8 Weakly interacting particles as dark matter

8.1 Light neutrinos (11 Jan. 2010)

From the Lee-Weinberg-curve we have seen that neutrinos, after relativistic freeze-out, can be the dark matter if

$$\sum m_{\nu} \sim 12 \text{ eV}$$

From oscillation experiments we know that the three ordinary neutrinos have equal masses up to quadratic differences of $\Delta m^2 \sim (50 \text{ meV})^2$.

Experimental limit on common neutrino mass scale from tritium beta-decay experiments

 $m_{\nu} \lesssim 2.2 \text{ eV}$ at 95% CL

Will be improved to about 0.2 eV by KATRIN experiment unless there is a detection.

Therefore ordinary neutrinos can not make up all of the dark matter.

Even before we knew that neutrino masses are degenerate (so tritium decay limits all of them), low-mass particles like neutrinos with masses in the eV-range have problems being the dark matter in galaxies because of limited phase space.

Assume galaxy properties

| r | linear dimension of galaxy |
|-----------------------------------|----------------------------|
| σ | velocity dispersion |
| $\sigma^2 = \frac{G_{\rm N}M}{m}$ | virial theorem |

Phase-space volume = volume in coordinate space \times volume in momentum space

$$\Gamma = V_{\mathbf{r}} \times V_{\mathbf{p}} \sim r^3 \times p^3 \sim r^3 \times (m\sigma)^3$$

In natural units this is a pure number (number of phase-space cells), multiply with \hbar for ordinary units.

Total mass = phase-space volume \times average occupation number \times particle mass

$$M = \langle f \rangle m \, \Gamma$$

Remove the total mass from the expression (with the virial theorem)

$$\frac{r\,\sigma^2}{G_{\rm N}} = \langle f \rangle m \times r^3 \times (m\sigma)^3$$

This implies

$$G_{\rm N} m^4 \, \sigma \, r^2 \sim \langle f \rangle^{-1}$$

involving as observational quantities the geometrical size and velocity dispersion of the bound object.

If the particles are fermions (neutrinos) the Pauli principle requires f < 1. More general, however: For freely streaming (collisionless) gas, Liouville theorem implies that average phase-space density remains unchanged. (Free streaming only mixes phase space.)

If particles were originally in thermal equilibrium, $f \sim 1$, so for bosons and fermions $\langle f \rangle \lesssim 1$.

Therefore limit on particle mass (Tremaine–Gunn–limit)

$$m \gtrsim (G_{\rm N}\sigma r^2)^{-1/4} = 20 \text{ eV} \left(\frac{100 \text{ km s}^{-1}}{\sigma}\right)^{1/4} \left(\frac{10 \text{ kpc}}{r}\right)^{1/2}$$

In detail modified by numerical factors.

For Milky Way roughly $m \gtrsim 30$ eV. Larger particle masses required for smaller systems such as dwarf galaxies.

Strange coincidence that small scale structures provide a lower limit on m very close to the upper limit for these particles not to provide too much dark matter.

Particles with masses of a few eV are no credible dark matter candidates from small-scale structure.

We will see later that certain dark matter candidates (axions) are very low-mass bosons. However, they were never in thermal equilibrium and have huge phase-space occupation numbers. In this case, the previous arguments do not apply.

8.2 Hot vs. cold dark matter

8.2.1 Free streaming length

Small-mass particles also have problems with structure formation. When the interaction rate of particles has become so small that their mean fee path exceeds the Hubble scale (freeze out), they henceforth stream freely (collisionless).

Density inhomogeneities imprinted previously will be smeared out ("collisionless phase mixing"). Inhomogeneities on small scales, needed to seed structure formation, will be erased.

How large is the free-streaming length, measured in the present-day universe? Structure formation begins at the epoch of matter-radiation equality. We found its redshift in the Homework (week 7) to be

$$z_{\rm eq} = 3570$$

We need to calculate the distance traveled between decoupling and this epoch.

The physical distance traveled during a short time interval dt at an epoch t is, assuming radial motion in suitably chosen polar coordinates,

$$v(t) \, \mathrm{d}t = a(t) \, \mathrm{d}r$$

where v is the particle velocity (usually smaller than the speed of light), a(t) the cosmic scale factor, and dr is the distance traveled in co-moving coordinates.

Measured in the present-day universe, the free-streaming distance traveled between epochs A and B is

$$D_{\rm fs} = a_0 (r_B - r_A) = a_0 \int_A^B dt \, \frac{v(t)}{a(t)} = \int_A^B dt \, \frac{v(t)}{y(t)}$$

where as usual $y = a/a_0$ is the cosmic scale factor in units of the present-day value, so at present y = 1.

Transform as usual with the help of the Friedman Eqn $H = \dot{y}/y$, so dt = dy/(Hy) and

$$D_{\rm fs} = \int_A^B \mathrm{d}y \, \frac{v}{y^2 H}$$

During radiation dominated epoch, valid until $y_{\rm eq}$, we have $H \propto y^{-4}$ and so

$$H = H_{\rm eq} \frac{y_{\rm eq}^2}{y^2} \qquad \Rightarrow \qquad D_{\rm fs} = \frac{1}{H_{\rm eq} y_{\rm eq}^2} \int_A^B \mathrm{d}y \, v$$

Before particle becomes non-relativistic v = 1, later decreases like momentum with cosmic expansion: $v = y_{nr}/y$ and so

$$D_{\rm fs} = \frac{1}{H_{\rm eq} y_{\rm eq}^2} \left(\int_0^{y_{\rm nr}} \mathrm{d}y + \int_{y_{\rm nr}}^{y_{\rm eq}} \mathrm{d}y \, \frac{y_{\rm nr}}{y} \right) = \frac{y_{\rm nr}}{H_{\rm eq} \, y_{\rm eq}^2} \left(1 + \log \frac{y_{\rm eq}}{y_{\rm nr}} \right)$$

Hubble expansion rate in simple Friedman models

$$H = H_0 \sqrt{\Omega_{\rm R} y^{-4} + \Omega_{\rm M} y^{-3} + \Omega_{\Lambda}}$$

At equality neglect Λ term and by definition the radiation and matter term are equal and equal to $\Omega_{\rm M} y_{\rm eq}^{-3}$. Therefore

$$H_{\rm eq} = H_0 \sqrt{2\Omega_{\rm M} y_{\rm eq}^{-3}}$$

Free-streaming length then

$$D_{\rm fs} = H_0^{-1} \frac{y_{\rm nr}}{\sqrt{2\Omega_{\rm M} y_{\rm eq}}} \left(1 + \log \frac{y_{\rm eq}}{y_{\rm nr}}\right)$$

A particle x becomes nonrelativistic roughly when $3T_x = m_x$ and so

$$y_{\rm nr} \sim \frac{3T_{x,0}}{m_x} = \frac{3T_{\gamma,0}}{m_x} \frac{T_{x,0}}{T_{\gamma,0}}$$

where the second factor accounts for heating of photons relative to x particles by annihilations of other particles between the epoch of decoupling and today.

For neutrino-like particles take $T_{x,0}/T_{\gamma,0} = (4/11)^{1/3}$, implying

$$z_{\rm nr} + 1 = \frac{1}{y_{\rm nr}} = 2.0 \times 10^6 \, \frac{m_x}{\rm keV}$$

So finally

$$D_{\rm fs} = 1.2 \,\,\mathrm{Mpc} \, \frac{\mathrm{keV}}{m_x} \, \left(1 + \frac{1}{7.3} \log \frac{m_x}{\mathrm{keV}} \right)$$

The matter from which a galaxy forms comes from a volume with a size of a fraction of a Mpc, so if $m_x \lesssim$ few keV, such structures erased by free streaming: hot dark matter. In the opposite extreme: cold dark matter. For masses of a few keV: warm dark matter.

In CDM scenarios, structure forms bottom up: smallest objects form first.

HDM is a top-down scenario: Large structures form first and fragment later.

Cold dark matter today the standard paradigm, usually referred to as Λ CDM models: dominant dark components are cosmological constant and cold dark matter.

8.2.2 Subdominant hot dark matter

Neutrinos exist and are known to have small masses, but are not the dominant dark matter: Even in a Λ CDM cosmology a small admixture of hot dark matter.

Free streaming somewhat erases small-scale structure, less seed power on small scales.

As an example we show N-body simulation, left with CDM alone, right with additional neutrinos with sum of the masses $\sum m_{\nu} = 6.9$ eV at experimental limit. Clearly small-scale structures are blurred. (For complete movie see: http://www.phys.au.dk/~haugboel)



The power spectrum of the observed cosmic density distribution reveals that neutrino free streaming must have been a subdominant effect: One derives very restrictive limits on the HDM fraction, translating into a limit of approximately

$$\sum m_{\nu} \lesssim 0.6 \text{ eV}$$

At present time, this is the most restrictive limit on the overall neutrino mass scale.

Can be translated to mass limit on other hypothetical low-mass particles such as axions.

8.3 Invisible width of the Z-boson

Ordinary neutrinos not possible as dark matter. Are there heavier neutrino-like particles, perhaps a fourth generation of elementary particles?

Count "sequential neutrinos" by Z^0 decay width. An unstable particle (or an excited level of an atom etc.) does not have a fixed mass or energy: the decay implies a "width." (Like a harmonic oscillator with friction: resonance frequency not exactly defined.

Amplitude of an unstable particle (or damped oscillator) varies in time as

$$\psi(t) = \mathrm{e}^{-\mathrm{i}Et - \Gamma t/2}$$

and probability declines as

$$p(t) = |\psi(t)|^2 = e^{-\Gamma t}$$
 with lifetime $\tau = \Gamma^{-1}$

Normalized "energy spectrum" of unstable particle at rest given by Lorentz resonance

$$f(E) = \frac{1}{\pi} \frac{\Gamma/2}{(E-m)^2 + (\Gamma/2)^2}$$
 with $\int_{-\infty}^{+\infty} dE f(E) = 1$

Has full width at half maximum ("width") of Γ .

Particle has no fixed energy because it can not exist forever. Exists only for roughly $\tau \sim \Gamma^{-1}$. "Energy time uncertainty relation": The energy of a particle existing for time Δt not better defined than $\Delta E \gtrsim 1/\Delta t \sim \Gamma$.

Unstable particle produced in a reaction, for example Z^0 in e^+e^- collision at old LEP storage ring at CERN (predecessor of LHC, with electrons and positrons in the pipes).



Can be viewed as pair production. But if e^+e^- energies on resonance, an "on-shell" or "real" Z-boson is produced: Production and decay a two-step process. If energies do not match, Z is "off shell" or "highly virtual" and the reaction a single coherent event.

In overall reaction, energy conserved to the extent of the energy definition of initial beams.

If we measure the Z, forget memory of production and set t = 0. Like for any decay process: no memory of previous history if we measure the particle at some t.



Width of resonance Γ determined by "total friction", i.e. by all decay channels, visible or invisible. Secondary neutrino pairs can not be measured \rightarrow invisible decays.

Properties of Z boson measured at LEP

 $m_Z = (91.1876 \pm 0.0021) \text{ GeV}$ and $\Gamma = (2.4952 \pm 0.0023) \text{ GeV}$

Invisible width (neutrinos or hypothetical others)

 $\Gamma_{\rm invis} = (499.0 \pm 1.5) \,\,{\rm MeV}$

Standard-model width contribution from one neutrino flavor

$$\Gamma_{\nu\bar{\nu}} = 167.2 \text{ MeV}$$

Comparing with invisible width yields

 $N_{\nu} = 2.984 \pm 0.008$

No room for new weakly interacting particles that couple to the Z boson!

A "neutrino-like" particle χ must fulfill at least one of the following conditions

$$m_{\chi} > \frac{m_Z}{2} = 46 \text{ GeV} \quad \text{and/or} \quad \frac{g_{\chi}^2}{g_{\nu}^2} \lesssim 0.008$$

where g is its effective gauge-coupling constant to the Z-boson. (Note that N_{ν} is 2σ below 3, so little room for extra species.)

8.4 Supersymmetric particles (12 Jan. 2010)

Weakly interacting massive particles (WIMPs) remain an interesting thermal relic particle for the CDM of the universe—but what is its natures? How would such particles fit into the overall picture of particle physics?

Supersymmetric partners to ordinary particles the front runners. Motivated by the "gauge hierarchy problem" of the standard model, essentially consisting of the question why the Fermi scale is so much smaller than the Planck scale.

Quantum field theory has mathematical problems, divergences, that need not have a simple interpretation.

Simplest example: Self-energy. Even classically: electrostatic energy in the Coulomb field of a point-like particle is infinite.



Usually divergences can be "renormalized." Physical answer obtained by subtracting divergences in judicious way, and only need to use physical electron mass.

Renormalization crucial ingredient for self-consistent quantum field theory.

Modern interpretation: Field theory only an effective description at low energies, beyond some scale (below some distance), something new must happen, e.g. string theory.

However problems in connection with scalar field (Higgs field). Its mass can not be renormalized, bubble graph too divergent.



Closely related to divergence of vacuum energy where all fields contribute.

One way out: every bosonic degree of freedom has a fermionic partner, contribute with opposite signs to divergence.

However, existing particles are not superpartners. Moreover, bosons are gauge bosons, fermions carry gauge charges.

Need new particles that pair off as follows:

Fermions (spin 1/2) \Leftrightarrow Sfermions (spin 0) Gauge bosons (spin 1) \Leftrightarrow Gauginos (spin 1/2)

So spin-1 gauge particles are Photon, Weak gauge bosons and gluons, whereas the spin-1/2 gauge particles are the photino, Wino, Zino and gluinos.

The "matter particles" are the spin-1/2 quarks and leptons as well as the higgsino, whereas the spin-0 ones are the squarks, sleptons and the Higgs.

SUSY not exact, mass of the partners are different (or else would have been detected a long time ago). "Good properties" to stabilize electroweak scale remain as long as

$$(m_{\rm boson}^2 - m_{\rm fermion}^2)g^2 \sim \delta M_H^2 \lesssim M_W^2$$

SUSY particles should exist with masses $\lesssim M_W/g \sim 1$ TeV. \Rightarrow Hoping to find SUSY particles at LHC.

Ordinary particles and SUSY partners distinguished by new multiplicative quantum number, R-parity.

$$R = \begin{cases} +1 & \text{Particles} \\ -1 & \text{S-Particles} \end{cases}$$

Assumption of conserved R-parity: At least one SUSY particle in final state of decays of SUSY particles

 \Rightarrow lightest supersymmetric particle (LSP) is stable

 \Rightarrow Good dark matter candidate, even if heavy.

Stability of DM particle important: All particles unstable unless protected by a conservation law. Only stable standard particles are proton, electron, lightest neutrino, photon.

Weakly interacting (electrically and color neutral) SUSY particles are Photino $\tilde{\gamma}$, Zino \tilde{Z} and Higgsino \tilde{H} (Majorana fermions).

Scattering for example by slepton exchange Cross section smaller, behavior like a massive neutrino.

Some superposition of Photino, Zino and Higgsino, called Neutralino, is an ideal WIMP candidate. Idea of neutralino today the main logic behind searches for WIMP dark matter.

Detailed interaction rates can be calculated using the DarkSUSY software package: http://www.physto.se/~edsjo/darksusy



From direct detection experiments: WIMP scattering cross section much smaller than ordinary weak cross section.

Early universe freeze-out: Annihilation rate somewhat more weakly than weak, depending on mass.

In SUSY models can be easily achieved. Annihilation and scattering can involve different particles, no simple crossing symmetry. E.g. Higgsino couples by Yukawa couplings, much stronger to third-generation fermions (early universe annihilation) than first generation (laboratory searches).

8.5 Direct WIMP Searches

8.5.1 Basic idea

Weakly interacting massive particle (WIMP) stands for a neutrino-like particle, however to be dark matter must couple to Z boson more weakly than neutrinos.

Often-discussed realization: Supersymmetric (SUSY) particles, to be discussed in more detail below.

If the dark matter in the galaxy consists of WIMPs, direct laboratory search possible. Perhaps most important effort in "experimental cosmology."

Main idea: A WIMP χ hits a nucleus (e.g. in a crystal), providing a small recoil energy that can be measured.



Velocity distribution of galactic dark matter particles: Assume virialized distribution and thus "isothermal" velocity distribution

$$\frac{\mathrm{d}n}{\mathrm{d}v} = n_0 \frac{4\,v^2}{\sqrt{\pi}\,\sigma^3} \,\mathrm{e}^{-v^2/\sigma^2}$$

One can show that σ is identical with galactic rotation velocity (for flat rotation curve)

$$\sigma = v_{\rm rot} = 220 \text{ km s}^{-1}$$
 for Milky Way

In this case velocity dispersion

$$\langle v^2 \rangle^{1/2} = \sqrt{\frac{3}{2}} \, \sigma = 270 \ \mathrm{km} \ \mathrm{s}^{-1} \sim 10^{-3} \, c$$

Recoil energy if WIMP of mass m_{χ} scatters on nucleus of mass m_A (mass number A), initial velocity v in laboratory system, scattering angle θ (in CM system)

$$\Delta E = \frac{m_A m_\chi^2}{(m_A + m_\chi)^2} v^2 \left(1 - \cos\theta\right)$$

Take Germanium as a typical nucleus with Z = 32 and $\langle A \rangle = 72.6$, for velocity $v \sim 10^{-3}$, and so for $m_{\chi} = m_A$

$$\Delta E \sim 20 \text{ keV}$$

Very small energy transfer!

8.5.2 Rate estimate

Momentum exchange in a typical collision is small, so add scattering amplitudes on individual nucleons coherently.



The phase difference between the scattered wave on two targets 1 and 2 is $\Delta \varphi = \Delta \mathbf{k} \cdot \mathbf{r}_{12}$ where $\Delta \mathbf{k}$ is the momentum transfer.

The scattered wave from both targets roughly in phase for $|\Delta \mathbf{k} \cdot \mathbf{r}_{12}| \lesssim 1$. Coherence condition therefore approximately

$$\Delta k \lesssim r^{-1}$$

where r is the geometric size of the overall target distribution.

Consider a nucleus with mass number A. Its volume is, assuming nuclear mass density of $3\times 10^{14}~{\rm g~cm^{-3}}$

$$V = \frac{A m_N}{\rho_{\rm nuc}} = \frac{4\pi}{3} \left(\frac{D}{2}\right)^3$$

where D is the diameter of the nucleus, assumed to be spherical. Therefore,

$$D^{-1} = \frac{0.09 \text{ GeV}}{A^{1/3}}$$

A typical momentum exchange is $\Delta k = m_{\chi} v$ which at maximum momentum transfer is $m_A v$. Coherence condition $D^{-1} \gtrsim \Delta k$ is therefore

$$\frac{0.09 \text{ GeV}}{A^{1/3}} \gtrsim 10^{-3} A m_N \qquad \Rightarrow \qquad A^{4/3} \lesssim 96 \qquad \Rightarrow \qquad A \lesssim 30$$

Usually full coherence assumed, although not a good approximation for higher-A nuclei that are often used.

Vector-current cross section with neutron number N and proton number Z is

$$\sigma = \frac{G_{\rm F}^2}{8\pi} \left[N - (1 - 4\sin^2\theta_{\rm w})Z \right]^2 \left(\frac{m_{\chi}m_A}{m_{\chi} + m_A}\right)^2$$

where $\sin^2 \theta_{\rm w} = 0.23$, so neutral-current cross section on protons strongly suppressed. Total cross section approximately $\propto N^2$, coherently enhanced. If one added the cross sections (not the amplitudes), total cross section proportional to N.

For axial-vector current (spin-dependent) scattering: Coherence implies that the cross section is proportional to the total spin of the nucleus, usually quite small, most nucleons pair off in a nucleus. So spin-dependent rate much smaller.

Scattering rate per target nucleus is

$$\Gamma = n_{\rm DM} \left\langle v \, \sigma \right\rangle = \frac{\rho_{\rm DM} \left\langle \sigma v \right\rangle}{m_{\chi}}$$

Canonical dark-matter mass density in typical galactic models near Earth is

$$\rho_{\rm DM} = 0.3 \ {\rm GeV} \ {\rm cm}^{-3}$$

For $v \sim 10^{-3}$ and the above velocity-independent cross section one finds for the rate per target mass

$$\frac{\text{Event rate}}{\text{Target mass}} = \frac{\Gamma}{m_A} \sim \underbrace{\frac{G_{\rm F}^2}{8\pi}}_{0.92 \text{ kg}^{-1} \text{ day}^{-1}} N^2 \frac{m_A m_\chi}{(m_A + m_\chi)^2}$$

For Germanium ($\langle N \rangle = 40.6$) at maximum cross section with $m_{\chi} = m_A$ the rate is

$$\frac{\text{Event rate}}{\text{Target mass}} \sim 380 \text{ kg}^{-1} \text{ day}^{-1}$$

To detect such small event rates need extremely pure materials. Radioactive background of natural contaminations largest problem. Experiments need to be underground to shield from cosmic rays, for example in Gran Sasso Laboratory ("underground physics").

In summary need experiments that detect

- Very small nucleus recoil energies (keV range)
- Very small rates, typical unit "events per kg and day".

8.5.3 Concrete projects

Small energy transfers can be detected in different ways. Three main approaches

• Heat (phonons)

Recoil excites lattice vibrations (phonons) ~ heat. At low T heat capacity very low (Debye freezing), so microscopic energy deposition enough to heat by measurable ΔT . Measurement for example by superconducting transition-edge thermometer, attached to macroscopic crystal. Cryogenic method: crystal needs to be at very low T, e.g. a few mK.

• Ionization (electric charge)

Recoil leads to ionization, electrons kicked into conduction band, e.g. of semi-conductor such as germanium. Charge pulled off by voltage \rightarrow current pulse when WIMP hits the crystal.

• Scintillation (light)

Some materials emit scintillation light when charged particle goes through. Recoil nucleus emits photons, can be detected

Many experiments use both techniques to reject background. (Radioactive events, e.g. caused by gammas, have different ratio of two signals than WIMP collision.)

Many projects worldwide. Names and used techniques are summarized here.



8.5.4 Best limits

Currently the best limits come from a liquid noble gas experiment (Xenon 10) and CDMS, an experiment using germanium as a target and using charge and heat to detect WIMP recoils.

The most recent CDMS germanium experiments (arXiv:0912.3592, 18 Dec. 2009) reports two events (probably background) in 612 kg days exposure. Compare with above expected rate: WIMP-scattering cross section with germanium about 4×10^{-6} weaker than ordinary neutrinos.

If WIMPs exist they interact far more weakly than ordinary weak interactions in scattering experiments!

The current exclusion plot for spin-independent scattering is shown here. To compare different experiments it is always assumed that the cross section scales with the atomic mass number as A^2 . (For neutrinos, the true scaling would be with N^2 , but anyway only a way to compare sensitivity of different experiments.)



8.5.5 Annual modulation

Assuming a WIMP signal was detected, how to recognize as WIMPs and not some new radioactive background?

One option is annual modulation caused by the Earth's relative motion to galactic WIMP distribution.



The WIMP halo is taken at rest on average, whereas the Sun moves around the galactic center with $v_{\rm rot} \sim 220$ km s⁻¹. The Earth moves around the Sun with 30 km s⁻¹. The projected velocity variation relative to halo is ± 15 km s⁻¹ over the year, leading to a signal variation of $\pm 1.2\%$ (see homework problem).

Effect is being observed in a scintillator experiment DAMA/LIBRA, using 250 kg of radiopure NaI (sodium iodide).



However, required cross section much larger than excluded by CDMS and others. In units of above picture, requires at least $\sim 2 \times 10^{-42}$ cm². So if correct, requires very peculiar cross section variation with nuclear properties.

8.6 Indirect searches by WIMP annihilation

Galactic DM WIMPs can annihilate, although cosmic average rate is small (no survival otherwise).

In galaxy DM density about 10^6 times cosmic average, so annihilation rate $\propto n^2$ roughly 10^{12} times larger.

Moreover, galactic halo probably clumpy, so "boost factor" from small-scale density variations.

In principle, can detect annihilation products, e.g. a photon line in TeV gamma ray telescopes.

WIMPs will be trapped in stars because they sometimes interact, e.g. in the Sun and lose enough energy to remain trapped.

Within Sun or Earth enhanced annihilation rate. Secondary high-E neutrinos can be detected in neutrino telescopes, e.g. IceCube at Southpole.