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Photon-photon dispersion of TeV gamma rays and its role for photon-ALP conversion

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The propagation of TeV gamma rays can be strongly modified by B-field induced conversion to axionlike particles (ALPs). We show that, at such high energies, photon dispersion is dominated by background photons—the only example where photon-photon dispersion is of practical relevance. We determine the refractive index for all energies and find that, for fixed energy density, background photons below the pair-production threshold dominate. The cosmic microwave background alone provides an “effective photon mass” of \( m_\gamma^2 = -(1.01 \text{ neV} \times \omega/\text{TeV})^2 \) for \( \omega \lesssim 1000 \text{ TeV} \). The extragalactic background light is subdominant, but local radiation fields in the Galaxy or the source regions provide significant contributions. Photon-photon dispersion is small enough to leave typical scenarios of photon-ALP oscillations unscathed, but big enough to worry about it case by case.

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I. INTRODUCTION

Astronomy with TeV gamma rays has opened a new window to the Universe, allowing us to study a plethora of fantastic sources of very high-energy photons [1–5]. In addition to the sources themselves, we can study intervening phenomena. In particular, the radiation emitted by all stars, the extragalactic background light (EBL), absorbs photons by \( \gamma \gamma \rightarrow e^+e^- \). As a result, the TeV \( \gamma \)-ray horizon is only some 100 Mpc and the observed source spectra are strongly modified. One can use this effect to explore the EBL which is hard to measure directly [6–8]. More fundamentally, the fast time structure of certain sources allows one to constrain novel dispersion effects, for example by Lorentz invariance violation [9–11].

We are here concerned with another effect at the low-energy frontier of elementary particle physics [12–15], the conversion of photons into axionlike particles (ALPs) in large-scale magnetic fields [16,17], enabled by the two-photon vertex of these hypothetical low-mass bosons. The conversion \( \gamma \rightarrow a \) modifies the source spectra. The conversion and subsequent back conversion \( \gamma \rightarrow a \rightarrow \gamma \) allows TeV gamma rays to “propagate in disguise” and evade absorption by \( e^+e^- \) pair production [18–43]. This effect is a possible explanation of the cosmic transparency problem, i.e., TeV gamma rays seem to travel further than allowed by typical EBL estimates. At the very least, this effect represents a systematic uncertainty when probing the EBL with TeV gamma rays.

Photon and ALP propagation and conversion is most easily studied in analogy to neutrino flavor oscillations [17,44]. A wave of frequency \( \omega \) and amplitude \( A \) evolves in the \( x \) direction according to \(-i\partial_x A = \gamma_{\text{ref}} \omega A\), where \( \gamma_{\text{ref}} \) is the refractive index which gives us the wave number by \( k = \gamma_{\text{ref}} \omega \). We write \( \gamma_{\text{ref}} = 1 + \chi + ik \) and assume \( |\chi + ik| \ll 1 \). The real part \( \chi \) describes dispersion and the imaginary part \( k \) absorption. \( A \) has three components, the photon amplitude \( A_\perp \) with polarization perpendicular to the transverse B-field, \( A_\parallel \) parallel to it, and the ALP amplitude \( a \), i.e., \( A = (A_\perp, A_\parallel, a) \), and \( \chi \) and \( k \) are now 3 \( \times \) 3 matrices. The off-diagonal \( \chi \) elements cause oscillations between different \( A \)-components such as the Faraday effect, where electrons in the longitudinal B-field instigate a rotation of the plane of polarization.

ALPs interact with photons by \( \mathcal{L}_{\gamma a} = g_{\gamma a} \mathbf{E} \cdot \mathbf{B} a \) in terms of the electric, magnetic, and ALP fields whereas \( g_{\gamma a} \) is a coupling constant of dimension inverse energy. An external transverse magnetic field \( B_T \) couples \( A_\parallel \) with \( a \) and provides an off-diagonal refractive index \( \chi_{\gamma a} = g_{\gamma a} B_T/2\omega \) which leads to ALP-photon oscillations. (We always use natural units with \( h = c = k_B = 1 \).) The ALP dispersion relation is \( \omega^2 - k^2 = m_a^2 \), providing the refractive index \( \chi_a = -m_a^2/2\omega^2 \). An analogous expression pertains to photons where the plasma frequency \( \omega_{\text{pl}}^2 = 4\pi n_e/m_e \) is the effective photon mass.

More important for TeV \( \gamma \)-ray dispersion is the B-field itself due to an effective photon-photon interaction mediated by virtual \( e^+e^- \) pairs. At low energies, it is described by the Euler-Heisenberg Lagrangian \( \mathcal{L}_H = (2\alpha^2/45m_e^2) \left[ (\mathbf{E}^2 - \mathbf{B}^2)^2 + 7(\mathbf{E} \cdot \mathbf{B})^2 \right] \), which however also pertains to background photons. The overall electromagnetic (EM) energy density \( \rho_{\text{EM}} = \frac{1}{2} (\mathbf{E}^2 + \mathbf{B}^2) \) produces [45–50]

\[
\chi_{\text{EM}} = \frac{44\alpha^2 \rho_{\text{EM}}}{135 m_e^2},
\]

implying spacelike dispersion \( \omega^2 - k^2 = -2\chi_{\text{EM}} \omega^2 \), i.e., a “negative effective mass squared.”
Large-scale fields or nonisotropic background photons imply further geometrical factors depending on direction of motion and polarization. If the background is homogeneous a B-field, the dispersion of $A_B$ receives a factor $(14/11) \sin^2 \theta$, whereas $A_\perp$ a factor $(8/11) \sin^2 \theta$ [50–54]. Here, $\theta$ is the angle between the photon and B-field directions, i.e., only the transverse field strength $B_T$ enters. These results correspond to what has been used in studies of TeV $\gamma$-ray propagation.

A minimal EM energy density everywhere is $\rho_{CMB} = (\pi^2/15) T_4^4 = 0.261$ eV cm$^{-3}$ provided by the cosmic microwave background (CMB), where we have used $T = 2.726$ K, leading to

$$\chi_{CMB} = 0.511 \times 10^{-42}. \quad (2)$$

A typical galactic B-field of $1 \mu$G corresponds to $\rho_{EM} = 0.0248$ eV cm$^{-3}$, so the CMB dominates by a factor of 10. Depending on the environment, dispersion of TeV gamma rays involves the CMB and possible larger local radiation densities. This insight is our main point.

II. PHOTON-ALP OSCILLATIONS

To develop a sense of the quantitative importance of this effect we consider $\gamma\alpha$ conversion in the Galaxy. We consider propagation in the $x$ direction in a transverse B-field, leading to

$$-i\partial_x \left( \frac{A_\parallel}{\rho} \right) = \left( \chi_{EM} \hat{\omega} - \frac{g_{\gamma B} B/2}{-m_\gamma^2/2\omega} \right) \left( \frac{A_\parallel}{\rho} \right). \quad (3)$$

Here, $\chi_{EM} = \chi_{CMB} + \chi_B$ with the CMB and B-field contributions. The oscillation probability (distance $L$) is

$$\mathcal{P}_{\gamma\rightarrow\alpha} = \left( \frac{\Delta_{\gamma B}}{2} \right)^2 \frac{\sin^2(\Delta_{osc} L/2)}{(\Delta_{osc} L/2)^2}, \quad (4)$$

where $\Delta_{\gamma B} = g_{\gamma B} B/2$ and the “oscillation wave number” is $\Delta_{osc} = [(\chi_{EM} \hat{\omega} + m_\gamma^2/2\omega)^2 + (g_{\gamma B} B)^2]^{1/2}$.

In the Galaxy, magnitudes of the matrix components in typical scenarios are

$$g_{\gamma B} B/2 = +1.52 \times 10^{-2} \text{ kpc}^{-1} g_{11} B_{\mu G}, \quad (5a)$$

$$\chi_{CMB} \hat{\omega} = +0.80 \times 10^{-4} \text{ kpc}^{-1} \alpha_{\text{TeV}}, \quad (5b)$$

$$\chi_B \hat{\omega} = +0.76 \times 10^{-5} \text{ kpc}^{-1} B_{\mu G} \alpha_{\text{TeV}}, \quad (5c)$$

$$-\frac{m_\gamma^2}{2\omega} = -0.78 \times 10^{-4} \text{ kpc}^{-1} m_{\text{ev}} \alpha_{\text{TeV}}, \quad (5d)$$

$$-\frac{\omega_{\text{pl}}^2}{2\omega} = -1.08 \times 10^{-10} \text{ kpc}^{-1} n_3/\alpha_{\text{TeV}}, \quad (5e)$$

where $g_{11} = g_{\gamma B} / (10^{-11} \text{ GeV}^{-1})$, $B_{\mu G} = B / (1 \mu G)$, $\alpha_{\text{TeV}} = \omega / (1 \text{ TeV})$, $m_{\text{ev}} = m_{\alpha} / (10^{-9} \text{ eV})$ and $n_3 = n_e / (10^{-3} \text{ cm}^{-3})$. For completeness we have included the electron contribution, which is completely negligible.

The term $g_{\gamma B} B/2$ exceeds all others, corresponding to maximal mixing. Indeed, the considered ALP masses are in the neV range to achieve this effect. The ALP and photon dispersion relations have opposite sign (timelike vs spacelike) so that the two effects add up in the expression for $\Delta_{osc}$. They cannot cancel each other and must be separately small to achieve large mixing.

Therefore, $\gamma\gamma$ dispersion will be unimportant only if $\chi_{EM} \hat{\omega} \ll g_{\gamma B} B$. The above parameters satisfy this condition, but maximal mixing can be lost, and $\gamma\gamma$ dispersion becomes important, for smaller $g_{\gamma B}$ or weaker $B$ (e.g. in intergalactic space). Likewise, $\omega \gtrsim 100$ TeV, keeping all else fixed, implies $\chi_{CMB} \hat{\omega} \sim g_{\gamma B} B$.

Moreover, the radiation fields in the Galaxy, in the TeV source regions, and the EBL provide additional contributions. However, these photons typically exceed the pair-production threshold so that we need to go beyond the low-energy limit to estimate their dispersive effect on TeV $\gamma$ propagation.

III. BEYOND EULER-HEISENBERG

So far, our results apply when pair creation can be neglected. In a static B-field, this is true when the dynamical parameter $eB_{\alpha\beta}/2m_\gamma^2 = 2.21 \times 10^{-14} B_{\mu G} \alpha_{\text{TeV}} \ll 1$ [52–54]. This condition is easily fulfilled in our context.

Pair production in the CMB becomes important for $\omega \gtrsim 100$ TeV. However, for TeV $\gamma$ rays propagating in the EBL or the galactic star light, the Euler-Heisenberg limit breaks down. The only pertinent literature is John Toll’s often-cited Ph.D. thesis (1952) [51] which we have actually found in our library. However, his results may be incorrect and we perform our own analysis.

The dispersive part $\chi$ of the refractive index is related to the imaginary part $\kappa = \Gamma/2\omega$ (absorption rate $\Gamma$) by the Kramers-Kronig relation [51,55,56]

$$\chi(\omega) = \frac{1}{\pi} \int_0^\infty \frac{\Gamma(\omega')}{\omega'^2 - \omega^2}, \quad (6)$$

where the integral denotes the Cauchy principal value. We assume background photons with number density $n_{\gamma}$, energy $\omega_{\gamma}$, and direction $\theta$ relative to the test photon. Then $\Gamma = (1 - \cos \theta)n_{\gamma}\sigma_{\gamma\gamma}$, where $\sigma_{\gamma\gamma} = (\pi a^2/2m_\gamma^2)J(u)$ is the total $\gamma\gamma \rightarrow e^+e^-$ cross section [57–59]. Here $u = \omega/\omega_0$ and $\omega_0$ is the threshold energy for pair production defined by $\omega_0\omega_0(1 - \cos \theta) = 2m_\gamma^2$ and

$$J(u) = \frac{1}{\pi} \frac{\omega_0}{u}, \quad u \leq 1 - \frac{n_{\gamma\gamma}}{n_{\gamma}}$$

where $n_{\gamma\gamma}$ is the number density of background photons.

Toll writes that he has solved the Kramers-Kronig integral analytically [his Eq. (2.2–10)], but the result on p. 54 fails to define his quantity $b$. The plot of $\delta r/d\Omega_{\text{forward}}$ in Fig. 2.2(B), which is proportional to the squared forward-scattering amplitude and thus to $|1 - n_{\gamma\gamma}/n_{\gamma}|^2$, does not have a zero. Our numerical integration of his equations does not reproduce his Fig. 2.2(B).
The radiation density in the Galaxy can far exceed the CMB. The main component is star light (SL) which, however, is partly processed by dust to form infrared radiation (IR). In Fig. 2 we show the estimated spectral energy distribution of these components in the Galaxy near the solar neighborhood [61–63]. The IR energy density is comparable to the CMB whereas the SL provides about 2.6 times more energy. At smaller galactocentric distances, the non-CMB contributions are much larger.
Another way of estimating the importance of star light is to use the total galactic luminosity of about $5 \times 10^{10} L_\odot$ and, if the source were concentrated at the Galactic center, would provide $\rho_{\text{EM}}/\rho_{\text{CMB}} \sim (12 \text{ kpc}/r)^2$. Of course, the disk geometry requires a detailed model, e.g., the one of the GALPROP code [63] that we used for Fig. 2.

The corresponding $\gamma\gamma$ refractive index is shown in Fig. 3 as a function of the test-photon energy. For $\omega \lesssim 200 \text{ GeV}$, all background radiations contribute essentially with their Euler-Heisenberg strength, whereas for higher energies, first the star light and then the infrared radiation drop out. The CMB contribution becomes small and finally negative only at $\omega \gtrsim 2000 \text{ TeV}$.

V. OTHER EFFECTS

Photon-photon refraction leads to deflection, e.g., in the radiation field of the Sun. In the Euler-Heisenberg limit and for photons grazing the Sun, we find an energy-independent deflection of $6.7 \times 10^{-24} \text{ arcsec}$, much smaller than the gravitational deflection of 1.75 arcsec. Photon-photon dispersion matters only in the context of $\gamma\gamma$-ALP oscillations where interference with the ALP dispersion enhances the effect.

In the early universe, there is a brief epoch when $\gamma\gamma$ dispersion dominates. As the Universe cools, the $e^+ e^-$ density is $n_{e^+ e^-} = 2^{1/2}(m_e T / \pi)^{3/2} e^{-m_e/T}$, producing $\omega_{\text{pl}}^2 = 4\pi n_{e^+ e^-}/m_e = 6.08 \times 10^8 \text{ eV}^2 (T/m_e)^{3/2} e^{-m_e/T}$. Photons provide $\chi_{\text{EM}} = 1.676 \times 10^{-16} P_{\text{keV}}$, corresponding to $m_{\text{eff}}^2 = -2\chi_{\text{EM}} \alpha^2$ and a thermal average $\langle m_{\text{eff}}^2 \rangle \sim -3.47 \times 10^{-9} \text{ eV}^2 T^6_{\text{keV}}$. This is similar to $-\omega_{\text{pl}}^2$ at $T = 30 \text{ keV}$, in agreement with the crossover shown in Fig. 3.6 of Ref. [50]. The cosmic $e/\gamma$ ratio is about $5.3 \times 10^{-10}$ so that $\omega_{\text{pl}}^2 = 2.32 \times 10^{-8} \text{ eV}^2 T^3_{\text{keV}}$. It takes over from $\gamma\gamma$ dispersion at $T \sim 2 \text{ keV}$.

Therefore, in the primordial plasma, $\gamma\gamma$ dispersion dominates when $2 \text{ keV} \lesssim T \lesssim 30 \text{ keV}$, providing photons with a spacelike dispersion relation. Note, however, that the photon gas does not support longitudinal excitations and does not contribute to Debye screening [49].

We also mention a recent study of the impact of photon-photon interaction on the polarization of CMB photons after recombination [65], although the effect looks extremely small. Photon-photon interaction is a polarization-dependent effect and therefore can lead to nontrivial birefringence effects [66,67].

VI. CONCLUSIONS

A photon gas is a dispersive medium for photon propagation. The ubiquitous CMB alone produces $n_{\text{eff}} = 1 + 0.511 \times 10^{-42}$, independently of energy if $\omega \lesssim 1000 \text{ TeV}$. This tiny effect dominates the dispersion of TeV gamma rays and, while it has always been ignored, can modify the oscillation between TeV gamma rays and axionlike particles in astrophysical magnetic fields.

If the energies of the background photons exceed the pair-creation threshold, the dispersion effect decreases, i.e., soft background photons are more important. Therefore, even though radiation in the Galaxy or the source regions can far exceed the CMB, their harder spectra prevent them from having a large impact on dispersion except for relatively small energies of $\omega \lesssim 100 \text{ GeV}$. On the other hand, $\gamma\gamma$ dispersion is weaker for smaller $\omega$, so while the relative importance of local radiation fields is larger for smaller $\omega$, the absolute importance of the overall effect decreases.

Photon-ALP oscillations depend on the $a\gamma$ interaction strength, the $B$-field strength and spatial distribution, the ALP mass, the photon energy, and, as a new ingredient, the density and spectrum of background photons. It is fortuitous that for many scenarios considered in the literature, $\gamma\gamma$ dispersion will be a benign effect and does not exclude that ALPs could be important for TeV gamma propagation in the Universe. On the other hand, the effect is large enough...
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that it cannot be summarily dismissed—its quantitative importance has to be evaluated in every individual case.

TeV gamma rays propagating in the Universe provide an intriguing example where $\gamma\gamma$ dispersion, despite its intrinsic weakness, can be of practical interest.

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The second sentence in the paragraph following Eq. (1) should read “If the EM background is a homogeneous $B$-field, the dispersion of $A_{\parallel}$ receives a factor $\frac{21}{11}\sin^2 \theta$, whereas $A_{\perp}$ a factor $\frac{12}{11}\sin^2 \theta$.” In the original version, we had given $\frac{14}{11}$ and $\frac{8}{11}$ for these coefficients, respectively, a factor $\frac{2}{3}$ smaller. We had mistakenly included $\langle \sin^2 \theta \rangle = \frac{2}{3}$ in these expressions. We thank M. Roncadelli for pointing out this error. Notice that for an isotropic gas of unpolarized test photons, averaging over directions leads to $\langle \sin^2 \theta \rangle = \frac{2}{3}$ and averaging over polarizations leads to $(\frac{1}{2}) \times (\frac{21}{11} + \frac{12}{11}) = \frac{3}{2}$, i.e., their average dispersion in a $B$-field is indeed given by Eq. (1).

By a similar token, notice that in Eq. (5c) one should include the coefficient $\frac{21}{11}$ for the described configuration, i.e., $0.76$ should be replaced with $1.44$.

Finally, in Eq. (3), the quantity $\chi_{\text{EM}}$ preferably should be denoted as $\chi_{\text{tot}} = \chi_{\text{CMB}} + \chi_{\text{B}}$ to avoid confusion with $\chi_{\text{EM}}$ given in Eq. (1).
To obtain the spectral average of the refraction effect in Eq. (11), we have erroneously averaged over the number distribution rather than the energy distribution of the background photons, i.e., we have used $x^2$ in the integrand of Eq. (11) instead of $x^3$ and the concomitant normalization factor $1/(2\zeta_3^3)$ instead of $15/\pi^3$. The correct equation is

$$
\text{FIG. 1.} \quad \text{The red line has changed as a result of the corrected Eq. (11).}
$$

$$
\text{FIG. 2.} \quad \text{The shape of the cosmic microwave background (CMB) curve has changed, reflecting the modified Eq. (11), whereas the other curves have remained unchanged.}
$$
We thank Hendrik Vogel and Ranjan Laha for spotting this error. This correction slightly changes the figures, but the overall conclusions of the paper remain the same.

\[ g_2(w) = \frac{15}{\pi^3} \int_0^\infty dx \frac{x^3}{e^x - 1} \left( wx \frac{30 \zeta_3}{\pi^4} \right). \] (11)

FIG. 3. The shape of the shoulder of the CMB curve has slightly changed as well as the corresponding region of the “total” curve.