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Introduction to Gauge/Gravity Duality

Examples VI

To hand in Thursday 3rd December in the examples class

I. **Relation between propagators in AdS**

Let us consider euclidean AdS in the Poincare patch.

a) Derive the equations of motion for a scalar field with mass m in euclidean AdS_{d+1}.

(2 points)

b) Use the ansatz $\phi(z) = z^{\Delta}$ near the boundary $z \to 0$ and determine the two possible values of $\Delta_{\pm}$, where $\Delta_{+} > \Delta_{-}$.

(2 points)

c) The bulk-to-boundary propagator $K(x, z; x')$ is the solution of the equations of motion which is regular in the interior and diverges like

$$\lim_{z \to \epsilon} K(z, x; x') = \epsilon^{\Delta_{-}} \delta(x - x')$$

near the boundary, i.e. for $\epsilon \ll 1$. The bulk-to-bulk propagator is given by the solution of the equation of motion with a pointlike source term,

$$\left(\Box_{x,z} - m^2\right) G(z, x; z', x') = \frac{1}{\sqrt{g}} \delta(x - x') \delta(z - z'),$$

where $\Box_{x,z}$ is the scalar Lapacian in euclidean AdS_{d+1} acting only on x and z. Moreover the bulk-to-bulk propagator is regular in the interior.

Show that the bulk-to-boundary propagator $K(x, z; x')$ can be calculated from the bulk-to-bulk propagator $G(z, x; z', x')$ by

$$K(z, x; x') = \lim_{z' \to \epsilon} \frac{\Delta_{+} - \Delta_{-}}{\epsilon^{\Delta}} G(z, x; z', x').$$

Hint: Do not use the explicit solution for $G(z, x; z', x')$, but Green's second identity!

(6 points)