Homework Set (Week 09)
Introduction to Astroparticle Physics

Georg G. Raffelt
Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)
Föhringer Ring 6, 80805 München
Email: raffelt(at)mppmu.mpg.de

`http://wwwth.mppmu.mpg.de/members/raffelt` → `Teaching`

12 January 2010

1 Energy transfer in WIMP-nucleus collision

A WIMP with mass m_χ and nonrelativistic velocity v strikes a nucleus at rest with mass m_A.

(i) Show that the energy transfer to the nucleus is

$$\Delta E = \frac{m_A\, m_\chi^2}{(m_A + m_\chi)^2}\, v^2\, (1 - \cos\theta)$$

where θ is the scattering angle in the CM frame.

(ii) For which scattering angle and WIMP mass is the transfer maximal? Interpretation?

(iii) Assume the galactic WIMP velocity distribution follows an isothermal halo model with

$$\frac{\mathrm{d}n}{\mathrm{d}v} = n_0\, \frac{4\, v^2}{\sqrt{\pi}\, \sigma^3}\, \mathrm{e}^{-v^2/\sigma^2}$$

where σ is the velocity dispersion. Assume further an isotropic scattering cross section that is independent of velocity. What is the average energy transfer per collision, assuming the Earth is at rest relative to an isotropic WIMP velocity distribution?

(iv) Give numerical values for the average energy transfer, assuming $\sigma = 220$ km s^{-1} and taking WIMP masses of 50, 100, and 200 GeV and the target nuclei oxygen ($A = 16$), silicon ($A = 28$), calcium ($A = 40$), germanium ($A = 74$), xenon ($A = 132$), or tungsten ($A = 184$).

2 Annual modulation of WIMP signal

Consider the same isothermal halo model of the previous exercise. The Earth moves in the halo with a velocity $\mathbf{v}_{\mathrm{E}}$. The halo itself is assumed to be non-rotating, so the WIMP velocity distribution is isotropic in a non-rotating frame.

(i) What is the velocity distribution in the laboratory frame, relevant for a WIMP detection experiment?

(ii) Assuming $v_{\mathrm{E}} = \sigma$, how much larger is the detection rate compared to $v_{\mathrm{E}} = 0$? How much larger is the average energy transfer per collision?

(iii) On its orbit around the Sun, the Earth velocity relative to the halo varies in a range ± 15 km s^{-1} relative to the average, taken to be $\langle v_{\mathrm{E}} \rangle = \sigma = 220$ km s^{-1}. How large is the annual variation of the detection rate?