9.4 Thermal axions from the early universe (25 Jan. 2010)

The collective axion oscillations excited by the "misalignment mechanism" survive only if the friction caused by the interaction with other particles does not erase the axion condensate.

For sufficiently strongly interacting axions, they will thermalize and form a "hot population" of particles, not a cold one.

The misalignment mechanism operates just before the QCD phase transition. Directly afterwards, the main and most generic interaction process is with thermal pions

$$a + \pi \leftrightarrow \pi + \pi$$

Since $T_{\rm QCD} \sim 170$ MeV and $m_\pi = 135$ MeV, pions are essentially massless at that epoch. In this limit one finds for the axion absorption rate

$$\Gamma = A \left(\frac{1-z}{3(1+z)} \right)^2 \frac{T_{\rm QCD}^5}{f_a^2 f_\pi^2}$$

where $z = m_u/m_d = 0.56$ and one finds numerically the coefficient $A = 0.215$.

The Hubble expansion rate at that time is as usual

$$H = \sqrt{g_{\rm QCD}^*}\, 1.66\, T_{\rm QCD}^2 / m_{\rm Pl}$$

with $g_{\rm QCD}^* \sim 23$.

Axions are in thermal equilibrium at that epoch if $\Gamma \gtrsim H$, implying

$$A \left(\frac{1-z}{3(1+z)} \right)^2 \frac{T_{\rm QCD}^5}{f_a^2 f_\pi^2} \gtrsim \sqrt{g_{\rm QCD}^*}\, 1.66\, \frac{T_{\rm QCD}^2}{m_{\rm Pl}}$$

Therefore axions are in thermal equilibrium after $T_{\rm QCD}$ if

$$f_a \lesssim 4 \times 10^7 \text{ GeV} \qquad \text{and} \qquad m_a \gtrsim 0.15 \text{ eV}$$

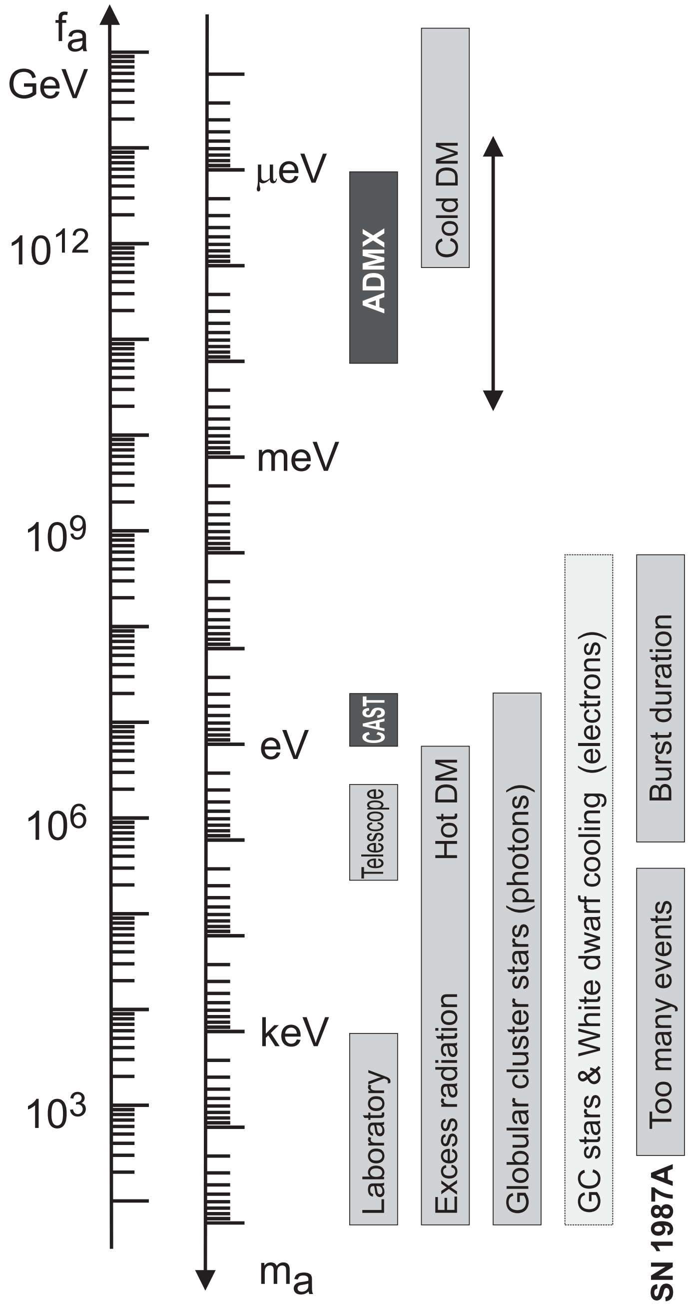
We conclude that

- Axions relevant for CDM are not thermalized.
- The HDM bounds on neutrino masses carry over to axions. In detail one finds that $m_a \gtrsim 1.0$ eV is excluded, taking into account the slightly different number densities of axions vs. neutrinos.

Cosmology alone significantly constrains the allowed range of axion parameters!

9.5 Astrophysical limits on axions

In addition to cosmological limits, there exist restrictive limits from stellar evolution. The main argument is that axions (like neutrinos) would be emitted from the interior of stars, escape, and drain energy. The observed properties of stars then allow one to further constrain axion parameters. These stellar evolution arguments will be discussed in a more general sense later.



9.6 Axion dark matter searches

9.6.1 Primakoff effect

The good news is that axion dark matter, if it exists, can be found in the laboratory. Main idea: axion-photon interaction.

$$\mathcal{L}_{a\gamma\gamma} = -\frac{G_{a\gamma\gamma}}{4} F_{\mu\nu}\widetilde{F}^{\mu\nu} = G_{a\gamma\gamma}\,\mathbf{E}\cdot\mathbf{B}\,a$$

Here F is the EM field-strength tensor, $\tilde{F}$ its dual, and a the axion field. The coupling constant (of dimension inverse energy) is written as

$$G_{a\gamma\gamma} = \frac{\alpha}{2\pi f_a}\left(\frac{E}{N} - \frac{2}{3}\frac{4+z}{1+z}\right)$$

where E and N are model-dependent coefficients quantifying the electromagnetic and color anomaly of the axion and $z = m_u/m_d$. In the simplest model $E/N = 0$.

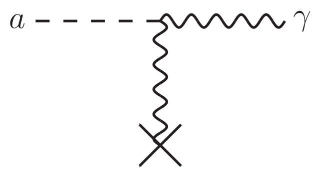
Like neutral pions, axions can decay into two photons, with a rate

$$\Gamma_{a\to\gamma\gamma} = \frac{G_{a\gamma\gamma}^2 m_a^3}{64\pi} = 1.1\times 10^{-24}\ \mathrm{s}^{-1}\left(\frac{m_a}{\mathrm{eV}}\right)^5$$

where for the numerical expression we used $E/N = 0$ and $z = m_u/m_d = 0.56$. Axions decay faster than the age of the universe if $m_a \gtrsim 20$ eV.

Free decay of axions very slow, so for dark matter axions impossible to detect. However, conversion between photons and axions possible in external field.

Simplest example: Primakoff process.



Differential cross section (no recoil of target particle)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha\, G_{a\gamma\gamma}^2}{8\pi}\frac{(\mathbf{k}_\gamma\times\mathbf{k}_a)^2}{|\mathbf{k}_\gamma - \mathbf{k}_a|^4}$$

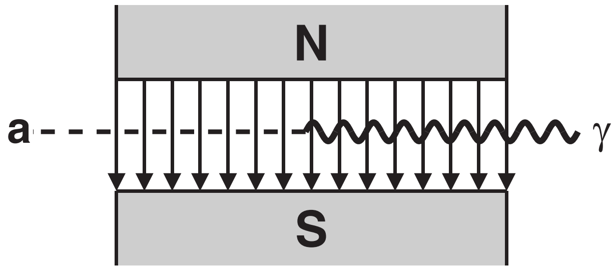
Total cross section therefore

$$\sigma \sim \frac{\alpha G_{a\gamma\gamma}^2}{8\pi} \sim \frac{\alpha}{8\pi}\left(\frac{\alpha}{2\pi f_a}\right)^2 \sim 10^{-61}\ \mathrm{cm}^2\left(\frac{10^{12}\ \mathrm{GeV}}{f_a}\right)^2$$

Hopelessly small for dark matter axions. However, interesting for producing axions in stars (see homework).

9.6.2 Coherent conversion in external B-field

Even for very small couplings, axion-photon conversion possible using coherent interaction in a macroscopic B-field.



The external B-field with oscillating axions field: Source for photons! Can be integrated over a large volume (macroscopic Primakoff effect). [P.Sikivie, Experimental tests of the invisible axion", PRL 51 (1983) 1415].

Add to standard EM Lagrangian the axion term from above

$$\mathcal{L}_{a\gamma\gamma} = -\frac{G_{a\gamma\gamma}}{4} F_{\mu\nu}\widetilde{F}^{\mu\nu} = G_{a\gamma\gamma}\, \mathbf{E}\cdot\mathbf{B}\, a$$

Modified Maxwell Eqs. become

$$\partial_\mu \tilde{F}^{\mu\nu} = 0$$
$$\partial_\mu F^{\mu\nu} = J^\nu + G_{a\gamma\gamma}\tilde{F}^{\mu\nu}\partial_\mu a$$

where J is EM four-current density. In non-covariant form this is

$$\nabla\cdot\mathbf{E} = \rho - G_{a\gamma\gamma}\mathbf{B}\cdot\nabla a \qquad \nabla\times\mathbf{E} + \dot{\mathbf{B}} = 0$$
$$\nabla\cdot\mathbf{B} = 0 \qquad \nabla\times\mathbf{B} - \dot{\mathbf{E}} = \mathbf{j} + G_{a\gamma\gamma}(\dot{a}\mathbf{B} + \nabla a\times\mathbf{E})$$

In the presence of a time- and space-varying axion field there is an extra effective charge density and current present in the inhomogeneous Maxwell Eqns.

In addition there is the modified Klein-Gordon Eqn for the axion field

$$\Box a = -G_{a\gamma\gamma}\mathbf{E}\cdot\mathbf{B} - m_a^2 a$$

9.6.3 Searches for galactic axions (26 Jan. 2010)

Typical properties of galactic axion dark matter

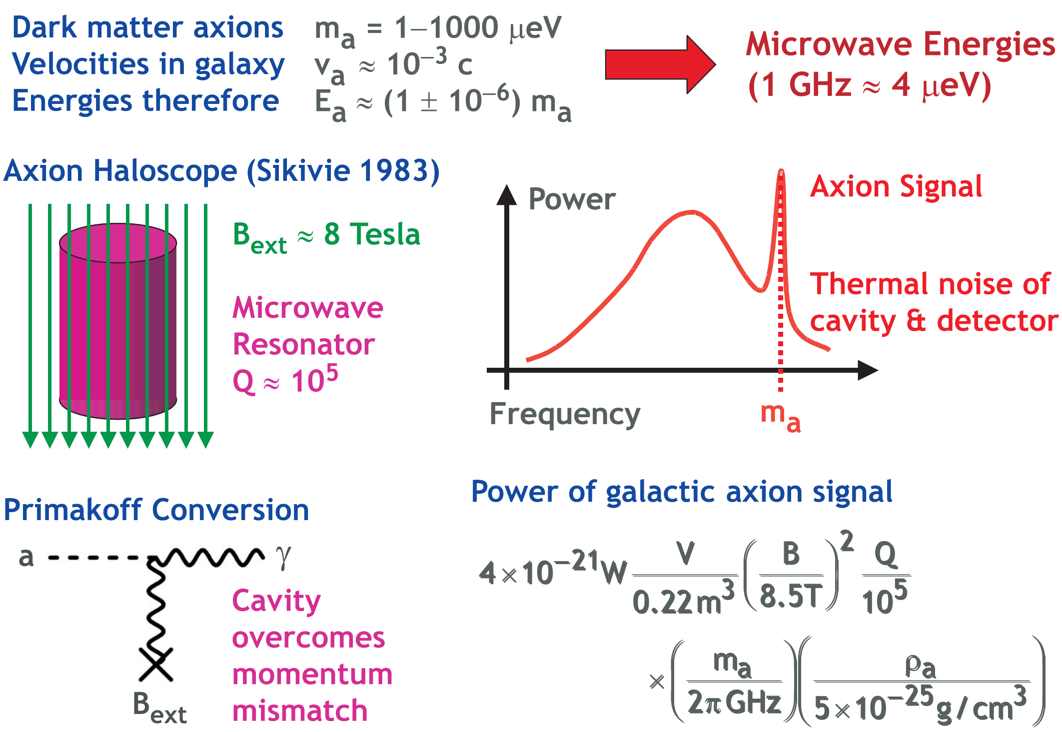
- Local density approximately 300 MeV cm^{-3}
- Local axion number density $\dfrac{\rho_{\rm DM}}{m_a} \sim 3\times10^{13}\ {\rm cm}^{-3}\ \dfrac{10\ \mu{\rm eV}}{m_a}$
- Typical velocity galactic virial velocity of $v \sim 10^{-3}\,c$
- Typical momentum $p = m_a v \sim 10^{-2}\ \mu$eV
- Corresponding wave length $\lambda = \dfrac{2\pi}{p} \sim 120$ m

Over laboratory scales, axion field is homogeneous, varies in time with mass m_a as frequency.

In the presence of a static B-field in the laboratory

$$\dot{\mathbf{E}} = -G_{a\gamma\gamma}\,\mathbf{B}\,\dot{a}$$

Produced photons massless: Wavelength much smaller than for axions, many wavelengths in an axion field region.



Units: 1 Watt = 10^7 erg s^{-1} = 6.24×10^{18} eV s^{-1}. So this power roughly 25 meV s^{-1}. For an axion mass of 10 μeV roughly 2500 axions per second. Not small!

Dissipation of photons in cavity with quality factor Q

$$\Gamma = \frac{\omega}{Q} = \frac{m_a}{Q}$$

For $m_a = 10\ \mu$eV and $Q = 2 \times 10^5$ and noting that a frequency of 1 GHz corresponds to an axion mass of 4 μeV we find a lifetime of a cavity mode to be

$$\tau = \Gamma^{-1} = \frac{2 \times 10^5}{10\ \mu\text{eV}} = 1.3 \times 10^{-5}\ \text{s}$$

Therefore, average occupation number from conversion

$$f_{\text{conversion}} \sim 0.03$$

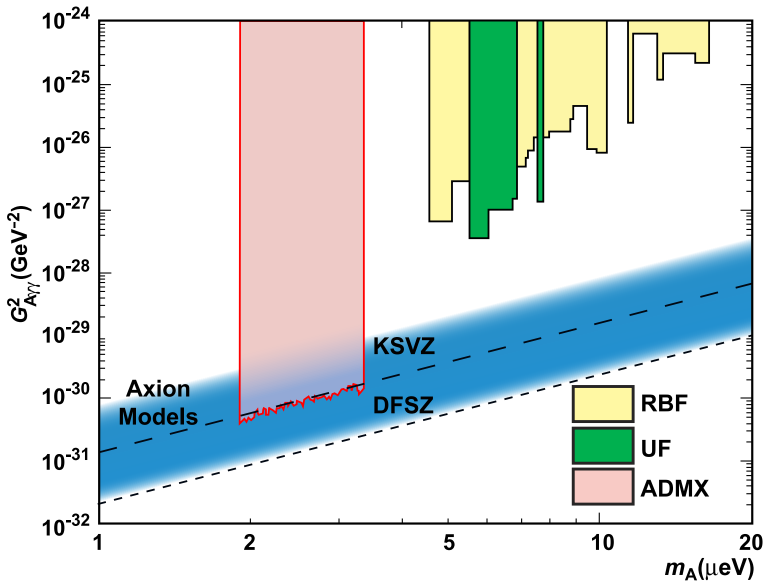
Compare with thermal occupation number at $T = 1$ K

$$f_{\text{thermal}} = \frac{1}{e^{\omega/T} - 1} = 8.13 \qquad \text{for} \qquad \frac{10\ \mu\text{eV}}{1\ \text{K}} = 0.116$$

Need low T to have large signal to noise

Cooling to $T \sim 10$ mK no real problem, but amplifier/detector also introduces noise $\rightarrow$ limiting factor.

Experiments of this sort have been performed since the 1980s, but only in a narrow range of masses has one reached a sensitivity coming close to the dark matter prediction.



Here KSVZ refers to a model where $E/N = 0$ in the axion-photon coupling and DFSZ to a model where $E/N = 8/3$.

Main problem: need to scan over broad range of masses and adjust the cavity frequency to the search mass. Slowly step through range of masses.

To improve speed need microwave amplifier with little internal noise. Recent development is to adjust SQUID amplifiers to GHz range and reach almost theoretical quantum limit.

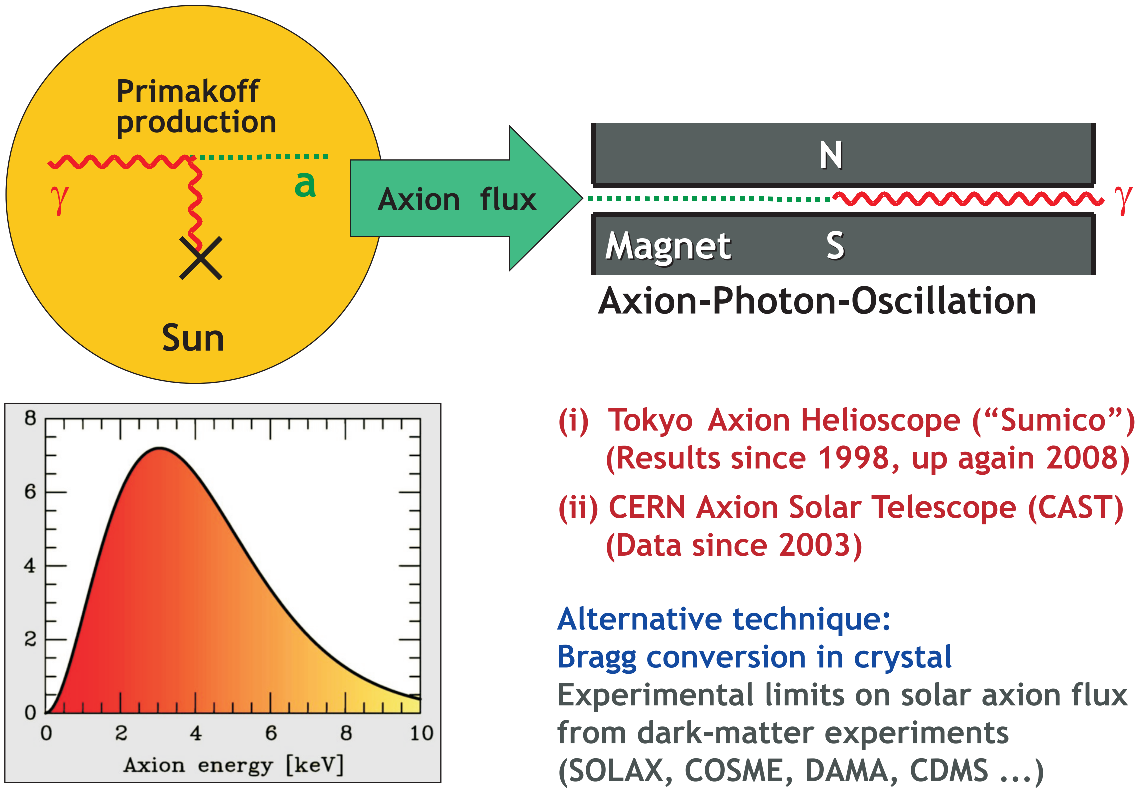
Has been proven recently. ADMX (axion dark matter experiment) will be refurbished to operate at cryogenic T of a few 10 mK.

With additional tunable cavities, this experiment is poised to find axion dark matter if it exists in the mass range 1–100 μeV.

9.7 Photon beam experiments

9.7.1 Photon-axion mixing

Assume relevant energies are sufficiently large that field region $\gg$ Compton wavelength for both photons and axions. Motivation by "helioscope" search for solar axions.



From the free Lagrangian and $\mathcal{L}_{a\gamma\gamma} = G_{a\gamma\gamma}\,\mathbf{E}\cdot\mathbf{B}\,a$ follows the wave eqn

$$\begin{aligned}\Box\mathbf{A} &= +G_{a\gamma\gamma}\,\mathbf{B}\,\partial_t a\\ (\Box + m_a^2)\,a &= -G_{a\gamma\gamma}\,\mathbf{B}\cdot\partial_t\mathbf{A}\end{aligned}$$

where in addition it was assumed that there is an external static field $\mathbf{B}$ that is strong compared to the time-varying and propagating field represented by $\mathbf{A}$.

Consider the propagation of a wave in the z–direction with fixed frequency ω. Assume $\mathbf{B}$ transverse to propagation direction yield wave equation

$$\left[-\omega^2 - \partial_z^2 + \begin{pmatrix} m_{\gamma\perp}^2 & 0 & 0\\ 0 & m_{\gamma,\parallel}^2 & \mathrm{i}G_{a\gamma\gamma}B\omega\\ 0 & -\mathrm{i}G_{a\gamma\gamma}B\omega & m_a^2\end{pmatrix}\right]\begin{pmatrix}A_\perp\\ A_\parallel\\ a\end{pmatrix} = 0$$

External B field mixes $A_\parallel$ with a.

9.7.2 Photon dispersion

In the presence of external fields or media, photon propagation is not identical to that of massless particles. Dispersion relation

$$k = n\omega \qquad \text{or} \qquad \omega^2 - k^2 = m_\gamma^2$$

where n is the refractive index and

$$\begin{aligned}K^2 &= m_\gamma^2 > 0 \qquad \text{time-like photon}\\ K^2 &= m_\gamma^2 < 0 \qquad \text{space-like photon}\end{aligned}$$

Connection to refractive index

$$m_\gamma^2 = \omega^2 - k^2 = \omega^2 - n^2\omega^2 = (1-n^2)\omega^2 = (1-n)(1+n)\omega^2$$

If $|n-1| \ll 1$ we have $1+n \approx 2$. Therefore

$$1 - n = \frac{m_\gamma^2}{2\omega^2}$$

In air or water $n > 1$, photons are space-like, allowing for Cherenkov effect.

In a nonrelativistic plasma the photon dispersion relation is

$$m_\gamma^2 = \omega_{\rm plas}^2 = \frac{4\pi\alpha}{m_e}\,n_e$$

where $\omega_{\rm plas}$ is the plasma frequency.

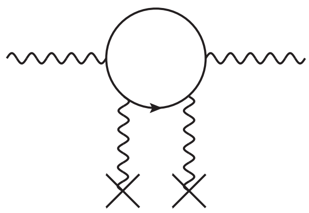
In an external strong B-field we also have a nontrivial dispersion relation.

$$n_\parallel = 1 + 7\xi \qquad \text{and} \qquad n_\perp = 1 + 4\xi$$

where

$$\xi = \frac{2\alpha^2}{45}\frac{B^2}{m_e^4} = 1.32 \times 10^{-32}\left(\frac{B}{\text{Gauss}}\right)^2 = 1.32 \times 10^{-22}\left(\frac{B}{10\ \text{Tesla}}\right)^2$$

1 Tesla $= 10^4$ Gauss. In the laboratory, 10 Tesla is a typical field strength that can be reached in superconducting magnets, e.g. for NMR tomography or for LHC bending magnets. The largest astrophysical B-fields are probably in pulsars with $B = 10^{12}$–10^{13} Gauss typical values, but much larger fields have been suggested in so-called magnetars.



Strong astrophysical B fields also allow for new processes such as photon splitting.

The effect of having different refractive indices for the linear photon polarizations in a transverse B-field is called the Cotton-Mouton effect. In vacuum with a B-field it is called the vacuum Cotton-Mouton effect. Photon birefringence in a longitudinal field in a medium is called the Faraday effect.

Natural unit for magnetic field strength is "critical field" where the cyclotron frequency $\hbar eB/m_e c$ of an electron equals its rest energy $m_e c^2$, or in natural units

$$B_{\text{crit}} = \frac{m_e^2}{e} = \frac{m_e^2}{\sqrt{4\pi\alpha}} = 4.413 \times 10^{13}\ \text{Gauss}$$

However, no upper limit to possible field strengths.

Different for electric fields. When field energy in a volume of dimension the electron's Compton wave length is comparable to the electron mass, spontaneous pair creation. In other words, the fluctuating e^+e^- pairs from the vacuum can be pulled apart (Schwinger process). Vacuum not stable for large electric fields which naturally neutralize themselves.