# 7 Relic particles from the early universe

# 7.1 Neutrino density today (14 December 2009)

We have now collected the ingredients required to calculate the density of relic particles surviving from the early universe. Besides photons and neutrinos these are ordinary matter (baryons), dark matter, and light nuclei such as deuterium and helium.

The surviving neutrino density today is determined by the following sequence of events

- Neutrinos are in thermal equilibrium up to  $T \sim 1$  MeV, for example by  $e^-e^+ \leftrightarrow \nu \bar{\nu}$ .
- Thermal distribution of electrons and positrons gets Boltzmann suppressed when T falls below  $m_e$ .
- Relevant reactions only  $e^-e^+ \leftrightarrow \gamma\gamma$ , but not  $e^-e^+ \leftrightarrow \nu\bar{\nu}$ .
- Entropy stored in the  $e^-e^+$  plasma gets transferred to photons if process is adiabatic.
- Photons are heated relative to neutrinos.
- Calculate relative number densities from entropy conservation.

Entropy density in relativistic gases

$$s = \frac{2\pi^2}{45} T^3 \left( \sum_{\text{bosons}} g_{\rm B} + \frac{7}{8} \sum_{\text{fermions}} g_{\rm F} \right)$$

Count relativistic degrees of freedom that are in equilibrium with each other before  $e^+e^-$  disappearance

Photons 
$$g_{\rm B} = 2$$
  
 $e^-e^+$   $g_{\rm F} = 4$   $\left\{ \begin{array}{c} g^* = 2 + 4 \times \frac{7}{8} = \frac{11}{2} \end{array} \right.$ 

After  $e^+e^-$  disappearance

$$\left. \begin{array}{c} g_{\rm B} = 2\\ g_{\rm F} = 0 \end{array} \right\} \quad g^* = 2$$

Adiabatic evolution implies entropy conservation among interacting particles:

$$s_{\rm int}a^3 = {\rm const.}$$

The cosmic scale factor evolves in the same way for all particles, so we infer

$$g_{\rm int}^* T^3 = {\rm const.} \qquad \Rightarrow \qquad \frac{11}{2} \times T_{\rm before}^3 = 2 \times T_{\rm after}^3 \qquad \Rightarrow \qquad \frac{T_{\rm before}}{T_{\rm after}} = \left(\frac{11}{4}\right)^{1/3}$$

Therefore after  $e^+e^-$  annihilation

$$\frac{T_{\nu}}{T_{\gamma}} = \left(\frac{4}{11}\right)^{1/3} \Rightarrow T_{\nu}^{\text{today}} = \left(\frac{4}{11}\right)^{1/3} \times 2.725 \text{ K} = 1.945 \text{ K}$$

To assign a T to neutrinos today, however, makes only sense if  $m_{\nu} \lesssim T_{\nu}$ .

Number densities in one neutrino flavor vs. photons

$$n_{\nu\bar{\nu}} = 2 \times \frac{3}{4} \frac{\zeta_3}{\pi^2} T_{\nu}^3$$
 and  $n_{\gamma} = 2 \times \frac{\zeta_3}{\pi^2} T_{\gamma}^3$ 

and therefore in one flavor

$$\frac{n_{\nu\bar{\nu}}}{n_{\gamma}} = \frac{3}{4} \left(\frac{T_{\nu}}{T_{\gamma}}\right)^3 = \frac{3}{4} \times \frac{4}{11} = \frac{3}{11} \qquad \Rightarrow \qquad n_{\nu\bar{\nu}}^{\text{today}} = 112.1 \text{ cm}^{-3}$$

Even with 3 flavors, the number density of all neutrinos is somewhat less than the total photon density.

Radiation energy density in neutrinos after  $e^+e^-$  annihilation

$$\rho_{\nu\bar{\nu}} = 3_{\text{flavors}} \times 2_{\text{spins}} \times \frac{7}{8} \times \frac{\pi^2}{30} T_{\nu}^4 = \frac{21}{8} \left(\frac{T_{\nu}}{T_{\gamma}}\right)^4 \rho_{\gamma} = \frac{21}{8} \left(\frac{4}{11}\right)^{4/3} \rho_{\gamma} = 0.6813 \,\rho_{\gamma}$$

Is a relatively small correction, but depending on neutrino mass the epoch of matterradiation equality is shifted (see homework).

# 7.2 Neutrinos with mass

If neutrino masses are significantly larger than a typical thermal energy of  $3 \times 1.95$  K  $\sim 0.5$  meV, then their contribution to present-day energy density determined by their rest mass. For one flavor

$$\rho_{\nu\bar{\nu}} = m_{\nu} n_{\nu\bar{\nu}} = m_{\nu} \frac{3}{11} n_{\gamma}$$

In units of critical density

$$\rho_{\rm crit} = h^2 \, 10.54 \, \rm keV \, cm^{-3}$$

we find

$$\Omega_{\nu\bar{\nu}}h^2 = \sum_{\text{flavors}} \frac{m_{\nu}}{94.0 \text{ eV}}$$

With modern cosmological data:  $\Omega_{\text{dark matter}} = 0.23$  and h = 0.74 this implies

$$M_{\nu} = \sum_{\text{flavors}} m_{\nu} < 12 \text{ eV}$$

Originally in the 1960's when  $\nu_{\mu}$  was first detected, cosmological limit on its mass from cosmological data (Gershtein and Zel'dovich).

Experimental limits from tritium  $\beta$  decay together with neutrino oscillations imply

$$M_{\nu} = \sum_{\text{flavors}} m_{\nu} < 6.6 \text{ eV}$$
 at 95% CL

so from this perspective alone neutrinos no longer suitable to account for all of dark matter.

### 7.3 Dilution of sterile particles

The reduced number density of neutrinos today relative to photons illustrates a general principle: A particle freezing out at some early epoch will get "diluted" relative to photons.

More precisely, photons will be heated by absorbing the entropy stored in the other disappearing d.o.f.

Consider a particle freezing out at an epoch where the effectively excited thermal d.o.f. (other than those from the particle) are  $g^*_{\text{freeze}}$  and assume this is long before neutrino decoupling.

Therefore, as other particles disappear, their entropy eventually ends up in neutrinos and photons.

However, these have different temperatures after  $e^+e^-$  annihilation. So the sum of the effective d.o.f. today, relative for entropy conservation, is

$$g_s^* = 2 + \frac{7}{8} \times 6 \times \left(\frac{T_\nu}{T_\gamma}\right)^3 = 2 + \frac{7}{8} \times 6 \times \frac{4}{11} = \frac{43}{11} = 3.909$$

Therefore the number density of these X particles relative to photons today compared to when they were in thermal equilibrium is

$$\frac{n_X}{n_\gamma}\Big|_{\text{today}} = \frac{g_s^*}{g_{\text{freeze}}^*} \frac{n_X}{n_\gamma}\Big|_{\text{thermal}} = \frac{43}{11} \frac{1}{g_{\text{freeze}}^*} \frac{n_X}{n_\gamma}\Big|_{\text{thermal}}$$

If a particle freezes out before the QCD transition,  $g^*_{\text{freeze}} > 61.75$  and therefore

$$\frac{n_X}{n_\gamma}\Big|_{\text{today}} < \frac{43}{11} \frac{1}{61.75} \frac{n_X}{n_\gamma}\Big|_{\text{thermal}} = \frac{1}{15.8} \left. \frac{n_X}{n_\gamma} \right|_{\text{thermal}}$$

If it freezes out much earlier, it gets even more diluted. This effect would apply, in particular, to sterile neutrinos (Dirac partners of ordinary neutrinos if these are Dirac particles, see homework assignment) or similar states or also to gravitons, assuming these were in equilibrium at some early epoch.

Sometimes it may be useful to express the number ratio today relative to one species of ordinary neutrinos. Their number density themselves is reduced by a factor 4/11 relative to early epochs, so drop this factor,

$$\frac{n_X}{n_{\nu\bar{\nu}}}\Big|_{\text{today}} = \frac{43}{4} \frac{1}{g_{\text{freeze}}^*} \frac{n_X}{n_{\nu\bar{\nu}}}\Big|_{\text{thermal}} < \frac{1}{5.74} \frac{n_X}{n_{\nu\bar{\nu}}}\Big|_{\text{thermal}}$$

where the inequality refers to particles freezing out just before the QCD transition.

### 7.4 Non-relativistic freeze-out: General picture

Neutrinos are an example where particles stop interacting (freeze out) while being relativistic.  $\Rightarrow$  Number density fixed by thermal number density at freeze out.

If particles still interact while becoming nonrelativistic, their number density gets Boltzmann suppressed. In the absence of a chemical potential, the number density of massive  $\frac{1}{2}$  fermions plus antifermions is, as derived earlier,

$$n_{f\bar{f}} = g \,\mathrm{e}^{-m/T} \,\left(\frac{mT}{2\pi}\right)^{3/2}$$

Here g number of degrees of freedom, for example g = 4 for a Dirac fermion.

To factor out general cosmic expansion, express density relative to entropy density (recall entropy is conserved in a comoving volume),

$$s = g^* \frac{(2\pi)^2}{90} T^3$$

Relative number density therefore

$$Y \equiv \frac{n_{f\bar{f}}}{s} = \frac{g}{g^*} \frac{\sqrt{2\pi}}{90} e^{-x} x^{3/2} \qquad \text{where} \qquad x = \frac{m}{T} \propto \sqrt{t}$$

is a useful measure of epoch. Like time, and in contrast to T, it increases in the direction of cosmic evolution.



Boltzmann suppressed number density frozen at  $x_{\rm F} = m/T_{\rm F}$  when the annihilation rate becomes smaller than the expansion rate (or rather, when mfp exceeds Hubble distance).

## 7.5 Boltzmann collision equation

### 7.5.1 Homogeneous, isotropic, expanding system

To make this argument more precise, study in more detail the freezing of particles when they are nonrelativistic.

To this end study the evolution of the distribution function in phase space  $f(\mathbf{x}, \mathbf{p})$ . Its evolution is governed by Boltzmann's Collision Equation

$$L[f] = C[f]$$

Left-hand side is Liouville operator, describing flow of particles by free motion and caused by external forces:  $\dot{\mathbf{p}} = \mathbf{F}$ 

$$L[f] = \left(\frac{\partial}{\partial t} + \dot{\mathbf{x}} \cdot \boldsymbol{\nabla}_{\mathbf{x}} + \dot{\mathbf{p}} \cdot \boldsymbol{\nabla}_{\mathbf{p}}\right) f(\mathbf{x}, \mathbf{p})$$

Collision operator C[f] changes the distribution by microscopic processes, e.g. collisions and annihilations.

Assuming homogeneous and isotropic universe:

$$f(\mathbf{x}, \mathbf{p}) \to f(p)$$
 where  $p = |\mathbf{p}|$ 

In thermal equilibrium

$$f_p = f(p) = \frac{1}{e^{E/T} \pm 1}$$
 where  $E = \sqrt{p^2 + m^2}$ 

In the expanding universe the Liouville term is provided by

$$\frac{\dot{p}}{p} = -\frac{\dot{a}}{a} = -H \qquad \Rightarrow \qquad \dot{p} = -Hp$$

causing the cosmic redshift of momenta.

Collision equation is finally

$$\frac{\partial}{\partial t}f(t,p) - H p \frac{\partial}{\partial p}f(t,p) = \frac{\mathrm{d}f(t,p)}{\mathrm{d}t}\Big|_{\mathrm{coll}}$$

#### 7.5.2 Integrated equation

It will be useful to study not the detailed phase-space distribution, but only the total number density

$$n = \int \frac{\mathrm{d}^{3}\mathbf{p}}{(2\pi)^{3}} f(p) = \int \mathrm{d}p \, \frac{p^{2}}{2\pi^{2}} f(p)$$

For the moment ignore spin degrees of freedom. Can be trivially included with a summation as long as polarization effects play no role.

Take phase space integral of entire collision equation and look at Liouville term

$$\int_0^\infty \mathrm{d}p \, p^2 \left( p \, \frac{\partial f}{\partial p} \right) = \Big|_0^\infty p^3 f - \int_0^\infty \mathrm{d}p \, 3p^2 f$$

First term vanishes if f(p) falls off fast enough at infinity, trivial for thermal distribution. Therefore Liouville term

$$\int \frac{\mathrm{d}^3 \mathbf{p}}{(2\pi)^3} \left( p \frac{\partial f}{\partial p} \right) = -3n$$

Integrated collision equation therefore

$$\dot{n} + 3Hn = \int \frac{\mathrm{d}^3 \mathbf{p}}{(2\pi)^3} \left. \frac{\mathrm{d}f(t,p)}{\mathrm{d}t} \right|_{\mathrm{coll}}$$

For collisionless gas this is

$$\dot{n} + 3Hn = 0$$

Conservation of particle number in cosmic co-moving volume

$$n \propto a^{-3}$$

and therefore

$$\dot{n} \propto \partial_t \left( a^{-3} \right) = -3a^{-2} \dot{a} = -3a^{-3} \frac{\dot{a}}{a} = -3Ha^{-3} \propto -3Hn$$

#### 7.5.3 Integrated equation for annihilations

Now consider collision equation for a system of particles and antiparticles (massive neutrinos and antineutrinos). Collisions of the form  $\nu + X \rightarrow X + \nu$  do not change the overall density. Therefore consider only annihilations of the form

 $\nu \bar{\nu} \leftrightarrow e^+ e^-$  or other channels

Number of collisions per unit time in terms of cross section

$$\Gamma = \sigma n_{\text{targets}} v_{\text{rel}}$$

Right-hand side of collision equation

$$\frac{\mathrm{d}f(t,p)}{\mathrm{d}t}\Big|_{\mathrm{coll}} = -\Gamma_p f(p) = -\sigma \underbrace{\langle n_{\mathrm{targets}} v_{\mathrm{rel}} \rangle}_{\int \frac{\mathrm{d}^3 \mathbf{q}}{(2\pi)^3} \bar{f}(q) v_{\mathrm{rel}}} f(p)$$

where overbarred quantities refer to antiparticles. Integrated collision term

$$\int \frac{\mathrm{d}^3 \mathbf{p}}{(2\pi)^3} \left. \frac{\mathrm{d}f(t,p)}{\mathrm{d}t} \right|_{\mathrm{coll}} = -\underbrace{\int \frac{\mathrm{d}^3 \mathbf{p}}{(2\pi)^3} \int \frac{\mathrm{d}^3 \mathbf{q}}{(2\pi)^3} \sigma v_{\mathrm{rel}} f_p \bar{f}_q}_{\langle \sigma v_{\mathrm{rel}} \rangle n \bar{n} \text{ for } \nu \bar{\nu} \to e^+ e^-} + \underbrace{\mathrm{Production term}}_{e^+ e^- \to \nu \bar{\nu}}$$

Pauli blocking or Bose stimulation factors not included in the final states. However, not necessary because massive particles at freeze-out very dilute.

Integrated Boltzmann Eqns together then

$$\dot{n} + 3Hn = -\langle \sigma v \rangle n\bar{n} + P$$
$$\dot{\bar{n}} + 3H\bar{n} = -\langle \sigma v \rangle n\bar{n} + \bar{P}$$

Symmetric situation (no chemical potential):  $\bar{n} = n$  etc.

$$\dot{n} + 3Hn = -\langle \sigma v \rangle n^2 + P$$

In thermal equilibrium, nothing changes by collisions, so r.h.s. = 0 and

$$P_{\rm eq} = \left< \sigma v \right> n_{\rm eq}^2$$

Assume that all particles except for  $\nu \bar{\nu}$  in equilibrium,

 $P = P_{\rm eq}$ 

So production rate by thermal medium expressed in terms of the total annihilation rate: Manifestation of "detailed balancing" in a thermal system. Rate of production and annihilation equal in thermal equilibrium. The integrated collision equation thus turns into

$$\dot{n} + 3Hn = -\langle \sigma v \rangle \left( n^2 - n_{\rm eq}^2 \right)$$

For massive particles the true density n always larger than equilibrium density, so r.h.s. is negative as expected.

Now express in terms of the variable Y = n/s (entropy density s). Entropy conservation in a comoving volume implies

$$\dot{s} = -3Hs$$

Therefore

$$s\dot{Y} = s\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{n}{s}\right) = s\frac{\dot{n}s - n\dot{s}}{s^2} = \dot{n} + 3Hn$$

and therefore Boltzmann Eqn

$$\dot{Y} = -\langle \sigma v \rangle s \, \left( Y^2 - Y_{\text{eq}}^2 \right) = -\langle \sigma v \rangle s \, \left( Y + Y_{\text{eq}} \right) \left( Y - Y_{\text{eq}} \right)$$

As long as distribution is close to equilibrium we have  $Y \sim Y_{eq}$  and therefore  $Y + Y_{eq} \sim 2Y_{eq}$ . With equilibrium annihilation rate

 $\Gamma_{\rm eq} = \langle \sigma v \rangle n_{\rm eq} = \langle \sigma v \rangle Y_{\rm eq} s$ 

we have close to equilibrium

 $\dot{Y} = -2\Gamma_{\rm eq}\left(Y - Y_{\rm eq}\right)$ 

The factor 2 arises because in each annihilation, a particle and an antiparticle disappear.

As expected, the rate of approaching equilibrium is driven by the deviation from equilibrium  $(Y - Y_{eq})$ .

On the other hand, at late times, after freezing the number density,  $Y_{\rm eq} \ll Y$  and

 $\dot{Y} = -\Gamma Y$  where  $\Gamma = \langle \sigma v \rangle n$ 

It stays approximately constant if the times considered are much smaller than the Hubble time.

Another way to write the collision equation uses a change of variables from time to x = m/T. With

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}t}\frac{\mathrm{d}}{\mathrm{d}x} = Hx\frac{\mathrm{d}}{\mathrm{d}x}$$

and using a prime to denote d/dx one finds

$$x Y' = -\frac{\Gamma_{\rm eq}}{H} \frac{Y + Y_{\rm eq}}{Y_{\rm eq}} \left(Y - Y_{\rm eq}\right)$$

Freezing out roughly when  $2\Gamma_{eq} = H$ . Since  $\Gamma_{eq}$  drops exponentially for nonrelativistic freeze out, the exact criterion not important for estimated freeze-out temperature  $T_{\rm F}$ .

### 7.6 Solution for s-wave annihilation (15 Dec. 2009

Particles annihilating to radiation by s-wave process (no relative angular momentum)

$$\sigma \propto \frac{1}{v_{\rm rel}} \qquad \Rightarrow \qquad \sigma v_{\rm rel} = \sigma_0 = {\rm const.}$$

Equilibrium annihilation rate therefore

$$\Gamma_{\rm eq} = \langle \sigma v_{\rm rel} \rangle n_{\rm eq} = \sigma_0 \frac{g}{2} \left( \frac{mT}{2\pi} \right)^{3/2} e^{-m/T} = \frac{g}{2} \sigma_0 \frac{m^3}{(2\pi)^{3/2}} x^{-3/2} e^{-x}$$

where x = m/T and g the number of degrees of freedom (4 for Dirac neutrinos).

Expansion rate

$$H = \frac{T^2}{m_{\rm Pl}} \sqrt{\frac{4\pi^3}{45} g^*} = \frac{1}{x^2} \frac{m^2}{m_{\rm Pl}} \sqrt{\frac{4\pi^3}{45} g^*}$$

Freezing-out condition

$$H = 2\Gamma_{\rm eq}$$

Therefore

$$x^{-1/2} e^x = \sigma_0 m_{\rm Pl} m g \left(\frac{45}{32\pi^6 g^*}\right)^{1/2}$$

Take natural logarithm on both sides

$$x_{\rm F} = \log(\sigma_0 m_{\rm Pl} m) + \frac{1}{2} \log\left(\frac{45 \, g^2 \, x_{\rm F}}{32 \, \pi^6 \, g^*}\right)$$

Zeroth-order result by ignoring second log term on r.h.s., and insert it on r.h.s., so find to first order

$$x_{\rm F} = \log(\sigma_0 m_{\rm Pl} m) + \frac{1}{2} \log\left[\frac{45 \, g^2 \log(\sigma_0 m_{\rm Pl} m)}{32 \, \pi^6 \, g^*}\right]$$

Particle density at freeze-out from freeze-out condition

$$H = 2\Gamma_{\rm eq} = \sigma_0 Y_{\rm eq} s$$

and therefore

$$Y_{\rm F} = \left. \frac{H}{s \,\sigma_0} \right|_{T=T_{\rm F}} = \frac{1}{x_{\rm F}^2} \, \frac{m^2}{m_{\rm Pl}} \, \sqrt{\frac{4\pi^3}{45} \, g^*} \frac{1}{\sigma_0} \, \frac{90}{(2\pi)^2 \, g^*} \, \frac{x_{\rm F}^3}{m^3} = \sqrt{\frac{45}{\pi g^*}} \, \frac{x_{\rm F}}{\sigma_0 m_{\rm Pl} m}$$

Upon insertion of our result for  $x_{\rm F}$  we find

$$Y_{\rm F} = \sqrt{\frac{45}{\pi g^*}} \, \frac{\log(\sigma_0 m_{\rm Pl} m) + \frac{1}{2} \, \log\left[\frac{45 \, g^2 \log(\sigma_0 m_{\rm Pl} m)}{32 \, \pi^6 \, g^*}\right]}{\sigma_0 m_{\rm Pl} m}$$

The surviving density roughly inversely proportional to the annihilation cross section.

Survival of the weakest: More weakly interacting particles freeze out earlier and survive in larger numbers.

## 7.7 Baryon catastrophe

Take protons/antiprotons as an example with  $m_p = 938$  MeV.

For the annihilation cross section assume they annihilate "when they touch," i.e. a cross section would be something like  $\pi r^2$  with  $r \sim 1$  fm =  $1 \times 10^{-13}$  cm, i.e.  $\sigma \sim 30$  mb (millibarn) with 1 mb =  $1 \times 10^{-27}$  cm<sup>2</sup>. So we estimate

 $\sigma_0 = \langle \sigma v_{\rm rel} \rangle \sim 30 \ {\rm mb}$ 

Therefore crucial small parameter

 $\sigma_0 m_{\rm Pl} m \sim 30 \text{ mb} \times 938 \text{ MeV} \times 1.22 \times 10^{19} \text{ GeV} = 0.88 \times 10^{21}$ 

Therefore, they would freeze out roughly at

 $x_{\rm F} \sim \log(0.88 \times 10^{21}) = 48.2$  or  $T_{\rm F} \sim 20 \,\,{\rm MeV}$ 

This is very late in the universe. Effective degrees of freedom roughly  $g^* = 10.75$  (neutrinos, electrons, photons). Therefore more precisely

$$x_{\rm F} = \log(0.88 \times 10^{21}) + \frac{1}{2} \log\left(\frac{45 \times 4^2 \times \log(0.88 \times 10^{21})}{32\pi^6 \times 10.75}\right) = 47.1$$

almost identical with the simplest estimate.

Surviving relative number density

$$Y_{\rm F} = \sqrt{\frac{45}{\pi g^*}} \frac{x_{\rm F}}{\sigma_0 m_{\rm Pl} m} = 0.6 \times 10^{-19}$$

Therefore, baryonic matter surviving today would be very dilute (and symmetric between matter and antimatter).

The actual baryon density (no antibaryons) is roughly  $10^{-9}$  and derives from a primordial baryon asymmetry: the creation of the Baryon Asymmetry of the Universe (BAU) from an initially symmetric plasma is an important issue of particle cosmology.

Relic particles from the early universe can survive if they freeze-out early enough to have a significant density today (weakly interacting relics) or if they have an asymmetry created in the early universe.

### 7.8 Weakly Interacting Massive Particles (WIMPs)

Consider particles like ordinary neutrinos that interact only by weak interactions. Call them  $\chi$  to distinguish from known "sequential" neutrinos.

Annihilation processes are of the following form, where the actual final states of course depend on the  $\chi$  mass and thus on the available energy



Assume mass of weakly interacting particles is below the Z-mass. Dirac annihilation cross section

$$\sigma_0 = \langle \sigma v_{\rm rel} \rangle_{\rm Dirac} \sim \frac{G_{\rm F}^2 m^2}{2\pi} \times {\rm channels} \times {\rm coupling \ constants} \sim \frac{5}{2\pi} \, G_{\rm F}^2 m^2 \sim G_{\rm F}^2 m^2$$

Assume freezes out before QCD transition, so  $g^* \sim 60$ . Freeze-out conditions, taking in the logarithmic terms m = 1 GeV,

$$\sigma_0 m_{\rm Pl} m \sim G_{\rm F}^2 m_{\rm Pl} m^3 = 1.7 \times 10^9 \, m_{\rm GeV}^3 \qquad \text{where} \qquad m_{\rm GeV} = \frac{m}{1 \,\,{\rm GeV}}$$

Freeze-out temperature

 $x_{\rm F} = 18.8 + 3\log m_{\rm GeV}$ 

The mass number density today roughly

$$Y_{\rm F} \sim 1.1 \times 10^{-8} \ m_{\rm GeV}^{-3} \left(1 + 0.16 \log m_{\rm GeV}\right)$$

The number density today is

$$n_{\chi\bar{\chi}} = Y_{\rm F} s_0$$

where the entropy density today is

$$s_0 = g_s^* \frac{2\pi^2}{45} T_0^3$$
 where  $T_0 = 2.725$  K

The effective entropy density today includes photons with full strength (bosons)  $g_{\gamma}^* = 2$ and 6 fermionic degrees for neutrinos, which however have reduced number density relative to photons of by the factor  $\frac{4}{11}$ , so they have  $g_{\nu\bar{\nu}}^* = \frac{7}{8} \times 6 \times \frac{4}{11} = \frac{21}{11}$ . So as discussed earlier

$$g_s^* = \frac{43}{11} \qquad \Rightarrow \qquad s_0 = \frac{43}{11} \frac{2\pi^2}{45} (2.725 \text{ K})^3 = 2894 \text{ cm}^{-3}$$

In units of the critical density

$$\rho_{\rm crit} = h^2 \, 10.54 \ {\rm keV} \ {\rm cm}^{-3}$$

we find

$$\Omega_{\chi\bar{\chi}} = \frac{mY_{\rm F}s_0}{\rho_{\rm crit}} \qquad \Rightarrow \qquad \Omega_{\chi\bar{\chi}}h^2 \sim 3.0 \, \frac{1 + 0.16 \log m_{\rm GeV}}{m_{\rm GeV}^2}$$

This result applies for nonrelativistic freeze out. Neutrinos freeze out relativistically at  $T_{\rm F} \sim 1$  MeV, so this result applies if

 $m \gtrsim 1 \text{ MeV}$ 

### 7.9 Majorana neutrinos

It is usually assumed that WIMPs are Majorana particles: If fermions carry no conserved gauge charge there is no need to have a four-component state.

Several consequences. All else equal, only two degrees of freedom, not four, so reduce surviving mass density by factor 1/2.

Majorana fermions are identical, so can be in relative s-wave only if spins are opposite (Pauli principle).



Majorana particles in relative s-wave: Can not annihilate to  $\nu\bar{\nu}$  pair if interaction is lefthanded. Remember that  $\pi^0 \to \nu\bar{\nu}$  was not possible because spin-0 state can not decay into particles with opposite helicity (their spins add up to 1 if they move in opposite directions).



However,  $Z^0$  coupling to charged leptons not purely left-handed (remember that  $Z^0$  is a superposition of underlying neutral gauge bosons  $W^0$  and B such that photon is the massless combination).

So Majorana annihilation is mixture of s- and p-wave annihilation. For p-wave annihilation note that amplitude involves a factor of velocity of incoming particles, and rate therefore a factor  $v^2$ . Cross section therefore

$$\langle \sigma v_{\rm rel} \rangle \sim \sigma_0 \langle v^2 \rangle \sim \sigma_0 \frac{T}{m}$$

so a further factor x = T/m in annihilation rate.

Overall effective annihilation cross section smaller  $\Rightarrow$  larger relic density. For Majorana neutrinos, relic mass density ~ factor 2–3 larger.

### 7.10 Lee–Weinberg–Curve

A single species of neutrino-like particles contributes to the dark matter density as a function of the assumed mass m. With our previous result for relativistic freeze out we have (for Majorana neutrinos and ignoring the logarithmic term)

$$\Omega_{\chi\bar{\chi}}h^2 \sim \begin{cases} m_{\rm eV}/94 & {\rm for} \ m \lesssim 1 \ {\rm MeV} \\ 6 \ m_{\rm GeV}^{-2} & {\rm for} \ m \gtrsim 1 \ {\rm MeV} \end{cases}$$

Increases with increasing mass, then decreases. "Lee–Weinberg–curve:" Dark-matter density as function of particle mass.



Dark-matter density today known to be  $\Omega_{\rm DM}h^2 = 0.13$ , so neutrino-like particles can be dark matter for

$$m_{\nu} \sim \begin{cases} 10 \text{ eV} & \text{Hot dark matter} \\ 10 \text{ GeV} & \text{Cold dark matter} \end{cases}$$

Two dark-matter solutions for weakly interacting particles.