E₈ and F-Theory GUTs

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Motivation: Part I

There are few clues or constraints on symmetries beyond the Standard Model – apart from a strong motivation for Grand Unification (minimally SU(5)).

Symmetries and their matter representations are some of the most robust and well-understood aspects of string theory – perhaps are the likeliest arena for extracting universal predictions/constraints?

F-theory is a good framework for addressing such questions: on the line between generality and simplicity.

Matter curves and Yukawa couplings are also associated to symmetries in F-theory.
• Top quark Yukawa coupling implies the existence of a point of $E_6$

• Have we extracted the full implications of this?

• Does the unification of Yukawa, matter and gauge fields through symmetries imply a role for exceptional groups in controlling also the matter representations and gauge symmetries?

• A seemingly key question is therefore: given the exceptional structure of the top-quark Yukawa coupling does $E_8$, as the maximal compact exceptional group, play a role in controlling the matter and gauge symmetries?

• In F-theory this translates to a question of whether the existence of an exceptional co-dimension 3 locus can limit the structure of co-dimension 2 and 1 physics?
Motivation: Part II

Over the last years there has been much work on understanding four-dimensional compactifications of F-theory which exhibit an SU(5) gauge symmetry extended by some Abelian symmetries.

The physics data extracted from a typical such fibration is:

• A divisor supporting the SU(5) gauge group and a section associated to a U(1)
• Curves on $S_{\text{SU}(5)}$ which support massless representations of the symmetry groups present
• An intersection structure of matter curves giving rise to associated Yukawa couplings

There are by now quite a few examples, but it is not clear what the possibilities are? Are they finite? Is there a controlling structure?

Motivation Parts I + II: Is there a connection to $E_8$?
Higgsing $E_8$ using its adjoint

A nice way to parameterise possible SU(5)$\times$U(1)$^n$ theories arising from $E_8$ is by considering the maximal decomposition to SU(5)$\times$U(1)$^4$ of the adjoint representation

$$E_8 \rightarrow SU(5)_{\text{GUT}} \times SU(5)_\bot$$

$$248 \rightarrow (24, 1) \oplus (1, 24) \oplus (10, 5) \oplus (5, 10) \oplus (\bar{10}, \bar{5}) \oplus (5, \bar{10})$$

$$U(1)_A = \sum_{i=1}^{5} a_i^A t^i, \quad \sum_i a_i = 0$$

$$10_i : t_i, \quad \bar{5}_{ij} : t_i + t_j, \quad 1_{ij} : t_i - t_j, \quad t_i t_j = \delta_i^j.$$ 

The possible Higgsing of the U(1)s is determined by vevs for the 10 differently charged GUT singlets

This leads to a Higgsing $E_8$ tree, where the different charged states are denoted {#10,#5,#1}

Note: we consider only D-flat Higgsing where pairs of singlets get equal vevs
A first guess for a relation to $E_8$ might be that the F-theory spectra should be embeddable in one of the spectra reached from Higgsing $E_8$ using its adjoint.

This is not possible.

For example, consider a theory with 1 $U(1)$. From $E_8$ we have 2 possibilities:

4-1 Theory: $10_4, 10_1, 5_3, 5_2, 1_5$

3-2 Theory: $10_2, 10_3, 5_6, 5_4, 5_1, 1_5$

An example fibration gives:

BGK: $10_{-1}, 5_{-8}, 5_{-3}, 5_2, 5_7, 1_5, 1_{10}$

[ Braun, Grimm, Keitel ’13 ]

Is there any connection between the spectra of fibrations in the literature and $E_8$ at all?

There is a set of elliptic fibrations which ‘globalise’ the Higgsed $E_8$ theories: Factorised Tate Models

The 3-2 Factorised Tate model can be written in 2 ways. As a Tate form fibration:

$$y^2 = x^3 + a_1 xyz + a_2 x^2 z^2 + a_3 yz^3 + a_4 xz^4 + a_6 z^6$$

$$a_1 = e_2 d_3, \quad a_2 = (e_2 d_2 + \alpha \delta d_3) w, \quad a_3 = (\alpha \delta d_2 + \alpha \beta d_3 - e_2 \delta \gamma) w^2,$$

$$a_4 = (\alpha \beta d_2 + \beta e_2 \gamma - \alpha \delta^2 \gamma) w^3, \quad a_6 = \alpha \beta^2 \gamma w^5.$$  

As a fibration in $P_{[1,1,2]}$:

$$P_T^{[1,1,2]} = z^2 + b_0 u^2 z + b_1 uv z + b_2 v^2 z + c_0 u^4 + c_1 u^3 v + c_2 u^2 v^2 + c_3 uv^3 = 0$$

$$b_0 = -wd_3 \alpha, \quad b_1 = -e_2 d_3, \quad b_2 = \delta, \quad c_0 = w^3 \alpha \gamma,$$

$$c_1 = w^2 (d_2 \alpha + e_2 \gamma), \quad c_2 = w e_2 d_2, \quad c_3 = w \beta.$$
Both forms share the following matter spectrum:

\[
10_{-2}^1 : \quad w = d_3 = 0 , \quad 10_{-3}^2 : \quad w = e_2 = 0 ,
\]

\[
5_{-6}^1 : \quad w = \delta = 0 , \quad 5_{-4}^2 : \quad w = \beta d_3 + d_2 \delta = 0 ,
\]

\[
5_{-1}^3 : \quad w = \alpha^2 c_2 d_2^2 + \alpha^3 \beta d_3^2 + \alpha^3 d_2 d_3 \delta - 2\alpha c_2^2 d_2 \gamma - \alpha^2 c_2 d_3 \delta \gamma + c_2^3 \gamma^2 = 0 .
\]

\[
1_{-5}^1 : \quad f_1 = f_2 = 0
\]

\[
1_{-10}^2 : \quad \beta = \delta = 0
\]

Comparing with the 3-2 E_8 model it is based on: 3-2 Theory: \(10_{-2}, 10_{-3}, 5_{-6}, 5_{-4}, 5_{-1}, 1_{-5}\)

We find an extra singlet field! Exactly the one needed to make the missing coupling \(1_{-10}^2 5_{-6}^1 5_{-4}^2\)

This is a general property of many fibrations – one finds for any pair of 5-matter curves there is a singlet which makes such a cubic coupling. Let us call the set of fibrations which exhibit this property “complete networks”.
Higgsing away from $E_8$

It is possible to deform this fibration in a way which corresponds to Higgsing the non-$E_8$ singlet

$$P_T^{[1,1,2]} \to P_T^{[1,1,2]} + c_{4,1} w v^4$$

And in Tate form:

$$a_4 \to a_4 - c_{4,1} \alpha \left( \alpha d_3^2 + 4 \gamma w \right) w^3, \quad a_6 \to a_6 + c_{4,1} \left( -\alpha d_2 + e_2 \gamma \right)^2 w^5.$$  

This leads to a $Z_2$ gauge theory with spectrum

$$10^1_0, 10^2_1, \bar{5}^1_0, \bar{5}^3_1, 1^1_1,$$

$$\bar{5}^1 : \delta (\beta d_3 + d_2 \delta) + e_2 c_{4,1} d_3^2 = w = 0.$$  

As expected there is no such theory in the Higgsed $E_8$ constructions
A proposition for extending $E_8$

In the decomposition $SU(5) \times U(1)^4$ the spectrum exhibits the property that there are not enough singlets to make all possible $1 \times 5 \times 5$ couplings

$$10_i : t_i, \quad \overline{5}_{ij} : t_i + t_j, \quad 1_{ij} : t_i - t_j,$$

A natural extension is then to include a new set of 15 GUT singlets with charges such that they allow for a $1 \times 5 \times 5$ coupling with all the fields

$$1_{ijkl} : t_i + t_j - t_k - t_l$$

Now Higgsing with these new singlets leads to an extended Higgsing tree

The combinatorics are that 4 choose 25 is 12650, but a computer analysis reveals a surprisingly compact structure
Embedding known models

Now coming back to our example fibration with no embedding in $E_8$

$$\text{BGK: } 10_{-1}, 5_{-8}, 5_{-3}, 5_2, 5_7, 1_5, 1_{10}$$

We find that it can be embedded in one of the new theories
Overall we looked at 30 fibrations constructed in the literature [Borchmann, Braun, Cvetic, Garca-Etxebarria, Grassi, Grimm, Kapfer, Keitel, Klevers, Kuntzler, Lawrie, Mayrhofer, EP, Piragua, Sacco, Schafer-Nameki, Till, Weigand]:

Find in total that $1/29$ and $27/29$ of flat fibrations embeddable in $E_8$ and our extended set respectively.

1 fibration not possible to turn off non-flat loci

2 fibrations can not be embedded even in the extended set
The 2 non-embeddable models were the only ones with the property that they did not form complete networks

\[ I_{5}^{(102)} \]

\[ (4,2,0,0,0,0,0,0) \]

\[ [\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9] \]

\[ \sigma_2 \sigma_4 - \sigma_3 \sigma_5 \]

\[ (B.9) \]

\[ (B.10) \]

The 5-curves \( \sigma_1 \) and \( \sigma_2 \) have no 1 5 5 coupling

So all 27 complete networks were embeddable in our extended set

Less generic configurations need further understanding...
SO(10) Fibrations

It turns out that the lack of singlets in the adjoint of $E_8$ to form $155$ coupling occurs only for $SU(5)$ as a GUT group.

For $SO(10)$ we have a singlet for each pair of $10$ representations:

$$248 \rightarrow (45, 1) \oplus (16, 4) \oplus (\overline{16}, \overline{4}) \oplus (10, 6) \oplus (1, 15)$$

We find, consistently, that $10$ $SO(10)$ top models are all embeddable in $E_8$ upon restriction of flatness.

<table>
<thead>
<tr>
<th>Model</th>
<th>10-matter</th>
<th>16-matter</th>
<th>16 16 10 coupling</th>
<th>Non-flat loci</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top 1</td>
<td>$b_2, c_{1,2}$</td>
<td>$c_{2,1}$</td>
<td>$c_{2,1} \cap b_2$</td>
<td>$c_{2,1} \cap c_{1,2}$</td>
</tr>
<tr>
<td>Top 2</td>
<td>$b_{0,2}, c_3$</td>
<td>$c_{2,1}$</td>
<td>$c_{2,1} \cap b_{0,2}$</td>
<td>$c_{2,1} \cap c_3$</td>
</tr>
<tr>
<td>Top 3</td>
<td>$b_{2,1} - b_{0,1} c_{3,1}$</td>
<td>$b_{0,1}, b_2$</td>
<td>$b_{0,1} \cap b_2, b_{0,1} \cap c_{1,2}$</td>
<td>$c_{3,1} \cap b_2$</td>
</tr>
<tr>
<td>Top 4</td>
<td>$c_{3,1}$</td>
<td>$b_2$</td>
<td>-</td>
<td>$c_{3,1} \cap b_2, b_{0,1}$</td>
</tr>
<tr>
<td>Top 5</td>
<td>$c_{1,2}$</td>
<td>$b_{0,1}$</td>
<td>$c_{1,2} \cap b_{0,1}$</td>
<td>$b_2$</td>
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<td>$d_0 \cap b_0, c_{1,1}$</td>
</tr>
<tr>
<td>Top 2</td>
<td>$c_2, b_{2,2} d_0 - c_{1,1} d_2,1$</td>
<td>$d_0, c_{1,1}$</td>
<td>$d_0 \cap c_{2,0} d_0 \cap c_{1,1}$</td>
<td>$d_0 \cap d_{2,1}, c_{1,1} \cap c_{1,1} \cap b_{2,2}$</td>
</tr>
<tr>
<td>Top 3</td>
<td>$b_{2,2}, c_2$</td>
<td>$c_{1,1}$</td>
<td>-</td>
<td>$c_{1,1} \cap b_{2,2}, c_{1,1} \cap c_{1,1} \cap d_0$</td>
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[See also Kuntzler, Schafer-Nameki ‘14]
Heterotic Duality – Bundle Data

Since perturbative Heterotic models are based on $E_8$ it is interesting to think about the Heterotic duals of the beyond-$E_8$ models.

Beyond $E_8$ in bundle data? Find that in Weierstrass form $f_0...f_3$, $g_0...g_5$ encode more data than an SU(5) spectral cover.

Concretely starting from a Tate fibration an mapping to Weierstrass one finds

\[
y^2 = x^3 + a_1xyz + a_2x^2z^2 + a_3yz^3 + a_4xz^4 + a_6z^6
\]

\[
f_0 = -1/48b_5^4, \quad f_1 = -1/12b_5^2b_4, \quad f_2 = -1/12(b_4^2 - 6b_5b_3), \quad f_3 = 1/24b_2,
\]
\[
g_0 = 1/864b_5^6, \quad g_1 = 1/144b_5^4b_4, \quad g_2 = 1/72b_5^2(b_4^2 - 3b_5b_3),
\]
\[
g_3 = -1/864(3b_2b_5^2 - 8b_4^3 + 72b_5b_4b_3), \quad g_4 = 1/144(-b_2b_4 + 36b_3^2) + \Delta g_4,
\]
\[
g_5 = 1/288b_0,
\]
Heterotic Duality - Bundle

\[ b_i \sim a_{6-i,5-i} \]

\[ \Delta g_4 = 1/576 b_5^2 \left( a_{1,1}^4 + 8a_{1,1}^2 + 16a_{2,2}^2 - 24a_{1,1}a_{3,3} - 48a_{4,4} + 2a_{1,1}a_{1,2}b_5 + 8a_{1,2}a_{2,2}b_5 - 24a_{3,4}b_5 + 2a_{1,2}b_5^2 + 4a_{1,1}a_{1,3}b_5^2 + 8a_{2,4}b_5^2 + 4a_{1,4}b_5^3 + 8a_{1,1}a_{1,2}b_4 + 16a_{2,3}b_4 + 8a_{1,3}b_5b_4 - 24a_{1,3}b_3 \right). \]  

The sub-leading terms in \( w \) are in Tate form are induced by Higgsing away form \( E_8 \)

\[ a_4 \rightarrow a_4 - c_{4,1} \alpha (\alpha d_3^2 + 4\gamma w) w^3, \]

For SO(10) or higher the degrees of freedom on each side of the duality are equal

A natural possibility is that \( \Delta g_4 \) relates to the degrees of freedom of Higgsing beyond \( E_8 \)
Heterotic Duality - Geometry

Non-perturbative gauge symmetries outside of $E_8$ can arise in the Heterotic string when the bundle degenerates over a singularity.

We studied the relation of the singlets extending $E_8$ with singularities in the Heterotic dual for the 4 top models in [Borchmann, Mayrhofer, EP, Weigand ‘13].

Find that the heterotic duals have singularities, and that restricting the fibration to turn off the singularities truncates the spectrum so that it is embeddable in $E_8$.

However for the 3-2 factorised Tate model we find no singularity associated to the non-$E_8$ singlet.
Summary

• Introduced an extension to $E_8$ which results in a new tree of $SU(5)xU(1)^n$ theories

• All 27 flat (complete network) fibrations in the literature could be embedded into this extended set

• 2 non-complete networks did not have an embedding – a further extended notion of $E_8$?

• Some comments on Heterotic duality: non-$E_8$ singlets possibly associated to additional degrees of freedom on the F-theory side of the bundle data, also possibly associated to singular loci on the Heterotic geometry

• Only a first step towards a possible programme of research to extract the full implications of the existence of a Yukawa $E_6$ point
Thanks
What do we know $E_8$ is not:

- $E_8$ is not a maximal gauge symmetry group in String Theory

$$E_8^{2561} \times F_4^{7576} \times G_2^{20168} \times SU(2)^{30200}$$  

[Candelas, Perevalov, Rajesh ’97]

But in considering a GUT all the charged matter comes from one divisor

- Even on a single $E_8$ divisor it is still possible to enhance the symmetry further

Heterotic Small Instantons $\leftrightarrow$ M-theory M5-M9($E_8$) Collisions $\leftrightarrow$ F-theory non-minimal singularities

But in string theory this seems to always be accompanied by an infinite number of massless states: tensionless strings and their excitations (4D still not fully understood?)