Homework Set (Week 05)
Introduction to Astroparticle Physics

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24 November 2009

1 Entropy conservation in the expanding universe

Show that in a comoving volume of the universe the total entropy is conserved,

$$\frac{\mathrm{d}}{\mathrm{d}t}(a^3 s) = 0 \qquad \text{where} \qquad s = \frac{\rho + p}{T}\,.$$

Use the results

$$\frac{\mathrm{d}p}{\mathrm{d}T} = \frac{\rho + p}{T} \qquad \text{and} \qquad \frac{\mathrm{d}}{\mathrm{d}t}\left(a^3\rho\right) = -p\,\frac{\mathrm{d}}{\mathrm{d}t}\left(a^3\right)$$

where the former follows from thermodynamic reasoning and the latter was discussed in the lectures for deriving the second Friedmann Eqn.

2 Chemical potential

(i) Consider a relativistic species (e.g. electrons/positrons) in the early universe and assume a small asymmetry between particles and antiparticles. Express the asymmetry parameter

$$\eta = \frac{n_{e^-} - n_{e^+}}{n_\gamma}$$

in terms of the relativistic chemical potential μ in the approximation $\mu \ll T$.

(ii) Consider the opposite limit of a degenerate Fermi gas in the $T \to 0$ limit, e.g. the neutrons in a neutron star. Express the number density in terms of the Fermi momentum.

(iii) How large is the Fermi momentum for nuclear density, $\rho_{\rm nuc} = 3 \times 10^{14}$ g cm^{-3}, assuming there are only neutrons or assuming there are equal densities of protons and neutrons ("symmetric nuclear matter")? How large is the nonrelativistic Fermi energy in symmetric nuclear matter? How large is the typical distance between nucleons if we use the length L of a cube that is on average available to one nucleon?

(iv) Compare the typical distance between nucleons with a typical distance over which pions as exchange particles provide a significant interaction potential.

3 Conditions at the deconfinement transition

The transition from the quark-gluon plasma to hadrons occurs in the early universe at $T_{\rm QCD} \sim 170$ MeV. (i) Taking pions to be relativistic, what is a typical distance between them immediately after the transition? (ii) What is the density of protons and neutrons, assuming a vanishing chemical potential? Is this approximation good, considering that today $\eta_B \sim 10^{-9}$?

4 Some particle properties

To answer the following simple questions, visit the homepage of the particle data group at `http://pdg.lbl.gov` and look up most of the answers. (i) What is the mass difference between protons and neutrons? (ii) Between π^0 and $\pi^\pm$? What are the dominant decay channels of neutral and charged pions, and what are the mean lifetimes for them? Can you imagine why the decay $\pi^0 \to \nu\bar{\nu}$ does not seem to occur? And why is the charged-pion decay $\pi^+ \to e^+ + \nu_e$ so much slower than the muon decay channel? Hint: Think of the left-handedness of weak interactions and of angular momentum conservation! (iii) What is the spin and mass of the Δ resonance? (iv) Baryons containing a strange quark are called hyperons. Which are the lightest hyperons and what is their quark content? Can you imagine what is the difference between the Λ and the Σ^0?

5 Cosmic-ray muons

Primary cosmic rays consist of protons and heavier nuclei, hitting the atmosphere with a steeply falling energy spectrum. The interactions with nuclei of the air leads to particle showers, containing large amounts of pions. Charged pions decay according to $\pi^+ \to \mu^+ + \nu_\mu$ and $\pi^- \to \mu^- + \bar{\nu}_\mu$. At sea level the muon flux is ~ 1 cm^{-2} min^{-1}. The production is at an altitude of ~ 15 km. (i) For an energy of 3 GeV, which fraction of muons reaches sea level before decaying? (ii) On the other hand, how far do $\pi^\pm$ of the same energy get after production before decaying? Can you imagine what would happen if the pion lifetime were much longer?