When are towers needed for the Weak Gravity Conjecture?

work with Cesar Cota, Alessandro Mininno, Max Wiesner: 2312.11611

see also: 2212.09758, 2208.00009

Timo Weigand, Geometry, Strings, and the Swampland, Ringberg, March 18-22, 2024

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Properties of species

+

Emergent String Conjecture

imply:

Towers of super-extremal particles are *required* by consistency of the Weak Gravity Conjecture under S^1 reduction only for

- **KK U(1) and heterotic perturbative U(1)**
- **and - possibly - certain strong coupling limits.**

Resolves parametric problems with Weak Gravity Conjecture for theories without known super-extremal towers.

Weak Gravity Conjecture

In a U(1) gauge theory coupled to quantum gravity in $D \geq 3$, there must exist a super-extremal state with (q, m_D) : [Arkani-Hamed,Motl,Nicolis,Vafa'06]

$$\frac{g_{\mathrm{U(1)}}^2 q^2}{m_D^2} \geq \gamma \frac{1}{M_{Pl,D}^{D-2}} \quad \text{where} \quad \left.\frac{g_{\mathrm{U(1)}}^2 Q^2}{M^2}\right|_{\text{ext. BH}} \equiv \gamma \frac{1}{M_{Pl,D}^{D-2}}$$

without massless scalars: $\gamma = \dfrac{D-3}{D-2}$

Original Motivation:

Guarantees that every charged black hole can decay $\Longrightarrow$ **No stable remnants**

Weak Gravity Conjecture

In a U(1) gauge theory coupled to quantum gravity in $D \geq 3$, there must exist a

super-extremal state: $$\frac{g_{\mathrm{U(1)}}^2 q^2}{m_D^2} \geq \gamma \frac{1}{M_{\mathrm{Pl,D}}^{D-2}}$$ *[Arkani-Hamed,Motl,Nicolis,Vafa'06]*

Two ways to satisfy the WGC:

i) super-extremal state is **particle** in EFT: $m \leq M_{\mathrm{BH,min}}$ most direct constraint on EFT

ii) super-extremal states is itself a **black hole**: only indirect constraint (higher-dim. operators)

[Kats,Motl,Padi'06][Cheung,Liu,Remmen'18][Hamada,Noumi,Shiu'18]

Tower Weak Gravity Conjecture

Heidenreich,Reece,Rudelius'15]

[Montero,Shiu,Soler'16]

[Andriolo,Junghans,Noumi,Shiu'18]

*A U(1) gauge theory coupled to quantum gravity possesses a **tower** of*

- *infinitely many super-extremal states*
- *of arbitrarily high charges*

$$\frac{g_{\mathrm{U}(1)}^2\, q^2}{m_D^2} \geq \gamma \frac{1}{M_{Pl,D}^{D-2}}$$

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$$\frac{g_{\mathrm{U}(1)}^2 q^2}{m_D^2} \geq \gamma \frac{1}{M_{Pl,D}^{D-2}}$$

Similar distinction:

i) tower of super-extremal particles:

In region $\mathscr{M}$ of moduli space: ∃ super-extremal particle with $m_n \leq M_{\mathrm{BH,min}}$ and charge $n\,q \quad \forall n \in \mathscr{I}_q$ ← infinite set

ii) tower of super-extremal states at/above BH threshold

Tower Weak Gravity Conjecture: Particle version

Tower of super-extremal particles:

In region $\mathscr{M}$ of moduli space:

∃ super-extremal particle with

$m_n \leq M_{\mathrm{BH,min}}$ and charge $n\, q \quad \forall n \in \mathcal{I}_q$

$$\frac{g_{\mathrm{U}(1)}^2\, q^2}{m_D^2} \geq \gamma \frac{1}{M_{Pl,D}^{D-2}}$$

Two possibilities (necessary conditions) *[Cota,Mininno,TW,Wiesner'23]*

Asymptotic weak coupling limit

$$g_{\mathrm{U}(1)}^2 M_{\mathrm{Pl,D}}^{D-4} \to 0 \qquad \text{and} \qquad \frac{g_{\mathrm{U}(1)}^2 M_{\mathrm{Pl,D}}^{D-2}}{M_{\mathrm{Pl},\infty}^2} \to 0$$

Strong coupling limit at finite distance

$$g_{\mathrm{U}(1)}^2 M_{\mathrm{Pl,D}}^{D-4} \to \infty \qquad \text{such that} \quad \gamma \to \infty$$

Tower Weak Gravity Conjecture: Motivations

- **Consistency under dimensional reduction** (see later)

[Heidenreich,Reece,Rudelius'15] [Montero,Shiu,Soler'16] [Andriolo,Junghans,Noumi,Shiu'18]

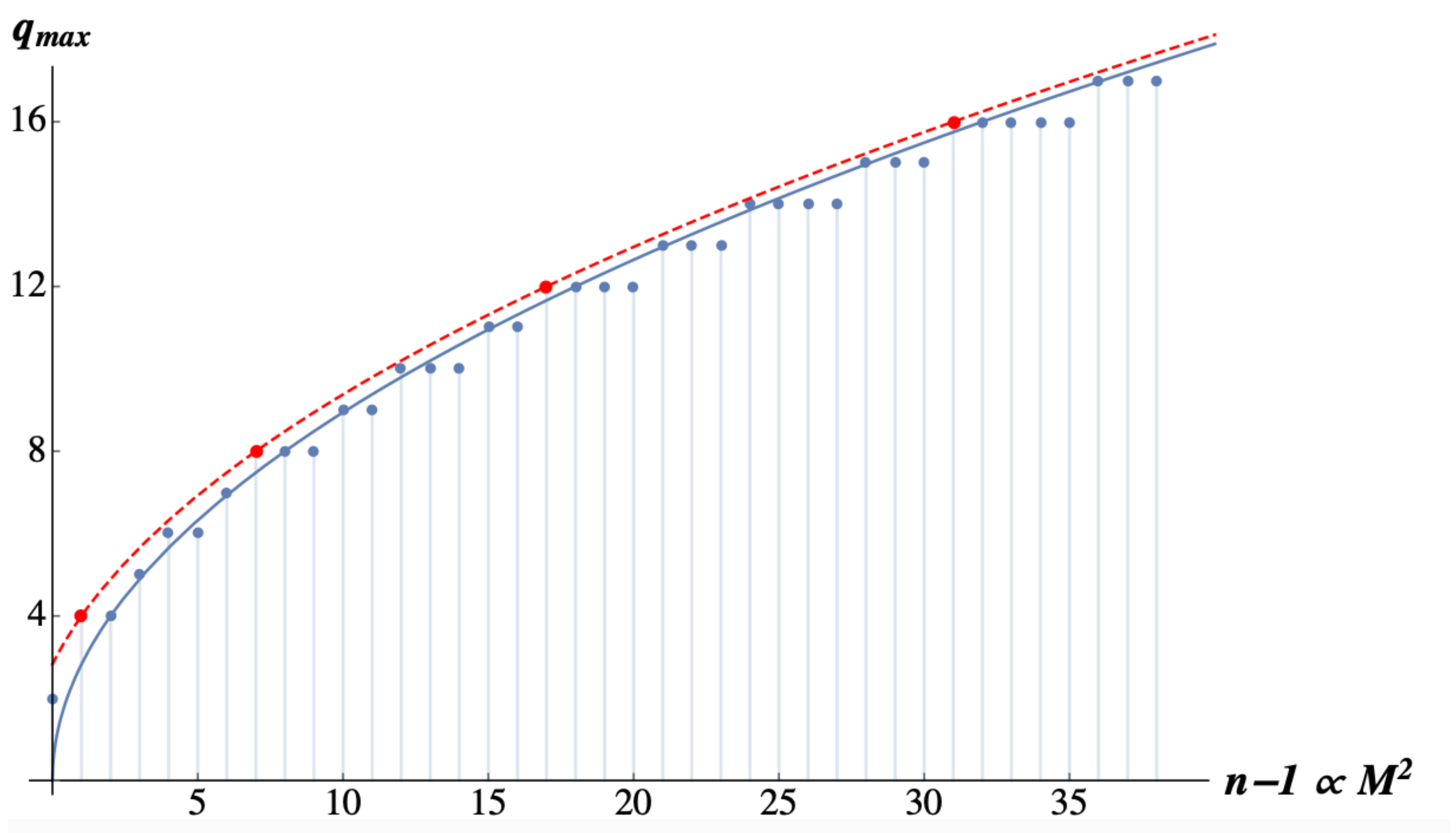
- Consistent with **absence of global symmetries**:

$$\frac{g_{\mathrm{U}(1)}^2\, q^2}{m_D^2} \geq \gamma \frac{1}{M_{Pl,D}^{D-2}}$$

In limit $g_{\mathrm{U}(1)} \to 0$ infinitely many states become massless

- **Passed many non-trivial tests**

Tower Weak Gravity Conjecture: Tests



1) Particle towers at asymptotic weak coupling

KK towers in (dual) decompactification limit:

- **KK U(1)s** *[Heidenreich,Reece,Rudelius'15]*
- **Type IIB on CY 3-fold in asymptotic complex structure regions** *[Grimm,Palti,Valenzuela'18] [Bastian,Grimm,Heisteeg'20] [Gendler,Valenzuela'21]*
- **M-theory on CY 3-fold in weakly coupled gauge sector** *[Lee,Lerche,TW'19] [Cota,Mininno,TW,Wiesner'22,23]*

String excitation towers:

- **Perturbative heterotic** *[AMNV'06] [Heidenreich,Reece,Rudelius'15]*
- **Closed perturbative bosonic** *[Heidenreich,Lotito'24]*
- **General F-theory in weakly coupled gauge sector** *[Lee,Lerche,TW'18,'19] [Kläwer,Lee,TW,Wiesner'20]*
- **M-theory on CY3 in weakly coupled gauge sector** *[Cota,Mininno,TW,Wiesner'22,23]*

Tower Weak Gravity Conjecture: Tests

1) Particle towers at weak coupling → **KK towers (up to duality)**

→ **String towers**

2) Towers away from weak coupling

- **BPS black hole towers in M-theory on CY 3-folds**

  in particular along directions where BPS = extremality

  [Alim,Heidenreich,Rudelius'21], [Gendler,Heidenreich,Moritz,Rudelius'23]

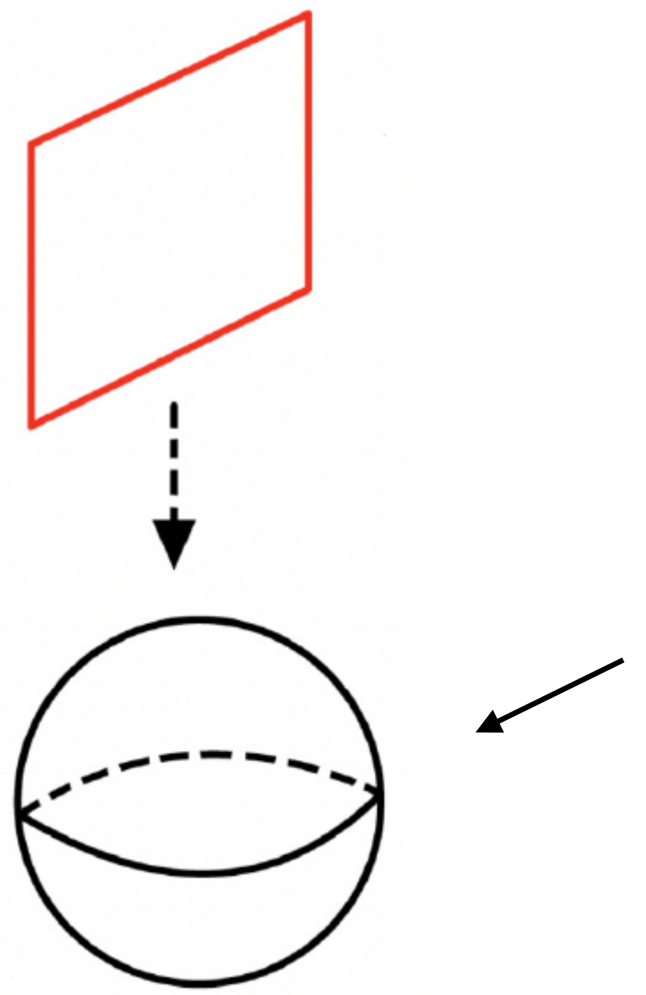
- Possibly: BPS SCFT sectors in M-theory on CY3 as particles

Tower Weak Gravity Conjecture: Counter-examples?

Examples:

- U(1)s associated with conifold transitions in M-theory: cf. *[Alim,Heidenreich,Rudelius'21]*

  no *known* *tower* of charged particles or BHs -
  but maybe non-BPS tower of BHs unknown to us?



- Open string U(1)s: *Heidenreich,Reece,Rudelius'21[Cota,Mininno,TW,Wiesner'22]*

  no *known* *tower* of charged particles - non-pert. towers at best at BH level

Consistency under dimensional reduction

This talk: *[Cota,Mininno,TW,Wiesner'23]*

Is absence of a super-extremal tower consistent with dimensional reduction of the theory along a circle?

1) **Review consistency under circle reduction**
2) **Loop hole: Minimal radius in generic circle reductions**
3) **Consequences for tower WGC**

Reminder: WGC under dimensional reduction

U(1) Theory on $\mathbb{R}^{1,D-2} \times S^1$: $U(1)_D \longrightarrow U(1)_{D-1} \times U(1)_{\rm KK}$

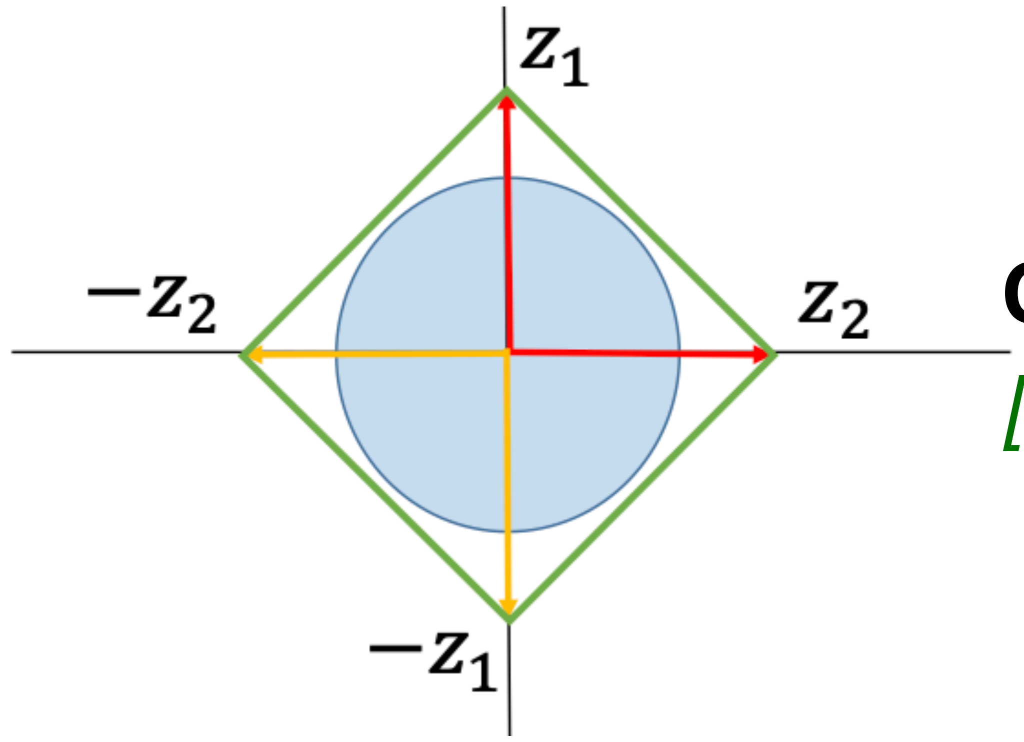
$U(1)_{\rm KK}$ coupling: $\frac{1}{g_{KK}^2} = \frac{1}{2} r_{S^1}^2 M_{Pl,D}^{D-2}$

Mass of state at KK level q_{KK}: $m_{D-1}^2 = m_D^2 + \frac{1}{r_{S^1}^2}\left(q_{KK}^2 - q\theta\right)$

$0 \leq \theta < 1$: $U(1)_D$ Wilson line parameter

$\Longrightarrow$ **Need to satisfy the CHC for $U(1)_{D-1} \times U(1)_{KK}$**

[Heidenreich,Reece,Rudelius'15]



CHC:
[Cheung,Remmen 14]

This requires existence of state in D dim. such that for all allowed values of r_{S^1} and θ:

$$(m_D r_{S^1})^2 \geq \frac{1}{4z_D^2(z_D^2-1)} + \frac{q\theta(1-q\theta)}{z_D^2}$$

$$z_D = g_D M_{Pl,D}^{\frac{D-2}{2}} \gamma^{1/2} \frac{|q|}{m_D}$$

Reminder: WGC under dimensional reduction

Consistency under S^1 reduction requires state such that for all allowed values of r_{S^1} and θ:

$$(m_D r_{S^1})^2 \geq \frac{1}{4z_D^2(z_D^2 - 1)} + \frac{q\theta(1 - q\theta)}{z_D^2} \qquad \left(z_D = g_D M_{Pl,D}^{\frac{D-2}{2}} \gamma^{1/2} \frac{|q|}{m_D}\right)$$

[Heidenreich,Reece,Rudelius'15]

➡ Problematic regime: $r_{S^1} \to 0$

Key Observation:

[Heidenreich,Reece,Rudelius'15/16]

Super-extremal Tower in D dimensions $\Longrightarrow$ **CHC even for $r_{S^1} \to 0$**

$\Longrightarrow$ **Bottom-up motivation for tower WGC**

[Heidenreich,Reece,Rudelius'15/16] [Montero,Shiu,Soler'16] [Andriolo,Junghans,Noumi,Shiu'18]

WGC under dimensional reduction

Analysis of dimensional reduction valid in field theory.

Potential loopholes include: *(as emphasized clearly in Heidenreich,Reece,Rudelius'15])*

- The quantum gravity theory may not admit a limit $r_{S^1} \to 0$.
- Quantum corrections near $r_{S^1} \to 0$ may become relevant.

Main message of this talk:

Dimensional reduction alone does not require a tower of super-extremal particles below BH threshold - away from weak coupling and - *possibly* - suitable strong coupling limits.

When is an EFT a KK reduction on S^1 ?

KK tower of mass $M_{\rm KK} \sim \frac{1}{2\pi r_{S^1}}$ detectable as particles (not black holes):

$$\frac{1}{2\pi r_{S^1}} \sim M_{\rm KK} \leq M_{\rm BH,min}$$

Minimal BH mass in D-1 dim:

$$\frac{M_{\rm BH,min.}}{M_{\rm Pl,D-1}} = \left(\frac{M_{\rm Pl,D-1}}{\Lambda_{\rm QG}}\right)^{D-4}$$

$\Lambda_{\rm QG} \sim r^{-1}_{\rm BH,min}$

Species Scale ≡ QG cutoff [*Dvali'07*]

In a **typical** theory expect for theory in D-1 dim: $\Lambda_{\rm QG} \sim M_{\rm Pl.,D-1}$

In **this** case require: $2\pi r_{S^1} \geq M^{-1}_{\rm Pl,D-1}$

Minimal radius for *typical* D-1 EFT to be a KK reduction

When could the minimal radius argument fail?

Loophole: QG cutoff scale may drop below Planck scale parametrically:

$$\Lambda_{\rm QG} \ll M_{\rm Pl.,D-1}$$

[Dvali'07]

This happens in presence of tower of light weakly coupled states at infinite distance in moduli space:

Swampland Distance Conjecture

[*Ooguri, Vafa`06*]

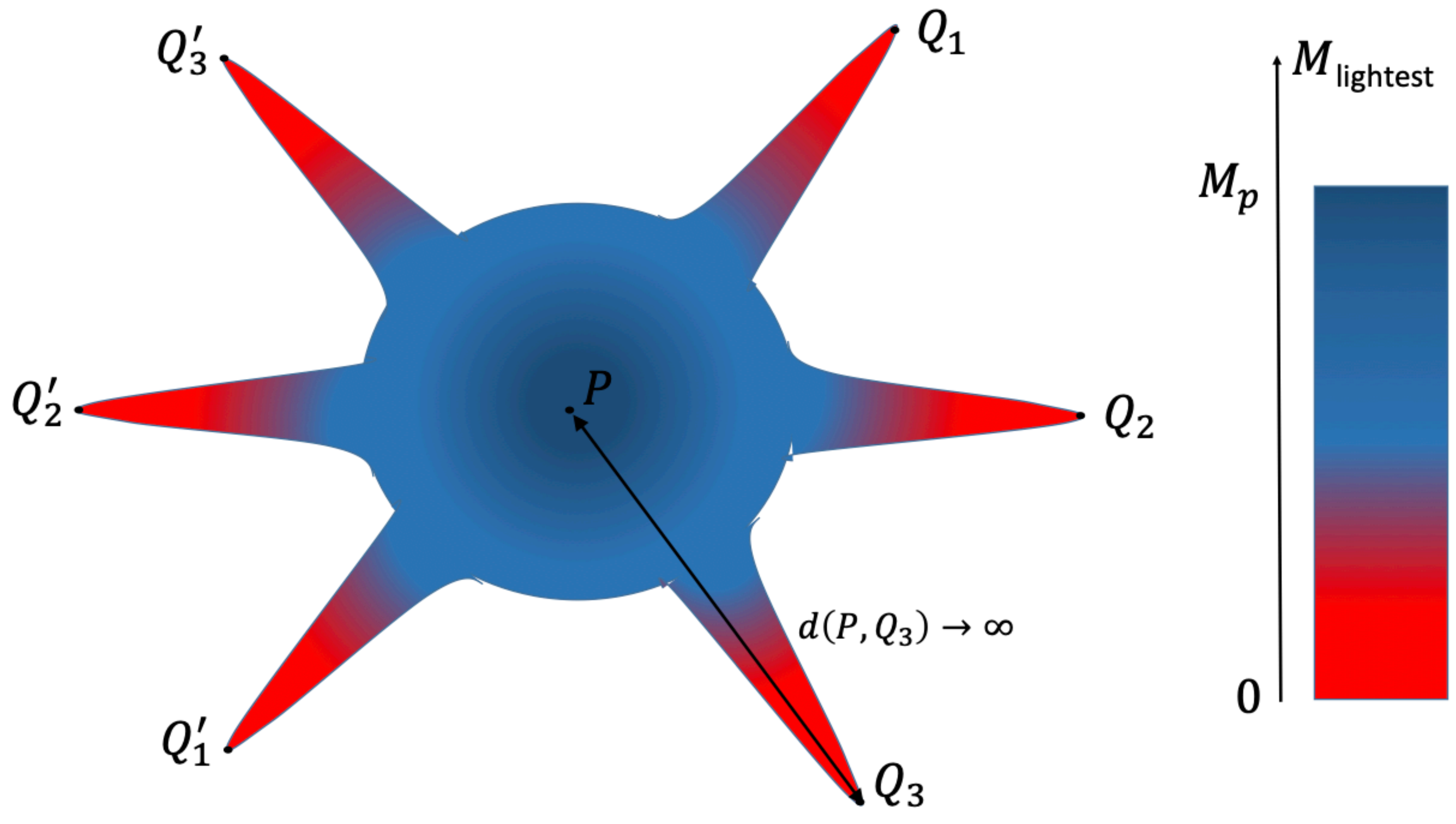


Image: Palti, 2019

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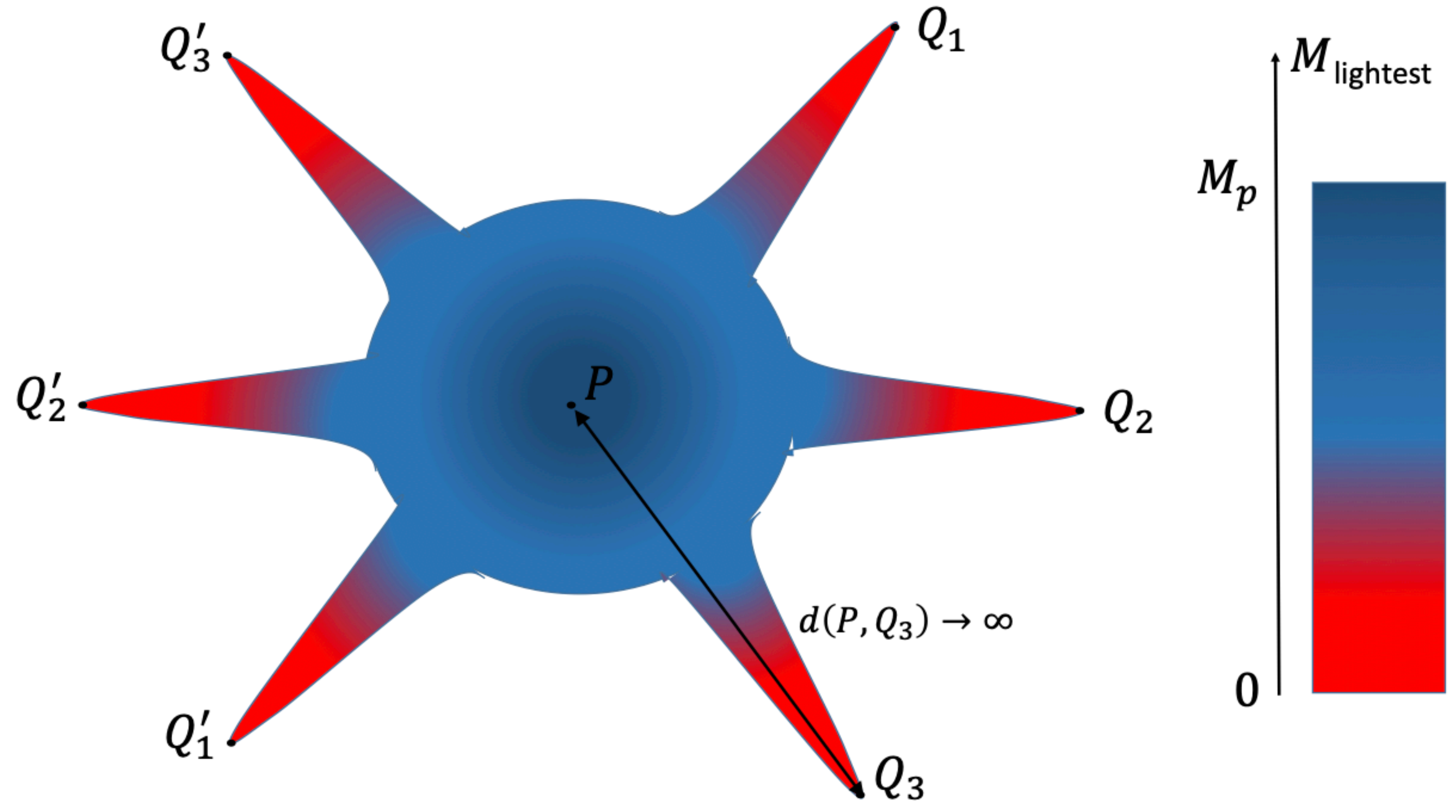


Emergent String Conjecture: *[Lee,Lerche,TW`19]*

Infinite distance physics is a decompactificaton limit

or a weakly coupled string theory

Decompactification: *[Long,Montero,Vafa,Valenzuela'21] [Marchesano,Melotti'22] [Castellano,Herraez,Ibanez'22] [Heisteeg,Vafa,Wiesner,Wu'22] [Cribiori,Lüst,Staudt'22] ….*

$\Lambda_{\rm QG}$ = higher-dim. $M_{\rm Pl}$

Emergent string limit:

$\Lambda_{\rm QG} \sim M_{\rm str.}$ *[Dvali,Lüst'09] [Dvali,Gomez'10]*

When could the minimal radius argument fail?

Case 1:

S^1 reduction of a D-dim theory

(a) in a decompactification limit

(b) in an emergent string limit

Case 2:

Limit $r_{S^1} \to 0$ itself corresponds to

(a) a (dual) decompactification limit

(b) an emergent string limit

When could the minimal radius argument fail?

Case 1:

S^1 reduction of a D-dim theory

(a) in a decompactification limit

$\Longrightarrow$ S^1 reduction of higher dim theory ✔

(b) in an emergent string limit

$\Longrightarrow$ S^1 reduction of a string theory

Case 2:

Limit $r_{S^1} \to 0$ itself corresponds to

(a) a (dual) decompactification limit

$\Longrightarrow$ S^1 reduction of a string theory

(b) an emergent string limit

$\Longrightarrow$ S^1 reduction of M-theory

When could the minimal radius argument fail?

S^1 reduction of M-theory:

- No minimal radius from BH argument:

$$r_{S^1} \geq M_{\text{Pl,D}-1}^{-1} \left(\frac{M_s}{M_{\text{Pl,D}-1}} \right)^{D-4} \to 0$$

($M_s = \Lambda_{\text{QG}}$ at small r_{S^1})

- A **different argument** does show **minimal radius for M-theory comp.** *generically*

✓ consistent with absence of known towers for generic theories

S^1 reduction of perturbative string theory:

- **No minimal radius** despite T-duality

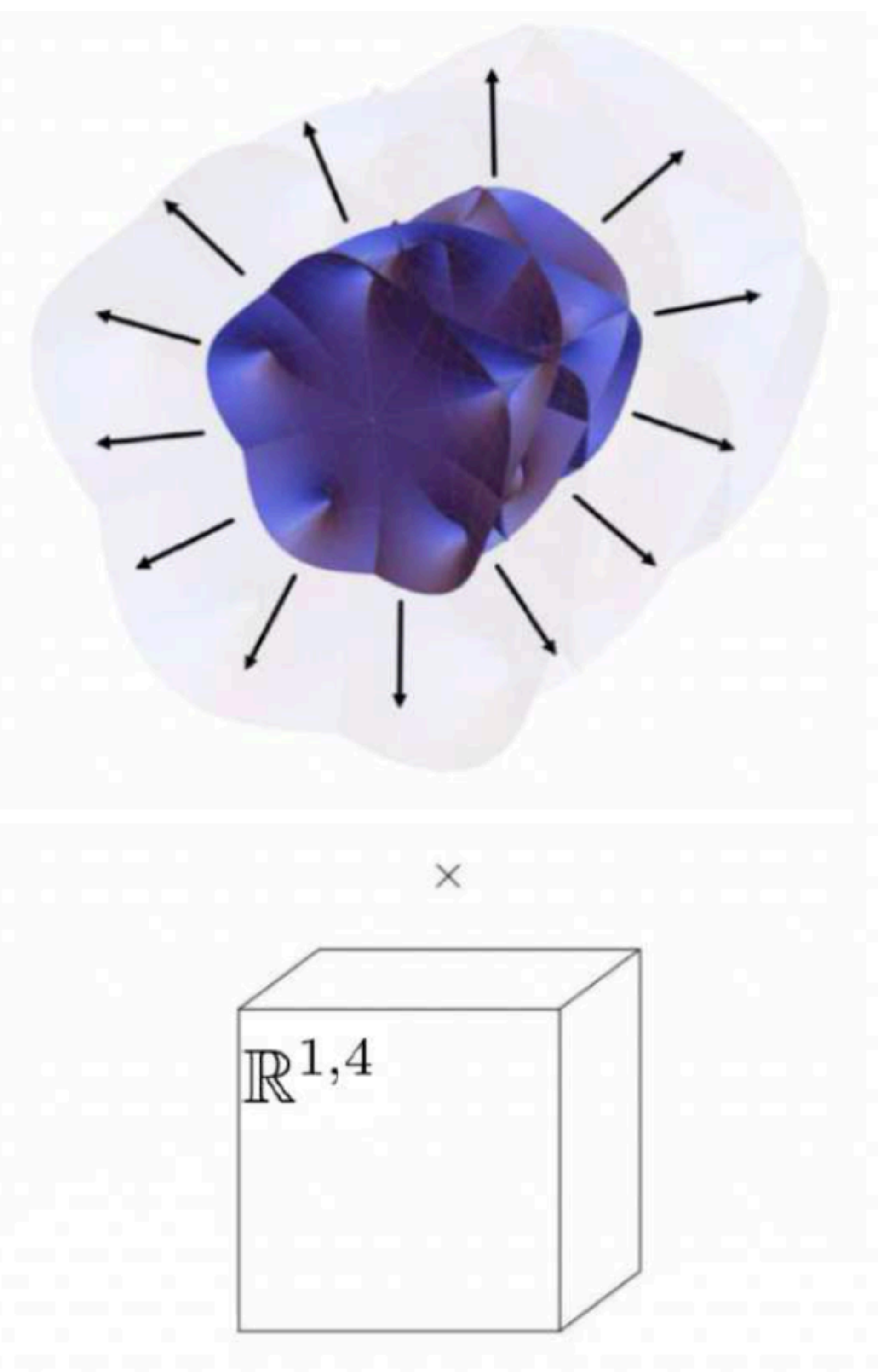
➡ for **heterotic string**, this necessitates tower of WGC states in agreement with spectrum

➡ for **open string theory**, no tower required:

(see later)

✓ consistent with absence of established towers for open string

Circle reduction of M-theory on CY



- $M_{\text{Pl},5}^3 = 4\pi M_{\text{11d}}^3 \mathscr{V}_{X_3}$ $\qquad$ $\mathscr{V}_{X_3}$ volume in units of M_{11d}

- KK reduction on S^1: Can we take $r_{S^1} M_{\text{Pl},5} \to 0$ at constant $M_{\text{Pl},5}$

  i.e. $r_{S^1} M_{\text{11d}} \to 0$ at $\mathscr{V}_{X_3}$ constant ?

M-theory-IIA duality:

Volume in string units

i) $g_{\text{IIA}}^{2/3} = 2\pi M_{\text{11d}} r_{S^1}$ and ii) $g_{\text{IIA}}^{1/3} = \dfrac{M_s}{M_{\text{11d}}}$ $\implies$ $\dfrac{\mathscr{V}_{X_3,s}}{g_{\text{IIA}}^2} = \mathscr{V}_{X_3}$: universal hyper

$\implies$ $2\pi r_{S^1} M_{\text{11d}} = \left(\dfrac{\mathscr{V}_{X_3,s}}{\mathscr{V}_{X_3}}\right)^{1/3} \to 0$ at $\mathscr{V}_{X_3}$ constant requires co-scaling $\mathscr{V}_{X_3,s} \sim (r_{S^1} M_{\text{11d}})^3 \to 0$

Circle reduction of M-theory on CY

Limit $\mathscr{V}_{X_{3},s} \sim (r_{S^1} M_{11\mathrm{d}})^3 \to 0$ obstructed by α' corrections:

Regime $\mathscr{V}_{X_3,s} \ll 1$ not in stringy quantum moduli space

- Quantum volume of CY X_3 in Type IIA frame:

$$\mathscr{V}_{X_3,\,s} = e^{-\mathscr{K}_{\mathrm{K}}(X_3)} \longrightarrow \frac{1}{6}\int_{X_3} J \wedge J \wedge J - \frac{\chi(X_3)\zeta(3)}{4\pi^3}$$

in large volume regime

- Mirror symmetry $e^{-\mathscr{K}_{\mathrm{K}}(X_3)} = e^{-\mathscr{K}_{\mathrm{c.s.}}(Y_3)} = \frac{1}{|X_0|^2}\int_{Y_3} \Omega \wedge \bar{\Omega} \geq \alpha = \mathscr{O}(1)$ Y_3: mirror 3-fold

- Minimal volume at finite distance degenerations of complex structure of Y_3

Circle reduction of M-theory on CY

Limit $\mathscr{V}_{X_{3},s} \sim (r_{S^1} M_{11\mathrm{d}})^3 \to 0$ obstructed by α' corrections:

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- Minimal volume at finite distance degenerations of complex structure of Y_3

Example: Y_3 = quintic $\mathbb{P}^4[5]$: $p(x_i, \phi) = \sum_{k=1}^{5} x_k^5 - \phi x_1 x_2 x_3 x_4 x_5 = 0$

Landau-Ginzburg point $|\phi| \to 0 : \frac{1}{|X_0|^2} \int_{Y_3} \Omega \wedge \bar{\Omega} \to 3.08$

Circle reduction of M-theory on CY

Interpretation:

$$2\pi\, r_{S^1}^{\text{min.}} M_{11d} = \frac{\alpha^{\frac{1}{3}}}{\left(\mathscr{V}_{X_3}\right)^{\frac{1}{3}}}$$ is bound on theory as a KK EFT

➡ This is not a bound on g_{IIA}, but below $g_{\text{IIA}}^{\text{min}} \sim \left(2\pi\, r_{S^1}^{\text{min.}} M_{11d}\right)^{3/2}$ KK reduction not a good description

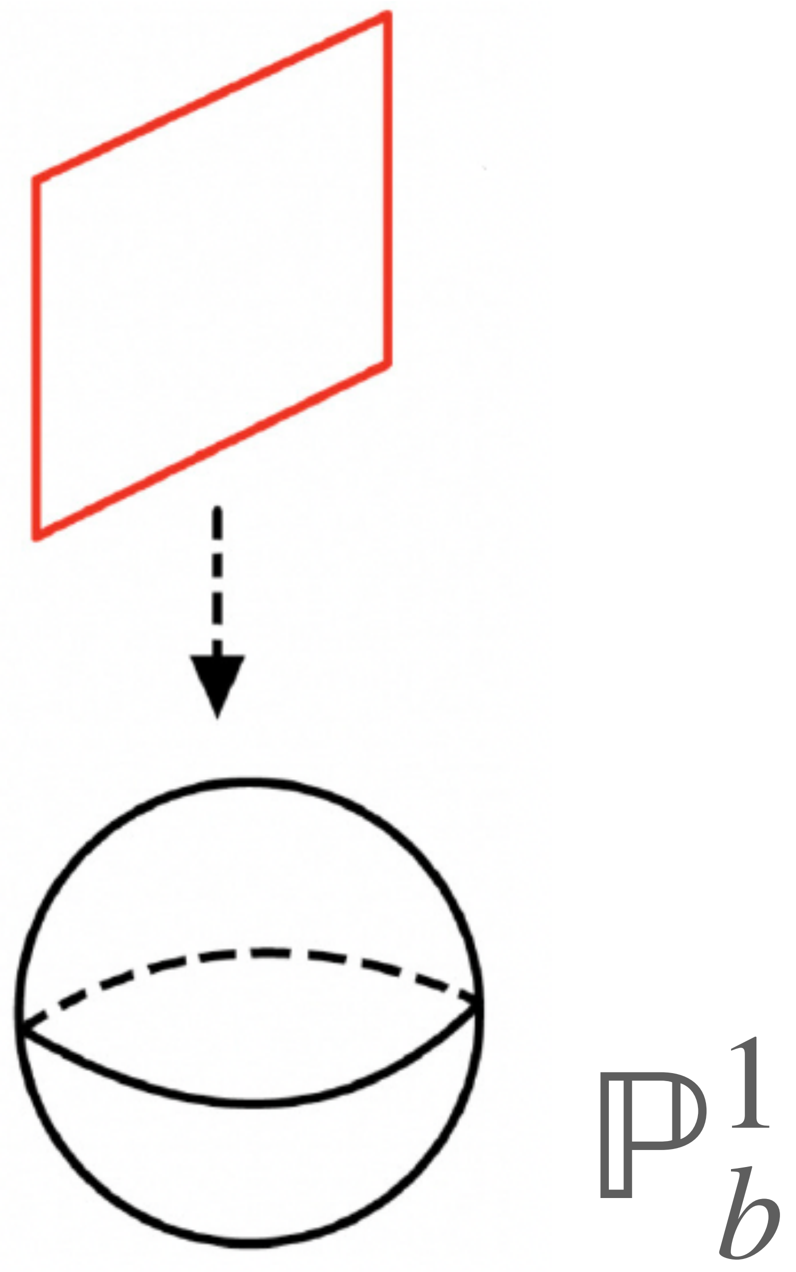
➡ This is a consequence of quantum geometry of compactification and does not occur for circle compactification of 11d M-theory

Circle reduction of M-theory on CY

Check CHC bound explicitly in regime $r \geq r_{\min}$ for U(1) with

- no weak coupling limit
- without a tower of charged BPS or known tower of charged non-BPS states

Example: $A = \int_{\mathbb{P}^1_b} C_3$ $\mathbb{P}^1_b$: base of K3-fibration

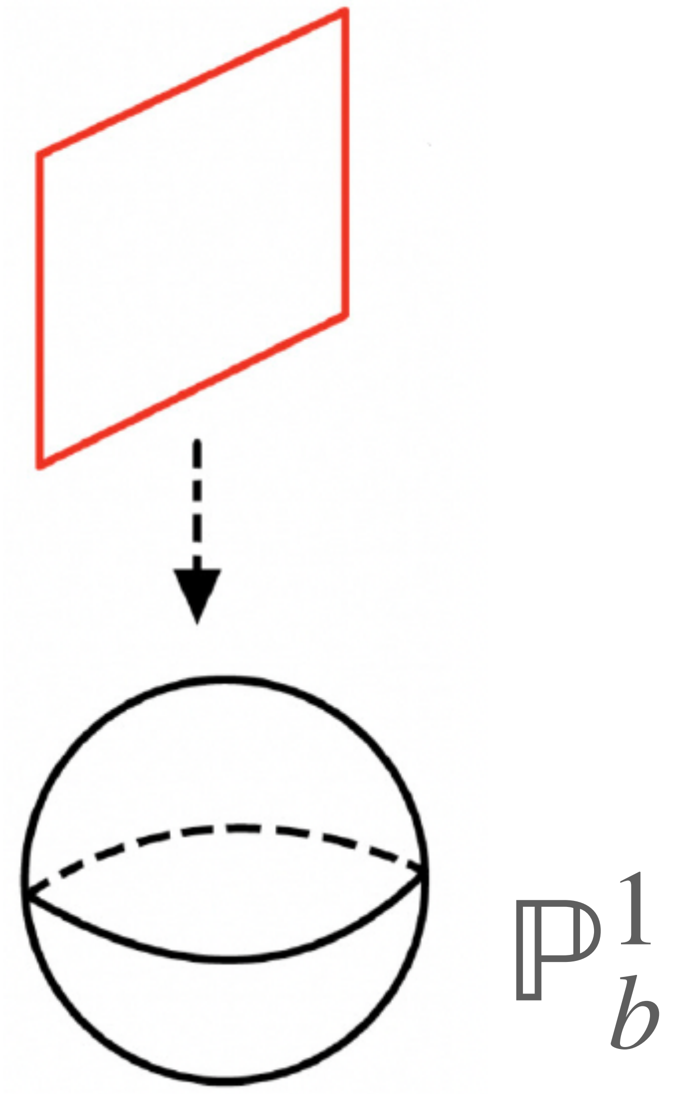


Concrete examples:

$$\mathbb{P}^4_{11222}[8] \qquad \mathbb{P}^4_{11226}[12]$$

$$\mathcal{I}(\mathbb{P}^4_{11222}[8]) = 8J_1^3 + 4J_1^2 J_2 \qquad \mathcal{I}(\mathbb{P}^4_{11226}[12]) = 4J_1^3 + 2J_1^2 J_2$$

Circle reduction of M-theory on CY



Evaluate CHC bound explicitly for U(1) with

- no weak coupling limit
- without a tower of charged BPS or known non-BPS states

Numerically: *cf. [Candelas,Font,Katz,Morrison'94] [Blumenhagen,Kläwer,Schlechter,Wolf'18]*

$$\alpha_{\mathbb{P}^4_{11222}[8]} \simeq 2.83 \qquad \alpha_{\mathbb{P}^4_{11226}[12]} \simeq 6.00$$

$$\text{RHS}_{\mathbb{P}^4_{11222}[8]} \leq 0.17 \qquad \text{RHS}_{\mathbb{P}^4_{11226}[12]} \leq 0.10$$

CHC at $r \geq r_{\text{min}}$ requires:

$$\alpha^{\frac{2}{3}} \geq \frac{\gamma}{2Q_\alpha f^{\alpha\beta} Q_\beta} \left(\frac{\gamma \mathscr{V}_C}{|q|^2 \left(2|q|^2 Q_\alpha f^{\alpha\beta} Q_\beta \mathscr{V}_{X_3}^{2/3} - \gamma \mathscr{V}_C\right)} + \frac{q\theta(1-\theta q)}{|q|^2} \right)$$

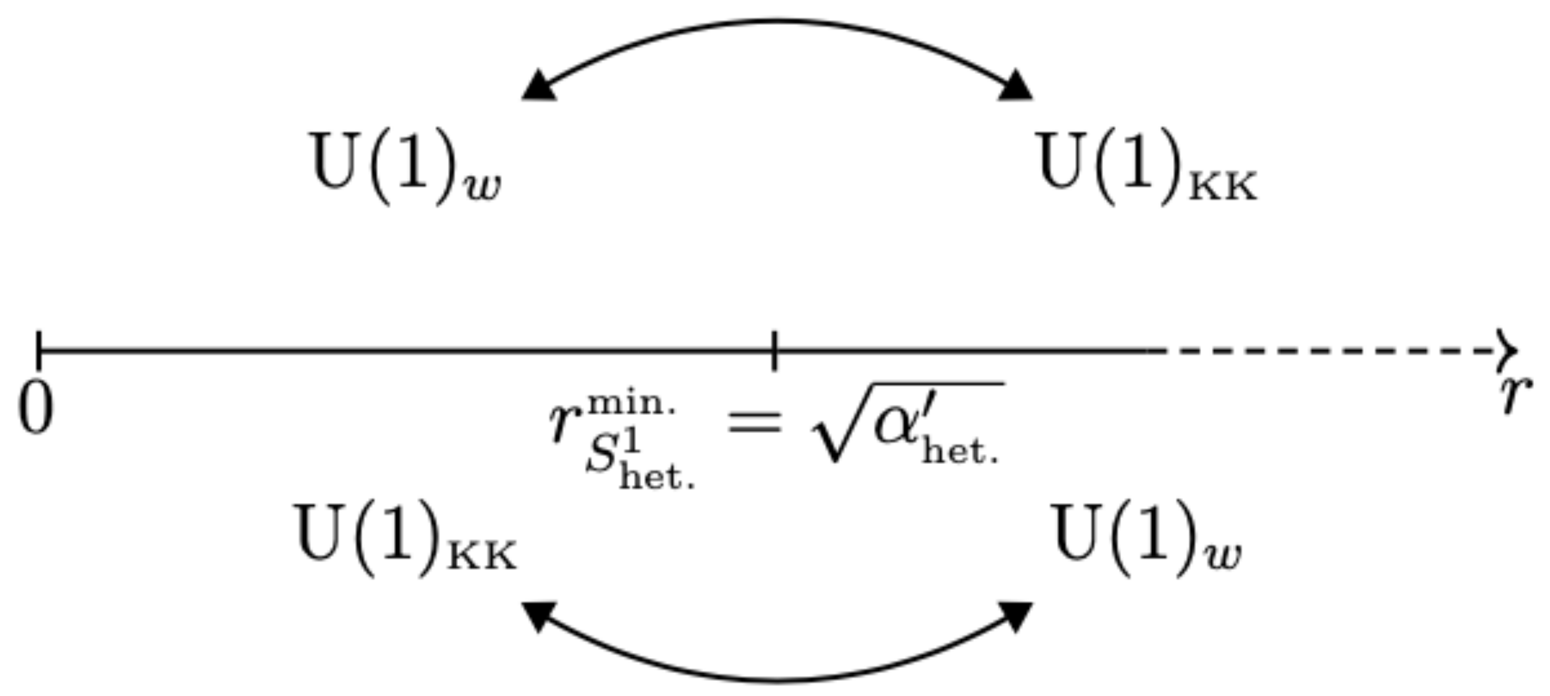
✔

Circle reduction of closed string theory

T-duality might seem to define minimal radius $r_{S^1}^{\min} = \sqrt{\alpha'}$

but if we restrict to $r \geq \sqrt{\alpha'}$ then must also consider

CHC for winding $U(1)_w$



Hence can **focus on $U(1)_{\text{KK}}$, but consider full regime $r_{S^1} \geq 0$** a priori

Minimal radius criterion:

$$\frac{1}{2\pi r_{S^1}} = M_{\text{KK}} \leq M_{\text{BH,min}} = \frac{M_s}{g_s^2} \implies r_{S^1}^{\min} \geq g_s^2 \sqrt{\alpha'} \to 0$$

as $g_s \to 0$: **No minimal radius!**

$$\frac{M_{\text{BH,min.}}}{M_{\text{Pl,D}}} = \left(\frac{M_{\text{Pl,D}}}{\Lambda_{\text{QG}}}\right)^{D-3}, \qquad \Lambda_{\text{QG}} = M_s$$

Circle reduction of string theory: heterotic

1) **Perturbative sector:** $g^2_{U(1)_{\text{pert}},6} M_{\text{het}}^{D-4} \propto g^2_{\text{het}}$

- superextremal states of charge $q^2 = 4\mathfrak{m}n$, excitation level n
- CHC after S^1 reduction: $4\frac{r^2_{S^1}}{\alpha'} \geq \frac{1}{n}\left(\frac{n-1}{4} + q\theta(1-q\theta)\right)$ clashes with $r^{\text{min}}_{S^1} \geq g_s^2\sqrt{\alpha'} \to 0$

$\Longrightarrow$ **Tower of super-extremal states required** - **in agreement with existing tower** ✓

2) **Non-perturbative sector:** E.g. from NS5-branes in comp. to 6d $g^2_{U(1)_{\text{n.p.}},6} M^2_{\text{het}} \propto g^{-2}_{\text{het}}$

- E.g. for massless charged sector: $\frac{r^2_{S^1}}{\alpha'} \geq \frac{q\theta(1-q\theta)}{z_6^2} \propto g^4_{\text{het}} q\theta_6(1-q\theta_6)$ no parametric clash

$\Longrightarrow$ **no tower needed in agreement with absence of known candidates!** ✓

Circle reduction of string theory: open

$g^2_{U(1)_{\text{pert}},D} M^{D-4}_{\text{het}} \propto g_s \implies$ **parametric clash for CHC and naively requires tower**

However:

No super-extremal particle tower in open pert. spectrum of increasing charge!

Solution:

$r \ll \sqrt{\alpha'}$ and
CHC with $U(1)_{\text{KK}}$

T-duality $\Longleftrightarrow$

$r \gg \sqrt{\alpha'}$ and
CHC with $U(1)_{\text{winding}}$
for theory localised along S^1

Furthermore: In limit $g_s \to 0$ gauge theory on brane decouples from gravity! ✔

WGC under dimensional reduction

Findings consistent with following pattern:

Consider a

- D-dimensional $U(1)_D$ gauge theory in a D-dimensional theory of quantum gravity such that
- the WGC is realized by a set of super-extremal particle-like states.

$\Longrightarrow$ In the (D-1)-dimensional theory after S^1 reduction, the CHC for $U(1)_D \times U(1)_{\mathrm{KK}}$ is satisfied

- by KK replicas of the D-dimensional super-extremal particle states
- for any value of the circle radius which allows for an interpretation as a circle reduction of the D-dimensional gauge theory coupled to gravity.

This holds **irrespective of whether the particles are part of a tower in the D-dimensional theory**.

Conclusions

✳ **WGC tower** of super-extremal particles **present and required** for consistency **for**:

i) perturbative heterotic string U(1)

ii) KK U(1)

Emergent String ⟷ Conjecture

all U(1) with a weak coupling limit

✳ All **known cases without** established super-extremal **tower** are **consistent**:

- conifold U(1) M-theory
- pert. open string U(1)
- non-pert. sector in 6d/4d heterotic
- generic F-theory away from emergent string limits

Conclusions

Open question:

Super-extremal particle tower present also for **strongly coupled BPS sectors** (5d SCFTs):

They would be required by circle reduction if these were strictly extremal.

Are they?

If so, this would motivate a

Minimal Tower Weak Gravity Conjecture:

*Super-extremal particle towers are present **if and only if** they are required by consistency of the WGC under circle reduction.*

Appendix

Weak Gravity Conjecture: Criterion for particles

Claim/Conjecture: *[Cota,Mininno,TW,Wiesner'23]*

The WGC must hold at the particle level for a genuine 0-form gauge theory coupled to gravity:

i) not a defect theory in a higher dimensional theory:

$\ell_{\text{perp.}}$: size of extra dimensions perpendicular to gauge brane

$\ell_{\text{min.}} = \frac{1}{\Lambda_{\text{QG}}}$: minimal length scale of QG $\Lambda_{\text{QG}} \sim r^{-1}_{\text{BH,min}}$: Species scale [Dvali,07]

hence require: $\ell_{\text{perp.}} \leq \ell_{\text{min}}$

ii) not secretly a higher-form symmetry:

$\ell_{\text{perp.}}$: size of cycle over which a higher-form was reduced : $\ell_{\text{perp.}} \leq \ell_{\text{min}}$

iii) gauge degrees of freedom not decoupled from gravitational sector