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Introduction to Gauge/Gravity Duality

Examples IX

To hand in Friday 14th December in the examples class

I. **Penrose-Brown-Henneaux Transformation**

Show that, when transforming the metric

$$ds^2 = L^2 \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} g_{\mu\nu} dx^\mu dx^\nu \tag{1}$$

according to the Penrose-Brown-Henneaux transformation

$$\rho = \rho'(1 - 2\sigma(x')) \qquad x^\mu = (x')^\mu + a^\mu(x', \rho') \tag{2}$$

and demanding that

$$g'_{55} = g_{55} \text{ and } g'_{\mu 5} = g_{\mu 5}\,, \tag{3}$$

one obtains

$$\partial_5 a^\mu = \frac{L^2}{2} g^{\mu\nu} \partial_\nu \sigma \tag{4}$$

and

$$g_{\mu\nu} \to g_{\mu\mu} + 2\sigma \left(1 - \rho \frac{\partial}{\partial \rho}\right) g_{\mu\nu} + \nabla_\mu a_\nu + \nabla_\mu a_\nu\,. \tag{5}$$

Note that the index 5 stands for the ρ direction. 5 points

II.**Operators and Fixed Points**

a) Explain what irrelevant, relevant and marginal operators are. 3 points

b) Comment on the stability conditions of fixed points. 2 points

References:
"Quantum Field Theory and Critical Phenomena" by Jean Zinn-Justin
"An Introduction to Quantum Field Theory" by Peskin and Schroeder