Solutions for Assignment of Week 10
Introduction to Astroparticle Physics

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1 Galactic axion dark matter and the local Θ value

The pseudoscalar axion field can be expressed in the form $\Phi = \Theta f_a$ where f_a is the Peccei-Quinn scale and $-\pi < \Theta < +\pi$ an angular variable that represents the Θ parameter of QCD. If axions are indeed the dark matter of our galaxy, how large is Θ in our neighborhood?

Hints: The local dark matter density is taken to be $\rho_{\rm DM} = 300~{\rm MeV~cm^{-3}}$. The energy density of the nonrelativistic axion field is $\frac{1}{2}(m_a^2\Phi^2 + \dot\Phi^2)$. The relation between the axion mass m_a and f_a was given in the lectures to be $m_a f_a = m_\pi f_\pi \sqrt{z}/(1+z)$ where $z = m_u/m_d = 0.56$ is the up/down quark mass ratio.

Solution

The nonrelativistic axion field oscillates as $\Phi = \Phi_0 \cos(m_a t)$ and thus also $\Theta = \Theta_0 \cos(m_a t)$. Therefore, the local axion energy density is

$$\rho_a = \frac{1}{2}\left(m_a^2\Phi^2 + \dot\Phi^2\right) = \frac{1}{2}\,m_a^2 f_a^2\left(\Theta^2 + \dot\Theta^2\right) = \frac{1}{2}\,m_a^2 f_a^2 \Theta_0^2 = \frac{1}{2}\,\Theta_0^2\, m_\pi^2 f_\pi^2\,\frac{z}{(1+z)^2}$$

With $m_\pi = 135$ MeV and $f_\pi = 93$ MeV and the above dark matter density one finds

$$\Theta_0 = \frac{\sqrt{2\rho_{\rm DM}}}{m_\pi f_\pi}\,\frac{1+z}{\sqrt{z}} = 3.6\times10^{-19}$$

This is very much smaller than the limit of $|\Theta| \lesssim 10^{-10}$ from the experimental limits on a neutron electric dipole moment.

2 Occupation number of galactic dark matter axions

If axions are the dark matter of the galaxy, how large are the occupation numbers of the axion field modes? Numerical value, assuming the axion mass is $m_a = 10\ \mu\text{eV}$?

Hint: Assume the local dark matter density $\rho_{\text{DM}} = 300\ \text{MeV cm}^{-3}$ and assume the velocity distribution corresponds to an isothermal halo model of the form

$$\frac{\mathrm{d}n}{\mathrm{d}v} = n_0 \frac{4\,v^2}{\sqrt{\pi}\,\sigma^3}\,\mathrm{e}^{-v^2/\sigma^2}$$

with $\sigma = v_{\text{rot}} = 220\ \text{km s}^{-1}$.

Solution

The phase space distribution $f_{\mathbf{p}}$ or occupation number of axions is

$$\mathrm{d}n = f_{\mathbf{p}} \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}$$

Assuming isotropy and integrating over the angular directions implies $\mathrm{d}^3\mathbf{p} = 4\pi p^2 \mathrm{d}p$. For nonrelativistic axions we have $p = m_a v$ and thus

$$\mathrm{d}n = f_v \frac{4\pi}{(2\pi)^3}\, m_a^3\, v^2 \mathrm{d}v = f_v \frac{m_a^3}{2\pi^2}\, v^2 \mathrm{d}v = n_0 \frac{4\,v^2}{\sqrt{\pi}\,\sigma^3}\,\mathrm{e}^{-v^2/\sigma^2}\,\mathrm{d}v$$

Noting that $\rho_{\text{DM}} = m_a n_0$ one finds

$$f_v = \frac{\rho_{\text{DM}}}{m_a^4}\,\frac{8\pi^{3/2}}{\sigma^3}\,\mathrm{e}^{-v^2/\sigma^2} = 2.6 \times 10^{25}\,\left(\frac{10\ \mu\text{eV}}{m_a}\right)^4\,\mathrm{e}^{-v^2/\sigma^2}$$

which is simply a Maxwell-Boltzmann distribution. For the numerical estimate, the value $\sigma = 220\ \text{km s}^{-1}$ was used.

The axion field modes thus are extremely highly occupied and the dark matter axions form an almost perfect classical condensate.