Conjectures on the Swampland

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Gravity and Quantum Field Theory are notoriously difficult to combine in a single consistent framework that is valid in the ultraviolet

However, from the perspective of low-energy effective theory, ultraviolet problems need not be of concern

A critical energy scale is the Planck mass $M_p \sim 10^{19} \text{ GeV}$

- For $E \sim M_p \rightarrow$ unique theory?
- For $E \ll M_p \rightarrow$ anything goes?

Any problem with GR + Maxwell below $M_p$? 

$$S = \int \sqrt{-G} \left( M_p^2 R + \frac{1}{4g^2} F^2 \right)$$
Within string theory, this apparent freedom, manifests as the **Landscape**
A valid question arose:

Does our **current knowledge** of String Theory imply any non-trivial **universal** predictions for low-energy theories?

- String Theory very much still work-in-progress – final rules not clear
- Specific vacua very predictive – can just follow phenomenological approach

Initial suggestions proposed with the introduction of the **Swampland:**

*The set of self-consistent effective theories that cannot be completed into quantum gravity in the ultraviolet*  

[Vafa ’05]
The Landscape might be huge, but it is small compared to the Swampland.
The Swampland programme is about extracting universal predictions from string theory not as a specific low-energy theory, but as rules governing such theories.

- **String Theory Constructions**
- **Explicit String Theoretic Origin**
- **A Swampland Conjecture**
- **Quantum Gravity (Black Holes)**
- **New Microscopic Physics**

“String Inspired”
Prototypical example: Einstein-Maxwell theory in the Swampland

\[ S = \int \sqrt{-G} \left( M_p^2 R + \frac{1}{4g^2} F^2 \right) \]

The **Weak Gravity Conjecture**

- **Must have a charged particle with mass smaller than charge**

  \[ g \, q \, M_p \geq m \]

  **Electric WGC**

- **The cutoff scale of the theory (infinite tower of new states) is at**

  \[ \Lambda \sim g \, M_p \]

  **Magnetic WGC**
Need a charged particle for charged extremal Black Holes to discharge.

- Electric field
- Positively charged black hole
The limit $g \to 0$ is one where the gauge symmetry becomes a global symmetry.

Gravity should be the weakest force acting on a particle.

No stable gravitationally bound states.

These are indirect, loose arguments: signposts, rather than microscopic physics.

Strongest evidence from String Theory [Dine et al ’03; ... ; Lee, Lerche, Weigand ‘18]
The **Swampland Distance Conjecture** [Ooguri, Vafa ’06]

For a massless scalar field $\phi$, which undergoes a variation $\Delta \phi$, there is an infinite tower of states whose mass scale as $\Delta \phi \to \infty$ goes as

$$m \sim M_P e^{-\frac{\Delta \phi}{M_P}}$$

for some $\alpha > 0$

It was further proposed that: [Baume, EP ‘16; Klaewer, EP ‘16]

- Exponential behavior appears precisely at $\Delta \phi \sim M_P$, and $\alpha \sim \mathcal{O}(1)$
- Also holds for fields with a potential $V(\phi)$
Prototypical example: compactification on a circle

$-\infty \leftarrow \phi \leftarrow \infty$

$m_W \sim e^\phi$

$m_{KK} \sim e^{-\phi}$

Highly non-trivial evidence this is general in String Theory (for 8 supercharges)

[Ooguri, Vafa ‘06; Cecotti ‘15; Grimm, EP, Valenzuela ‘18; Lee, Lerche, Weigand ‘18; Grimm, Li, EP ‘18]

Model-independent general results – highly mathematical
The evidence for fields with a potential has two aspects:

- **Simple**: the behaviour is set by the field-space metric (the kinetic terms)
  \[ g_{\phi\phi}(\phi)(\partial \phi)^2 + V(\phi) \quad \phi \text{ non-canonical} \]

- **Highly non-trivial**: the potential can itself change appropriately the field-space metric

Axion monodromy models:

[Silverstein, Westphal ’08; ...]

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Gravitational backreaction

[EP ’15; Baume, EP ’16]
Consider the WGC in the presence of massless scalar fields

\[ g^2 q^2 M_p^2 \geq m^2 + \mu^2 M_p^2 \]

The coupling to scalar fields is

\[ \mu = \partial \phi m \]

Non-trivial evidence in string theory

[EP ‘17; Lee, Lerche, Weigand ‘18]
Imposing that gravity should be the weakest force gives a \textbf{Scalar WGC}

\[ \partial_\phi m > m \]  

[EP ‘17]

Can be proven that BPS states satisfy this.

This can only hold for large variations of $\phi$ if we have

\[ m \sim e^{-\phi} \]

The distance conjecture can then be thought of as a magnetic version of the Scalar Weak Gravity Conjecture.
We find an inter-related collection of ideas, which hints at underlying physics.

\[ g^2 q^2 M_p^2 \geq m^2 + (\partial_\phi m)^2 M_p^2 \]

\[ \Lambda \sim g M_p \]

\[ \Lambda \sim e^{-\frac{\phi}{M_p}} M_p \]

Gravity Weakest Force

Electric-Magnetic duals

Asymptotic limit \( g \to 0 \)

Asymptotic limit \( \phi \to \infty \)

Global Symmetries
Proposal: the Swampland conjectures are consequences of the emergent nature of dynamical fields in quantum gravity

[Grimm, EP, Valenzuela ‘18]

See also [Harlow ’15; Heidenreich, Reece, Rudelius ‘17+‘18]
Emergent gauge field toy model $\mathbb{CP}^N$

\[ \mathcal{L} = \partial z_i^* \partial z^i + (z_i^* \partial z^i)(z_j^* \partial z^j) \quad \text{with} \quad z_i^* z^i = \frac{N}{g^2} \]

Contains a gauge symmetry $z_i \rightarrow e^{i\alpha(x)} z_i$, with a gauge field ‘variable’

\[ A \equiv \frac{g^2}{2iN} (z_i^* \partial z^i - z^i \partial z_i^*) \]

The charged scalars develop a mass $m_z$, can integrate them out, and in the IR find an emergent gauge field

\[ \mathcal{L}_{IR} = \frac{1}{4g_{IR}^2} F^2 \]

The gauge coupling behaves as if it comes purely from 1-loop threshold effects

\[ \frac{1}{g_{IR}^2} = \frac{1}{g_{UV}^2} + \frac{N}{12\pi^2} \log \frac{\Lambda}{m_z} \quad \text{(Like QED + N massive fields)} \]
**Emergent behavior:** IR coupling given by integrating down from scale $\Lambda$

\[
\frac{1}{g_{IR}^2} \sim 1\text{-loop}
\]

**More conservative:** Scale $\Lambda$ where reach strong coupling

\[
\frac{1}{g_{IR}^2} = \frac{1}{g_{UV}^2} \quad \text{+1-loop}
\]

\[
\frac{1}{g_{UV}^2} = \frac{1}{g_{IR}^2} \quad \text{−1-loop}
\]

$\varnothing(1)$

[Harlow ’15; Grimm, EP, Valenzuela ‘18]  
[Heidenreich, Reece, Rudelius ‘17+’18]
Integrating out a tower of states can generate dynamics for gravity/gauge/scalar.

Integrate down from a UV scale $\Lambda$.

$$M_p^2 \bigg|_{IR} = M_p^2 \bigg|_{UV} + N \Lambda^2$$

Fixes the UV cut-off scale as the Species scale

$$\Lambda = \frac{M_p}{\sqrt{N}}$$

[ ...; Dvali ’07 ]
Take equally spaced tower $\Delta m \sim m$, gives $N \sim \left(\frac{M_p}{m}\right)^{\frac{2}{3}}$

For the gauge coupling we have

$$\frac{1}{g_{IR}^2} = \frac{1}{g_{UV}^2} + \sum_{i}^{N} \frac{q_i^2}{6\pi^2} \log \frac{\Lambda}{m_i}$$

$$\frac{1}{g_{IR}^2} \sim N^3 \sim \frac{M_p^2}{m^2}$$

The mass scale of the tower $m \sim g_{IR} M_p$

Magnetic WGC

“Approximate” Electric WGC
For scalar field, the 1-loop wavefunction renormalization is

\[ g_{\phi\phi}^{IR} = g_{\phi\phi}^0 + \sum_{i}^{N} \left( \frac{\partial_{\phi} m_i}{4\pi^2} \right)^2 \log \frac{\Lambda}{m_i} \]

\[ g_{\phi\phi}^{IR} \sim N^3 \left( \partial_{\phi} m \right)^2 \sim \left( \frac{M_p \partial_{\phi} m}{m} \right)^2 \]

Proper distance

\[ \Delta \phi = \int \sqrt{g_{\phi\phi}^{IR}} \, d\phi \sim M_p \int \frac{\partial_{\phi} m}{m} \, d\phi \sim -M_p \log m \]

Find

\[ m \sim e^{-\frac{\Delta \phi}{M_p}} \qquad \text{Distance Conjecture} \]

\[ g_{1R}^{\phi\phi} \left( \partial_{\phi} m \right)^2 \sim m^2 \qquad \text{“Approximate” Scalar WGC} \]
Works for all tested **string theory** settings (8 supercharges):

For compactifications of type IIB string theory on a Calabi-Yau manifold, have towers of D3 branes wrapping 3-cycles

Integrating them out at 1-loop precisely recovers the behavior of the gauge couplings and scalar fields at any weak-coupling or large distance regime

[Grimm, EP, Valenzuela ‘18]

Similar results found for F-Theory on Calabi-Yau

[Lee, Lerche, Weigand ‘18]

In String Theory it is sometimes better not to think of fundamental and emergent but rather as a **duality**

\[ \phi \]
\[ A_\mu \]
\[ g_{\mu \nu} \]
Recently, the **Swampland de Sitter Conjecture** was proposed

\[
|\nabla V(\phi)| > c \, V(\phi) \quad c \sim \mathcal{O}(1)
\]

[Obied, Ooguri, Spodyneiko, Vafa ‘18]

In particular, this forbids de Sitter minima

**Experimentally testable!** Euclid, Dark Energy Survey, …
The conjecture was proposed based on a seeming conspiracy against de Sitter in the best-understood string theory constructions.

The position of the “de Sitter line” is under debate – but it is far enough to the right that one might seriously consider the conjecture.
Can the de Sitter conjecture be implemented into the coherent picture of the Swampland we have proposed? [Ooguri, EP, Shiu, Vafa ‘18]

It is natural expect some connection between the potential and tower of states

In $N = 2$ supergravity, the potential is in 1-to-1 correspondence with the gauge coupling matrix: if gauge fields are emergent so must the potential be
de Sitter space has a finite horizon for an observer, of radius $R$

$$S_{ds} = \log \dim \mathcal{H} = R^2$$

[ Hochs, Hawking ’77]

Can be interpreted as the number of states in the Hilbert space

[ Banks ’00; Witten ‘01]

In de Sitter space the potential can be associated with an entropy

$$S_{ds}(\phi) = \frac{1}{V(\phi)}$$
The **distance conjecture**:  

As we move in field space a tower of $N$ states becomes light and so the dimension of the Hilbert space of the effective theory increases

$$N(\phi) \sim e^{b\phi} \quad b \sim O(1)$$

We can assign an entropy to the tower below a cut-off scale

$$S_{\text{tower}}(\phi) \sim N(\phi)^\gamma R(\phi)^\delta$$

If the tower dominates the Hilbert space, then we can equate the two notions of entropy

$$\frac{1}{V(\phi)} \sim R(\phi)^2 \sim N(\phi)^\gamma R(\phi)^\delta \quad V(\phi) \sim N(\phi)^{-\frac{2\gamma}{2-\delta}}$$
Utilising the expression for $N(\phi)$ from the distance conjecture gives

$$\frac{\partial V}{\partial \phi} = \frac{\partial V}{\partial N} \frac{\partial N}{\partial \phi} \sim b \left( \frac{2\gamma}{2 - \delta} \right) V$$

This is the de Sitter conjecture

$$\left| \frac{\partial V}{\partial \phi} \right| > c \, V \quad \text{with} \quad c \sim \frac{2b\gamma}{2 - \delta}$$

Determining the exponents $\gamma$ and $\delta$ amount to the microstates of the tower in quantum gravity – this is a difficult problem

For free fields in a box of size $R$ we find $\gamma = \frac{1}{4}$ and $\delta = \frac{3}{2}$.
The argument relied on three assumptions:

• The distance conjecture for fields with a potential
• The states of the tower dominate the Hilbert space
• We can assign an entropy to the potential
The states of the tower dominate the Hilbert space

This follows at large distances in field space:

i) From the duality with the tower of states

ii) The exponentially large number of states in the tower expect to dominate the Hilbert space

Couplings in String Theory are scalar fields \((g_s, t, u, g_I, f_I, \ldots)\)

Weak Coupling \(g \to 0\) \quad \text{Large distance } \phi \to \infty

So the assumption holds in any weakly-coupled parametrically controlled regime of string theory
We can assign an entropy to the potential

Consider a potential which is away from a minimum so the field is rolling

An **apparent horizon** exists if the universe is accelerating

\[ \left| \frac{\partial V}{\partial \phi} \right| \leq \sqrt{2} V \]

The theory is stable on horizon scales (and over a Hubble time) if

\[ \frac{\partial^2 V}{\partial \phi^2} \geq -\mathcal{O}(1) V \]

Finite temperature lifting of mass

\[ m_{\phi}^2 = \frac{\partial^2 V}{\partial \phi^2} + H^2 = \frac{\partial^2 V}{\partial \phi^2} + V \]
The apparent horizon has area $R^2$ as utilized in the computation.

An entropy can be associated this horizon using the Covariant Entropy Bound:

Light sheets sweep out a volume $\mathcal{V}$

$$S(\mathcal{V}) \leq R^2$$

[Fischler, Susskind ’98; Bousso ‘99]
Imposing the assumptions utilized in the derivation we then arrive at a Refined de Sitter Conjecture:

\[ |\nabla V| \geq \frac{c}{M_p} V \quad \text{or} \quad \min(\nabla_i \nabla_j V) \leq -\frac{c'}{M_p^2} V \]

[ Ooguri, EP, Shiu, Vafa ‘18]

We have derived this from the distance conjecture, at any parametrically controlled regime of string theory

It is natural to conjecture that it holds even away from this regime
It is interesting to note that the refinement we were led to also avoids some counter examples to the original conjecture

- The top of the Higgs potential has

\[ |\nabla V| \sim 10^{-55} \ V \quad \text{min}(\nabla_i \nabla_j V) \sim -\frac{10^{35}}{M_p^2} \ V \]

[Denef, Hebecker, Wrase ‘18]

- The top of the potential for any axion (including QCD axions and pion)

\[ \text{min}(\nabla_i \nabla_j V) \sim -\frac{1}{f^2} \ V \]

The Weak Gravity Conjecture applied to axions gives \( f \leq M_p \)
Summary

• There are a number of existing conjectures about the Swampland, and they form a coherent interlinked framework

• Discussed a proposal for the underlying microscopic physics behind the conjectures: emergence of dynamical fields in quantum gravity

• The de Sitter conjecture can be tied to the distance conjecture in any parametrically controlled regime of string theory
Thank You