1 What is Astroparticle Physics (26 Oct. 2009)

No lecture notes.

2 Expanding Universe (27 Oct. 2009)

2.1 Basic picture

2.1.1 Galaxies and their distribution

Galaxies are the basic building blocks of the visible matter distribution in the universe.

Edwin Hubble (1889–1953) observed Cepheid variable stars in Andromeda (M31) with the 100 inch Hooker Telescope on Mt. Wilson ($\sim$ 1924) and estimated the distance.

→ Andromeda is an external galaxy, not some "nebula"

→ End of "Great cosmological debate about the scale of the universe" (cf. April 1920 Harlow Shapley and Heber D. Curtis at the Smithonian in Washington, DC)

Today we know that on cosmological scales galaxies are roughly uniformly distributed.

Figure 1: *Left:* Spiral galaxy NGC 2997. *Right:* Edwin Hubble (1889–1953) who formulated the law of systematic galactic recession velocities.

2.1.2 Redshift of spectral lines

Spectral lines emitted by distant galaxies are systematically redshifted (Wirtz 1918, Lundmark 1920–21, Slipher 1912–mid 1920s)

Redshift of a spectral line with wavelenth λ (E = Emission, A = Absorption)

$$z + 1 = \frac{\lambda_{\rm A}}{\lambda_{\rm E}} = \frac{\omega_{\rm E}}{\omega_{\rm A}}$$

Simplest interpretation as a Doppler effect.

If the relative velocity between E and A is v, the Doppler shift is

$$z + 1 = \sqrt{\frac{1+v}{1-v}} = 1 + v + \mathcal{O}(v^2)$$

We always use natural units with the speed of light $c = 1$, otherwise use v/c in this formula.

So for small redshifts $z = v/c = v$ in natural units.

In natural units, velocities are dimensionless like the redshift. Conversely, in the cosmological literature one often finds redshifts expressed as a velocity in units of km/s.

The stellar object with the largest redshift is the gamma-ray burst GRB090423 (23 April 2009) with $z = 8.3$, see `http://arXiv.org/abs/0906.1577`. (The corresponding Doppler effect is $v = 0.977$, so evidently a relativistic interpretation is necessary.)

Redshift can be caused by three different physical effects

- Kinematical (Doppler effect)
- Gravitational
- Cosmological (expanding universe)

For a given object, all three effects can be simultaneously important.

See first homework assignment for a derivation of the kinematical and gravitational redshift.

2.1.3 Hubble's law

The redshift of every galaxy is on average proportional to its distance (valid for small z, the only ones observed by Hubble)

$$z = v = H_0 D$$

with H_0 the Hubble constant. Its value was very uncertain for a long time and so it was often written in terms of a fudge factor h as

$$H_0 = h\,100 \text{ km s}^{-1} \text{ Mpc}^{-1}\,.$$

From a new cosmic distance ladder the most recent value is [arXiv:0905.0695]

$$H_0 = (74.2 \pm 3.6) \text{ km s}^{-1} \text{ Mpc}^{-1}\,.$$

Units: 1 Mpc = 10^6 pc and 1 pc = 3.26 lyr = 3.08×10^{18} cm.

1 Mpc is a typical distance between galaxies, e.g. the distance to Andromada (closest external galaxy) is about 0.75 Mpc.

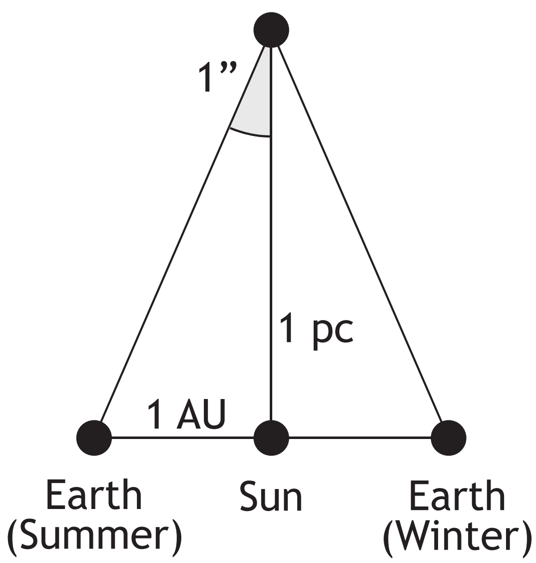

Figure 2: Definition of parsec by parallax of 1 arcsec of a star relative to the fixed stars. The mean Earth-Sun distance is 1 Astronomical Unit (1 AU = 149.60×10^6 km).

2.1.4 Hubble time

Hubble's law is obvious if all galaxies began moving at the same time at the same place: the faster galaxies have traveled further and thus have a larger distance today. In this picture the motion began in the past at a time

$$t_{\rm H} = \frac{D}{v} = \frac{D}{H_0 D} = H_0^{-1} = 4.2 \times 10^{17}\ {\rm s} = 13.2\ {\rm Gyr}\,. \qquad (1)$$

In linear approximation this is the expansion age of the universe.

It agrees well with other age determinations, e.g. radioactive dating of meteorites, the ages of the oldest globular clusters etc.

The solar system and Earth are approx. 4.6 Gyr old, i.e. not very much younger than the universe.

Hubble's original value of 500 km $\rm s^{-1}$ $\rm Mpc^{-1}$ was much too large due to a number of errors, e.g. the confusion of Cepheid variable stars with W Virginis stars.

The resulting expansion age was far too short, so assuming a beginning of the universe seemed implausible, leading to the "steady-state cosmology" of Bondi, Hoyle and Gold (1948). This scenario assumed the "Perfect Copernican Principle", i.e. the universe looks roughly the same at all times. It was assumed that matter and new galaxies were spontaneously produced from empty space.

"Big bang" was a derogatory term coined by Fred Hoyle (an outstanding cosmologist of his time) in about 1950 in a BBC broadcasting.

The discovery of the cosmic microwave background by Penzias and Wilson in 1965 was the crucial observation making the big-bang theory the standard scenario of the early universe.

A beginning of the universe solves Olbers' paradox of why the night sky is dark.

2.1.5 Hubble distance

This is the distance light can travel in a Hubble time, setting the spatial scale of the visible universe

$$D_{\rm H} = t_{\rm H} \sim 4\ {\rm Gpc}$$

where we have used, as ever, natural units with $c = 1$.

This distance is only a rough scale. The concept of distance requires some thought in an expanding universe.

We do not have any information about what is going on outside of the Hubble horizon.

2.1.6 Expansion without center

The recession of all galaxies away from us may seem like we are at the center of some great explosion.

The linearity of the Hubble expansion law implies that an observer on a different galaxy likewise sees himself at the center of a uniform expansion. (See first homework assignment.) Everybody may think he is at the center of the universe.

2.1.7 Rubber-balloon picture of cosmic expansion

The uniform Hubble expansion can be interpreted in two different ways.

- In the Newtonian picture, space is static. The Hubble motion of galaxies is like an explosion into space. Redshift is interpreted as a kinematical Doppler effect.
- Galaxies are fixed in space (except for "peculiar motions" relative to an average coordinate system), and the space between galaxies grows. This "inflating rubber balloon" picture of the universe is the one borne out from Einstein's general theory of relativity (GR). Redshift is interpreted as a "stretching" of light waves by the expansion of space.

On small scales both pictures are equivalent, but on large scales only the second picture provides a consistent description in the framework of GR.

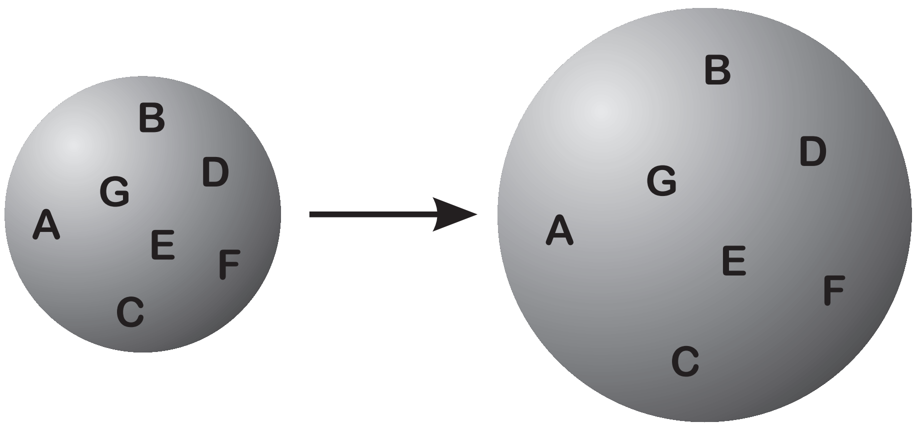

Figure 3: The apparent recession of galaxies from each other interpreted by the analogy of an inflating balloon. The galaxies themselves, represented by letters, do not expand, they only separate from each other.

2.1.8 Forces relevant in cosmology

We know about four fundamental forces of nature, which are

1. *Electromagnetic interaction.*
   Coulomb force is long range, but is screened if on average there are equal numbers of positive and negative charges.

2. *Weak interaction.*
   It is not weak, but short-ranged because of massive Z and W gauge bosons (as opposed to massless photon).

3. *Strong interaction ("Color interaction").*
   At distances larger than roughly a nucleon radius all objects must be color neutral ("confinement") → no macroscopic long range force (perfect screening).

4. *Gravitation.*
   Weakest of all interactions, but can not be screened because all mass and energy gravitates, no "negative charges," adds up coherently over large distances.

The electromagnetic, weak and strong forces are all understood as gauge theories in the spirit of Maxwell's theory of electromagnetism and we have a complete quantum theory.

They can be unified at different distance scales, being different low-energy manifestations of the same grand-unified interaction.

Gravitation does not fit into this pattern, can not be unified with the other interactions, and no quantum theory of gravity exists.

2.1.9 Does gravity really dominate?

To compare the strengths of the two possible long-range forces (Coulomb and gravitation), consider two masses (galaxies) $M_{1,2} = N_{1,2} m_u$ where m_u is the atomic mass unit and $N_{1,2}$ the "baryon numbers."

Assume these galaxies carry electric charges $Q_{1,2} = L_{1,2} e$ with $L_{1,2}$ integers and e the elementary unit charge.

Newtonian potential at distance D

$$\Phi_{\rm N} = -\frac{G_{\rm N} M_1 M_2}{D} = -\left(\frac{m_u}{m_{\rm Pl}}\right)^2 \frac{N_1 N_2}{D}$$

Coulomb potential

$$\Phi_{\rm C} = -\frac{Q_1 Q_2}{4\pi D} = -\alpha \, \frac{L_1 L_2}{D} \qquad \text{where} \qquad \alpha = e^2/4\pi \approx 1/137$$

The requirement that the Coulomb potential should be much smaller reads

$$\left|\frac{\Phi_{\rm C}}{\Phi_{\rm N}}\right| = \alpha \left(\frac{m_{\rm Pl}}{m_u}\right)^2 \frac{L_1 L_2}{N_1 N_2} \ll 1$$

Assuming the excess charge density is the same in both galaxies, $L/N \equiv L_1/N_1 = L_2/N_2$ and with $m_u = 0.931$ GeV and $m_{\rm Pl} = 1.22 \times 10^{19}$ GeV this means

$$L/N \ll 10^{-18} \, .$$

Theoretically one expects $L/N = 0$ because charge conservation implies that it is difficult to imagine the creation of a charge asymmetry in the universe.

Experimentally, the isotropy of cosmic rays implies the absence of large-scale electric fields and seems to imply [Orito and Yoshimura, PRL 54 (1985) 2457]

$$L/N \lesssim 10^{-30} \, .$$

Therefore, gravitation indeed is the dominating force in the universe.

2.1.10 Gravitation

Gravitational interaction can not be screened.

Newtonian $1/r$ potential leads to infinite potential in an infinite universe.

The Hubble recession velocities become relativistic at approx. the Hubble distance.

→ Need a relativistic theory of gravitation

→ Einstein's General Theory of Relativity (GR) the prime candidate.

Basic idea: Equivalence between inertial and gravitational mass, between acceleration and gravitation.

Locally there is a "freely falling" frame (inertial frame) where gravitational effects disappear

→ Uniform motion of test bodies.

Can mimic homogeneous gravitational fields by an accelerated frame ("Einstein elevator")

True gravitational fields can not be transformed away globally

→ Curved trajectories of test bodies by curvature of space–time

Test bodies move on geodesics (shortest distance) in curved space–time

Conversely: Masses (or rather energy-momentum tensor of all matter and fields) curves space–time.

2.2 Curved space–time

2.2.1 The metric

The metric is used to describe the internal geometric properties of curved space–time.

In an inertial frame special relativity applies and the Minkowski metric is

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2\,.$$

The "distance" $\mathrm{d}s$ between neighboring events is invariant against coordinate transformations.

Invariance of the speed of light in all frames

$$\mathrm{d}s^2 = 0 \qquad \Rightarrow \qquad \left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 = c^2 = 1$$

In a non-inertial frame $\mathrm{d}s^2$ becomes a general quadratic form of the coordinates.

Example: Coordinate system rotating with angular speed ω

$$\begin{aligned}\xi &= +x\,\cos\omega t + y\sin\omega t\\ \eta &= -x\,\sin\omega t + y\cos\omega t\\ \zeta &= z\end{aligned}$$

has the metric

$$\mathrm{d}s^2 = \left[1-\omega^2(\xi^2+\eta^2)\right]\,\mathrm{d}t^2 - \mathrm{d}\xi^2 - \mathrm{d}\eta^2 - \mathrm{d}\zeta^2 + 2\omega(\eta\mathrm{d}\xi - \xi\mathrm{d}\eta)\,\mathrm{d}t\,.$$

In general the proper distance is a quadratic form

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu$$

with summation over μ, $\nu = 0$, 1, 2, 3 implied.

Inertial frame: $x^0 = t$, $x^1 = x$, $x^2 = y$, $x^3 = z$.

4×4 matrix $g_{\mu\nu}$ and its derivatives contains all geometric information.

In flat (Minkowski) space–time we have

$$g_{\mu\nu} = \eta_{\mu\nu} = \begin{pmatrix} +1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1\end{pmatrix}$$

In general $g_{\mu\nu} = g_{\nu\mu}$ (symmetric)

- 10 functions of coordinates
- 1 positive, 3 negative eigenvalues: $\det g < 0$

Locally the metric can always be transformed to $\eta_{\mu\nu}$, approximating space–time locally by Minkowski space.

Effect of real gravitational fields: No global transformation $g_{\mu\nu} \to \eta_{\mu\nu}$ (10 functions, 4 coordinates!)

Special choice is possible: "Synchronized" or "Gaussian normal" coordinates:

$$g_{0i} = g_{i0} = 0 \quad (i = 1, 2, 3) \qquad \text{and} \quad g_{00} = 1 \qquad \text{(4 conditions)}$$

implying

$$g_{\mu\nu} = \eta_{\mu\nu} = \begin{pmatrix} +1 & 0 \\ 0 & -\gamma_{ij} \end{pmatrix}$$

with γ_{ij} the spatial metric and the invariant distance (i, $j = 1$, 2, 3)

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \gamma_{ij}\, \mathrm{d}x^i \mathrm{d}x^j \,.$$

Gravitational effects manifest themselves in a non-trivial spatial metric.

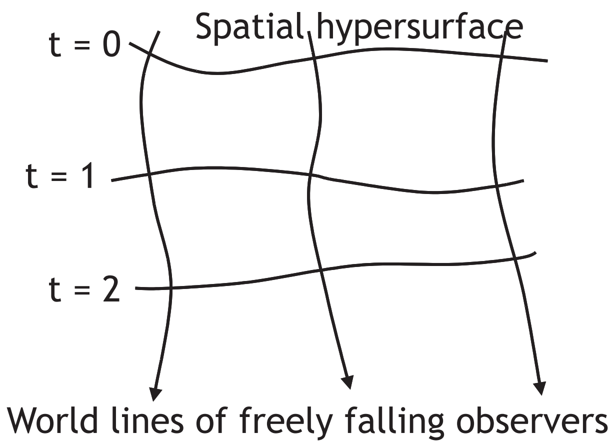

Figure 4: Synchronous system given by world lines of freely falling observers. The universal time coordinate corresponds to the clock time of the freely falling observers.

2.2.2 Curved space

What does one mean with curved space?

For example, a rolled-up sheet of paper is not curved in the following sense. Need to consider intrinsic properties.

Simple example: 2-surfaces. Draw circle (locus of all points with the same distance ℓ from the center) and measure circumference C.

Euclidean space: $C/\ell = 2\pi$

On the surface of a globe: $C/\ell < 2\pi$

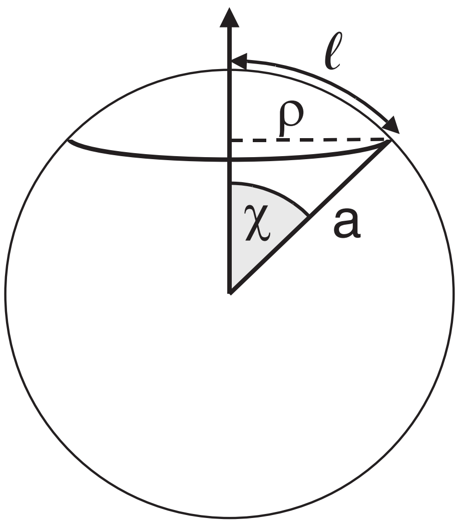

Figure 5: Globe with radius a. On it is a circle with radius ℓ within the 2-surface formed by the globe, while it has radius ρ in the embedding 3-space.

For a sphere the Gaussian curvature is $K = a^{-2}$.

The general definition for any 2-dimensional hypersurface is

$$K = \frac{3}{\pi} \lim_{\ell \to 0} \frac{2\pi\ell - C}{\ell^3}$$

For the globe we recover $K = a^{-2}$ noting that $\rho = a \sin\chi$, $\chi = \ell/a$, and the expansion $\sin\chi = \chi - \chi^3/6 + \mathcal{O}(\chi^5)$.

This concept of curvature can be generalized to higher dimensional spaces, for example our three-dimensional space.

Another possibility is to use the sum of angles in a triangle

$$\text{Sum of angles} \begin{cases} > 180^\circ & \text{positive curvature} \\ = 180^\circ & \text{Euclidean space (flat space)} \\ < 180^\circ & \text{negative curvature} \end{cases}$$

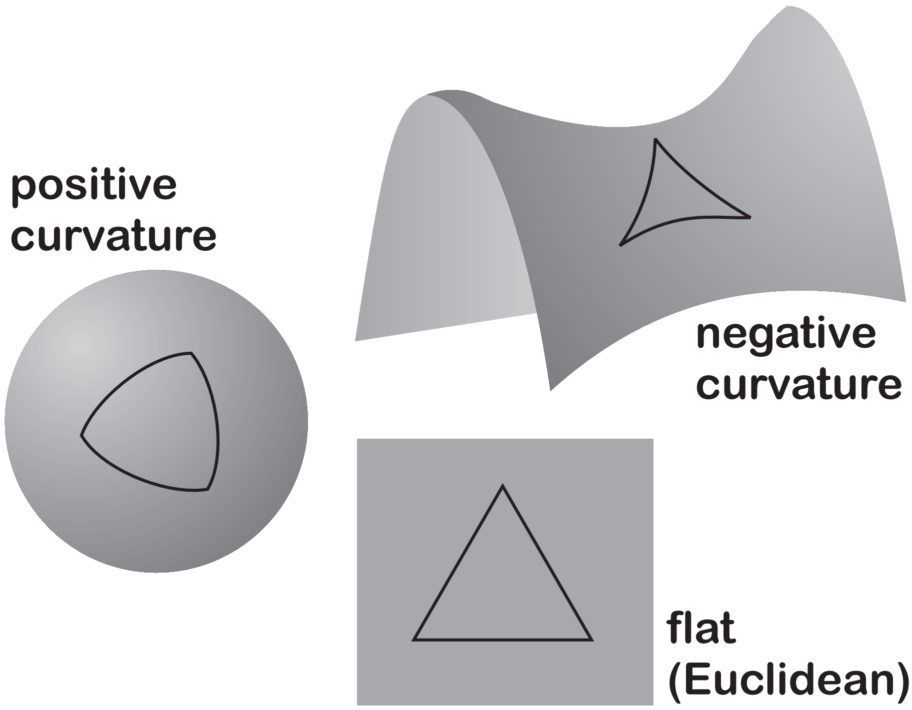

Figure 6: Three cases of curvature for a 2-dim hypersurface embedded in 3-dim Euclidean space.

2.2.3 Homogeneous and isotropic curved spaces

Observations indicate isotropy and homogeneity of the universe on large scales.

Copernican principle: all points are equivalent (no special location).

Isotropy about every point → homogeneity (translational invariance).

Cosmological principle: Universe is spatially homogeneous and isotropic around every point.

The "perfect Copernican principle" (including time invariance) is no longer assumed for the universe—evidently there was a beginning and the universe.

Use synchronized coordinates and find the most general spatial metric γ_{ij} for isotropic and homogeneous spaces.

Will depend on only one parameter, the radius of curvature a ("scale factor of the universe").

Our heuristic derivation begins with a spherical space of constant positive curvature.

Embed this space in a 4-dim Euclidean space with Cartesian coordinates $x_1, \ldots, x_4$ (time not included!).

A three-dimensional spherical space of radius a is defined by

$$a^2 = \sum_{i=1}^{4} x_i^2 \, .$$

The distance between neighboring points in 4-dim space is (Pythagoras)

$$\mathrm{d}\ell^2 = \sum_{i=1}^{4} \mathrm{d}x_i^2 \, .$$

Eliminate the 4th dimension by

$$x_4^2 = a^2 - \sum_{i=1}^{3} x_i^2 \, , \qquad \text{noting that} \quad \sum_{i=1}^{3} x_i^2 \leq a^2 \, .$$

Differentiation provides

$$2x_4 \mathrm{d}x_4 = -2 \sum_{i=1}^{3} x_i \mathrm{d}x_i \qquad \text{or} \qquad \mathrm{d}x_4 = -\frac{1}{x_4} \sum_{i=1}^{3} x_i \mathrm{d}x_i$$

Therefore

$$\mathrm{d}x_4^2 = \frac{1}{x_4^2}\left(\sum_{i=1}^{3} x_i \mathrm{d}x_i\right)^2 = \frac{1}{x_4^2}\sum_{i,j=1}^{3} x_i x_j \mathrm{d}x_i \mathrm{d}x_j$$

so that we can eliminate x_4 completely from the differential distance

$$\mathrm{d}\ell^2 = \sum_{i=1}^{3} \mathrm{d}x_i^2 + \frac{\sum_{i,j=1}^{3} x_i x_j \mathrm{d}x_i \mathrm{d}x_j}{a^2 - \sum_{i=1}^{3} x_i^2} = \mathrm{d}x_i \mathrm{d}x^i + \frac{x_i x_j \mathrm{d}x^i \mathrm{d}x^j}{a^2 - x_i x^i}$$

using the summation convention and co- and contravariant 3-dim vectors (no difference in cartesian coordinates). All elements of the spatial metric are in this way fixed.

Isotropy $\rightarrow$ polar coordinates $0 \leq \rho \leq a$, $0 \leq \theta \leq \pi$, and $0 \leq \varphi \leq 2\pi$ with

$$x_1 = \rho \sin\theta \cos\varphi$$
$$x_2 = \rho \sin\theta \sin\varphi$$
$$x_1 = \rho \cos\theta$$

This provides the differential

$$\mathrm{d}\ell^2 = \frac{a^2}{a^2 - \rho^2}\mathrm{d}\rho^2 + \rho^2 \mathrm{d}\Omega^2 \qquad \text{where} \qquad \mathrm{d}\Omega^2 \equiv \mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2$$

Flat space corresponds to the limit $a \rightarrow \infty$ and thus to

$$\mathrm{d}\ell^2 = \mathrm{d}\rho^2 + \rho^2 \mathrm{d}\Omega^2$$

A 3-dim space with constant negative curvature can not be obtained by a simple embedding in 4-dim space, actually one needs a 5-dim Euclidean embedding space. In the end one finds the same expression for the metric of the 3-dim space with $a^2 \rightarrow -a^2$ or

$$\mathrm{d}\ell^2 = \frac{a^2}{a^2 + \rho^2}\mathrm{d}\rho^2 + \rho^2 \mathrm{d}\Omega^2$$

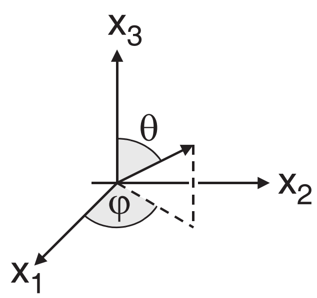

Figure 7: The usual polar coordinates in 3 dimensions.

One often uses a dimensionless coordinate $r = \rho/a$ with

$$\begin{array}{ll} 0 \le r \le 1 & \text{positive curvature} \\ 0 \le r < \infty & \text{flat or negative curvature} \end{array}$$

The metric is then

$$\mathrm{d}\ell^2 = a^2 \left(\frac{\mathrm{d}r^2}{1 - k\,r^2} + r^2 \mathrm{d}\Omega^2 \right) \qquad \text{with} \qquad k = \begin{cases} +1 & \text{positive curvature} \\ \ \ 0 & \text{flat space} \\ -1 & \text{negative curvature} \end{cases}$$

Positive and negatively curved spaces have an absolute length scale associated with them, the curvature radius a.

For a flat space, a is an arbitrary length scale.

The polar coordinates (ρ, θ, φ) can be viewed as being polar coordinates in a Euclidean space that is tangential to the curved space. For the globe of Fig. 5, a point on its surface can be described by ρ and the azimuth angle φ.

These coordinates are not unique—the upper and lower half spheres have the same coordinates.

Instead of ρ one can use the unique coordinate χ with $\rho = a \sin\chi$ and the range $0 \le \chi \le \pi$. It plays the role of a zenith angle in the higher-dimensional embedding space.

For $\chi \ll 1$ we have $r = \rho/a = \chi$.

In (χ, θ, φ) coordinates, the metric is

$$\mathrm{d}\ell^2 = a^2 \left(\mathrm{d}\chi^2 + r_\chi^2 \, \mathrm{d}\Omega^2 \right) \qquad \text{with} \qquad r_\chi = \begin{cases} \sin\chi & \text{positive curvature} \\ \chi & \text{flat space} \\ \sinh\chi & \text{negative curvature} \end{cases}$$

2.2.4 Coordinate distance

The distance between the coordinate origin and a point at (r, θ, φ) is

$$\ell = a\,\chi = a \int_0^r \frac{\mathrm{d}r'}{\sqrt{1 - kr'^2}} = a \begin{cases} \mathrm{asin}(r) & \text{positive curvature } (k = +1) \\ r & \text{flat space } (k = 0) \\ \mathrm{Asinh}(r) & \text{negative curvature } (k = -1) \end{cases}$$

2.2.5 Topology

Positively curved space is always closed (compact).

Other spaces can be closed by nontrivial topology, e.g. torus.

It is an observational question if our space is periodic, e.g. by searching for ghost images of galaxies.

It seems that a possible periodicity must be on scales exceeding our visible universe because periodic features have not been detected.

Some speculations hold that true space has more than 3 dimensions. The extra ones are assumed to be compactified at some small scale. In high-energy particle collisions one could probe the additional dimensions of space.