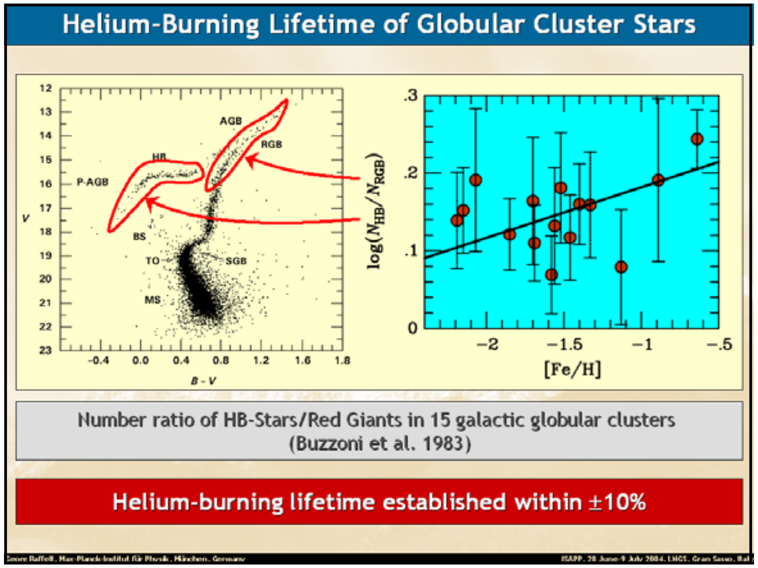
$M < 0.08\,M_{sun}$	Never ignites hydrogen → cools ("hydrogen white dwarf")		Brown dwarf
$0.08 < M \lesssim 0.8\,M_{sun}$	Hydrogen burning not completed in Hubble time		Low-mass main-squence star
$0.8 \lesssim M \lesssim 2\,M_{sun}$	Degenerate helium core after hydrogen exhaustion		• Carbon-oxygen white dwarf • Planetary nebula
$2 \lesssim M \lesssim 5\text{–}8\,M_{sun}$	Helium ignition non-degenerate		
$5\text{–}8\,M_{sun} \lesssim M <$???	All burning cycles → Onion skin structure with degenerate iron core	Core collapse supernova	• Neutron star (often pulsar) • Sometimes black hole? • Supernova remnant (SNR), e.g. crab nebula

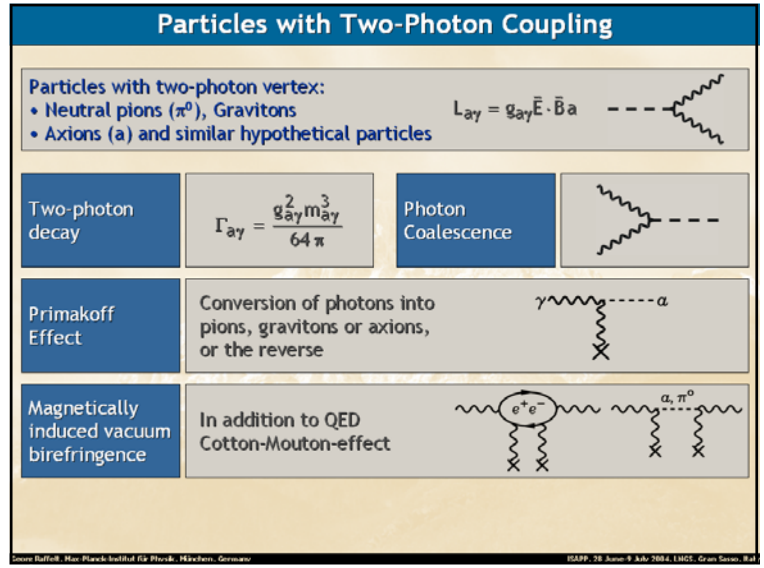

Globular Clusters of the Milky Way
Globular Clusters
Sun
Disk
Bulge
http://www.dartmouth.edu/~chaboyer/mwgc.html
Globular clusters on top of the FIRAS 2.2 micron map of the Galaxy
The galactic globular cluster M3

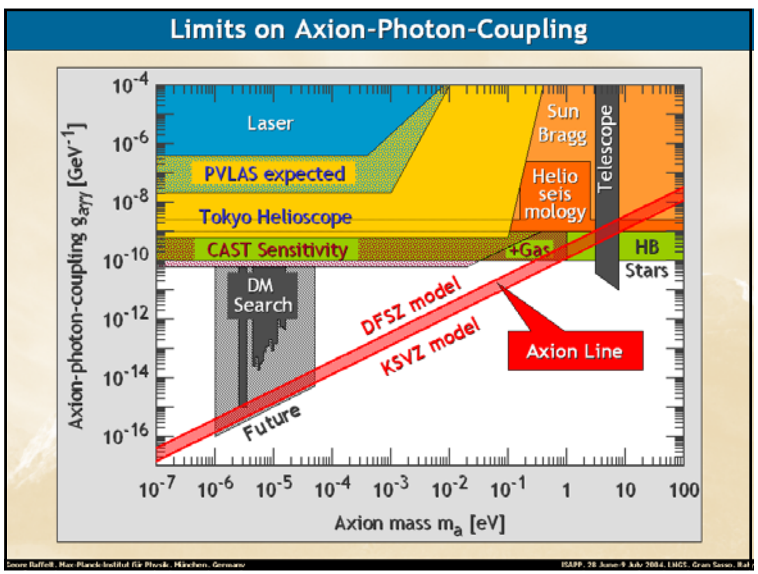

Color-Magnitude Diagram for Globular Clusters
• Stars with M so large that they have burnt out in a Hubble time
• No new star formation in globular clusters
Isochrones for 14 Gy, [Fe/H] = −2
Mass
H
Main-Sequence
Hot, blue
cold, red
Color-magnitude diagram synthesized from several low-metallicity globular clusters and compared with theoretical isochrones (W.Harris)

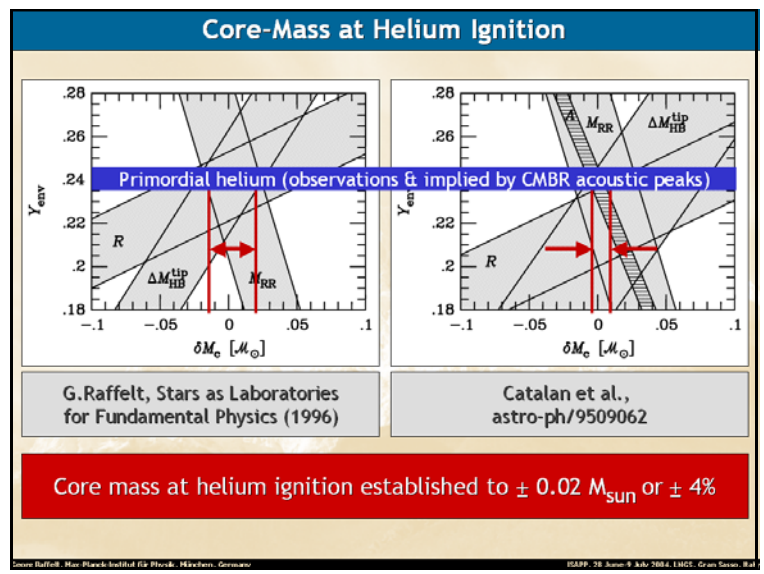

Color-Magnitude Diagram for Globular Clusters
H
He
C
O
Asymptotic Giant
H
He
Horizontal Branch
Isochrones for 14 Gy, [Fe/H] = −2
C
O
White Dwarfs
H
He
Red Giant
H
Main-Sequence
Hot, blue
cold, red
Color-magnitude diagram synthesized from several low-metallicity globular clusters and compared with theoretical isochrones (W.Harris)


Color-Magnitude Diagram for Globular Clusters
Particle emission delays He ignition, i.e. core mass increased
Particle emission reduces helium burning lifetime, i.e. number of HB stars
Asymptotic Giant
Horizontal Branch
Red Giant
Main-Sequence
Hot, blue
cold, red
Color-magnitude diagram synthesized from several low-metallicity globular clusters and compared with theoretical isochrones (W.Harris)


Helium-Burning Lifetime of Globular Cluster Stars
AGB
RGB
HB
P-AGB
BS
TO
SGB
MS
V
B - V
log(N_HB/N_RGB)
[Fe/H]
Number ratio of HB-Stars/Red Giants in 15 galactic globular clusters
(Buzzoni et al. 1983)
Helium-burning lifetime established within ±10%

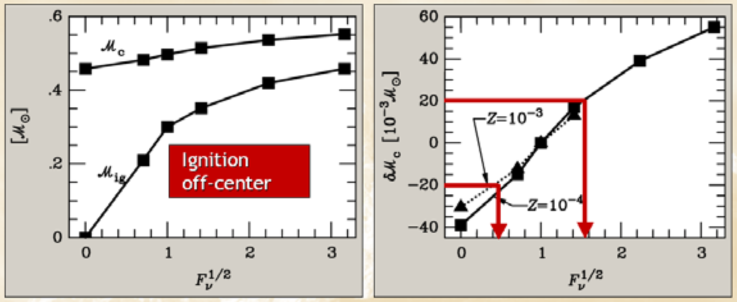

Particles with Two-Photon Coupling
Particles with two-photon vertex:
• Neutral pions (π⁰), Gravitons
• Axions (a) and similar hypothetical particles
L_aγ = g_aγ E·B a
Two-photon decay
Γ_aγ = g²_aγ m³_aγ / 64π
Photon Coalescence
Primakoff Effect
Conversion of photons into pions, gravitons or axions, or the reverse
Magnetically induced vacuum birefringence
In addition to QED Cotton-Mouton-effect


Limits on Axion-Photon-Coupling
Laser
PVLAS expected
Tokyo Helioscope
CAST Sensitivity
+Gas
Sun Bragg
Helio seis mology
Telescope
HB Stars
DM Search
DFSZ model
KSVZ model
Axion Line
Future
Axion-photon-coupling g_aγγ [GeV⁻¹]
Axion mass m_a [eV]


Core-Mass at Helium Ignition
Primordial helium (observations & implied by CMBR acoustic peaks)
Y_env
δM_c [M_⊙]
R
ΔM_HB^tip
M_RR
G.Raffelt, Stars as Laboratories for Fundamental Physics (1996)
Catalan et al., astro-ph/9509062
Core mass at helium ignition established to ± 0.02 M_sun or ± 4%

Core-Mass Dependence on Neutrino Cooling

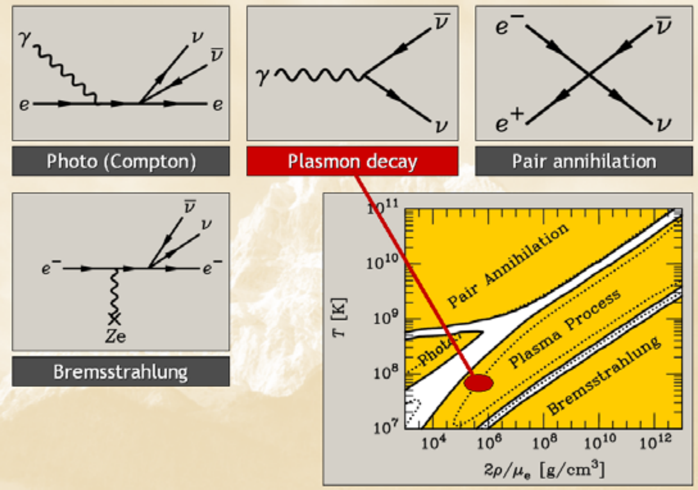
Multiply standard neutrino loss rates with a factor F_ν



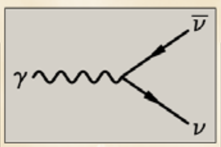
- Core mass (upper curve)
- Radial coordinate of helium ignition (lower curve)

Change of core mass

Neutrinos from Thermal Plasma Processes



Plasmon Decay in Neutrinos



Vacuum:
- Photon massless
- Can not decay into other particles, even if they themselves are massless

Vacuum:
- Massless neutrinos do not couple to photons
- May have dipole moments or even "millicharges"

Propagation in a medium:
- Photon acquires a "refractive index"
- In a non-relativistic plasma (e.g. Sun, white dwarfs, core of red giant before helium ignition, ...) behaves like massive particle:

$\omega^2 - k^2 = \omega_{pl}^2$

Plasma frequency (electron density n_e) $\omega_{pl}^2 = \frac{4\pi\alpha n_e}{m_e}$

- Degenerate helium core $\omega_{pl} = 18$ keV ($\rho = 10^6$ g cm^{-3}, T = 8.6 keV)

In a medium:
- Neutrinos interact coherently with the charged particles which themselves couple to photons
- Induces an "effective charge"
- In a degenerate plasma (electron Fermi energy E_F and Fermi momentum p_F)

$\frac{e_\nu}{e} = 16\sqrt{2}\, C_V G_F E_F p_F$

- Degenerate helium core (and C_V = 1) $e_\nu = 6\times10^{-11} e$

Neutrino Dipole Moments

Effective coupling of electromagnetic field to a neutral fermion		
	$L_{eff} = -F_1 \bar\Psi \gamma_\mu \psi A^\mu$	Charge $e_\nu = F_1(0) = 0$
	$-G_1 \bar\Psi \gamma_\mu \gamma_5 \psi \partial_\nu F^{\mu\nu}$	Anapole moment $G_1(0)$
	$-\frac{1}{2} F_2 \bar\Psi \sigma_{\mu\nu} \psi F^{\mu\nu}$	Magnetic dipole moment $\mu = F_2(0)$
	$-\frac{1}{2} G_2 \bar\Psi \sigma_{\mu\nu} \gamma_5 \psi F^{\mu\nu}$	Electric dipole moment $\varepsilon = G_2(0)$

- Charge form factor $F_1(q^2)$ and anapole $G_1(q^2)$ are short-range interactions if charge $F_1(0) = 0$
- Connect states of equal chirality
- In standard model they represent radiative corrections to weak interaction

- Dipole moments connect states of opposite chirality
- Violation of individual flavor lepton numbers (neutrino mixing)
  → Magnetic or electric dipole moments can connect different flavors or different mass eigenstates ("Transition moments")
- Usually measured in "Bohr magnetons" $\mu_B = e/(2m_e)$

Plasmon Decay And Stellar Energy Loss Rates

Assume photon dispersion relation like a massive particle (nonrelativistic plasma)

$$E_\gamma^2 - p_\gamma^2 = \omega_{pl}^2 = \frac{4\pi\alpha n_e}{m_e}$$

Decay rate of photon (transverse plasmon) with energy E_γ

$$\Gamma(\gamma\to\nu\bar\nu) = \frac{4\pi}{3}\frac{1}{E_\gamma}\times\begin{cases}\alpha_\nu\left(\omega_{pl}^2/4\pi\right) & \text{Millicharge}\\ \frac{\mu_\nu^2}{2}\left(\omega_{pl}^2/4\pi\right)^2 & \text{Dipole moment}\\ \frac{C_V^2G_F^2}{\alpha}\left(\omega_{pl}^2/4\pi\right)^3 & \text{Standard model}\end{cases}$$

Energy-loss rate of stellar plasma (temperature T and plasma frequency ω_{pl})

$$Q(\gamma\to\nu\bar\nu) = \int\frac{2d^3\vec p}{(2\pi)^3}\frac{E_\gamma\Gamma}{e^{E_\gamma/T}-1} = \frac{8\zeta_3}{3\pi}T^3\times\begin{cases}\alpha_\nu\left(\omega_{pl}^2/4\pi\right)\\ \frac{\mu_\nu^2}{2}\left(\omega_{pl}^2/4\pi\right)^2\\ \frac{C_V^2G_F^2}{\alpha}\left(\omega_{pl}^2/4\pi\right)^3\end{cases}$$

Globular Cluster Limits on Neutrino Dipole Moments

Compare magnetic-dipole plasma emission with standard case

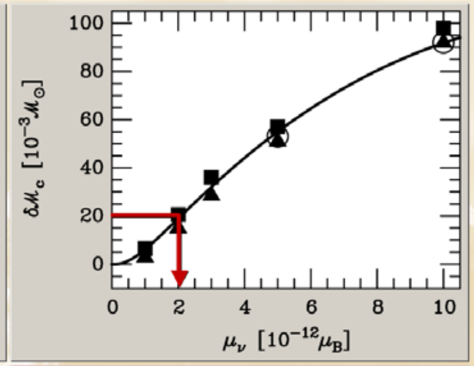
$$\frac{Q_\mu}{Q_{SM}} = \frac{2\pi\alpha\mu_\nu^2}{C_V^2G_F^2\omega_{pl}^2}$$

For red-giant core before helium ignition $\omega_{pl} = 18$ keV

$$\frac{Q_\mu}{Q_{SM}} = 9\times10^{22}\left(\frac{\mu_\nu}{\mu_B}\right)^2$$

Require this to be < 1

$\mu_\nu < 3\times10^{-12}\mu_B$

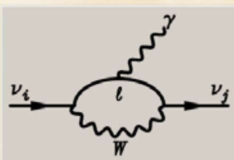


Globular-cluster limit on neutrino dipole moment

$\mu_\nu < 2\times10^{-12}\mu_B$

Standard Dipole Moments for Massive Neutrinos

In standard electroweak model, neutrino dipole and transition moments are induced at higher order



Massive neutrinos ν_i (i = 1,2,3), mixed to form weak eigenstates

$$\nu_\ell = \sum_{i=1}^{3}U_{\ell i}\nu_i$$

Explicit evaluation for Dirac neutrinos (Magnetic moments μ_{ij} electric moments ε_{ij})

$$\mu_{ij} = \frac{e\sqrt{2}G_F}{(4\pi)^2}(m_i+m_j)\sum_{\ell=e,\mu,\tau}U_{\ell j}U_{\ell i}^* f\left(\frac{m_\ell}{m_W}\right)$$

$$\varepsilon_{ij} = \ldots(m_i - m_j)\ldots$$

$$f\left(\frac{m_\ell}{m_W}\right) = -\frac{3}{2}+\frac{3}{4}\left(\frac{m_\ell}{m_W}\right)^2+O\left(\left(\frac{m_\ell}{m_W}\right)^4\right)$$

Standard Dipole Moments for Massive Neutrinos

Diagonal case (Magnetic moments of Dirac neutrinos)

$$\mu_{ii} = \frac{3e\sqrt{2}G_F}{(4\pi)^2}m_i = 3.20\times10^{-19}\mu_B\frac{m_i}{\text{eV}} \qquad \mu_B = \frac{e}{2m_e}$$

$$\varepsilon_{ii} = 0$$

Off-diagonal case (Transition moments)

First term in $f(m_\ell/m_W)$ does not contribute ("GIM cancellation")

$$\mu_{ij} = \frac{3e\sqrt{2}G_F}{4(4\pi)^2}(m_i+m_j)\left(\frac{m_\tau}{m_W}\right)^2\sum_{\ell=e,\mu,\tau}U_{\ell j}U_{\ell i}^*\left(\frac{m_\ell}{m_\tau}\right)^2$$

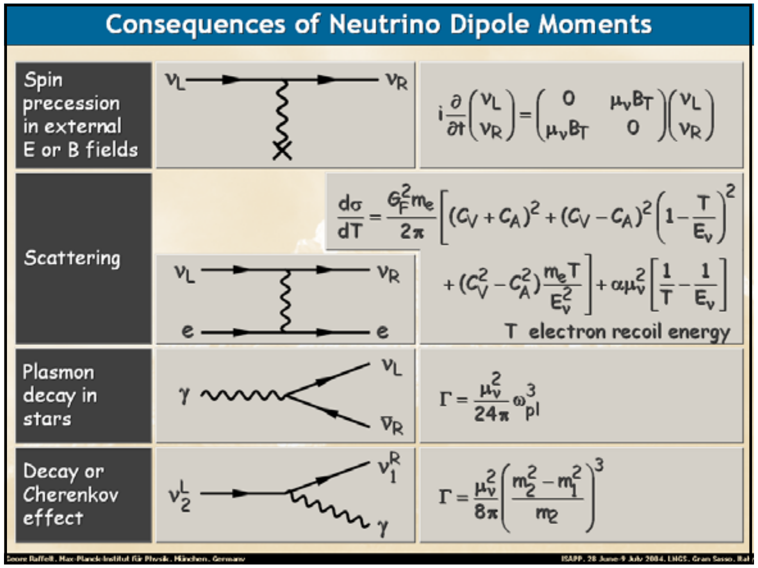
$$= 3.96\times10^{-23}\mu_B\frac{m_i+m_j}{\text{eV}}\sum_{\ell=e,\mu,\tau}U_{\ell j}U_{\ell i}^*\left(\frac{m_\ell}{m_\tau}\right)^2$$

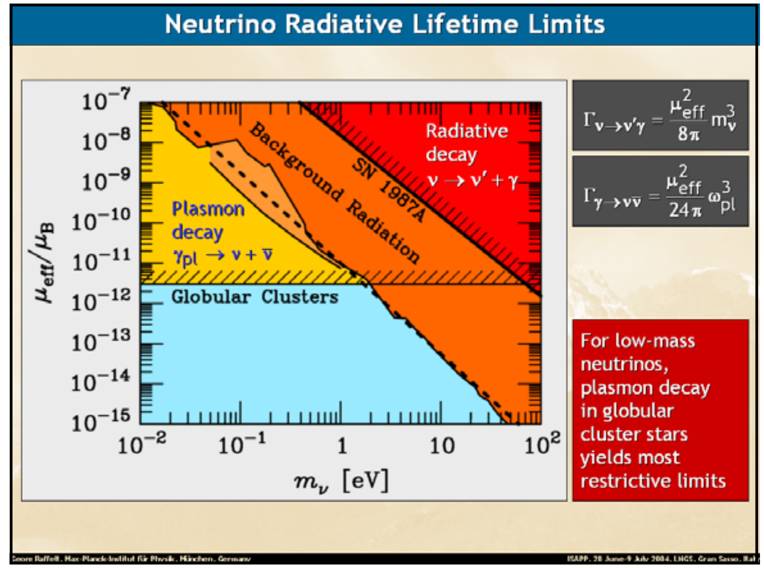
$$\varepsilon_{ij} = \ldots(m_i - m_j)\ldots$$

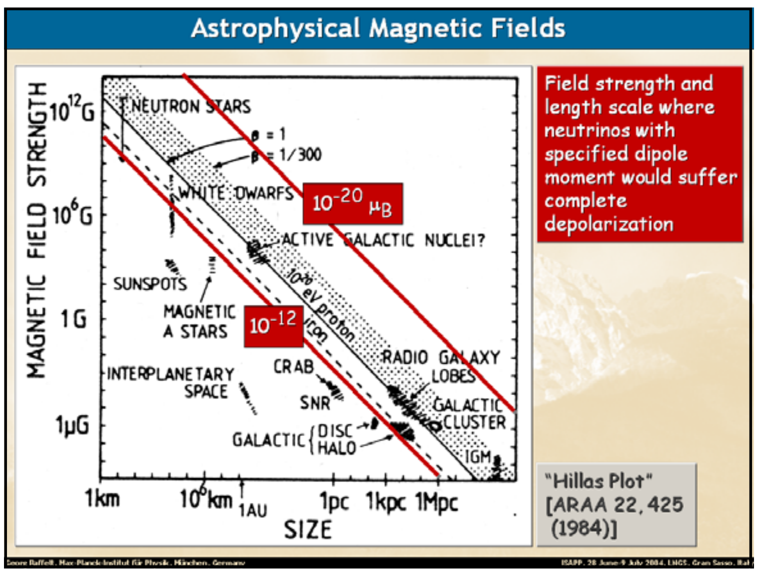
Largest neutrino mass eigenstate $0.05\ \text{eV} < m < 0.7\ \text{eV}$

For Dirac neutrino expect

$1.6\times10^{-20}\mu_B < \mu_\nu < 2.2\times10^{-19}\mu_B$


Consequences of Neutrino Dipole Moments
Spin precession in external E or B fields
Scattering
T electron recoil energy
Plasmon decay in stars
Decay or Cherenkov effect


Neutrino Radiative Lifetime Limits
Radiative decay
Background Radiation
SN 1987A
Plasmon decay
Globular Clusters
μ_{eff}/μ_B
m_ν [eV]
For low-mass neutrinos, plasmon decay in globular cluster stars yields most restrictive limits


Astrophysical Magnetic Fields
MAGNETIC FIELD STRENGTH
SIZE
NEUTRON STARS
WHITE DWARFS
ACTIVE GALACTIC NUCLEI?
SUNSPOTS
MAGNETIC A STARS
INTERPLANETARY SPACE
CRAB
SNR
RADIO GALAXY LOBES
GALACTIC CLUSTER
IGM
GALACTIC DISC HALO
Field strength and length scale where neutrinos with specified dipole moment would suffer complete depolarization
"Hillas Plot" [ARAA 22, 425 (1984)]