Homework Set (Week 11)
Introduction to Astroparticle Physics

Georg G. Raffelt
Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)
Föhringer Ring 6, 80805 München
Email: raffelt(at)mppmu.mpg.de

`http://wwwth.mppmu.mpg.de/members/raffelt` → `Teaching`

Assignment of 26 January 2010

1 Sun as an axion source

We consider axions that have a two-photon vertex, characterized by the coupling constant $G_{a\gamma\gamma}$ of dimension $(\text{energy})^{-1}$. As discussed in the lectures, the cross section $\gamma + p \to p + a$ due to the Primakoff process is very roughly

$$\sigma \sim \frac{\alpha G_{a\gamma\gamma}^2}{8\pi}$$

(i) Estimate the energy loss of the Sun due to axion emission, assuming axions can freely escape once produced. Treat the Sun as consisting purely of hydrogen and assume an average temperature of 1 keV. Express the result as a fraction of the solar photon luminosity which is roughly $L_\odot = 4 \times 10^{33}$ erg s^{-1}. Note also that the solar mass is $M_\odot = 2 \times 10^{33}$ g. In other words, the average nuclear energy generation rate in the Sun is 2 erg g^{-1} s^{-1}.

A more rigorous treatment, including screening effects in the Primakoff rate and integrating over a realistic solar model yields

$$L_a \sim G_{10}^2\, 1.85 \times 10^{-3}\, L_\odot\,,$$

very similar to the simple dimensional estimate. Here we have used $G_{10} = G_{a\gamma\gamma}/(10^{-10}\ \text{GeV}^{-1})$.

(ii) Assuming the solar axion production can not exceed its normal photon luminosity (why?), which limit on $G_{a\gamma\gamma}$ is implied?

(iii) Verify that for the relevant range of axion-photon couplings it is indeed true that axions can escape freely once produced, noting that the radius of the Sun is $R_\odot = 6.96 \times 10^{10}$ cm.

2 Plasma frequency and photon dispersion

In a gas of free electrons, photons propagate as if they had a mass, $\omega^2 - k^2 = \omega_{\rm plas}^2$, that is given by the plasma frequency

$$\omega_{\rm plas}^2 = \frac{4\pi\alpha}{m_e}\, n_e$$

where $\alpha = 1/137$ is the fine-structure constant, $m_e = 0.511$ MeV the electron mass, and n_e the electron density.

(i) Near the center of the Sun, the matter density is around 100 g cm^{-3}. Assuming it consists of hydrogen, how large is the plasma frequency? How does it compare with a typical blackbody photon energy, assuming the temperature is 1 keV?

(ii) The interstellar medium (ISM) consists to a large degree of ionized gas, i.e. free electrons, typically of order 1 cm^{-3} in the galaxy. How large is the corresponding plasma frequency?

(iii) If we observe a radio pulsar at a distance of 100 pc with photons of frequency $\nu = 1$ GHz (corresponding to an angular frequency of $2\pi \times 10^9$ s^{-1}), how large is the photon time-of-flight delay caused by the presence of the ISM?

(iv) The pulsar radio emission is pulsed with a typical period in the range 0.2–2 s, corresponding to the rotation period. The spectrum of radio frequencies is broad. What is the impact of the ISM? How can the dispersion effect be used to measure the interstellar electron density?