Exercise 5.1: DIS

Consider deeply inelastic scattering (DIS) in the parton model, \( e(k) + q(p) \to e(k') + q'(p') \), where the quark \( q \) struck by the electron has momentum \( p^\mu = \xi P^\mu \), so carries a fraction \( \xi \) of the proton’s momentum. The momentum transfer between the electrons is \( q^\mu = k^\mu - k'^\mu \), with \( q^2 = -Q^2 \). In the lectures we also introduced the scaling variable \( x = Q^2 / (2 P \cdot q) \) and the relative energy loss \( y = (P \cdot q) / (P \cdot k) = Q^2 / \hat{s} \), where \( \hat{s} = (p + k)^2 \). We have shown that

\[
\frac{1}{4} \sum_{\text{spins}} |\mathcal{M}|^2 = \frac{e^2 q e}{Q^4} L_{\mu \nu} Q_{\mu \nu} = 2 e^2 q e^4 \hat{s}^2 Q^4 (1 + (1 - y)^2)^2 . \tag{1}
\]

Starting from this expression, calculate the partonic differential cross section

\[
\frac{d^2 \hat{\sigma}}{dx dy} = \frac{4 \pi \alpha_s^2}{y Q^2} \left[ 1 + (1 - y)^2 \right] \frac{1}{2} e^2 q \delta(\xi - x) .
\]

Hint:

Use

\[
d\Phi_2 = \frac{1}{(2\pi)^3} \frac{d^3 k'}{2 E'} \frac{d^4 p'}{(2\pi)^4} 2 \pi \delta(p'^2) (2\pi)^4 \delta^4(k + p - k' - p') = \frac{d\phi}{(4\pi)^2} dx dy \delta(\xi - x) .
\]

Exercise 5.2: Splitting functions

The so-called “plus-prescription” for a function \( g(x) \) which is divergent at \( x = 1 \) is defined by

\[
\int_0^1 dx f(x) [g(x)]_+ = \int_0^1 dx \left( f(x) - f(1) \right) g(x) .
\]

where \( f(x) \) is an arbitrary (smooth) function. Example:

\[
\int_0^1 dx \frac{f(x)}{[1 - x]_+} = \int_0^1 dx \frac{f(x) - f(1)}{1 - x} .
\]

The regularised splitting function \( P_{q \to qg}(x) \) is given by

\[
P_{q \to qg}(x) = C_F \left( \frac{1 + x^2}{[1 - x]_+} + K \delta(1 - x) \right) . \tag{2}
\]

(a) Calculate \( K \) using the fact that

\[
\int_0^1 dx P_{q \to qg}(x) = 0 . \tag{3}
\]

(b) What is the physical meaning of eq. (3)?