Homework Set (Week 07)
Introduction to Astroparticle Physics

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1 Maximum weak interaction cross section and transparency of the universe to high-energy neutrinos

Consider a very high-energy neutrino emitted by a source of cosmic rays. Is the universe transparent to neutrinos of any energy or is there a cutoff as we have found for high-energy photons? (i) To solve this problem, first recall that for $s \ll m_{Z,W}^2$ the cross section varies roughly as $\sigma \sim \alpha^2 s/m_{Z,W}^4$, for very high energies with $s \gg m_{Z,W}^2$ as $\sigma \sim \alpha^2/s$ where s is the square of the CM energy. So roughly what is the largest possible cross section? (ii) Considering that the density of cosmic background neutrinos is of order 300 cm^{-3}, what is the minimum of the mean free path? How does this compare with the Hubble distance?

2 Is the Earth opaque for high-energy neutrinos?

High-energy neutrino telescopes, such as IceCube at the Southpole or ANTARES in the Mediterranean search for high-energy cosmic-ray neutrinos. In particular, they look for those coming through the Earth (upward going neutrinos) because from the other direction there is a huge signal from ordinary cosmic rays coming down from the atmosphere. What are the highest energy neutrinos that one can see through the Earth? (i) To solve this problem, first estimate the scattering cross section of a high-energy neutrino on a nucleon: The interaction is with the constituent quarks which have energies corresponding to their momentum uncertainty (they are localized in the nucleon). (ii) Then estimate the mean free path in the Earth and compare with its diameter of 12700 km.

3 Freeze-out of Dirac neutrinos

If neutrinos are Dirac particles, each family has four states, similar to electrons. However, the r.h. Dirac states have no gauge interactions and are in this sense sterile. They only couple with the mass to other particles. Their interaction rate, relative to active neutrinos, is suppressed by an approximate factor $(m/E)^2$. Which approximate value of the Dirac mass would be necessary to equilibrate these states after the QCD phase transition which happens at $T_{\rm QCD} \sim 170$ MeV?

4 Matter-radiation equality including neutrinos

(i) Determine the redshift of matter-radiation equality, assuming neutrinos are nearly massless. What is the photon temperature at that time and what the neutrino temperature?
(ii) How large should neutrino masses be to act as "matter" at this epoch?