Homework Set (Week 03)
Introduction to Astroparticle Physics

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09 November 2009

No lectures or exercise session on 10 Nov. 2009 due to other commitments of lecturer. The solutions will be discussed in the exercise session on Tuesday, 17 Nov. 2009.

1 Second Friedmann Equation

The Friedmann equation and continuum equation are

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3}\,G_{\rm N}\rho - \frac{k}{a^2} \qquad \text{and} \qquad \dot\rho = -3(\rho+p)\,\frac{\dot a}{a}\,.$$

Differentiating the Friedmann equation with respect to time and using the continuum equation, derive the second Friedmann equation

$$\frac{\ddot a}{a} = -\frac{4\pi}{3}\,G_{\rm N}(\rho+3p)\,.$$

2 Measures of distance

Assume that a cosmological model is perfectly characterized by $\Omega_{\rm M}$ and Ω_Λ, i.e. ignore the contribution of radiation. An observer receives light from a distant source (emitter E) and measures that the spectral lines have suffered the redshift $z_{\rm E}$. (i) Write down general integral expressions for the lookback time (how long ago was this signal emitted) and for the coordinate, luminosity and angle distances of the source. These quantities will depend on $\Omega_{\rm M}$, Ω_Λ and $z_{\rm E}$. (ii) Solve them explicitly for a flat, matter-dominated universe ($\Omega_{\rm M} = 1$ and $\Omega_\Lambda = 0$). (iii) For this model, find the redshift where the angular size of an object is smallest. (iv) What is this redshift in a realistic flat model ($\Omega_{\rm M} = 0.27$ and $\Omega_\Lambda = 0.73$)? (v) For this realistic model, plot the different distance measures as a function of redshift.

3 FLRW models models of the universe with matter and vacuum energy

Assume that a cosmological model in the post-radiation epoch is perfectly characterized by $0 \leq \Omega_{\rm M} < \infty$ and $-\infty < \Omega_\Lambda < +\infty$. In the plane defined by these two parameters, identify the regions where (i) the universe is flat, positively curved and negatively curved, (ii) is accelerating, coasting, or decelerating, (iii) the locus where a static Einstein universe is possible, and (iv) the universe expands forever or eventually recollapses.