Exploring the Interior of N=1 Field Spaces

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Work in progress + 2210.14238

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Introduction

String Theory (and its compactifications) come with a number of **scalar fields** whose vacuum expectation values determine the **properties of the effective theory**

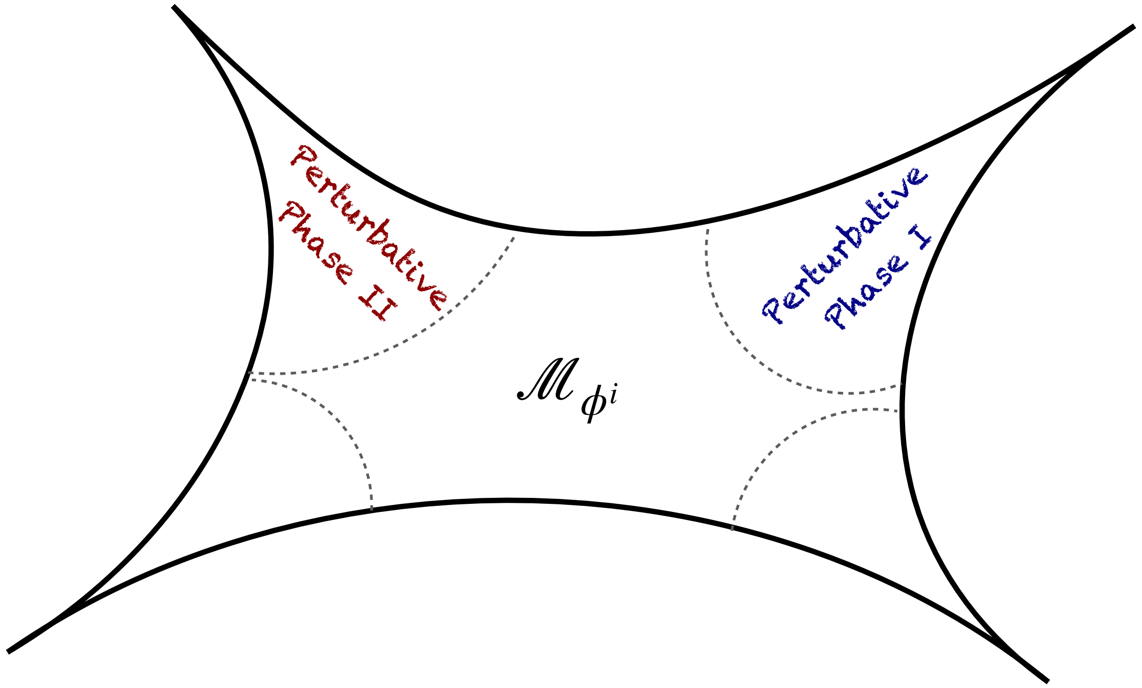
$\rightarrow$ values of couplings, masses of states, value of the EFT cut-off …

Families of EFTs from string theory parametrized by the values of the scalar fields

$\rightarrow$ scalar field space $\mathscr{M}_{\phi^i}$

Structure of $\mathscr{M}_{\phi^i}$ gives information about general properties of the theory

$\rightarrow$ allowed values for ϕ^i, different perturbative descriptions, dualities … .



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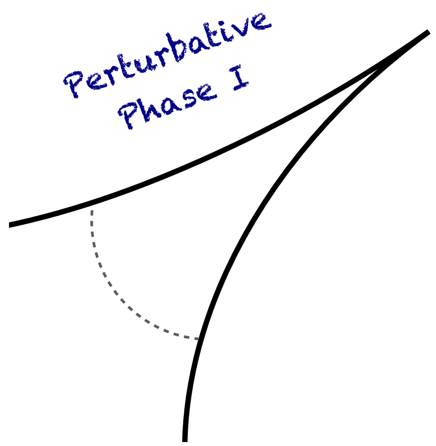

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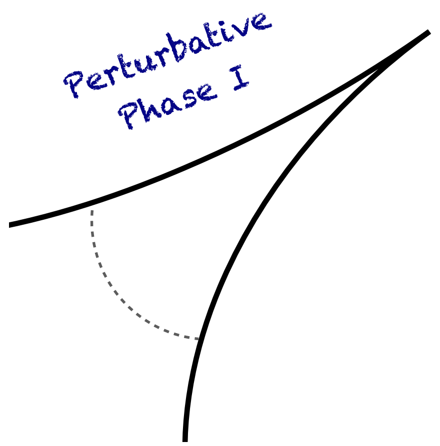

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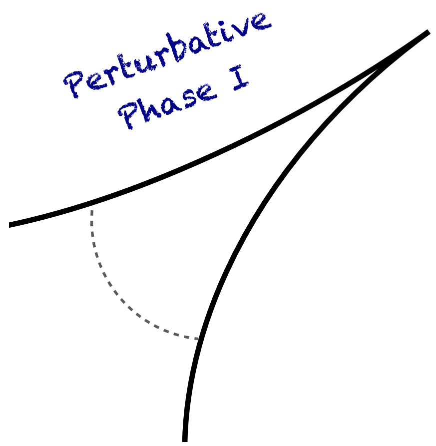

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Example 4d N=2: Moduli space factorizes into vector- and hypermultiplet sector and only one factor contains the string coupling → tree-level exact.

Question: What about the more realistic cases in 4d with minimal (or no) supersymmetry?

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Gather some intuition from 4d N=2 first — Specifically Type IIA Compactifications on CY 3-fold X_3

- Moduli space spanned by:
  - *Type II dilaton + axionic partner*
  - *Complex structure moduli of X_3 + axionic partners*

  hypermultiplets

  - *(complexified) Kähler moduli of X_3*

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- N=2 supersymmetry ensures factorization $\mathscr{M} = \mathscr{M}_{\text{HM}} \times \mathscr{M}_{\text{VM}}$.

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→ can trust the structure derived from string CFT

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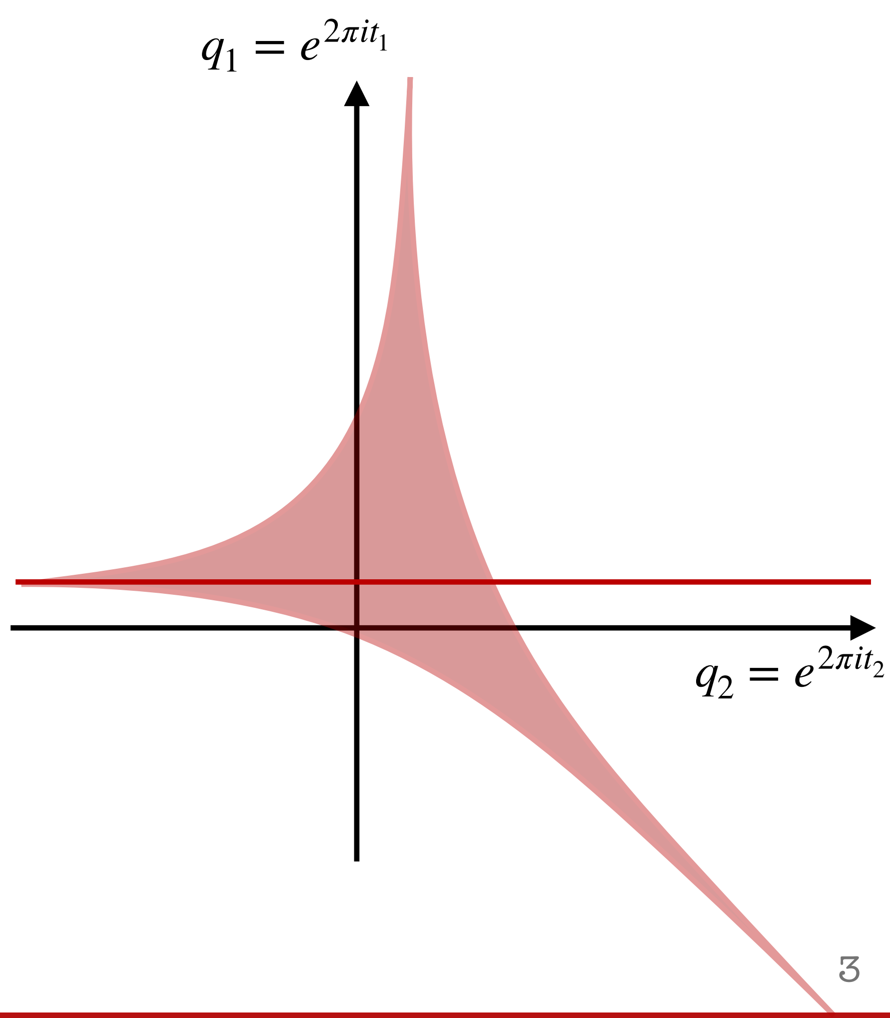

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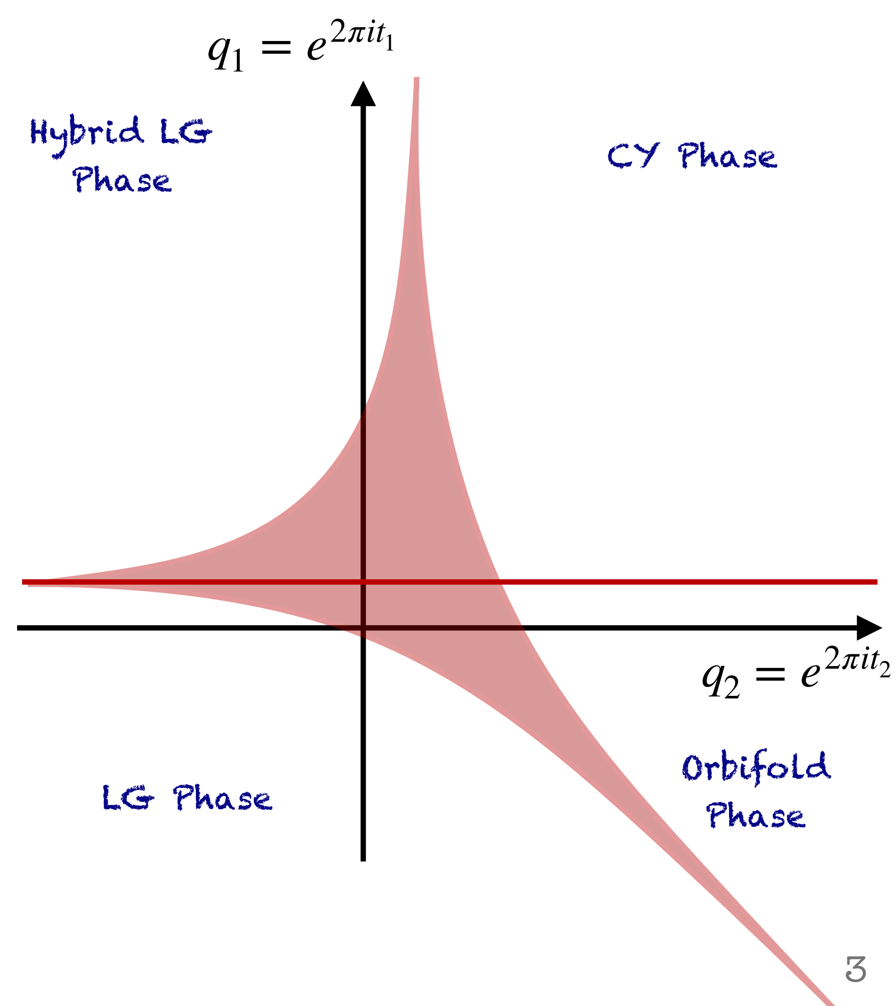

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  - *at small volume get phases different from CY phase, e.g. orbifold phases, Landau-Ginzburg or hybrid phases.*

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$$T_i = \frac{1}{2}\int_{D_a} J \wedge J + i \int_{D_a} C_4$$

J: Kähler form on B_3
D_a: Generators of $\mathrm{Eff}^1(B_3)$
C_4: Type IIB RR four-form

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- What happens away from the overall large volume limit?
  1. *small curve limit for <u>some</u> curves in B_3*
  2. *Mixing between c.s. and Kähler sector*

Structure of Kähler field space

- Consider first small curve limits in B_3.

- Naively might expect a similar pattern as in Type IIA → shrinking genus-0 curves also fall in three classes?

IIA on CY3:
[Witten '96]

- $\mathcal{N}_{C|B_3} = \mathcal{O}(-1) \oplus \mathcal{O}(-1)$
- $\mathcal{N}_{C|B_3} = \mathcal{O}(-2) \oplus \mathcal{O}(0)$
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- What happens in the small volume limit for such curves inside the base B_3 of F-theory?

- From Type IIA perspective: expect a conifold singularity $\Delta_C = 0$ at $\text{vol}(C) = 0$.

- Curve has $\bar{K} \cdot C = 0$ such that locally (for small C) see enhanced supersymmetry.

  $\rightarrow$ can we use this to our benefit?

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- Enhanced SUSY manifests itself in $X_3^{(2)} = T^2 \to \mathbb{F}_2$ having additional (non-polynomial) deformations

$$F_2 \subset \mathbb{P}^3 : \qquad \zeta\eta + \xi^2 = 0 \quad \to \quad \zeta\eta + \xi^2 = \epsilon\tau^2$$

- For $\epsilon \neq 0$ obtain $X_3^{(0)} : T^2 \to \mathbb{F}_0$ and additional complex structure deformations associated to ϵ give four scalar fields that pair up with Kähler modulus to give 5 scalar fields in N=(2,0) matter multiplet.

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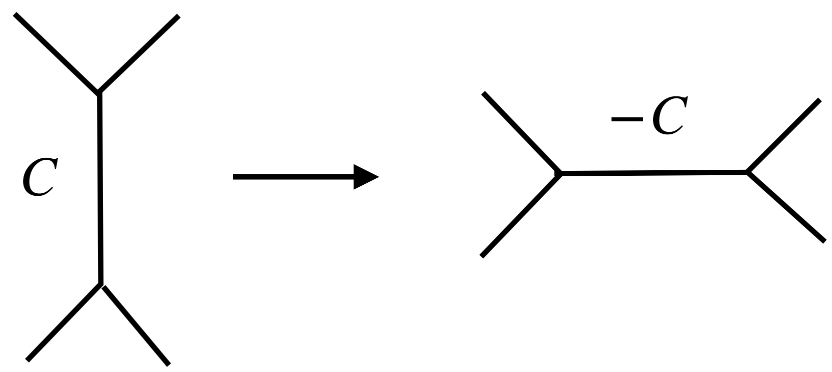

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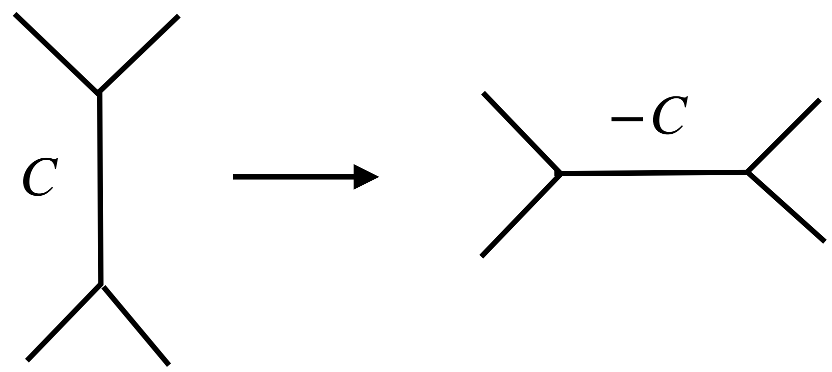

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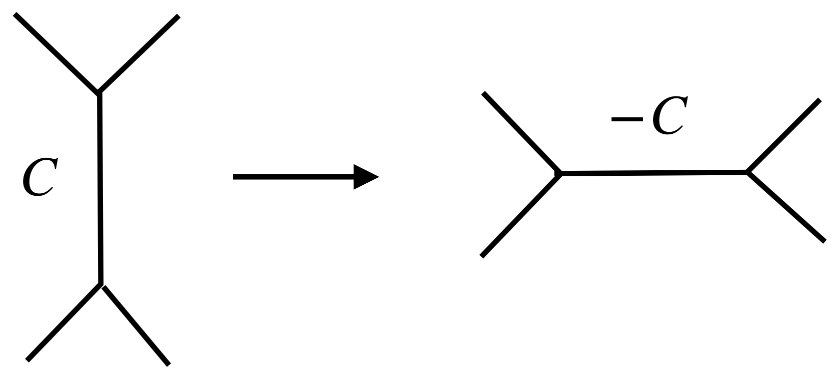

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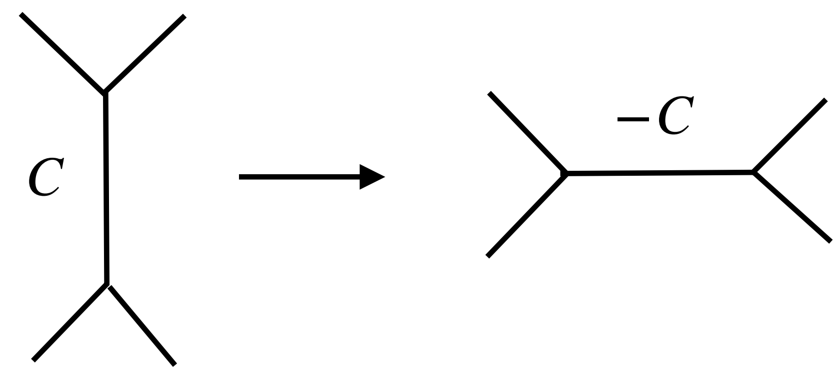

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→ hypermultiplet moduli space has constant curvature.

→ conifold singularity resolved at quantum level.

[Ooguri, Vafa '96]

Local description of small volume limit

- What are the scalar fields making up the N=2 hypermultiplet associated to C?

  → N=1 effective theory only gives two scalar fields

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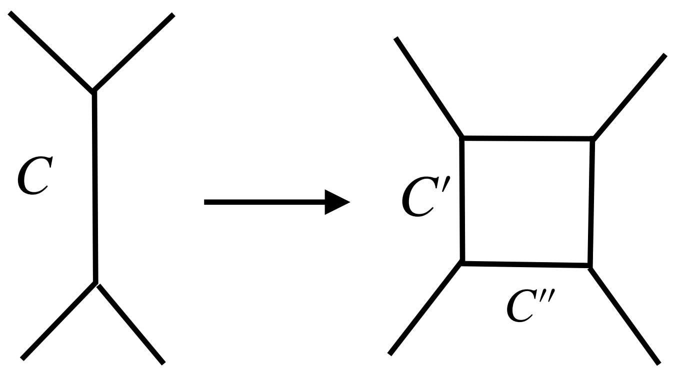

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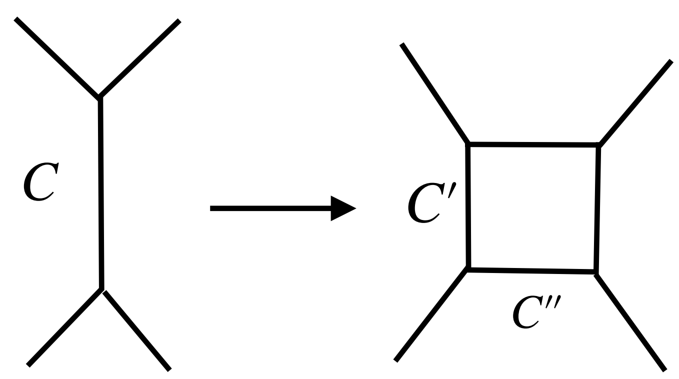

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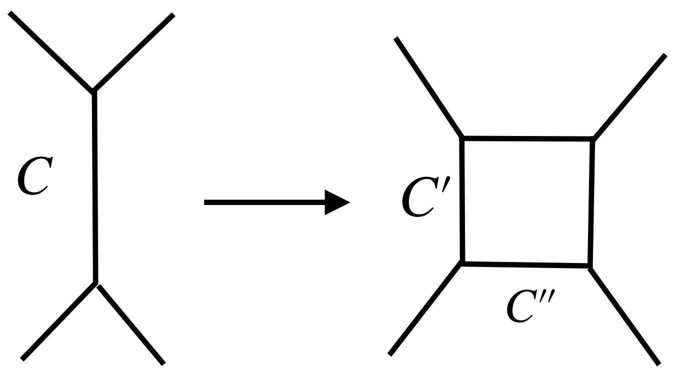

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(analogue of splitting in 6d)

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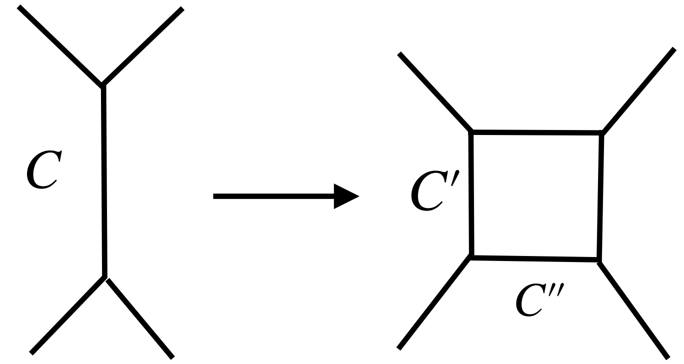

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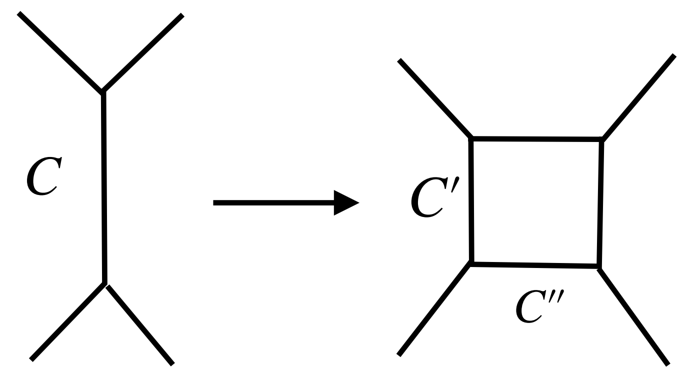

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- Theory on X_4 should be realized on sublocus in deformation space of $\hat{X}_4$

$$\Longrightarrow \quad \delta\chi = \chi(\hat{X}_4) - \chi(X_4) = 6(\delta h^{1,1} + \delta h^{3,1} - \delta h^{2,1}) = 0$$

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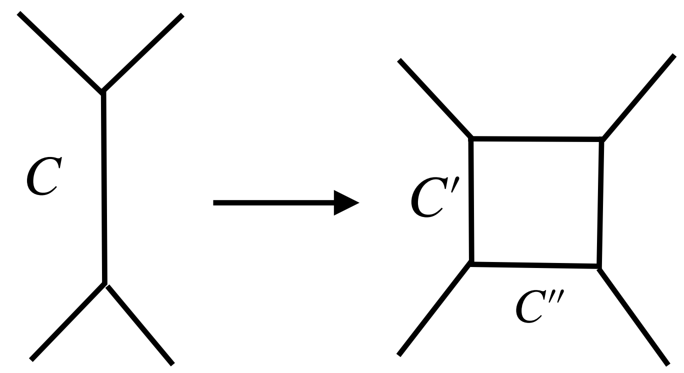

- Get a new fourfold $\hat{X}_4$ such that $\delta h^{1,1} = h^{1,1}(\hat{X}_4) - h^{1,1}(X_4) = 1$ and volumes of curves are identified as $t_C = t_{C'} - t_{C''}$

$$D3|_C \longrightarrow D3|_{C'} - D3|_{C''}$$

'N=2 string' $\longrightarrow$ **'N=1 string'** **'N=1 string'**

- Theory on X_4 should be realized on sublocus in deformation space of $\hat{X}_4$

$$\Longrightarrow \quad \delta\chi = \chi(\hat{X}_4) - \chi(X_4) = 6(\delta h^{1,1} + \delta h^{3,1} - \delta h^{2,1}) = 0$$

- This suggests $\delta h^{2,1} = \delta h^{1,1} = 1 \rightarrow$ from Type IIB perspective $h^{2,1}(\hat{X}_4)$ associated to periods of C_2 and B_2. [(Greiner), Grimm '14-'17]

  $\rightarrow$ these are the deformations that complete the would-be N=2 hypermultiplet.

Embedding of $\mathscr{M}_{X_4}$ in $\mathscr{M}_{\hat{X}_4}$

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  $\rightarrow$ at large $t = \int_C J$ turning on deformations associated to $\delta h^{2,1}$ brings us from X_4 to $\hat{X}_4$

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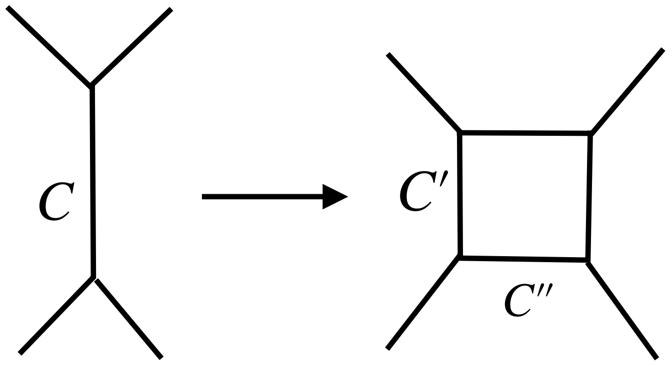

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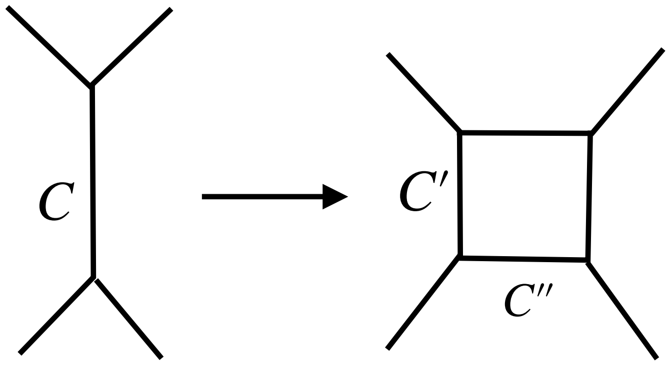

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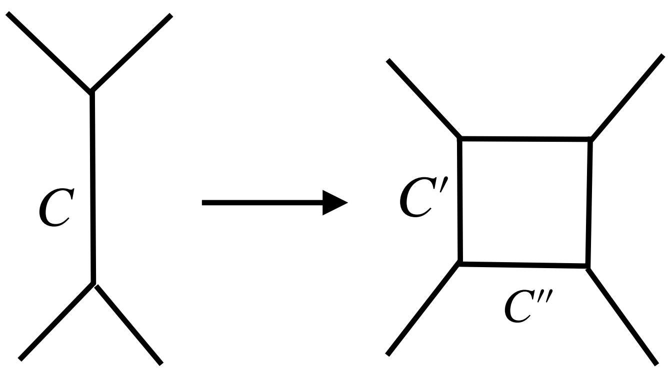

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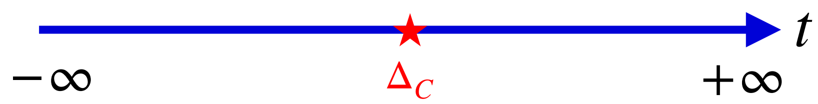
- $\mathscr{M}_{\hat{X}_4}$ locally described by N=2 hypermultiplet moduli space → no singularity

  → tension $T_{D3|C}$ finite at the quantum level.

Singularities: Local vs. Global

- Classical field space $\mathscr{M}_{X_4}$:



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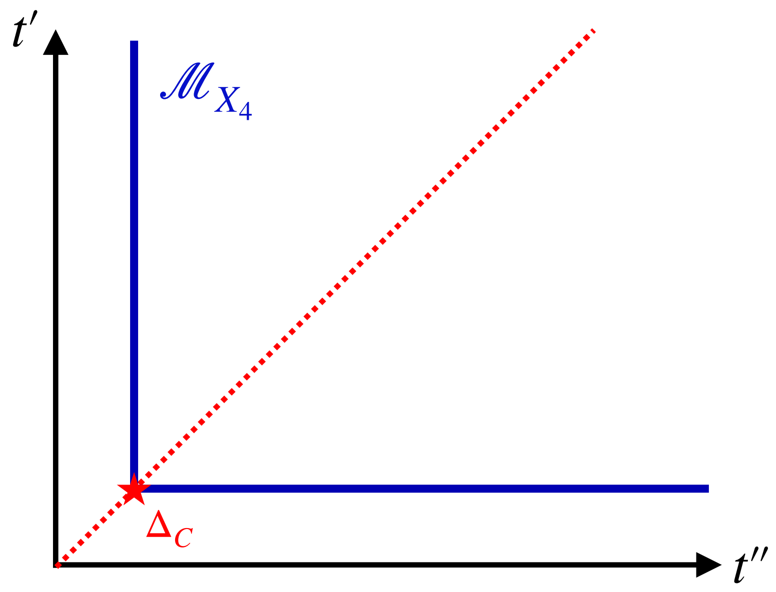

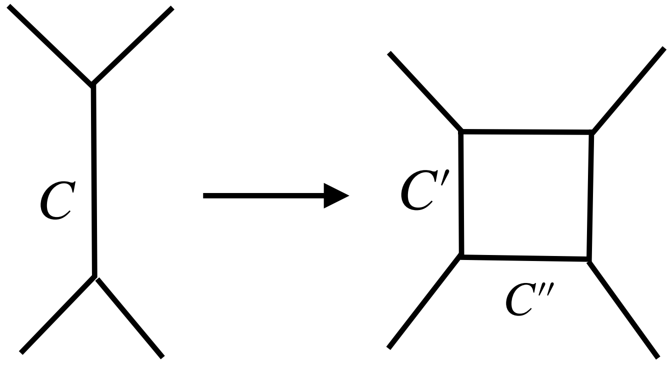

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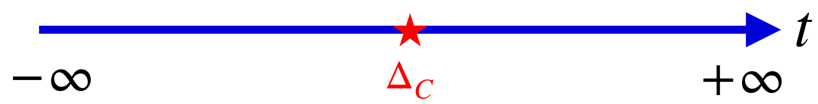

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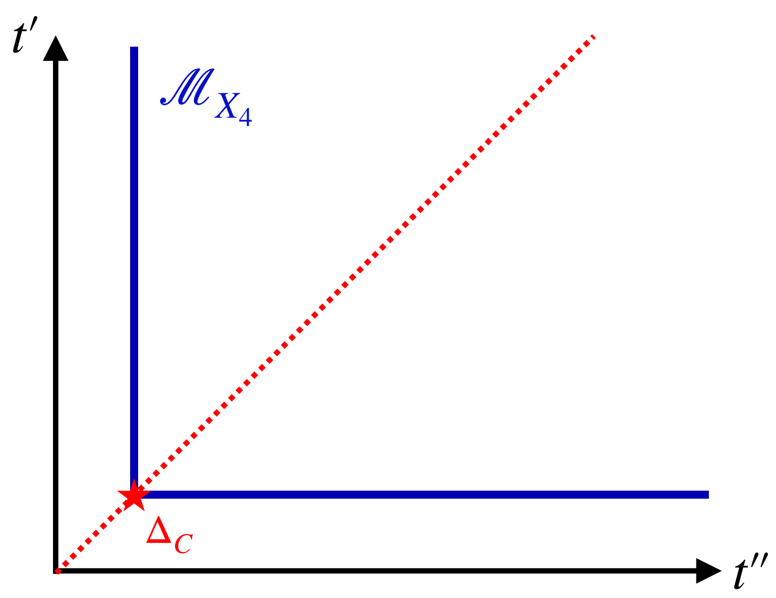

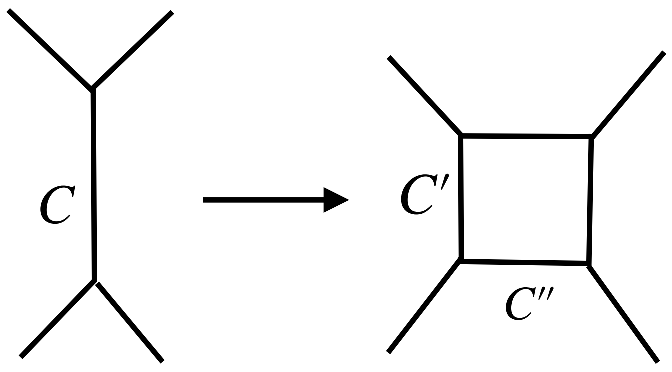

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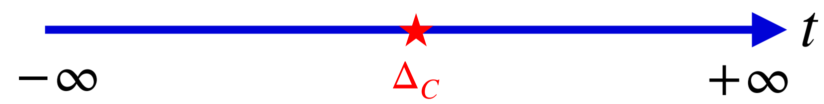

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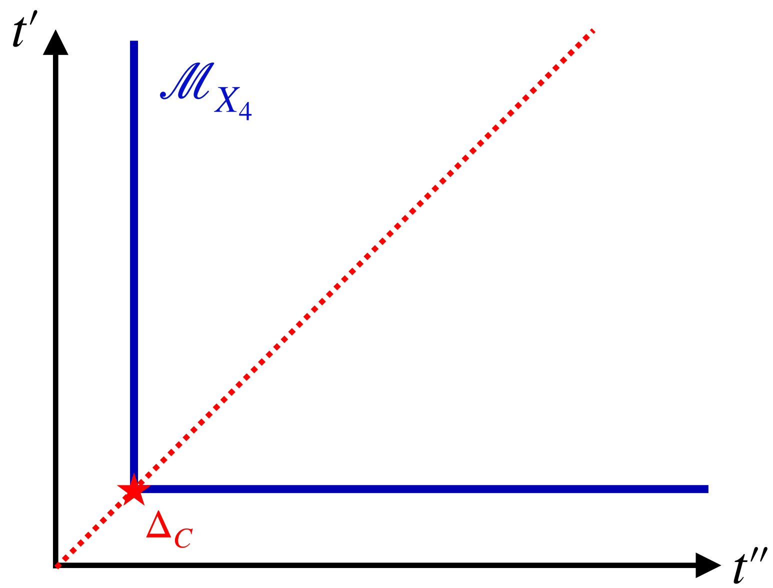

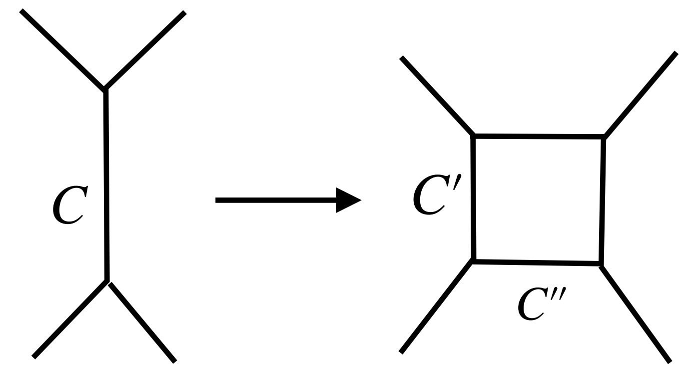

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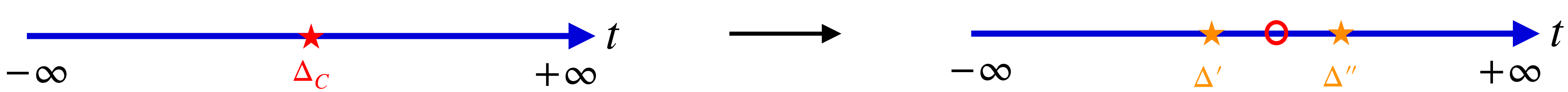
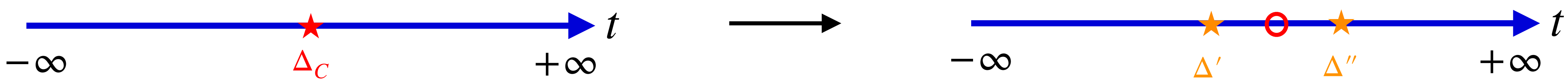
  → complex moduli space (instead of quaternionic) and still expect $\dim_{\mathbb{C}} = 1$ singularities



Singularities: Local vs. Global

- From perspective of $\hat{X}_4$ singularities Δ' and Δ'' associated to shrinking exceptional divisor E



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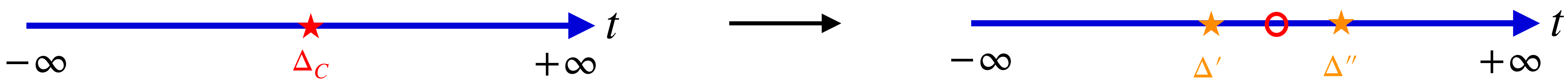

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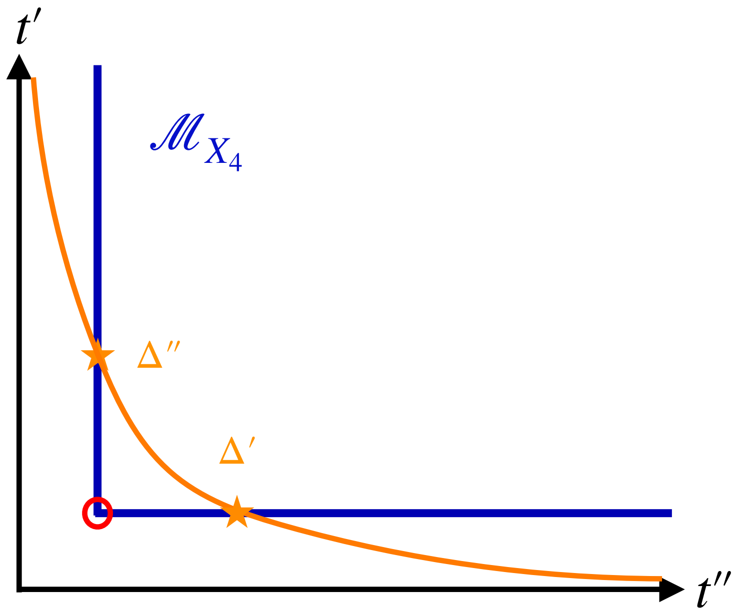

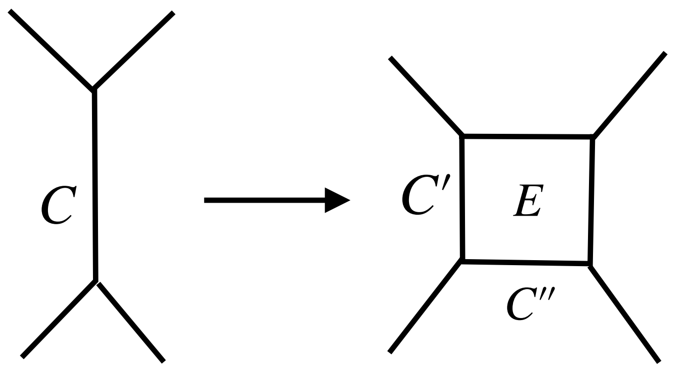

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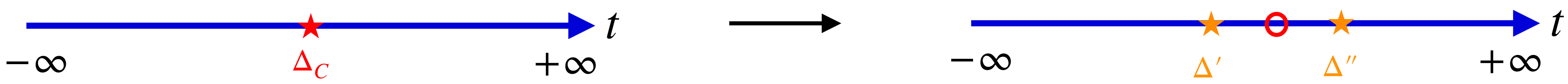

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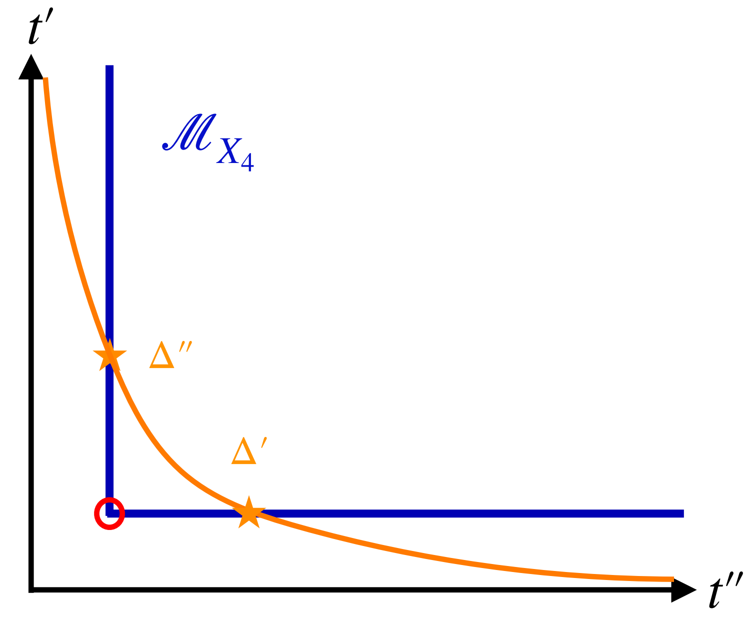
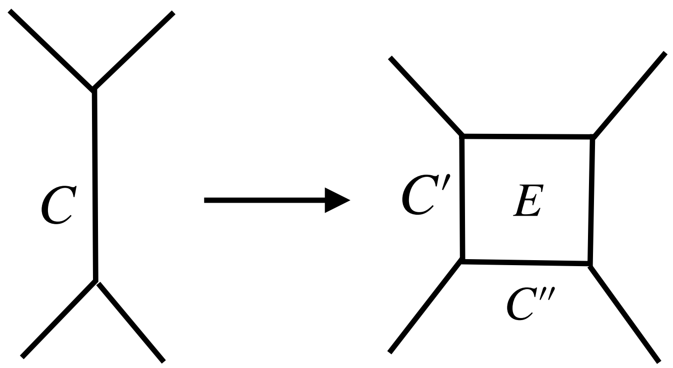
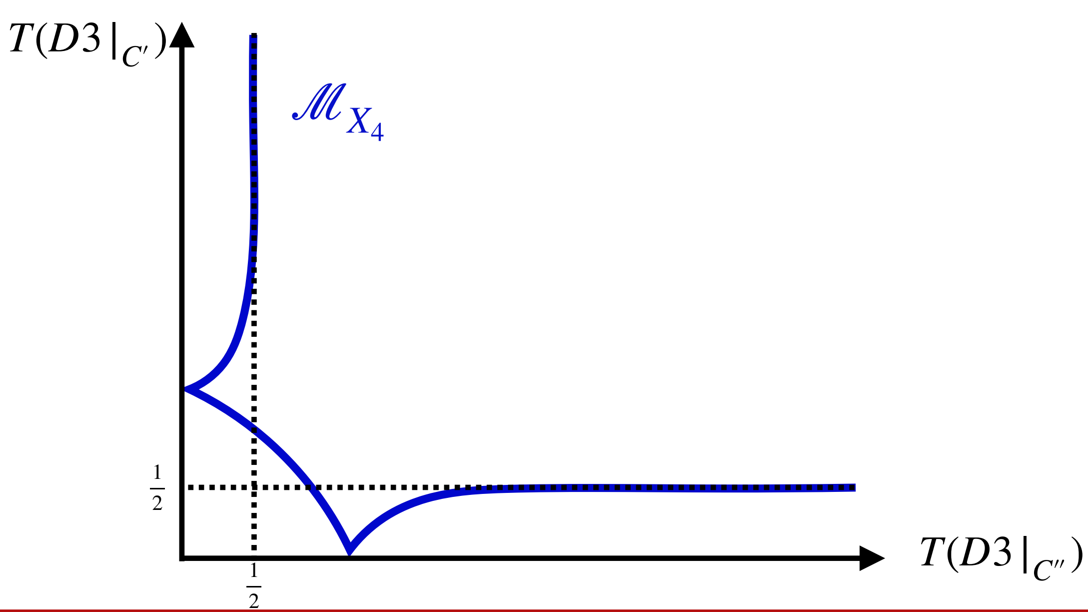




- Possible reason for singularity Δ'' (Δ'): N=1 strings $D3|_{C''}$ ($D3|_{C'}$) becomes tensionless



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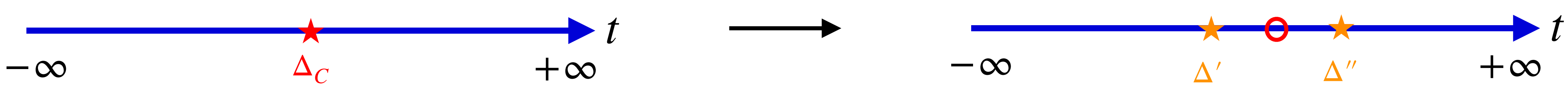

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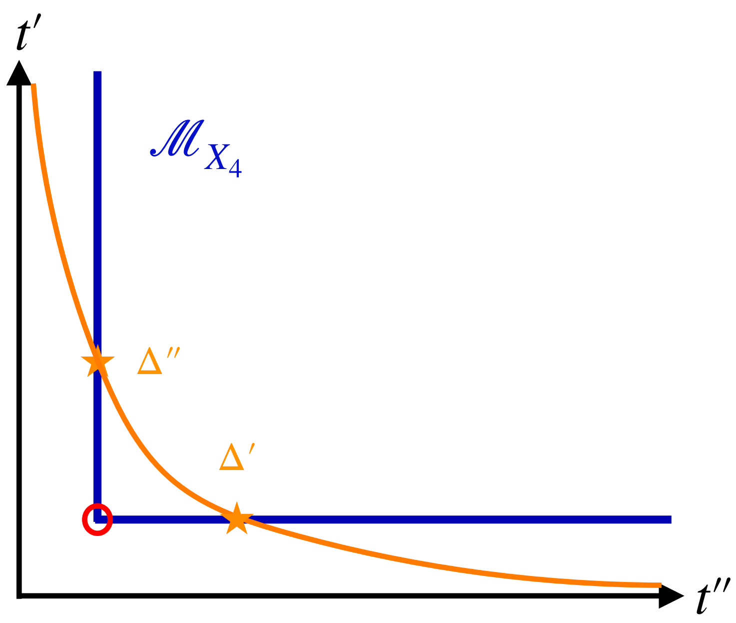

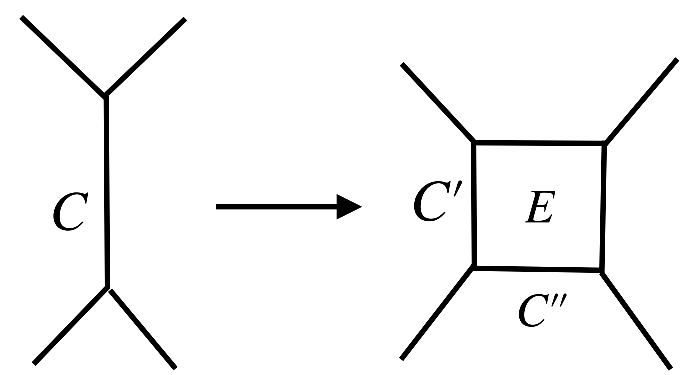

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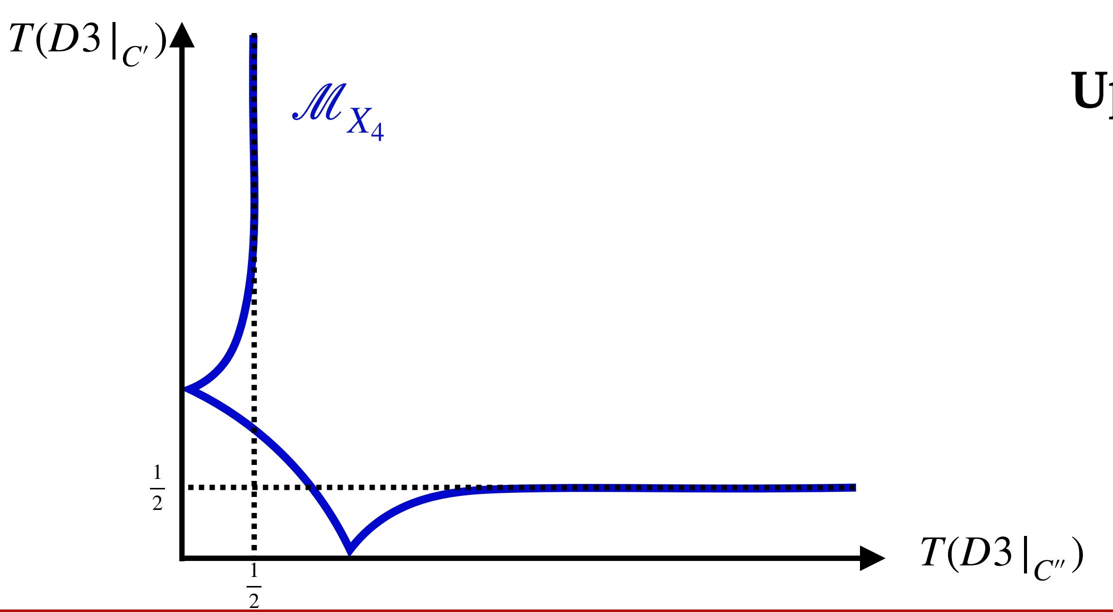

Upshot:

1. Small volume limit already for N=2 curves quite interesting.

2. Singularity structure different from naive Type IIA vector multiplet moduli space

What about other types of curves?

- So far: only considered very simple case of curves of "Type $\mathtt{F}$"

- Should also consider small volume limit for other type of curves and can differentiate

$\bar{K}(B_3) \cdot C < 0$ $\bar{K}(B_3) \cdot C = 0$ $\bar{K}(B_3) \cdot C > 0$

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Type $\mathtt{F}$ **(flop curves)**

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Type $\mathtt{M}$ with normal bundle $\mathcal{O}(-2) \oplus \mathcal{O}$

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- Shrinking Type $\mathtt{M}$ curve $\rightarrow$ divisor shrinks to curve
- Supersymmetry even further enhanced compared to Type $\mathtt{F}$ string (analogue of curve on (-2)-curve in 6d)
- Can shrink the curve without encountering any quantum correction.

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Main example: flip curves in the base B_3

[Denef, Douglas, Florea, Grassi, Kachru '05]

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Main example: flip curves in the base B_3

[Denef, Douglas, Florea, Grassi, Kachru '05]

- Similar in spirit to Type F curves but involves going through an orbifold singularity.
- No additional supersymmetry preserving deformations.
- **But:** Splitting of string $D3_C$ can be described in the same spirit as splitting of N=2 string — though qualitatively different.

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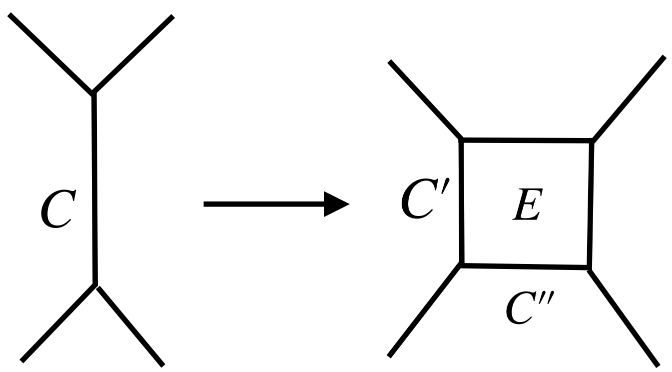

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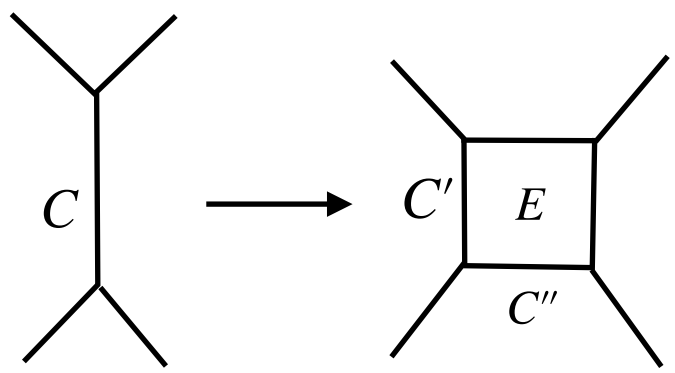

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$\mathcal{N}_{C|B_3} = \mathcal{O}(-1) \oplus \mathcal{O}(0)$ *divisor shrinks to curve*

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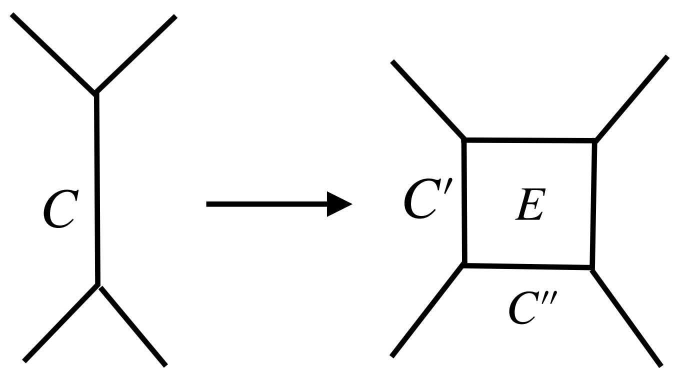

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→ look at corrections to effective action

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$$\mathscr{Z}_D = c_3(X_4) \cdot_{X_4} \pi^*(D) = c_1(B_3)^2 \cdot_{B_3} D = \ldots = 4\, c_1(B_3) \cdot_{B_3} C$$

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Genuine N=1 effects — Case I

- $\mathscr{V}_D$ receives corrections at $\mathcal{O}(\alpha'^2)$: [Grimm, Keitel, Mayer, Pugh, Savelli, Weissenbacher '13-'19]

$$\mathscr{V}_D^{\text{corr.}} = \mathscr{V}_D \left[1 + \alpha^2 \left((\kappa_3 + \kappa_5) \frac{\mathscr{Z}}{\mathscr{V}_{B_3}} \right) \right] + \alpha^2 \left(\tilde{\mathscr{Z}}_i \log \mathscr{V}_{B_3}^{(0)} + \kappa_7 \mathscr{Z}_D \right) . \qquad \mathscr{Z}_D = \int_{X_4} c_3(X_4) \wedge \pi^*(D)$$

$\mathscr{V}_D \to 0$ for $t_C \to 0$

Suppressed at sufficiently large $\mathscr{V}_{B_3}$

Relevant correction

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- For curve with $\bar{K} \cdot_{B_3} C \neq 0$ dominates → cannot trust the classical field space geometry for $\mathscr{V}_C \to 0$!

- *Consistency check: for curve with $\mathcal{N} = \mathcal{O}(-2) \oplus \mathcal{O}(0)$ correction vanish and we can still trust the geometric picture.* "Type M string"

Genuine N=1 effects — Case II

[MW '22]

Consider now $\bar{K}_{B_3} \cdot C = 2$ **and** $\mathcal{N}_{C|B_3} = \mathcal{O}(0) \oplus \mathcal{O}(0)$.

→ C is fiber of rationally-fibered $B_3 : C \to B_2$ ↔ theory dual to heterotic string on CY3.

[Morrison, Vafa '97; Lee, Lerche, Weigand '19]

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What happens in the limit of small C at constant volume $\mathcal{V}_{B_3}$?

- All divisor volumes receive corrections as

$$\mathcal{V}_D^{\text{corr.}} = \mathcal{V}_D \left[1 + \alpha^2 \left((\kappa_3 + \kappa_5) \frac{\mathcal{Z}}{\mathcal{V}_{B_3}} \right)\right] + \alpha^2 \left(\tilde{\mathcal{Z}}_i \log \mathcal{V}_{B_3}^{(0)} + \kappa_7 \mathcal{Z}_D \right) .$$

Diverges in the limit [Klaewer, Lee, Weigand, MW '20]

- Via duality can argue that (at least in simple cases) a strong coupling singularity is reached for gauge theory on $D = B_2$.

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- All other (vertical) divisors have minimal quantum volume:

$$\frac{1}{\alpha^2} \text{Re}\, T_a \Big|_{\text{sing.}} = - \frac{\text{Re}\, T_a^{(0)}}{\mathcal{V}_{B_2}^{(0)}} \left(\frac{b}{8\pi} \log \xi + \text{const} . \right) + \text{Re}\, T_a^*$$

ζ : Complex structure parameter of X_4

Genuine N=1 effects — Case II

[MW '22]

Shrinking of curve with $\mathcal{N} = \mathcal{O}(0) \oplus \mathcal{O}(0)$ is even worse than for $\bar{K} \cdot_{B_3} C = 1$.

- Get a strong coupling singularity at finite distance.
- Mixing between complex structure sector and Kähler sector → $\mathcal{M} \neq \mathcal{M}_{\text{c.s.}} \times \mathcal{M}_{\text{Kahler}}$

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→ corrections are big and field space does not necessarily factorize anymore.

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In general: Field space geometry for small genuine $\mathcal{N} = 1$ curves not describable by classical geometry

→ corrections are big and field space does not necessarily factorize anymore.

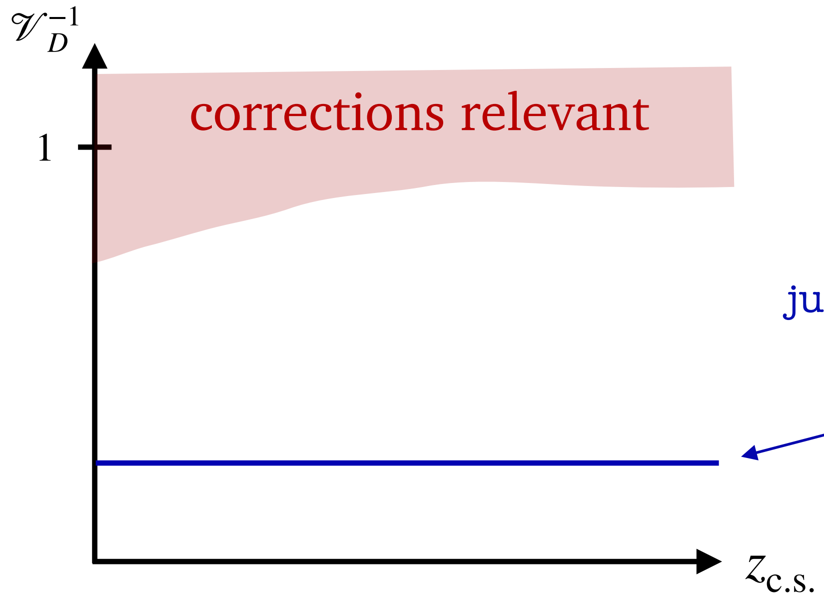
Question: Away from small curve limits can I still trust the classical field space structure?

→ does $\mathcal{M} \simeq \mathcal{M}_{\text{c.s.}} \times \mathcal{M}_{\text{Kahler}}$ only break down for very small volumes?

→ or corrections important for large complex structure?

Mixing in the Complex Structure Sector

Might expect that the mixing between Kähler and complex structure sectors is sufficiently suppressed as long as divisor volumes $\mathscr{V}_D \gg 1$:



For any finite value of $\mathscr{V}_{B_3}$ can treat this just as the c.s. field space with Kähler potential $K = -\log \int_{X_4} \Omega \wedge \bar{\Omega}$

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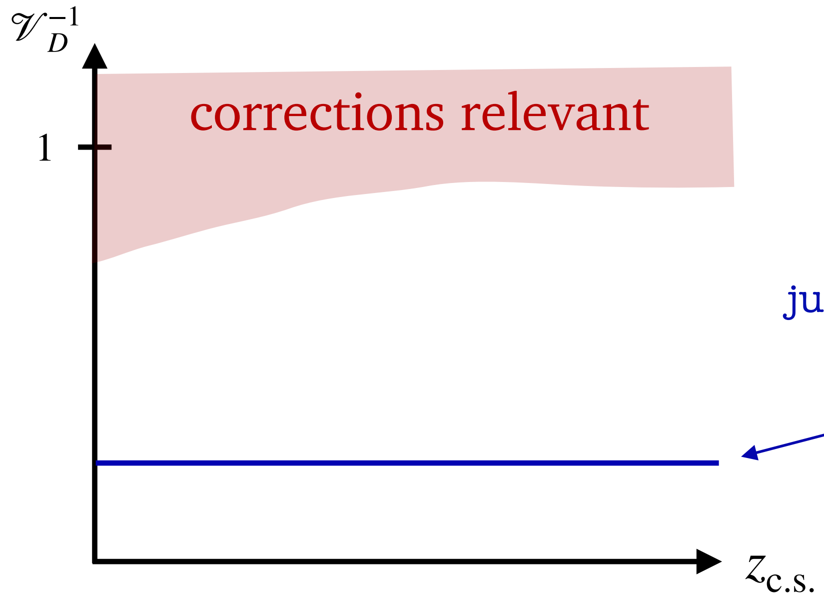

Motivated by viewing F-theory via IIB orientifolds:

→ For Type IIB CY compactifications the complex structure is classically exact.

→ Can evaluate periods of X_4 reliably to infer structure of $\mathcal{M}_{c.s.}$.

→ Period integrals simplify close to boundaries of $\mathcal{M}_{c.s.}$ ⇒ good setting for e.g. searches for flux vacua.

Is this picture correct?

A simple Calabi—Yau fourfold

Consider a **very simple** elliptically-fibered Calabi-Yau fourfold

$$X_4 = (T^2 \to B_2) \times T^2 \qquad \Longrightarrow \qquad B_3 = B_2 \times T^2$$

Elliptically-fibered Calabi-Yau threefold

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Elliptically-fibered Calabi-Yau threefold

F-theory on X_4 leads to a four-dimensional theory with $\mathcal{N} = 2$ supersymmetry.

Question: Can we already see in this theory what to expect got the mixing between complex structure sector and $\mathcal{V}_{B_3}$?

Therefore consider vector- and hypermultiplet sector of this F-theory comapctification:

- *complex structure moduli of $(T^2 \to B_2)$ and overall volume of B_2 + axionic partners* — hypermultiplets
- *(complexified) Kähler moduli of B_2 + moduli of T^2* — vector multiplets

Hypermultiplet Corrections to CY3 x T2

Focus on hypermultiplet sector of F-theory on $(T^2 \to B_2) \times T^2$

→ contains precisely the **volume modulus** and (part of) **the complex structure sector of** X_4.

F-theory on $(T^2 \to B_2) \times T^2$ dual to Type IIA on $T^2 \to B_2$.

→ hypermultiplet moduli spaces can be identified via

F-theory		IIA
complex structure moduli of $(T^2 \to B_2)$ *overall volume modulus of B_2*	↔	*complex structure moduli of $(T^2 \to B_2)$* *4d dilaton*

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- Type IIA hypermultiplet sector receives corrections due to D2-brane instantons
- D2-brane instanton contributions to moduli space metric have been computed in

[Alexandrov, Banerjee '14]; see [Robes-Llana, M. Rocek, F. Saueressig, U. Theis, S. Vandoren, '06] for mirror dual Type IIB.

$$S_{4d}^{\text{corr.}} = S_{4d}^{(0)} + \sum \text{D2-instantons}$$

- effect on (mirror dual of) large complex structure limit moduli space has been investigated in [(Baume), Marchesano, MW '19]; see also [Alvarez-Garcia, Klaewer, Weigand '21]

→ **effectively obstruct large complex structure limits!**

Consequence for N=1 Theories

- Can break supersymmetry to N=1 e.g. through non-trivial fibration $X_4: \; X_3 \to \mathbb{P}^1 \qquad B_3 = B_2 \to \mathbb{P}^1$

  $\to$ classically $\mathscr{M}_{c.s.}(X_3) \subset \mathscr{M}_{c.s.}(X_4)$

- Expectation: corrections present in N=2 also correct N=1 theory

  $\to$ asymptotic regimes in $\mathscr{M}_{c.s.}(X_4)$ also receive corrections at finite $\mathscr{V}_{B_2}$ due to corrections to action of D3-brane instantons on $D = B_2 \subset B_3$

$$S_{4d}^{\text{corr.}} = S_{4d}^{(0)} + \sum \text{D2-instantons} \quad \longrightarrow \quad \mathcal{S}_{D3|_{D=B_2}} = \mathscr{V}_{D=B_2} - f(z_{c.s.}) \int_{D=B_2} c_1(B_3)^2$$

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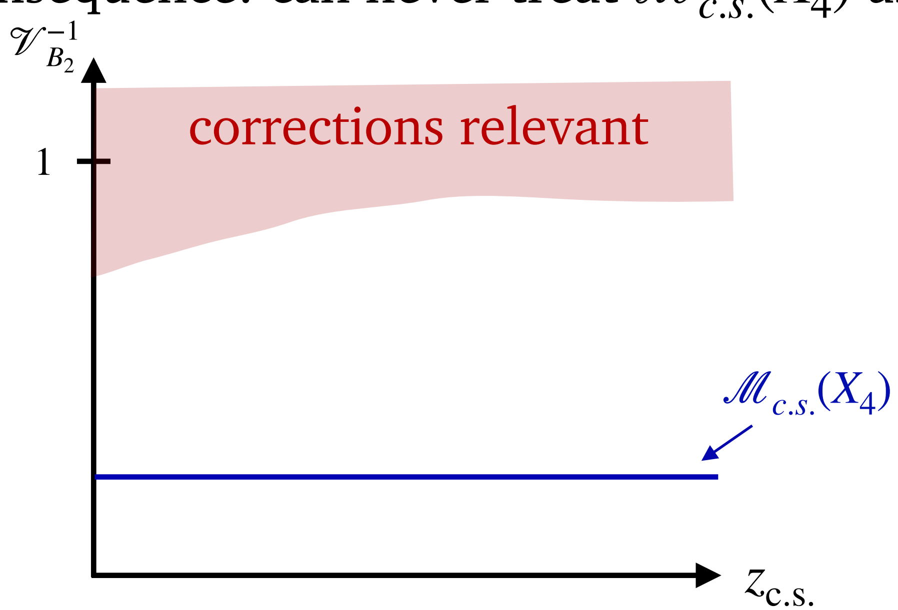

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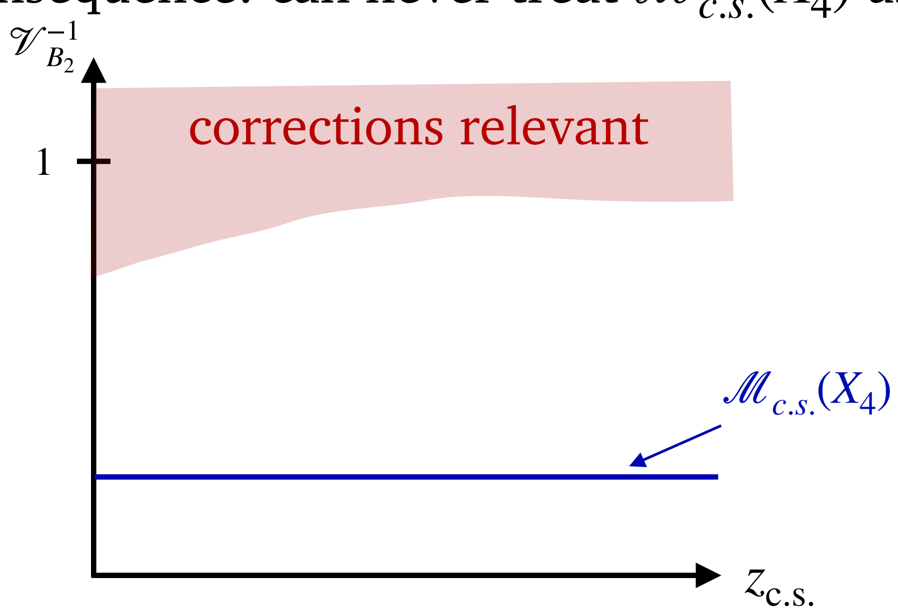

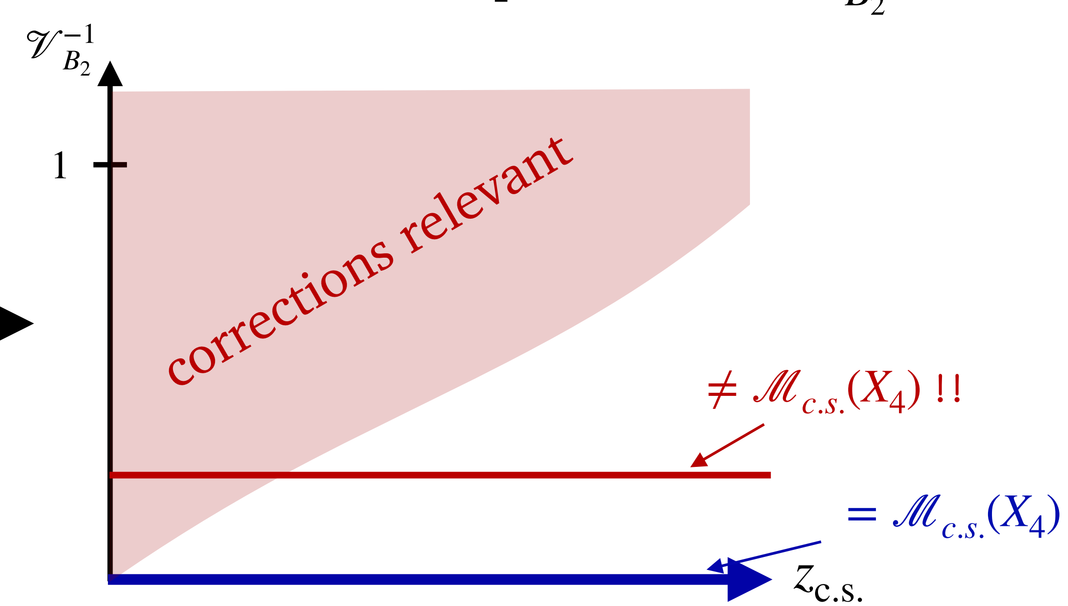

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- N=2 intuition useful to explore regimes in the field space with *local* supersymmetry enhancement → even here global N=1 breaking effect are important!

- Explicitly considered F-theory compactifications on four-folds
  - Hypermultiplet useful to describe local moduli space in small volume limit for $\bar{K} . C = 0$ curves. → not the full story!!
  - genuine N=1 effects become large if curves intersected by anti-canonical divisor become small → N=2 breaking not diluted.
  - Mixing between complex structure and Kähler sector becomes important away from $\mathscr{V}_D = \infty$.
  - asymptotic regions in c.s. sector only describable through classical geometry in double-scaling limit *(where N=2 supersymmetry is restored…)* → similar effects to N=2 hypermultiplet sector at finite string coupling …

Thank you!!