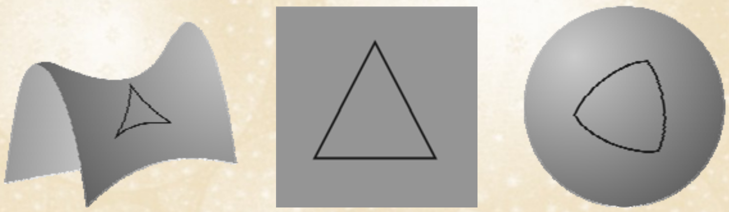
Friedmann-Lemaître-Robertson-Walker Cosmology

- On scales $\gtrsim$ 100 Mpc, space is maximally symmetric (homogeneous & isotropic)
- The corresponding Robertson-Walker metric is

$$ds^2 = dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

Clock time of co-moving observer	Cosmic scale factor	Curvature $k = 0, \pm 1$	r, θ, ϕ, co-moving spherical coordinates; r is dimensionless



$k = -1$ | $k = 0$ | $k = +1$

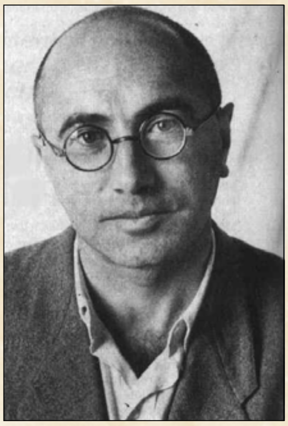
Einstein's "Greatest Blunder"

Density of gravitating mass & energy

Newton's constant

Curvature term is very small or zero (Euclidean spatial geometry)

Friedmann equation for Hubble's expansion rate $H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$



Yakov Borisovich Zeldovich 1914-1987

Cosmological constant Λ (new constant of nature) allows for a static universe by "global anti-gravitation"

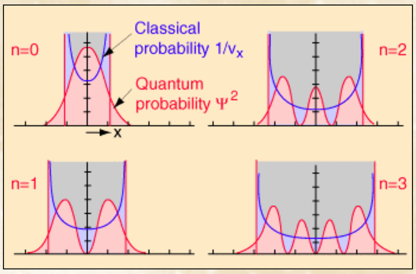
- Quantum field theory of elementary particles inevitably implies vacuum fluctuations because of Heisenberg's uncertainty relation, e.g. E and B fields can not simultaneously vanish
- Ground state (vacuum) provides gravitating energy
- Vacuum energy ρ_{vac} is equivalent to Λ

Zero Point Energy of Quantum Fields

Energy levels of the harmonic oscillator

$$E_n = \left(\frac{1}{2} + n\right)\hbar\omega$$

- Non-vanishing zero-point energy because of Heisenberg's uncertainty relation
- Location and momentum not simultaneously determined and therefore not both zero



Electromagnetic field: E and B not simultaneously zero because of uncertainty relation

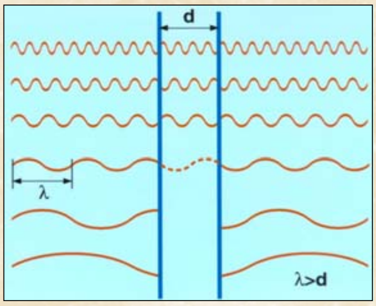
Energy density in the ground level (vacuum) is sum over infinitely many oscillators

$$\rho = \left\langle \frac{E^2 + B^2}{2} \right\rangle = \sum_n \frac{\hbar\omega_n}{2} = \infty$$

Nominal vacuum energy of the quantum fields
$+\infty$ for every bosonic degree of freedom (photons etc.)
$-\infty$ for every fermionic degree of freedom (electrons etc.)
How to interpret ???

Casimir Effect (1948)

A measurable manifestation of the zero-point energy of the electromagnetic field



Long-wavelength field modes between the plates are "displaced," causing a reduction of the vacuum energy compared with free space

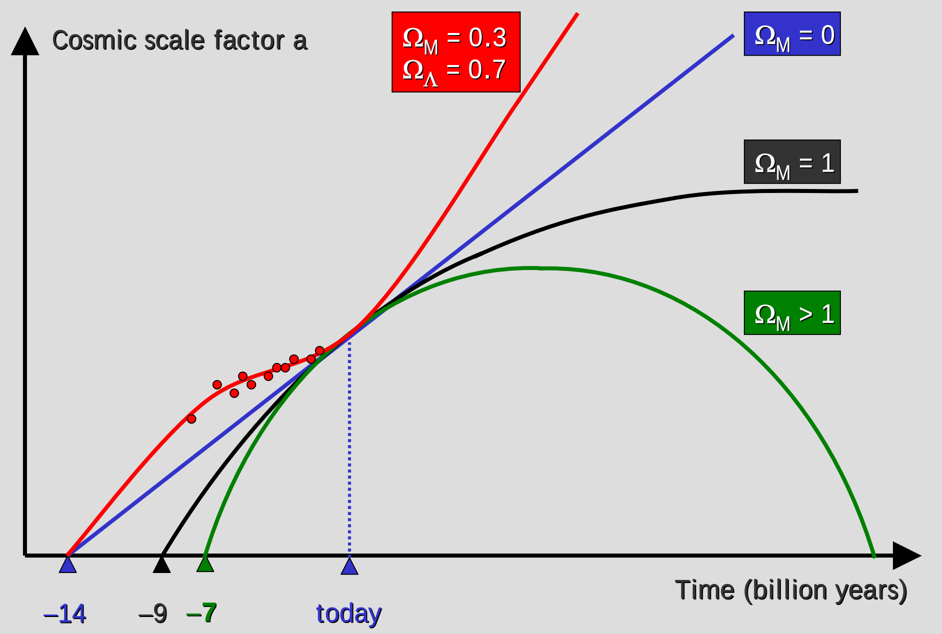
Hendrik Bugt Casimir (1909 - 2000)

$$F = \frac{\pi^2}{240}\frac{\hbar c}{d^4}A \approx 1.3\times10^{-7}\,\mathrm{N}\left(\frac{1\mu\mathrm{m}}{d}\right)^4\left(\frac{A}{1\mathrm{cm}^2}\right)$$

Casimir force between parallel plates (distance d, area A)

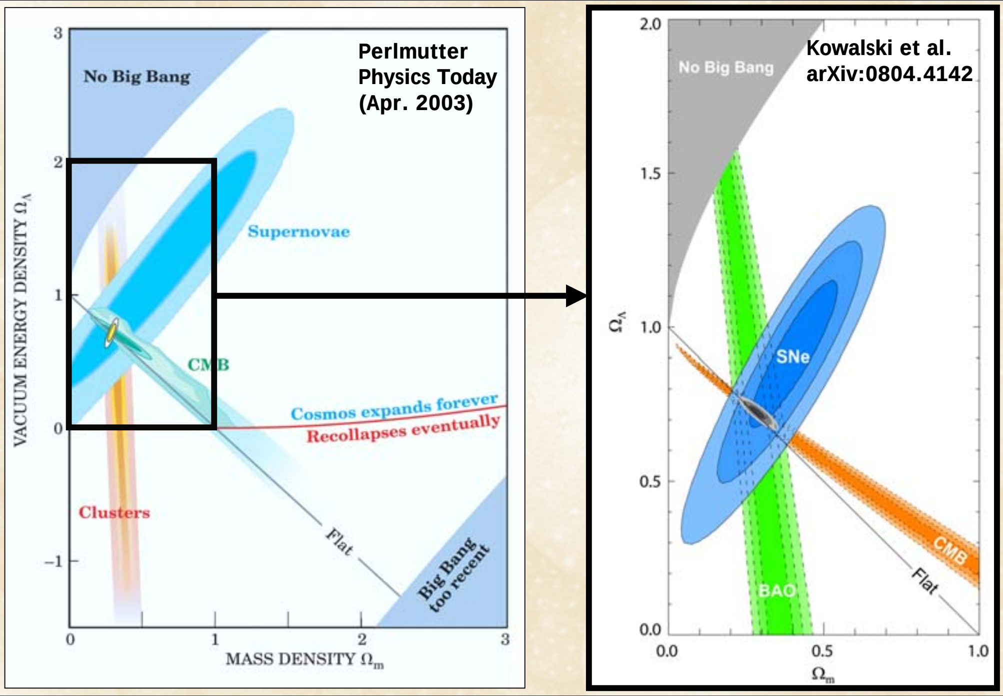
Bordag et al., New Developments in the Casimir Effect, Phys. Rept. 353 (2001)

Expansion of Different Cosmological Models

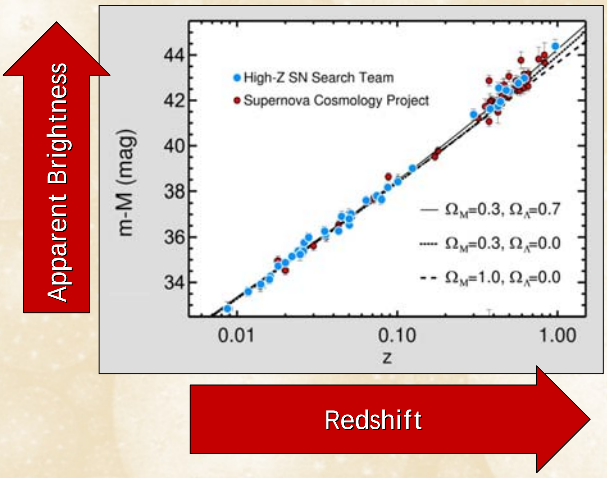


Adapted from Bruno Leibundgut

Best-Fit Universe

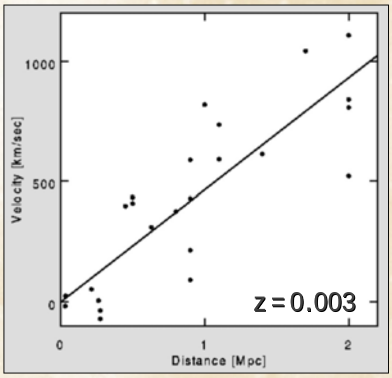


Hubble Diagram

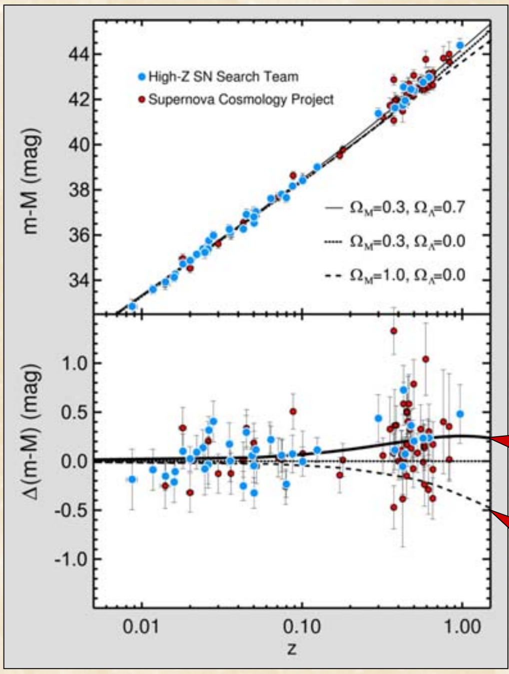


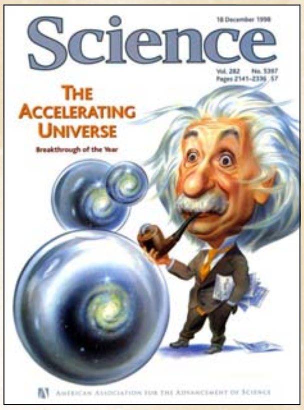
Supernova Ia as cosmological standard candles

Hubble's orginal data (1929)



Hubble Diagram

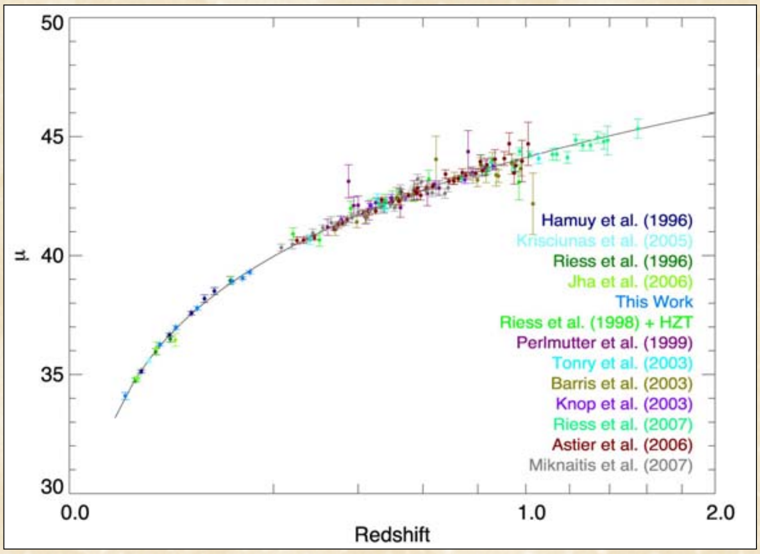




Accelerated expansion ($\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$)

Decelerated expansion ($\Omega_M = 1$)

Latest Supernova Data



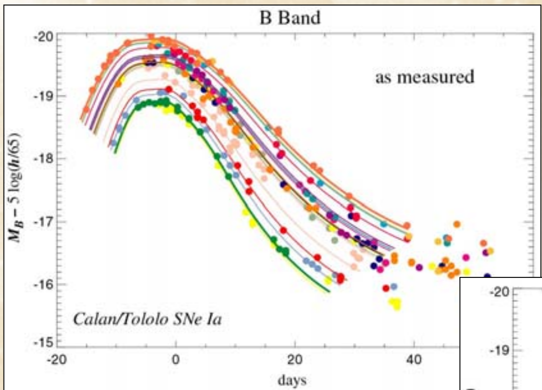
Kowalski et al.,
Improved cosmological constraints from new, old and combined supernova datasets, arXiv:0804.4142

Supernovae – Almost as Bright as Galaxies

SN 1998S in NGC 3877

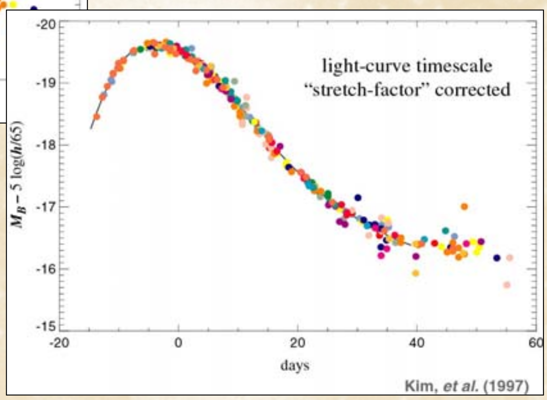
SN 1994D in NGC 4526

Universal Supernova Ia Light Curve

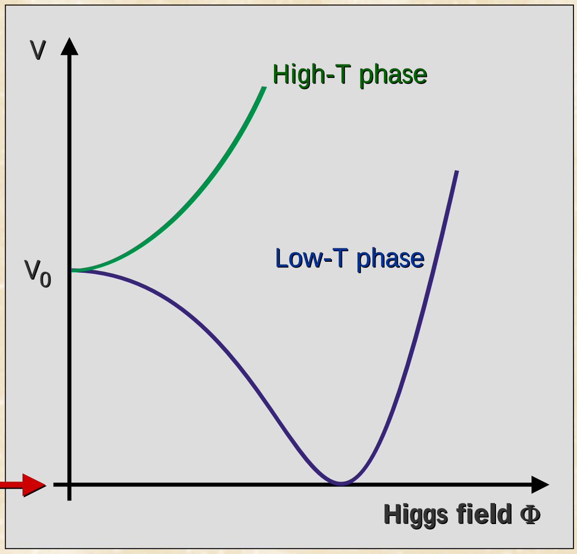


Supernova Ia lightcurves are empirically a 1-parameter family

After transformation
a universal light curve,
i.e. a de-facto standard candle

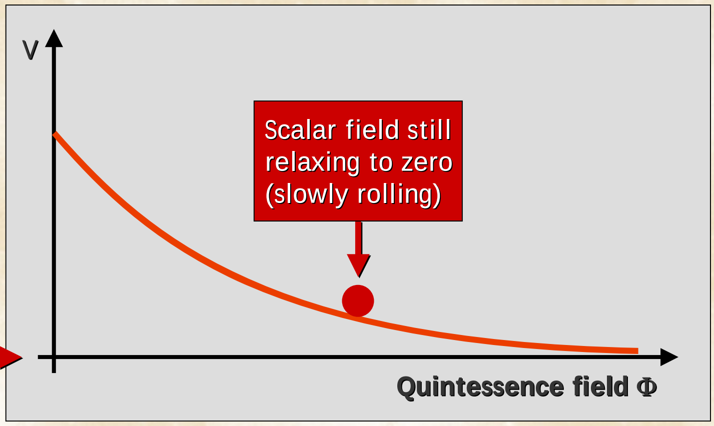


Scalar Fields and Cosmological Constant



Why is zero of all scalar field potentials (almost) exactly at V = 0

Quintessence



Absolute zero determined by unknown effect

Lagrangian	$L = \sqrt{g}\,[\frac{1}{2}\partial^\mu\Phi\,\partial_\mu\Phi + V(\Phi)]$	
Equations of motion for homogeneous mode ($\nabla\Phi$=0)	$H^2 = \frac{8\pi}{3} G_N [\frac{1}{2}(\partial_t\Phi)^2 + V(\Phi)]$ $\partial_t^2\Phi + 3H\partial_t\Phi + V'(\Phi) = 0$	System of coupled nonlinear equations

Quintessence as a Perfect Fluid

Energy-momentum tensor of homogeneous Φ-mode that of an isotropic perfect fluid	$\rho = \frac{1}{2}(\partial_t\Phi)^2 + V(\Phi)$ $p = \frac{1}{2}(\partial_t\Phi)^2 - V(\Phi)$
General equation of state	$p = w\rho$
Example: Exponential potential	$V(\Phi) = V_0\, e^{-\lambda\, 8\pi G_N \Phi}$
Explicit solution of eqs of motion imply	$w = \frac{p}{\rho} = \frac{\lambda^2}{3} - 1$

Like vacuum energy	$\lambda^2 = 0$	$w = -1$
Accelerated expansion	$\lambda^2 < 2$	$w < -1/3$
Like matter	$\lambda^2 = 3$	$w = 0$
Like radiation	$\lambda^2 = 4$	$w = 1$

Observational evidence for equation of state with “nonstandard” w-parameter?

Phantom Energy

What is the meaning of $w < -1$?

- Violates “dominant energy condition” $\rho + 3p > 0$
- Signals vacuum instability
  (e.g. Cline, Jeon & Moore, hep-ph/0311312)

Singularity of scale factor in the finite future (“big rip”)

$$t \sim t_0 + \frac{2}{3}\frac{1}{H_0}\frac{1}{|1+w|}\frac{1}{(1-\Omega_M)^{1/2}}$$

(Caldwell, Kamionkowski & Weinberg, astro-ph/0302506)

Constraints on Quintessence

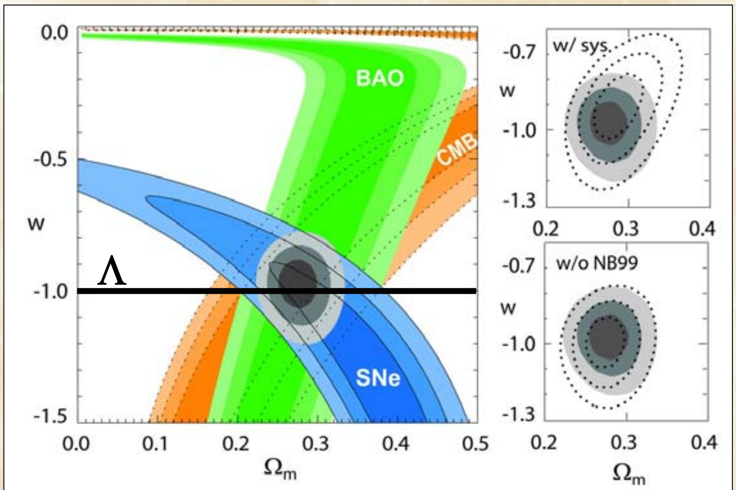
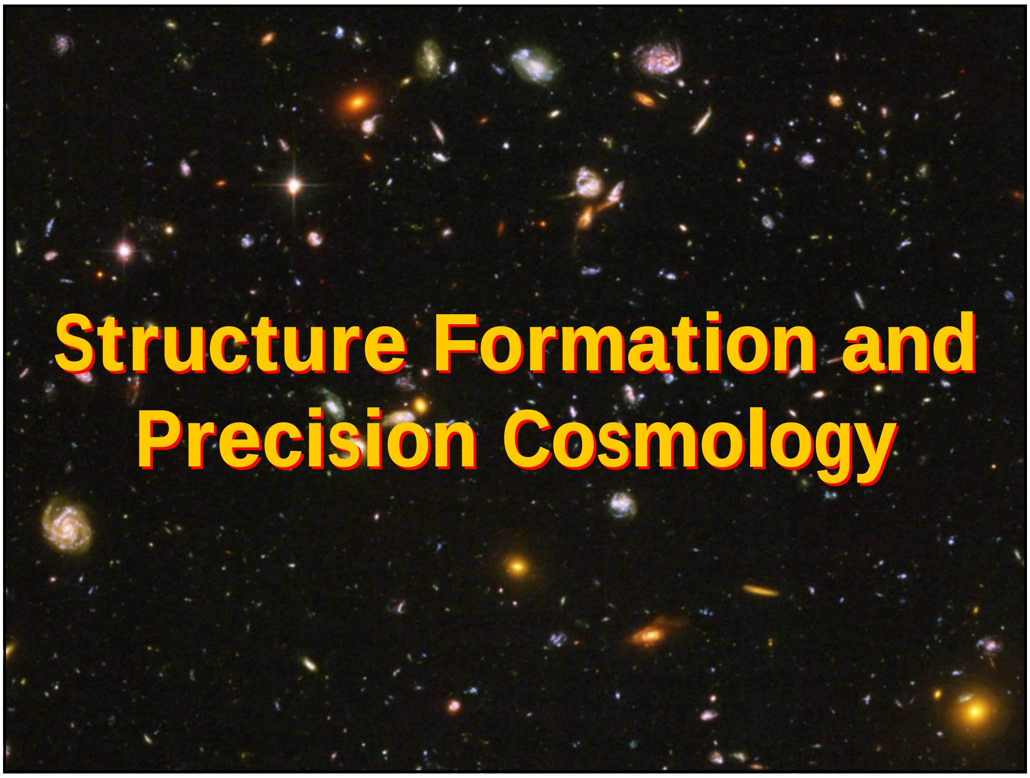
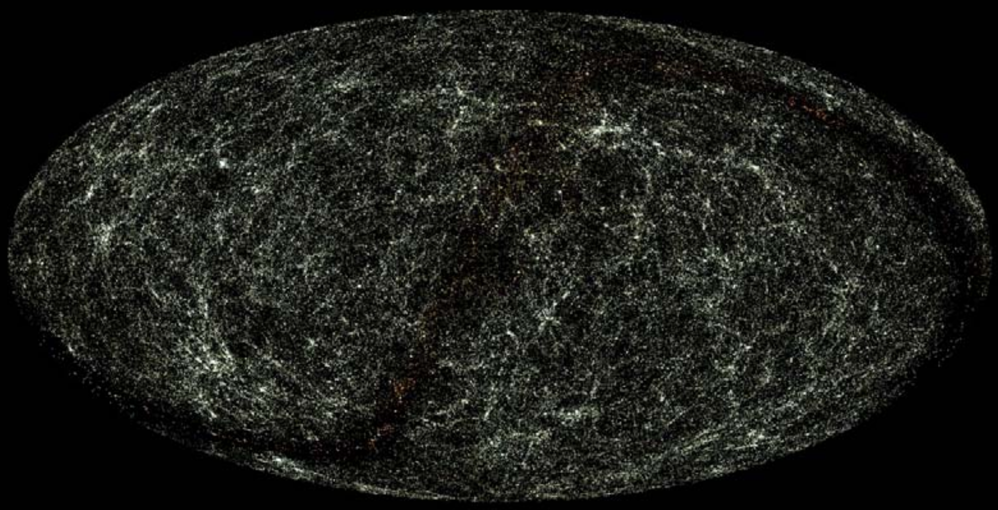


FIG. 14.— 68.3 %, 95.4 % and 99.7% confidence level contours on w and Ω_M, for a flat Universe. The top plot shows the individual constraints from CMB, BAO and the Union SN set, as well as the combined constraints (filled gray contours, statistical errors only). The upper right plot shows the effect of including systematic errors. The lower right plot illustrates the impact of the SCP Nearby 1999 data.

Kowalski et al., arXiv:0804.4142

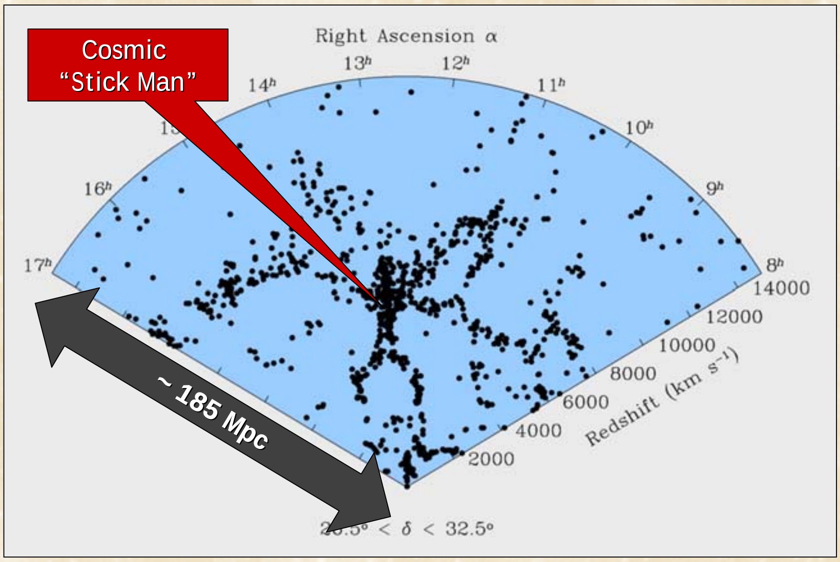


Sky Distribution of Galaxies (XMASS XSC)

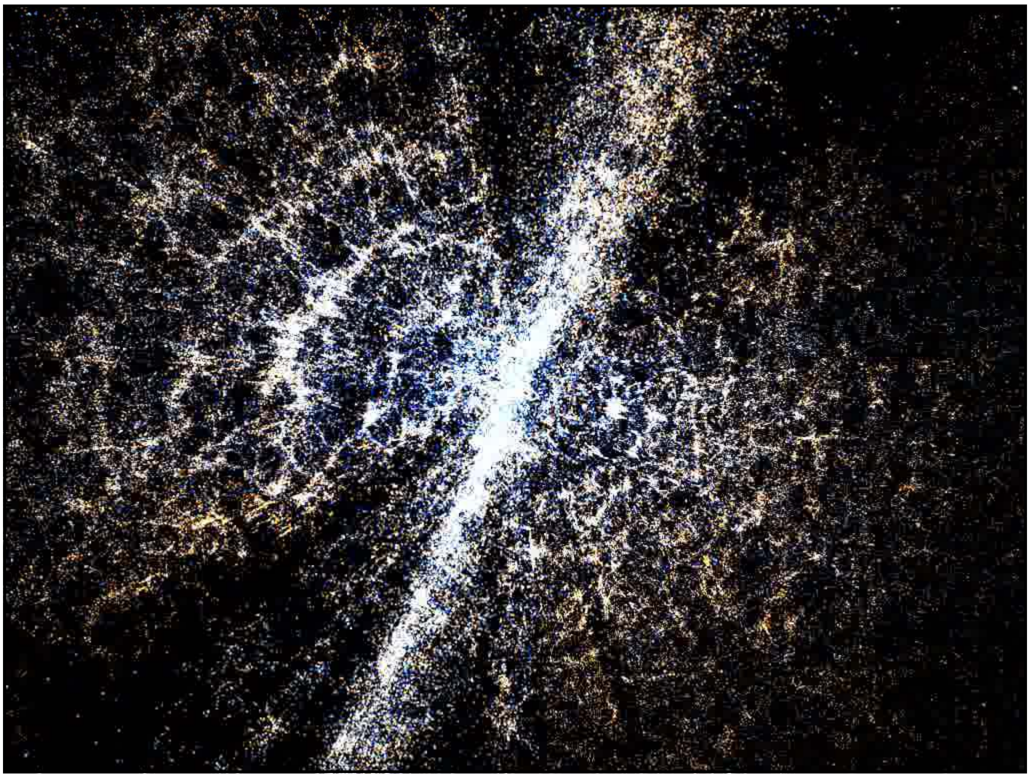


http://spider.ipac.caltech.edu/staff/jarrett/2mass/XSC/jarrett_allsky.html

A Slice of the Universe

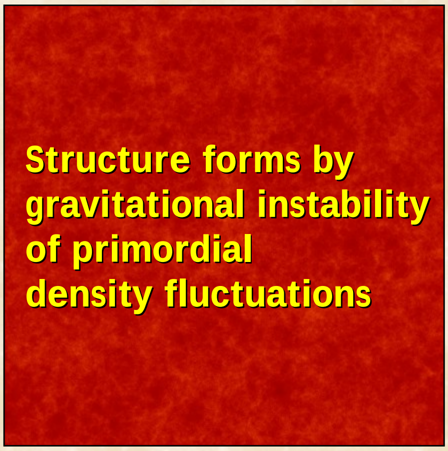


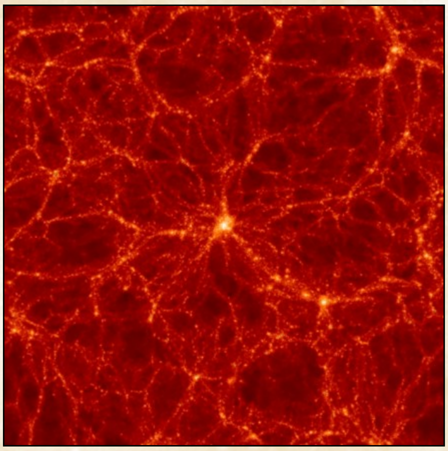
Galaxy distribution from the CfA redshift survey
[ApJ 302 (1986) L1]



Structure Formation in the Universe

Smooth	→	Structured





Generating the Primordial Density Fluctuations

Early phase of exponential expansion (Inflationary epoch)

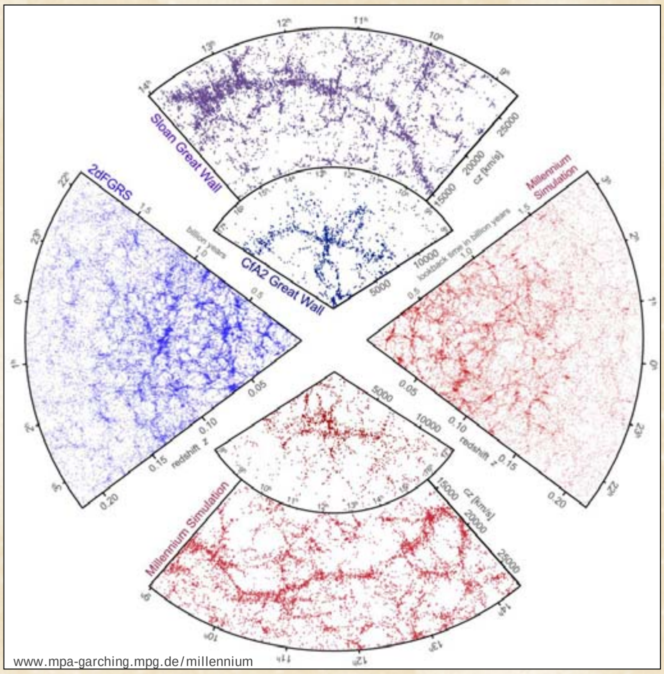
↓

Zero-point fluctuations of quantum fields are stretched and frozen

↓

Cosmic density fluctuations are frozen quantum fluctuations

Redshift Surveys vs. Millenium Simulation



Power Spectrum of Density Fluctuations

Field of density fluctuations

$$\delta(x) = \frac{\delta\rho(x)}{\bar\rho}$$

Fourier transform

$$\delta_k = \int d^3x\, e^{-ik\cdot x}\, \delta(x)$$

Power spectrum essentially square of Fourier transformation

$$\langle \delta_k \delta_{k'} \rangle = (2\pi)^3 \delta(k-k')\, P(k)$$

with δ the δ-function

Power spectrum is Fourier transform of two-point correlation function ($x = x_2 - x_1$)

$$\xi(x) = \langle \delta(x_2)\delta(x_1) \rangle = \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot x} P(k)$$

$$= \int \frac{d\Omega}{4\pi}\frac{dk}{k} e^{ik\cdot x} \underbrace{\frac{k^3 P(k)}{2\pi^2}}_{\Delta^2(k)}$$

Gaussian random field (phases of Fourier modes δ_k uncorrelated) is fully characterized by the power spectrum

$$P(k) = |\delta_k|^2$$

or equivalently by

$$\Delta(k) = \left(\frac{k^3 P(k)}{2\pi^2}\right)^{1/2} = \frac{k^{3/2}|\delta_k|}{\sqrt{2\pi}}$$

Gravitational Growth of Density Perturbations

The dynamical evolution of small perturbations

$$\delta(x) = \frac{\delta\rho(x)}{\bar\rho} << 1$$

is independent for each Fourier mode δ_k

- For pressureless, nonrelativistic matter (cold dark matter) naively expect exponential growth
- Only power-law growth in expanding universe

	Sub-horizon $\lambda \ll H^{-1}$	Super-horizon $\lambda \gg H^{-1}$
Radiation dominates $a \propto t^{1/2}$	$\delta_k \approx$ const	$\delta_k \propto a^2 \propto t$
Matter dominates $a \propto t^{2/3}$	$\delta_k \propto a \propto t^{2/3}$	

Processed Power Spectrum in Cold Dark Matter Scenario

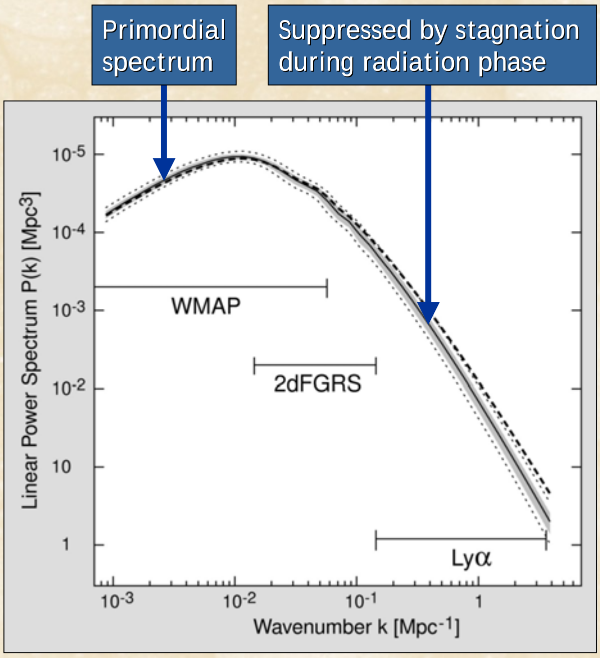
Primordial spectrum usually assumed to be of power-law form

$P(k) = |\delta_k|^2 \propto k^n$

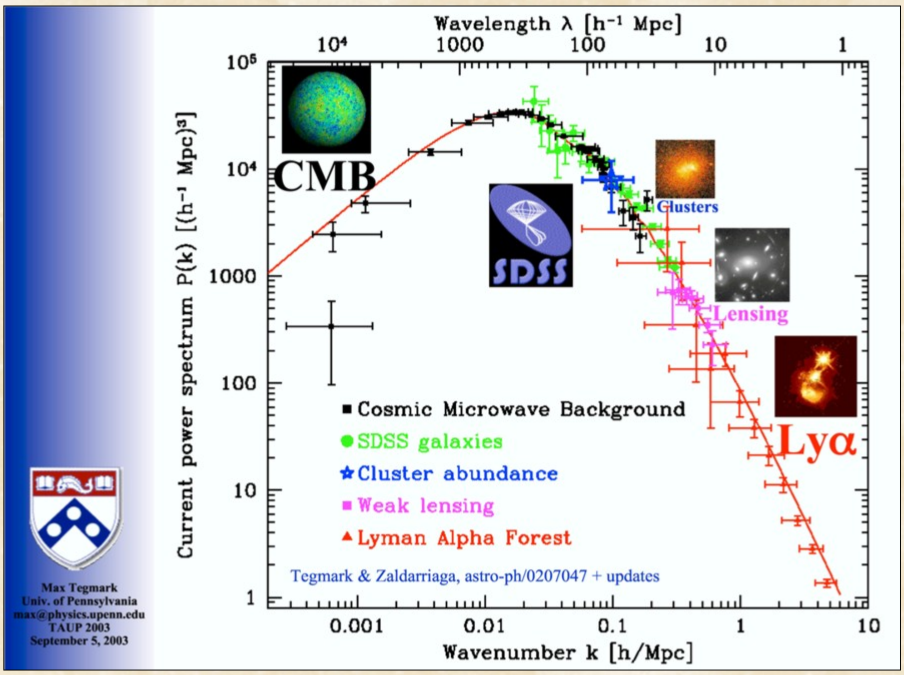
Harrison-Zeldovich ("flat") spectrum

$n = 1$

expected from inflation (may be slightly less than 1, depending on details of inflationary phase)



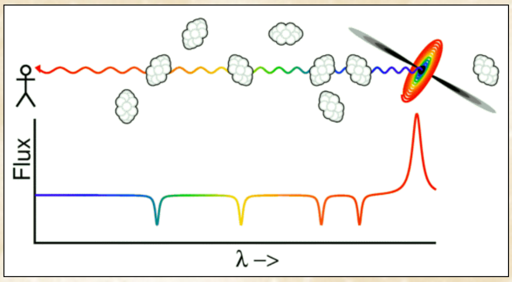
Power Spectrum of Cosmic Density Fluctuations

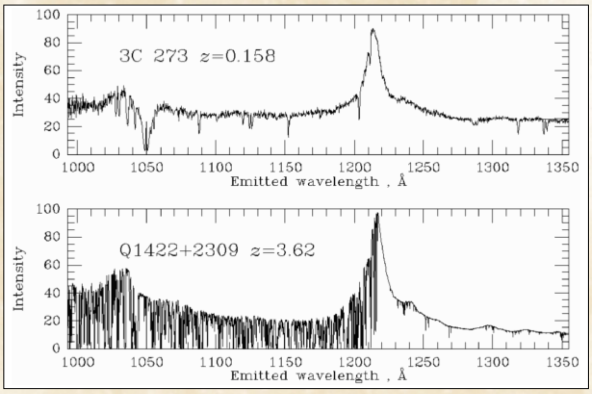


Lyman-alpha Forest

- Hydrogen clouds absorb from QSO continuum emission spectrum
- Absorption dips at Ly-α wavelengh corresponding to redshift

www.astro.ucla.edu/~wright/Lyman-alpha-forest.html

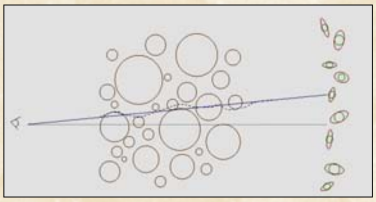


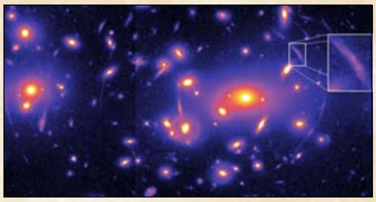


Examples for Lyman-α forest in low- and high-redshift quasars

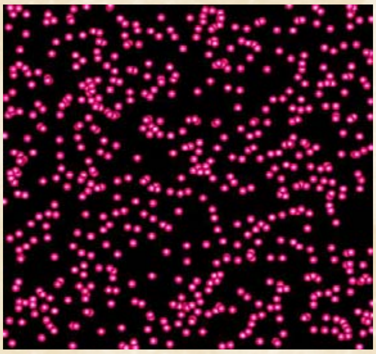
http://www.astr.ua.edu/keel/agn/forest.gif

Weak Lensing – A Powerful Probe for the Future

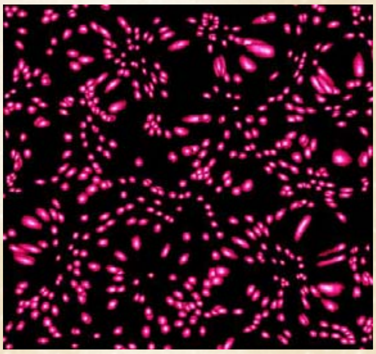




Distortion of background images by foreground matter

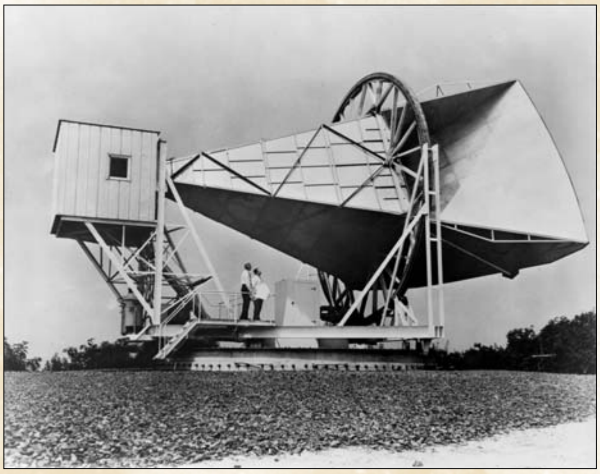


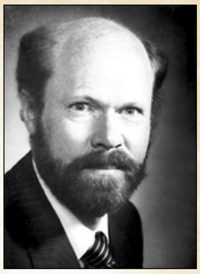
Unlensed



Lensed

Discovery of the Cosmic Microwave Background Radiation





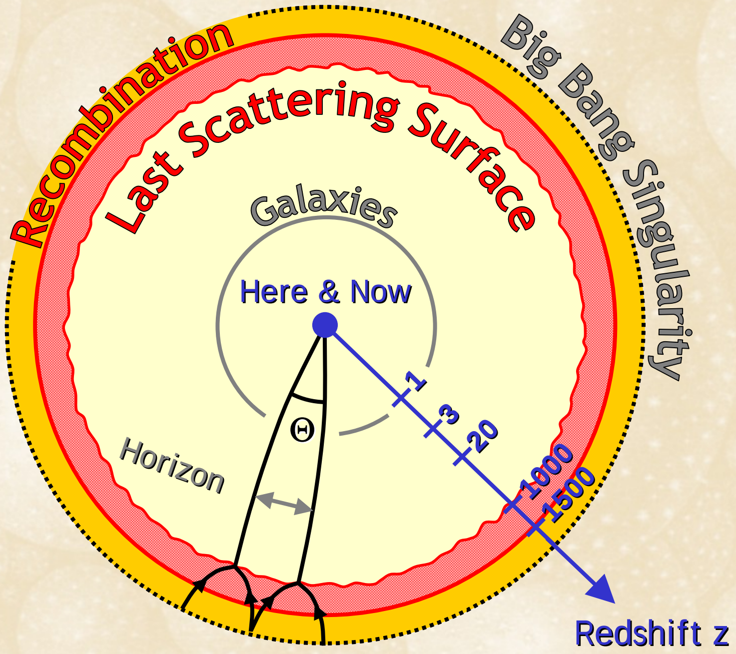
Robert W. Wilson
Born 1936

Arno A. Penzias
Born 1933

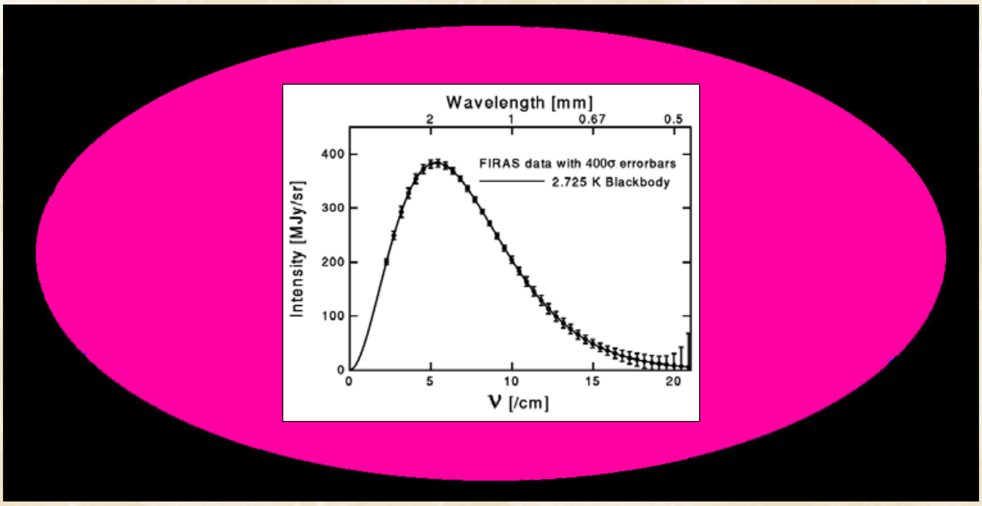
Discovery of 2.7 Kelvin
Cosmic microwave background radiation
by Penzias and Wilson in 1965
(Nobel Prize 1978)

Beginning of “big-bang cosmology”

Last Scattering Surface

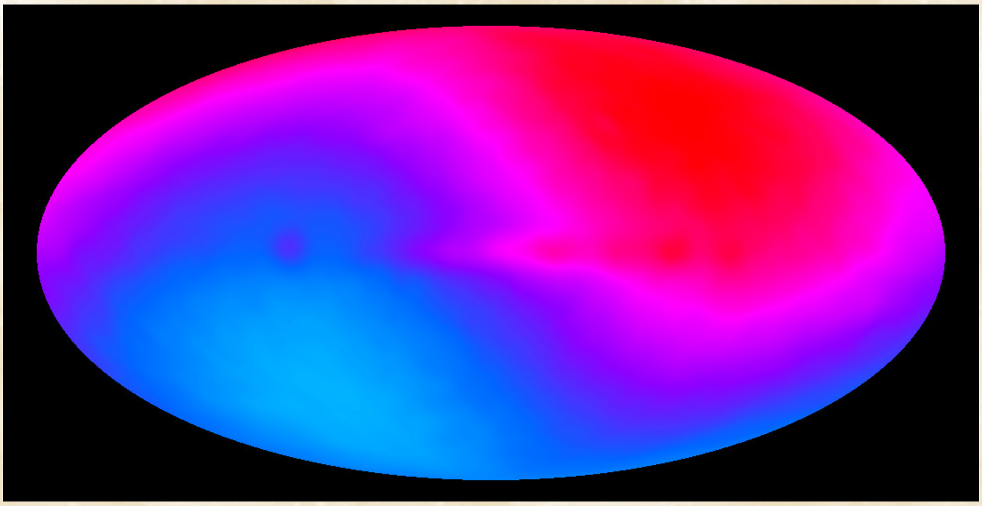


COBE Temperature Map of the Cosmic Microwave Background



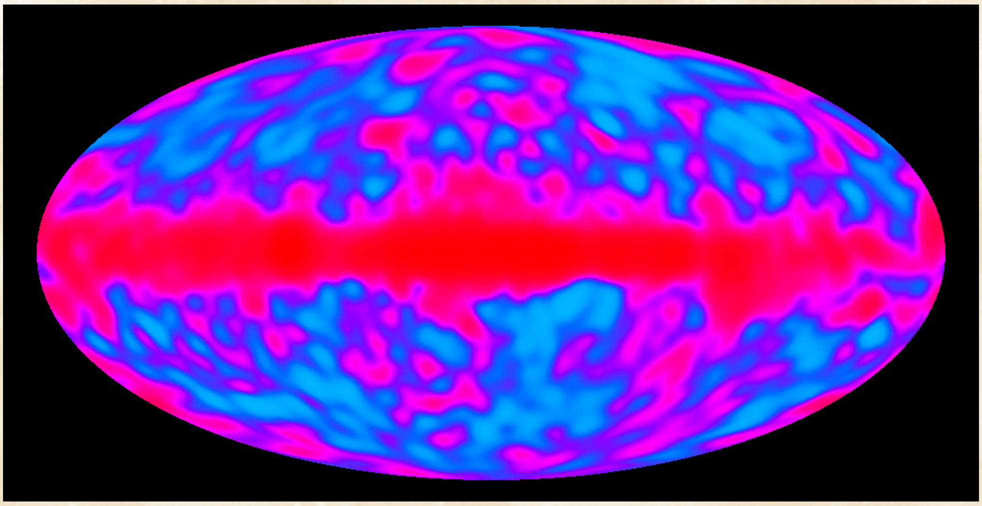
T = 2.725 K (uniform on the sky)

COBE Temperature Map of the Cosmic Microwave Background



Dynamical range $\Delta T = 3.353$ mK ($\Delta T/T \approx 10^{-3}$)
Dipole temperature distribution from Doppler effect caused by our motion relative to the cosmic frame

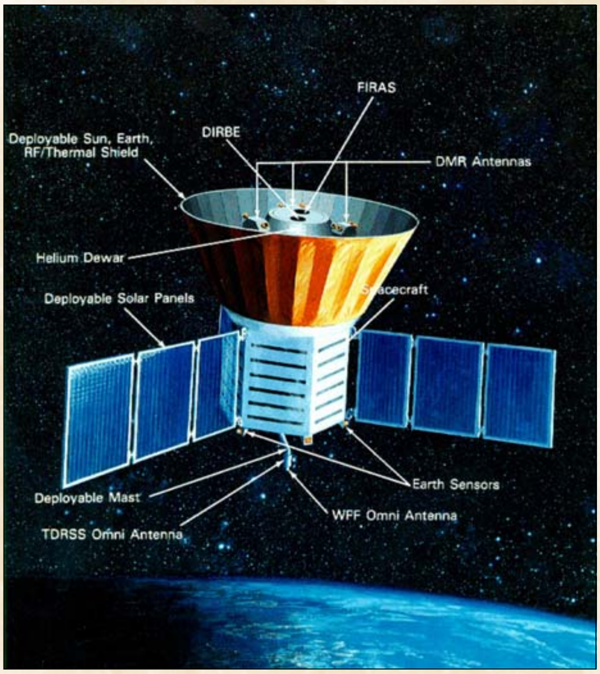
COBE Temperature Map of the Cosmic Microwave Background



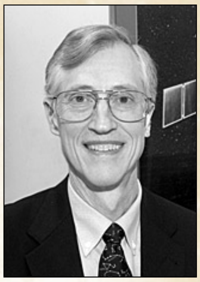
Dynamical range $\Delta T = 18\ \mu K$ ($\Delta T/T \approx 10^{-5}$)

Primordial temperature fluctuations

COBE Satellite

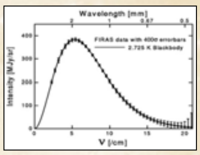


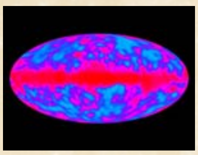
Nobel Prize 2006



John C. Mather
Born 1946

George F. Smoot
Born 1945

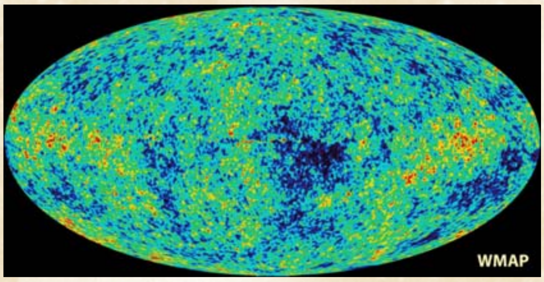




Power Spectrum of CMBR Temperature Fluctuations

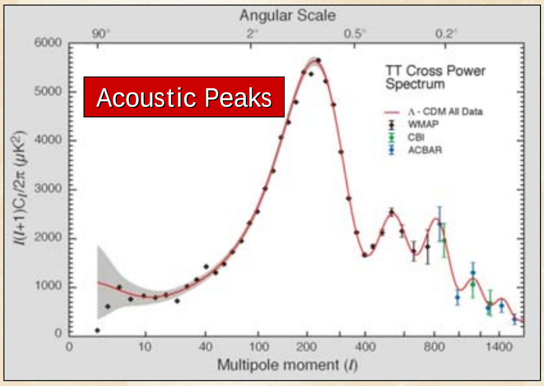
Sky map of CMBR temperature fluctuations

$$\Delta(\theta,\varphi) = \frac{T(\theta,\varphi) - \langle T \rangle}{\langle T \rangle}$$



Multipole expansion

$$\Delta(\theta,\varphi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta,\varphi)$$

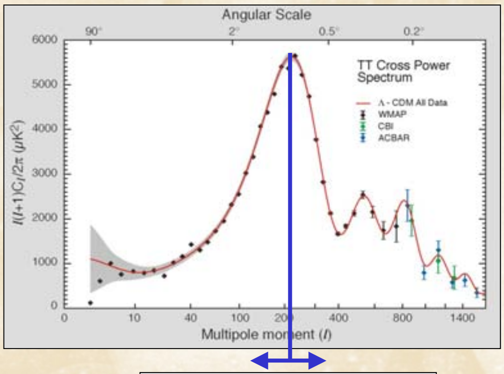


Angular power spectrum

$$C_\ell = \langle a^*_{\ell m} a_{\ell m} \rangle = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} a^*_{\ell m} a_{\ell m}$$

Flat Universe from CMBR Angular Fluctuations

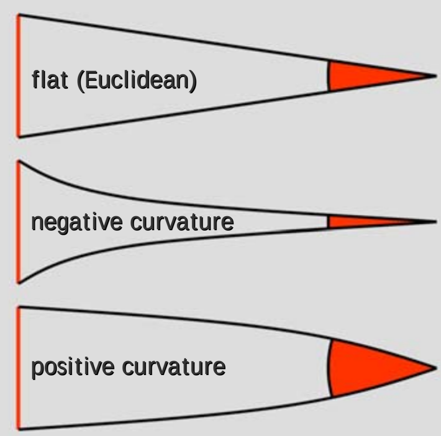
Spergel et al. (WMAP Collaboration) astro-ph/0302209



$$\ell_{max} \approx 200/\sqrt{\Omega_{tot}}$$

$$\Omega_{tot} = 1.02 \pm 0.02$$

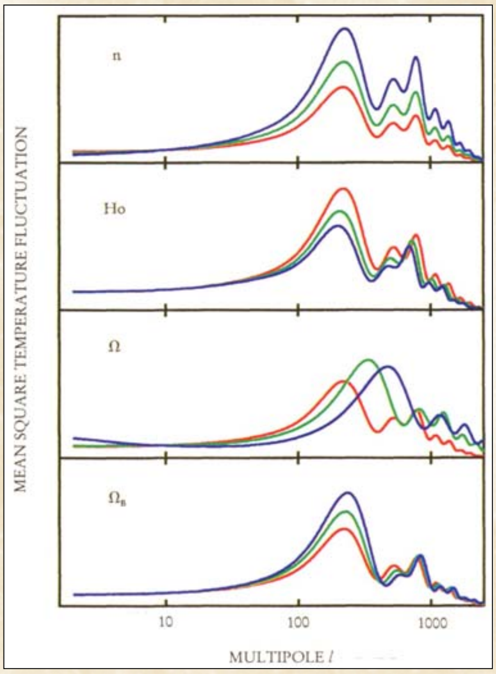
Triangulation with acoustic peak



Known physical size of acoustic peak at decoupling ($z \approx 1100$)

Measured angular size today ($z = 0$)

CMBR – The Cosmic Rosetta Stone



Power-law index (tilt)
n = 1.0, 1.1, 1.2

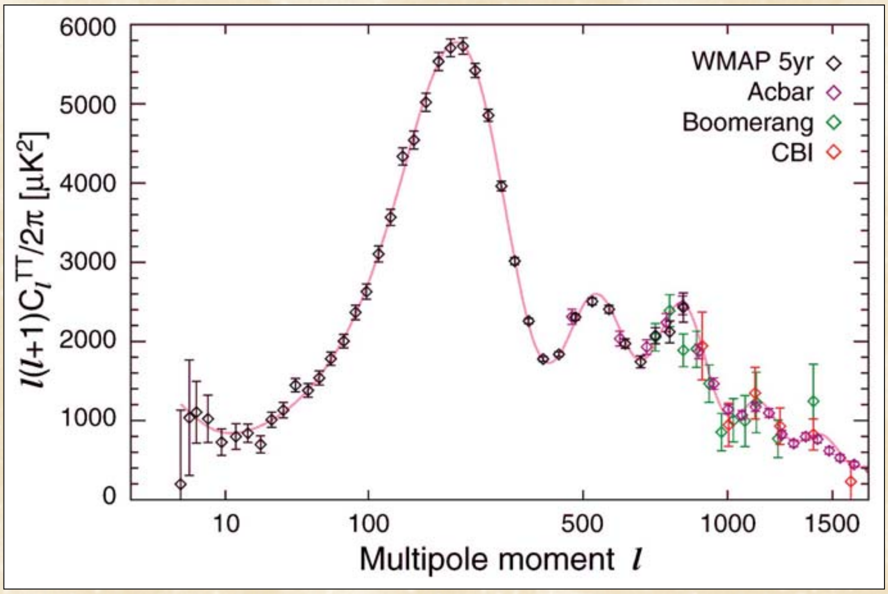
Hubble constant
H_0 = 50, 60, 70

Total density
Ω_{tot} = 1.0, 0.5, 0.3

Baryon density
Ω_B = 5, 7.5, 10×10^{-3}

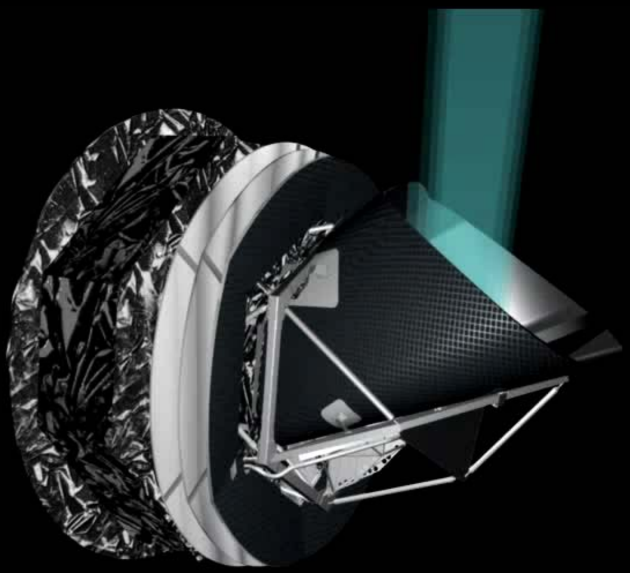
Physics Today 1997:11, 32

Latest CMB Results (WMAP-5 and Others)



Komatsu et al., arXiv:0803.0547

PLANCK Satellite



Concordance Model of Cosmology

A Friedmann-Lemaître-Robertson-Walker model with the following parameters perfectly describes the global properties of the universe

Expansion rate	$H_0 = (70.1 \pm 1.3)\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$	
Spatial curvature	$\lvert R_{curv} \rvert > 33\,\mathrm{Gpc}$	$-0.018 < \Omega_k < 0.009$
Age	$t_0 = (13.73 \pm 0.12) \times 10^9\ \mathrm{years}$	
Vacuum energy	$\Omega_\Lambda = 0.721 \pm 0.015$	
Cold Dark Matter	$\Omega_{CDM} = 0.233 \pm 0.013$	
Baryonic matter	$\Omega_B = 0.0462 \pm 0.0015$	

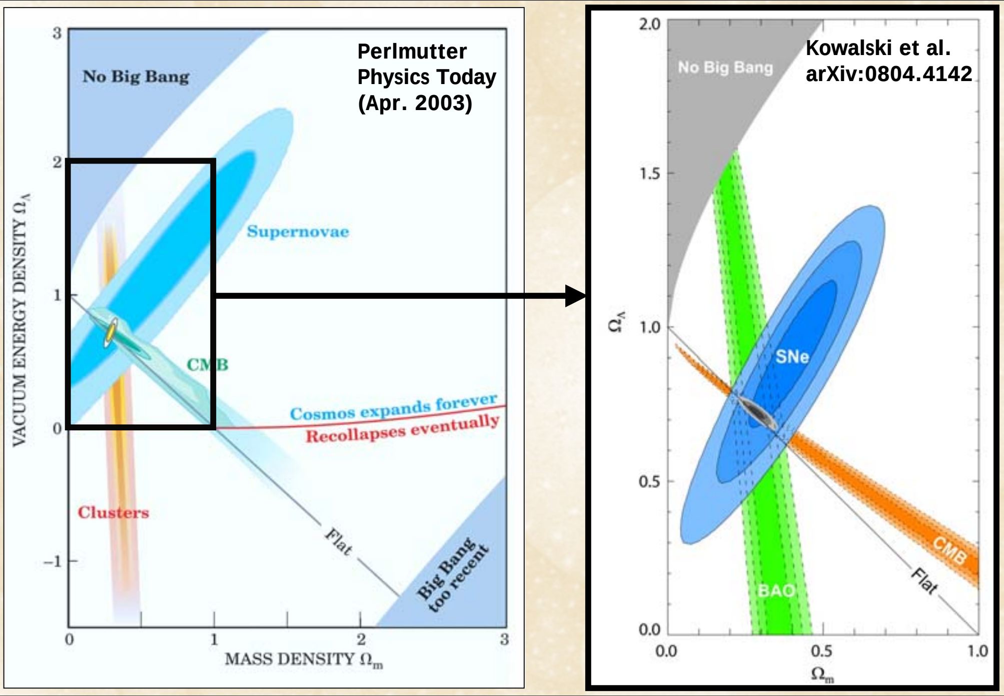
The observed large-scale structure and CMBR temperature fluctuations are perfectly accounted for by the gravitational instability mechanism with the above ingredients and a power-law primordial spectrum of adiabatic density fluctuations (curvature fluctuations) $P(k) \propto k^n$

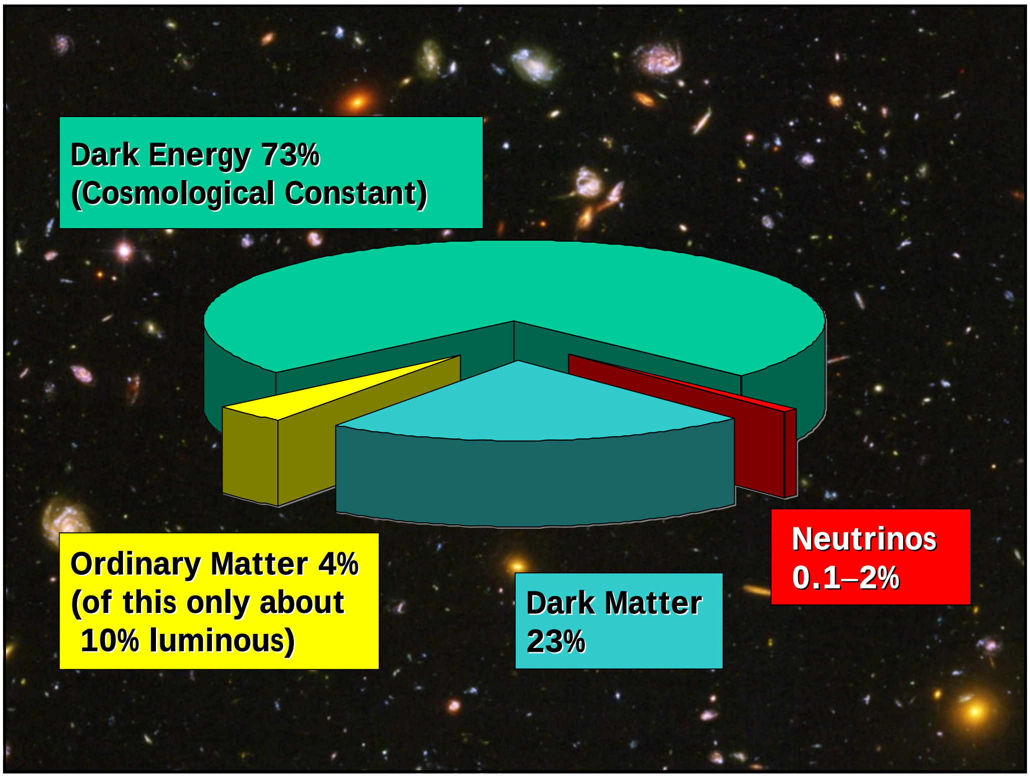
Power-law index	$n = 0.960 \pm 0.014$

Georg Raffelt, Max-Planck-Institut für Physik, München

Introduction to Astroparticle Physics, WS 2009/10, LMU

Best-Fit Universe





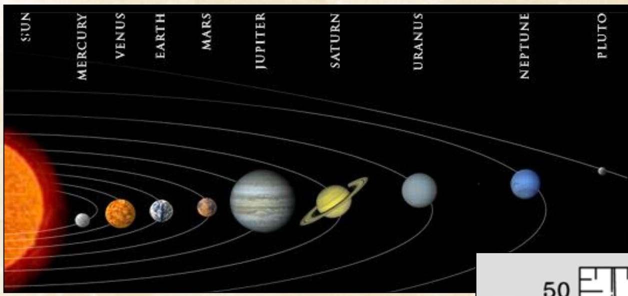
Evidence for Dark Matter on Small Scales

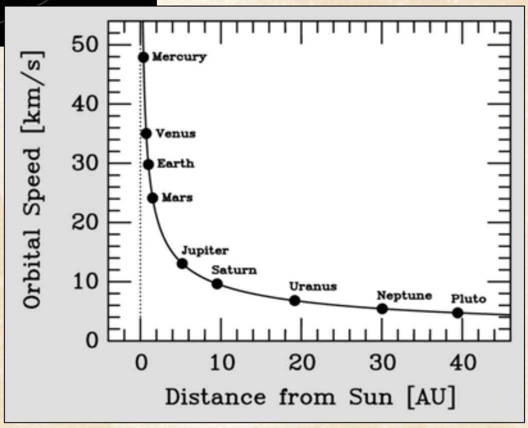
Structure of Spiral Galaxies

Spiral Galaxy NGC 2997

Spiral Galaxy NGC 891

“Rotation Curve” of the Solar System

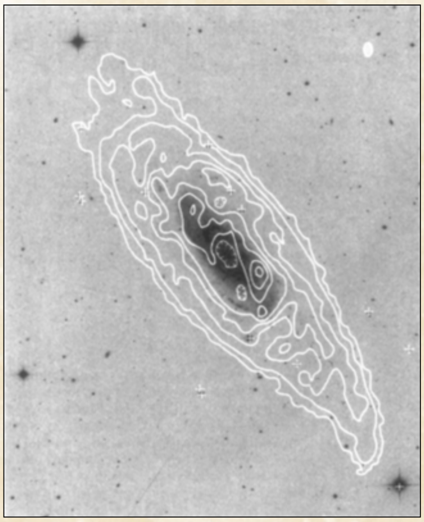


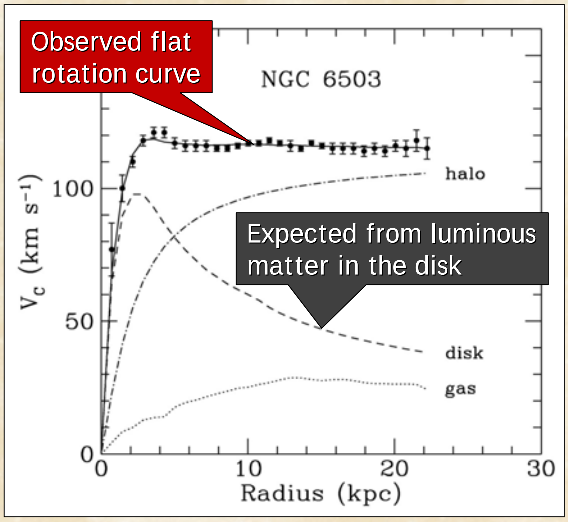


Kepler’s Law

$$v_{\rm rotation} = \sqrt{\frac{G_{\rm Newton} M_{\rm central}}{\rm radius}}$$

Galactic Rotation Curve from Radio Observations



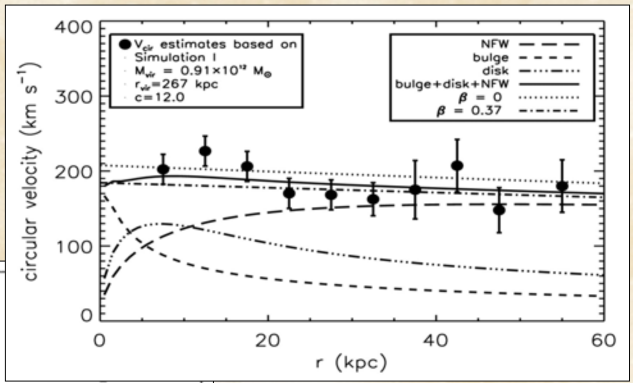


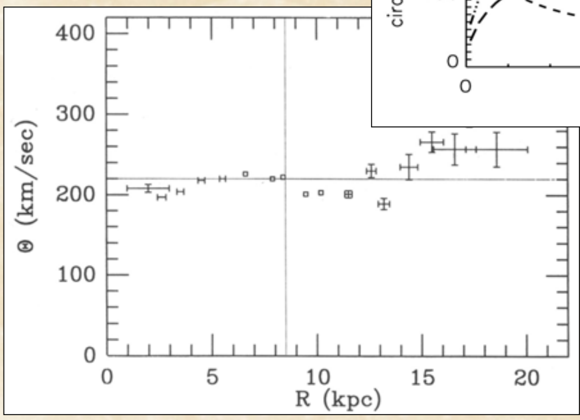
Spiral galaxy NGC 3198 overlaid with hydrogen column density [ApJ 295 (1985) 305]

Rotation curve of the galaxy NGC 6503 from radio observations of hydrogen motion [MNRAS 249 (1991) 523]

Rotation Curve of the Milky Way

Xue et al., arXiv:0801.1232v5 (2008)





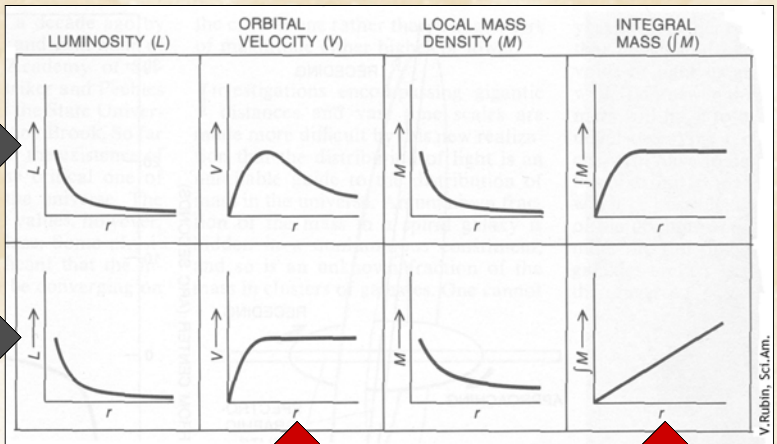
Expect dark matter density near solar system of about 300 MeV cm^{-3}

Finch & Tremaine ARAA 29 (1991) 409

Dark Matter in Spiral Galaxies - Summary

Expected from luminosity distribution

Infered from rotation curve



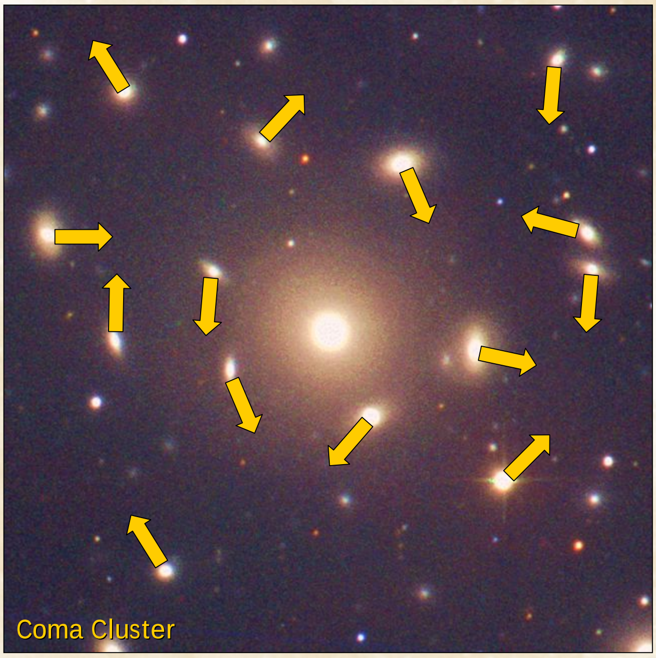
Flat rotation curve instead of Keplerian

No obvious limit to total mass

Structure of a Spiral Galaxy

Structure of a Spiral Galaxy

Dark Matter in Galaxy Clusters



A gravitationally bound system of many particles obeys the virial theorem

$$2\langle E_{kin} \rangle = -\langle E_{grav} \rangle$$

$$2\left\langle \frac{mv^2}{2} \right\rangle = \left\langle \frac{G_N M_r m}{r} \right\rangle$$

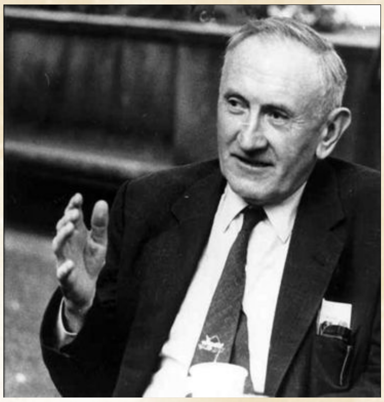
$$\langle v^2 \rangle \approx G_N M_r \langle r^{-1} \rangle$$

Velocity dispersion from Doppler shifts and geometric size

Total Mass

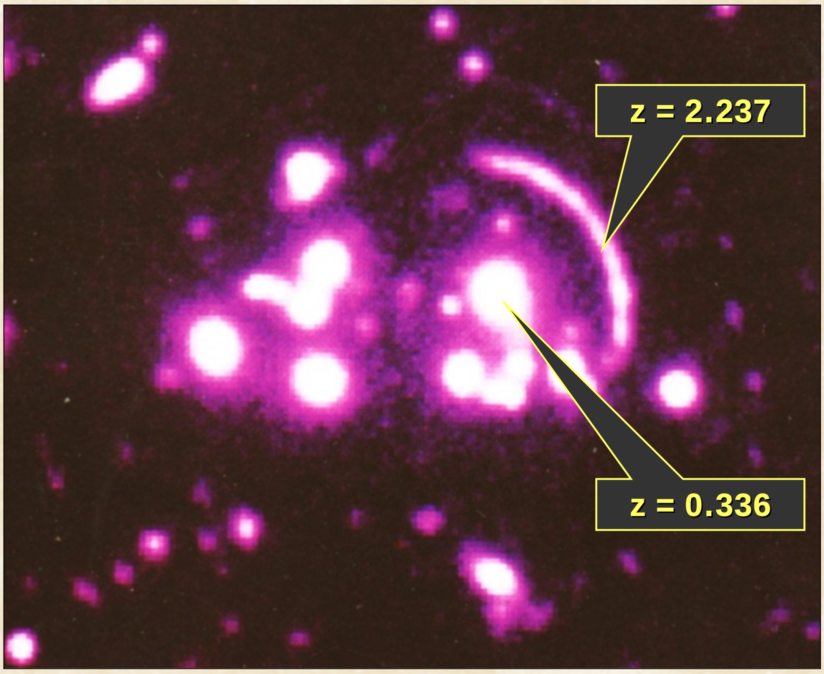
Dark Matter in Galaxy Clusters

Fritz Zwicky:
Die Rotverschiebung von Extragalaktischen Nebeln
(The redshift of extragalactic nebulae)
Helv. Phys. Acta 6 (1933) 110

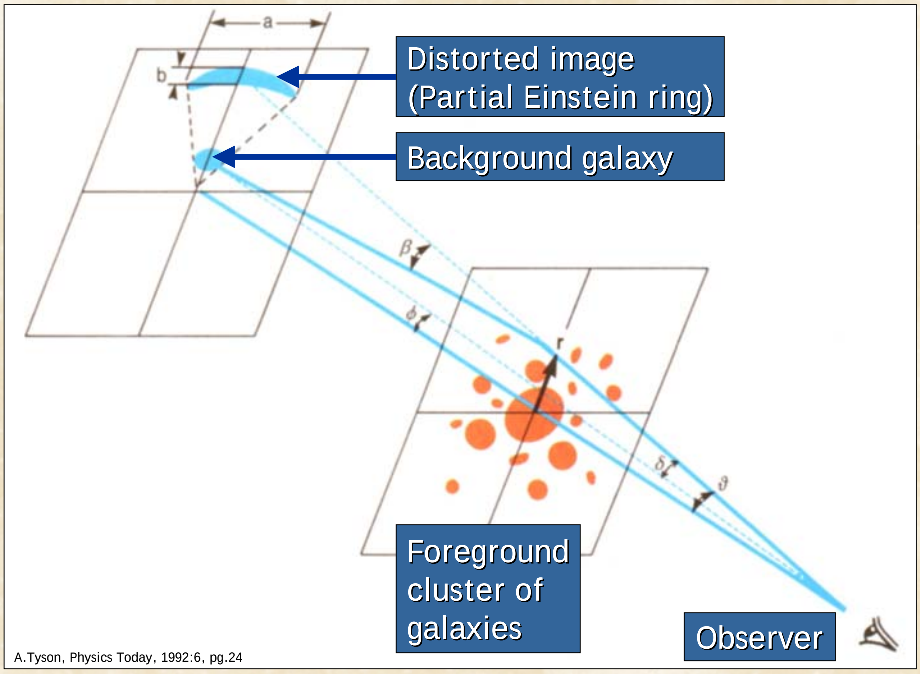


In order to obtain the observed average Doppler effect of 1000 km/s or more, the average density of the Coma cluster would have to be at least 400 times larger than what is found from observations of the luminous matter.
Should this be confirmed one would find the surprising result that dark matter is far more abundant than luminous matter.

Giant Arc in Cluster Cl 2244-02



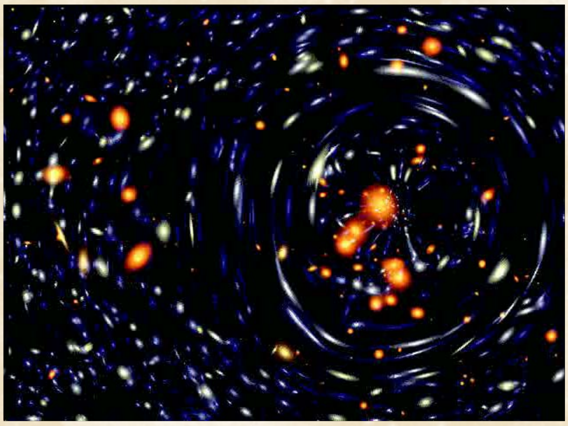
Giant Arcs – Gravitationally Lensed Background Galaxies



A.Tyson, Physics Today, 1992:6, pg.24

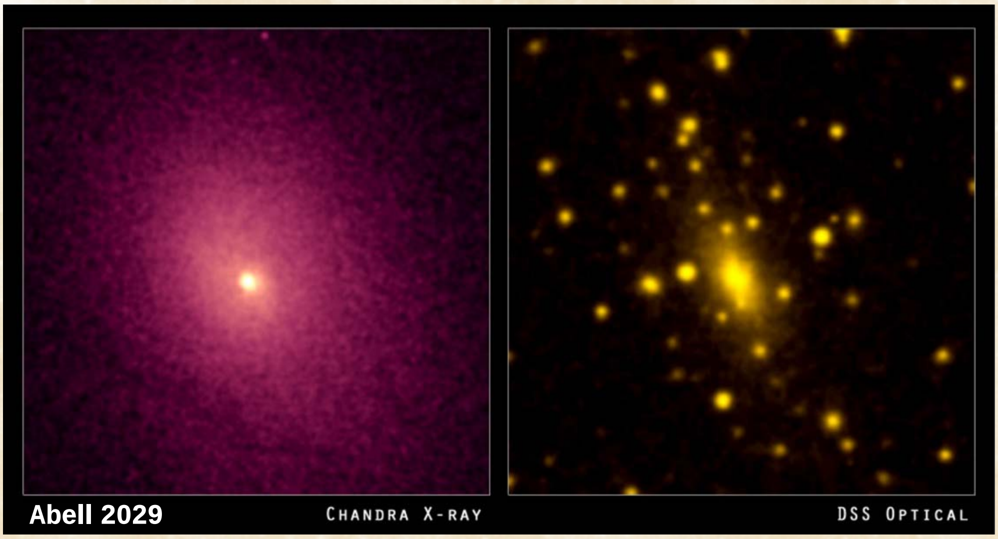
Gravitational Lensing in Clusters of Galaxies

Galaxy cluster Cl 0024+1654
[Hubble Space Telescope]



Numerical Simulation

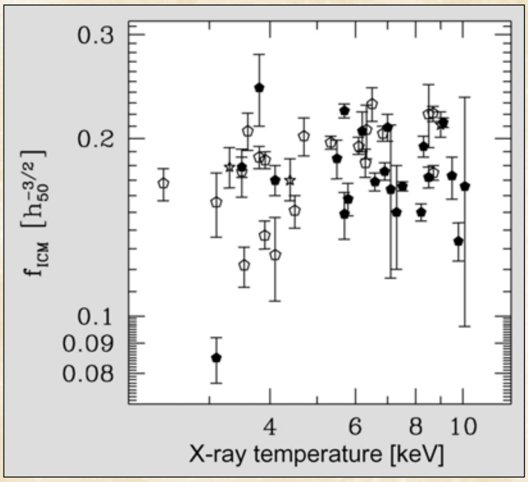
Hot X-Ray Gas in Clusters of Galaxies



Most of the baryonic mass in a typical galaxy cluster
resides in hot, x-ray emitting intergalactic gas

Galaxy Cluster Abell 2029 (composite optical & x-ray)

Cluster Gas Fraction



Mohr et al., astro-ph/9901281

Gas fraction in clusters

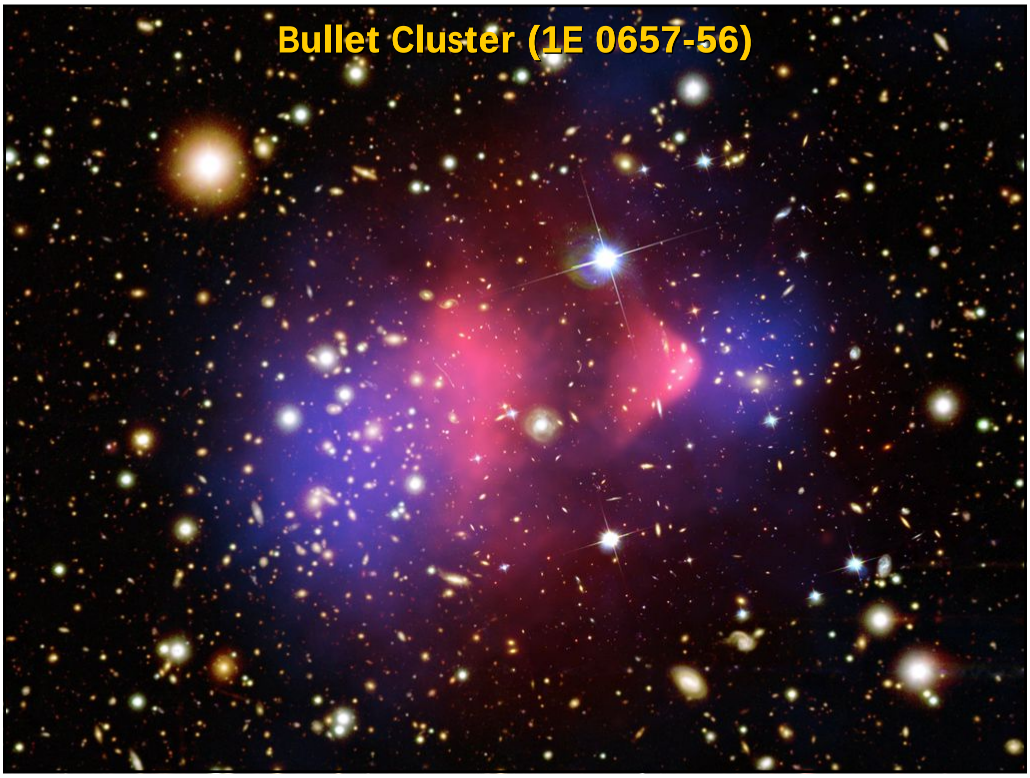
$f_{gas}\, h^{3/2} = 0.075 \pm 0.002$

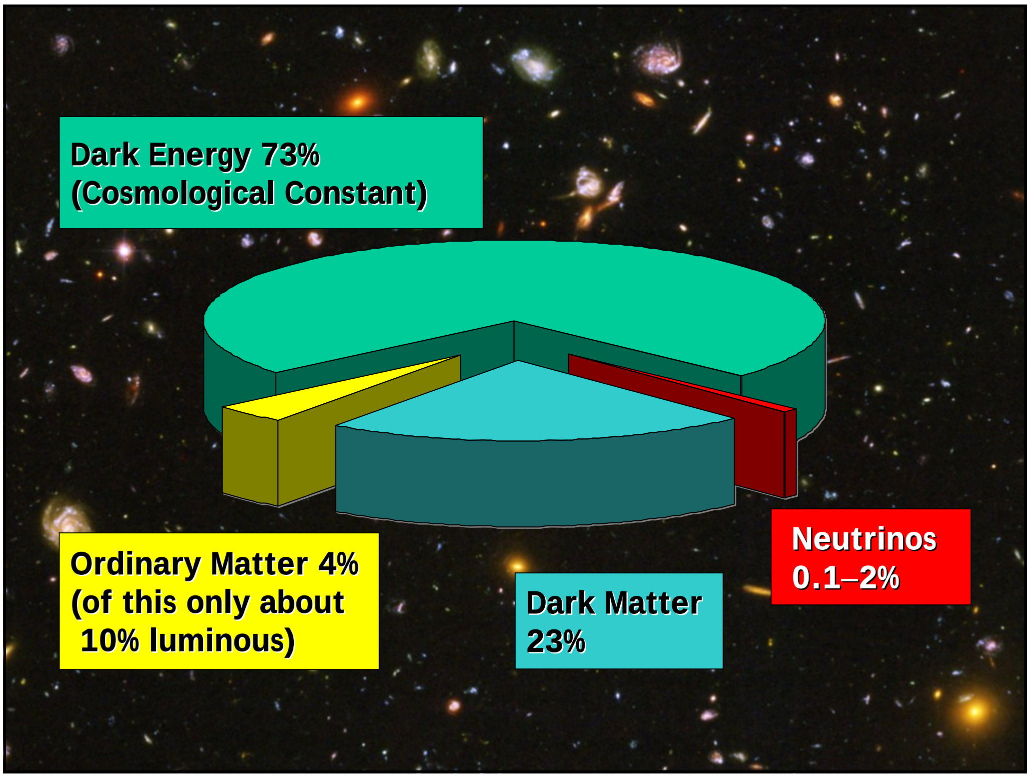
- Assume the cluster matter inventory represents a fair sample of the universe
- Use the measured baryon content Ω_B (BBN, CMBR)
- $h = 0.72 \pm 0.08$

Cosmic matter content

$\Omega_M = \Omega_B / f_{gas} = 0.325 \pm 0.034$

(Fabian, astro-ph/0304020)

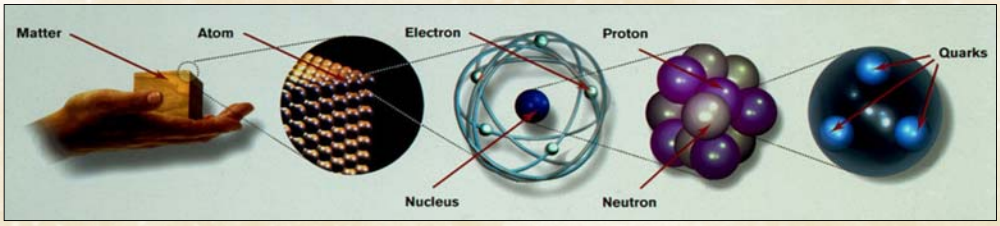

Bullet Cluster (1E 0657-56)


Dark Energy 73%
(Cosmological Constant)
Ordinary Matter 4%
(of this only about
10% luminous)
Dark Matter
23%
Neutrinos
0.1–2%

Dark Matter vs. Dark Energy

Dark Matter	Dark Energy
Acts graviationally like ordinary matter (attractive force)	• Provides “negative pressure” • “Anti-gravitation of the universe”
Probably new form of weakly interacting particles	• Cosmological constant (classical GR)? • Vacuum energy of quantum fields? • Quintessence (new scalar field)?
Dominates dynamics of galaxies, clusters, larger structures	Plays no role on small scales (homogeneous, does not cluster)
Decelerates cosmic expansion	Accelerates cosmic expansion
Possibly just an experimental problem (detect the dark matter particles!)	Probably a fundamental theory problem

Periodic System of Elementary Particles



	Quarks		Leptons	
	Charge +2/3	Charge −1/3	Charge −1	Charge 0
1. Family	Up u	Down d	Electron e	e-Neutrino ν_e
2. Family	Charm c	Strange s	Muon μ	μ-Neutrino ν_μ
3. Family	Top t	Bottom b	Tau τ	τ-Neutrino ν_τ
	Gravitation			
	Weak Interaction			
	Electromagnetic Interaction			
	Strong Interaction			