# 9 Axions (19 Jan. 2010)

## 9.1 General remarks

In principle there exist many possible dark matter candidates. It is usually assumed that dark matter should consist of new particles because one can not think of other plausible possibilities. Primordial black holes, for example, would be ok, but no plausible mechanism for producing them.

Particles as dark matter probably must have the following properties.

- Probably weakly interacting.
- Stable over cosmic time scales.
- Cold, i.e. small free streaming length until matter-radiation equality so they do not erase primordial structures.
- Produced with the appropriate abundance in the early universe.

Ideally the particles are also

- Well motivated from a particle-physics perspective.
- Can be discovered in realistic experiments.

WIMPs in the form of supersymmetric neutralinos fulfill these conditions. They are the prime example for **thermal relics**.

The other most often discussed "good candidate" is the axion. It fulfills all of these conditions, except that it is a **non-thermal** relic from the early universe.

Axions are special in many ways. They are

- Pseudoscalar particles, like $\pi^0$.
- Very low mass, typically for dark matter $m_a \sim 10\ \mu\text{eV}$.
- In spite of small mass cold dark matter, essentially a condensate.
- Ongoing experiments to detect dark matter axions.

## 9.2 Basic idea

Motivation for axions deeply in the guts of low-energy QCD. Ground-state of QCD not unique: Nontrivial topologically different gauge-field configurations. Each ground state (vacuum) characterized by the angle parameter

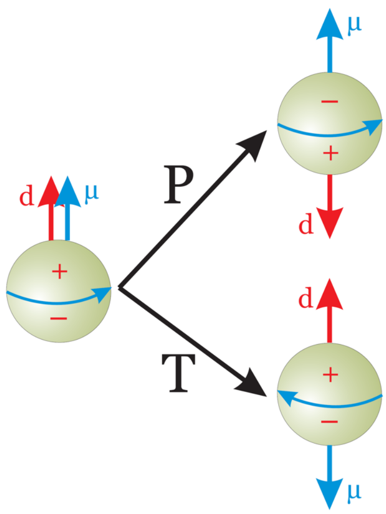
$$-\pi \leq \Theta \leq +\pi$$

Induces a piece of the QCD Lagrangian of the form

$$\mathcal{L}_\Theta = \Theta \, \frac{\alpha_s}{8\pi} \, G_{\mu\nu a} \tilde{G}_a^{\mu\nu} \propto \mathbf{E}_{\rm gluon} \cdot \mathbf{B}_{\rm gluon}$$

where $G$ is the gluonic field strength tensor and $\tilde{G}$ its dual. The r.h.s. is odd under CP transformations and induces a neutron electric dipole moment (EDM).

An EDM is a CP-odd quantity, allowing one to distinguish between neutrons and antineutrons in an absolute sense because a magnetic and electric dipole moment both reverse under charge conjugation C but opposite under spatial reflection P.




The natural scale for an EDM is the elementary charge, displaced roughly by the size (Compton wave length) of the particle, i.e,

$$\frac{e}{2m_N} = 1.06 \times 10^{-14} \, e \text{ cm}$$

The experimental limit is

$$|d_n| < 0.63 \times 10^{-25} \, e \text{ cm}$$

Implying a limit on the coefficient implied by $\mathcal{L}_\Theta$

$$\Theta \, \frac{m_q}{m_N} \lesssim 10^{-11}$$

The smallness of this parameter that could take on any value is called the CP problem of QCD or the strong CP problem.

In other words, the problem is that QCD is fundamentally a CP violating theory through $\mathcal{L}_\Theta$, yet no CP-violating effects have been observed.

The Peccei-Quinn mechanism to solve this problem appeals to "dynamical symmetry restoration." It amounts to re-interpreting the coefficient $\Theta$ as a physical field, the axion field, or more exactly

$$\mathcal{L}_\Theta = \Theta\,\frac{\alpha_{\rm s}}{8\pi}\,G_{\mu\nu a}\tilde{G}_a^{\mu\nu} \to \mathcal{L}_a = \frac{a}{f_a}\,\frac{\alpha_{\rm s}}{8\pi}\,G_{\mu\nu a}\tilde{G}_a^{\mu\nu}$$

It turns out that this term induces a potential for the axion field which is exactly such that the energy is minimized at the CP-conserving value.

The energy scale $f_a$ ("axion decay constant") is the only free parameter of the model, except for some model-dependent coefficients.

The axion mass arises from mixing with the $\pi^0$ and has the value

$$m_a = \frac{\sqrt{m_u m_d}}{m_u + m_d}\frac{m_\pi f_\pi}{f_a} = \frac{0.60\ \text{meV}}{f_a/10^{10}\ \text{GeV}}$$

where $m_\pi = 135$ MeV, the pion decay constant is $f_\pi = 93$ MeV, and $m_u$ and $m_d$ are the up and down quark masses with the canonical ratio $z = m_u/m_d = 0.56$ although a range 0.35–0.60 is possible.

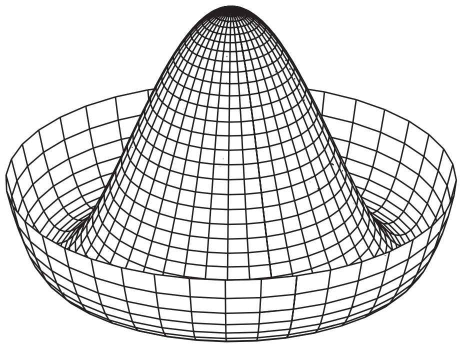
All coupling constants of axions to ordinary matter and photons are essentially those of the neutral pions times $f_\pi/f_a$. Axions can be very light and very weakly interacting, even though they arise from QCD.

## 9.3 Axion dark matter: Early universe creation

To implement the idea of axions one needs a new Higgs field with a Mexican hat potential, where the axion field is the angular mode of the field

$$\Phi(x) = \frac{f_a + \rho(x)}{\sqrt{2}}\, \mathrm{e}^{\mathrm{i}a(x)/f_a}$$

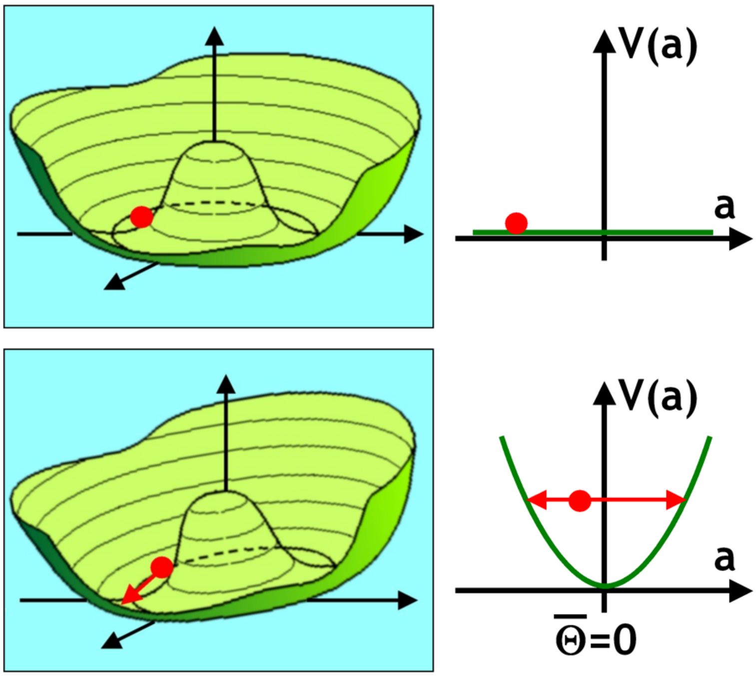
Essentially the axion field is a periodic variable.



Actually, the Mexican hat is tilted by QCD effects, so in the rim of the Mexican hat the axion field rolls to the CP conserving minimum.

This concept already has all ingredients for the cosmological role of axions.

- In the very early universe, $T \gg f_a$, the PQ symmetry is unbroken and the minimum of the Higgs potential at $\Phi = 0$.
- When $T$ falls below $f_a$, spontaneous breaking occurs and a minimum is selected somewhere in the Mexican hat. The potential for the axion $V(a)$, being the slope in the rim of the Mexican hat, is flat.
- When $T$ reaches $\Lambda_{\mathrm{QCD}} \sim 170$ MeV, QCD instanton effects induce a potential for the axion, corresponding to the Mexican hat being tilted in such a direction that the minimum is at the CP-conserving position.
- The axion began in some random position in the Mexican hat, so normally it will not be at the minimum position.
- The slope of the potential drives the axion field towards the minimum, but without significant friction it simply begins oscillating.
- Such classical oscillations of the axion field are tantamount to a boson condensate: Many axions are in the same highly-occupied state.
- From the very beginning, axions are nonrelativistic and thus form cold dark matter (CDM).




The calculation of the axion density today involves very few basic ingredients. Scalar field evolution in the expanding universe

$$\ddot{\Phi} + 3H\dot{\Phi} + (k^2 + m^2)\Phi = 0$$

Hubble expansion provides "friction term," i.e. for modes with $\omega = \sqrt{k^2 + m^2} \lesssim H$ the time evolution is strongly affected by expansion term.

For axions initially in the zero-momentum (homogeneous)mode:

$$\ddot{\Phi} + 3H\dot{\Phi} + m^2\Phi = 0$$

For $3H \gg m$ no motion, whereas for $3H \ll m$

$$\Phi(t) = \underbrace{\Psi(t)}_{\text{Slowly varying amplitude}} \sin(mt)$$

and therefore

$$\dot{\Phi} = \dot{\Psi}\sin(mt) + m\Psi\cos(mt)$$
$$\ddot{\Phi} = \ddot{\Psi}\sin(mt) + 2m\dot{\Psi}\cos(mt) - m^2\Psi\sin(mt)$$

Insert in equation of motion

$$\ddot{\Psi}\sin(mt) + 2m\dot{\Psi}\cos(mt) - m^2\Psi\sin(mt) + m^2\Psi\sin(mt)$$
$$+ 3H\left[\dot{\Psi}\sin(mt) + m\Psi\cos(mt)\right] = 0$$

Terms $\propto m^2$ cancel, collect fastest-moving terms $\propto m$

$$2m\dot{\Psi} + 3Hm\Psi = 0 \,\Big|\, \times \frac{\Psi}{2}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\left(\frac{1}{2}\,m\Psi^2\right)}_{\text{Number density}} = -3H\left(\frac{1}{2}\,m\Psi^2\right)$$

Quantity $\frac{1}{2}\,m\,\Psi^2$ evolves in the same way as a number density of particles.

The energy density in the homogeneous field mode is

$$\rho = \frac{1}{2}\left(m^2\,\Phi^2 + \dot{\Phi}^2\right)$$

Using again $\Phi = \Psi\,\sin(mt)$ and using only the dominant piece in the time derivative that is proportional to $m$ we have

$$\rho = \frac{1}{2}\,\left[m^2\,\Psi^2\sin^2(mt) + \Psi^2 m^2\cos^2(mt)\right] = \frac{1}{2}\,m^2\,\Psi^2$$

The energy density is of course conserved in static space. The number density is once more

$$n = \frac{\rho}{m} = \frac{1}{2}\,m\,\Psi^2$$

Axion number in comoving volume is conserved since the field started oscillating at the epoch denoted as 1. Number density today relative to that epoch

$$n_0 = \underbrace{\left(\frac{a_1}{a_0}\right)}_{\text{Ratio of cosmic scale factors}}^3 n_1$$

Adiabatic evolution: Comoving entropy density conserved, i.e.

$$g_s^* T^3 a^3 = \text{const.} \qquad \Rightarrow \qquad \left(\frac{a_1}{a_0}\right)^3 = \frac{g_{s,0}^* T_0^3}{g_{s,1}^* T_1^3}$$

Axion field when it starts oscillating: $a_1 = \Theta_1 f_a$

$$n_1 = \frac{1}{2}\,m_1\Theta_1^2 f_a^2$$

Here $m_1$ is not the vacuum axion mass, but the effective mass it has when it starts oscillating (to be determined).

Axion field starts oscillating when the oscillating term overcomes Hubble friction, i.e. when

$$m_1 \sim 3H_1$$

Therefore axion density at that epoch

$$n_1 = \frac{1}{2}\,3H_1\,\Theta_1^2 f_a^2$$

Here $H_1$ does not depend much on details because mass switches on within a short epoch. Take at first $H_1$ independent of axion parameters.

Present-day axion density then in terms of present-day axion mass $m_a$

$$\rho_0 = m_a n_0 = \frac{1}{2}\,m_a 3H_1\Theta_1^2 f_a^2\,\frac{g_{s,0}^* T_0^3}{g_{s,1}^* T_1^3} \propto m_a f_a^2 = \frac{m_a^2 f_a^2}{m_a} \sim \frac{m_\pi^2 f_\pi^2}{m_a} \propto m_a^{-1}$$

Inversely proportional to axion mass!

To find $H_1$ need to know how Mexican hat tilts, i.e. the $T$-dependent axion mass. Calculation of instanton effects at high $T$ gives estimate for the potential $V(\Theta)$.

Most recent evaluation (Bae et al. arXiv:0806.0497) with latest results for $\Lambda_{\rm QCD}$ and quark masses:

$$V(\Theta)\big|_{T\sim 1~\rm GeV} = (1.41~\text{MeV})^4 \left(\frac{\text{GeV}}{T}\right)^{6.878} \underbrace{(1-\cos\Theta)}_{\frac{1}{2}\frac{\Phi_a^2}{f_a^2}} = \frac{1}{2}\,m_a^2(T)\,\Phi_a^2$$

Therefore

$$m_a(T) = \frac{(1.41~\text{MeV})^2}{f_a}\left(\frac{\text{GeV}}{T}\right)^{3.439}$$

Hubble parameter around that time

$$H = \sqrt{g^*}\,1.66\,\frac{T^2}{m_{\rm Pl}}$$

The criterion that axions start oscillating when $m_a(T) \sim H(T)$ and using $g^* \sim 60$ provides

$$T_1 \sim 0.92~\text{GeV}\,f_{12}^{-0.184} \qquad \text{where} \qquad f_{12} = \frac{f_a}{10^{12}~\text{GeV}}$$

At that epoch

$$3H_1 = 3\sqrt{g^*}\,1.66\,\frac{T_1^2}{m_{\rm Pl}} \sim 3~\text{neV}$$

Without anharmonic corrections and using

$$m_a = 6~\mu\text{eV}\,\frac{1}{f_{12}}$$

and with $g^*_{s,0} \sim 3.91$ (including neutrinos) one finds

$$\rho_a = 1.15 \text{ keV cm}^{-3}\ \Theta_1^2\, f_{12}^{1.184}$$

In units of the critical density $\rho_{\rm crit} = h^2\, 10.54$ keV cm$^{-3}$ this is

$$\Omega_a h^2 = 0.109\,\Theta_1^2 \left(\frac{f_a}{10^{12}\text{ GeV}}\right)^{1.184} = 0.059\,\Theta_1^2 \left(\frac{10\ \mu\text{eV}}{m_a}\right)^{1.184}$$

It depends on $-\pi \leq \Theta_1 \leq +\pi$ which is a random variable. One expects $\Theta_1$ to be of order unity, but in principle can be very small.

For $\Theta_1$ of order unity, axions with mass of order 10 $\mu$eV would be the dark matter.

Assume no inflation after PQ symmetry breaking: $\Theta_1$ settles at different values in $-\pi < \Theta_1 < +\pi$. On average one finds

$$\langle \Theta_1^2 \rangle = \frac{\pi^2}{3}$$

Ignoring anharmonic effects and using $h = 0.74$ one finds

$$\Omega_a \sim 0.36 \left(\frac{10\ \mu\text{eV}}{m_a}\right)^{1.184}$$

If axions are the dark matter: $\Omega_a \sim 0.22$ and $m_a \sim 15\ \mu$eV.