Solutions for Assignment of Week 11
Introduction to Astroparticle Physics

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1 Sun as an axion source

We consider axions that have a two-photon vertex, characterized by the coupling constant $G_{a\gamma\gamma}$ of dimension (energy)$^{-1}$. As discussed in the lectures, the cross section $\gamma + p \to p + a$ due to the Primakoff process is very roughly

$$\sigma \sim \frac{\alpha G_{a\gamma\gamma}^2}{8\pi}$$

(i) Estimate the energy loss of the Sun due to axion emission, assuming axions can freely escape once produced. Treat the Sun as consisting purely of hydrogen and assume an average temperature of 1 keV. Express the result as a fraction of the solar photon luminosity which is roughly $L_\odot = 4 \times 10^{33}$ erg s^{-1}. Note also that the solar mass is $M_\odot = 2 \times 10^{33}$ g. In other words, the average nuclear energy generation rate in the Sun is 2 erg g^{-1} s^{-1}.

A more rigorous treatment, including screening effects in the Primakoff rate and integrating over a realistic solar model yields

$$L_a \sim G_{10}^2\, 1.85 \times 10^{-3}\, L_\odot\,,$$

very similar to the simple dimensional estimate. Here we have used $G_{10} = G_{a\gamma\gamma}/(10^{-10}\ \text{GeV}^{-1})$.

(ii) Assuming the solar axion production can not exceed its normal photon luminosity (why?), which limit on $G_{a\gamma\gamma}$ is implied?

(iii) Verify that for the relevant range of axion-photon couplings it is indeed true that axions can escape freely once produced, noting that the radius of the Sun is $R_\odot = 6.96 \times 10^{10}$ cm.

Solution

(i) Solar axion luminosity

The scattering rate (or inverse mean free path) of a photon in a proton gas of density n_p is

$$\Gamma = \sigma n_p$$

where we have used that the relative velocity is the speed of light (unity in natural units) and we have assumed that the protons are nonrelativistic. In each collision, the energy ω_γ of the photon is lost in the form of axions and the density photons is n_γ. Since the approximate cross section is independent of energy, the energy loss rate per unit volume is

$$Q = \sigma n_p \, \rho_\gamma$$

where $\rho_\gamma = (\pi^2/15)\, T^4$ is the thermal photon energy density in the solar interior. For pure hydrogen, the mass solar mass density is approximately $\rho = n_p m_p$ and therefore

$$Q = \sigma \, \frac{\rho}{m_p} \, \rho_\gamma \, .$$

Therefore, the energy loss rate per unit mass is

$$\epsilon = \frac{Q}{\rho} = \frac{\sigma \rho_\gamma}{m_p} \sim \frac{\alpha G_{a\gamma\gamma}^2}{8\pi} \, \frac{1}{m_p} \, \frac{\pi^2}{15} T^4 = \frac{\pi}{120} \, \frac{\alpha G_{a\gamma\gamma}^2 T^4}{m_p}$$

With $T = 1$ keV and $m_p = 0.935$ GeV one finds

$$\epsilon \sim G_{10}^2 \; 3 \times 10^{-3} \text{ erg g}^{-1} \text{ s}^{-1} \qquad \text{or} \qquad L_a \sim G_{10}^2 \; 1.5 \times 10^{-3} \, L_\odot \, .$$

(ii) Energy-loss limit

The Sun is halfway through its normal lifetime, so it would have burnt out already if it had lost twice the usual amount of energy all along. With the more exact energy loss rate we have the criterion

$$L_a = G_{10}^2 \, 1.85 \times 10^{-3} \, L_\odot \lesssim L_\odot \, ,$$

implying

$$G_{10} \lesssim 20 \, .$$

Using globular cluster stars one can actually derive a more restrictive limit from them not burning too fast, corresponding to $G_{10} \lesssim 1$.

(iii) Mean free path

Approximate the Sun as a homogeneous body with mass $M_\odot$, consisting purely of hydrogen, with radius $R_\odot$. Therefore, the average proton density is

$$n_p = \frac{M_\odot}{m_p} \, \frac{1}{(4\pi/3) \, R_\odot^3} = \frac{2 \times 10^{33} \text{ g}}{1.661 \times 10^{-24} \text{ g}} \, \frac{1}{(4\pi/3) \, (6.96 \times 10^{10} \text{ cm})^3} = 8.5 \times 10^{23} \text{ cm}^{-3}$$

The cross section for axion to photon conversion is

$$\sigma \sim \frac{G_{a\gamma\gamma}^2}{8\pi} = G_{10}^2\, 1.55 \times 10^{-49}\ \text{cm}^2$$

Therefore, the mean free path is

$$\lambda_{\text{mfp}} \sim \frac{1}{\sigma n_p} = G_{10}^{-2}\ 7.6 \times 10^{24}\ \text{cm} = G_{10}^{-2}\ 1 \times 10^{14}\ R_\odot$$

This is much larger than the solar radius, so axions escape freely once produced.

2 Plasma frequency and photon dispersion

In a gas of free electrons, photons propagate as if they had a mass, $\omega^2 - k^2 = \omega_{\text{plas}}^2$, that is given by the plasma frequency

$$\omega_{\text{plas}}^2 = \frac{4\pi\alpha}{m_e}\, n_e$$

where $\alpha = 1/137$ is the fine-structure constant, $m_e = 0.511$ MeV the electron mass, and n_e the electron density.

(i) Near the center of the Sun, the matter density is around 100 g cm^{-3}. Assuming it consists of hydrogen, how large is the plasma frequency? How does it compare with a typical blackbody photon energy, assuming the temperature is 1 keV?

(ii) The interstellar medium (ISM) consists to a large degree of ionized gas, i.e. free electrons, typically of order 1 cm^{-3} in the galaxy. How large is the corresponding plasma frequency?

(iii) If we observe a radio pulsar at a distance of 100 pc with photons of frequency $\nu = 1$ GHz (corresponding to an angular frequency of $2\pi \times 10^9\ \text{s}^{-1}$), how large is the photon time-of-flight delay caused by the presence of the ISM?

(iv) The pulsar radio emission is pulsed with a typical period in the range 0.2–2 s, corresponding to the rotation period. The spectrum of radio frequencies is broad. What is the impact of the ISM? How can the dispersion effect be used to measure the interstellar electron density?

Solution

(i) Plasma frequency in the solar interior

If the medium consists of hydrogen, the electron density is equal to the proton density and we have roughly $n_e = \rho/m_p$. With a density of 100 g cm^{-3} we find an electron density of $n_e = 6.0 \times 10^{25}\ \text{cm}^{-3}$. Noting that $1\ \text{cm}^{-1} = 1.973 \times 10^{-5}$ eV we find $\omega_{\text{plas}} = 0.3$ keV. For $T = 1$ keV, a typical photon energy is $3T = 3\ \text{keV} \sim 10\,\omega_{\text{plas}}$.

(ii) Plasma frequency in the galactic interstellar medium

Repeating the same calculation with $n_e = 1\ \text{cm}^{-3}$ yields a plasma frequency of $\omega_{\text{plas}} = 3.71 \times 10^{-11}$ eV.

(iii) Time-of-flight delay

The time to travel a distance D is $t = D/v$ and the velocity is $v = p/E = \sqrt{1 - m^2/E^2}$ for a particle with mass. Expanding to lowest order reveals a delay relative to a massless particle of

$$\Delta t = \frac{m^2}{2E^2}\, D\,.$$

For photons where the effective mass is caused by the plasma frequency, this is numerically

$$\Delta t = 0.41\ \mathrm{s}\, \frac{n_e}{1\ \mathrm{cm}^{-3}} \left(\frac{1\ \mathrm{GHz}}{\nu}\right)^2 \frac{D}{100\ \mathrm{pc}}\,.$$

(iv) Pulsar dispersion

The time delay is of the same order as the pulsar period, so a broad band radio signal will be smeared out. However, this dispersion effect has a known frequency dependence and therefore can be removed by a single fit parameter, the integrated electron density along the flight path, also known as the "dispersion measure." Therefore, the pulsed signal can be recovered and the electron column density can be determined.