Living with Ghosts

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OUTLINE

- Motivation
- What is stability?
- Our mission
- (Bonus slide)

MOTIVATION

- Higher order settings generically lead to the generation of ghostly states (negative kinetic energy modes) [Kobayashi '19]

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  - change of variables $x = \frac{q+Q}{\sqrt{2}}; y = \frac{q-Q}{\sqrt{2}}$
  - "new" Lagrangian $L = \frac{1}{2}\dot{x}^2 - \frac{1}{2}\dot{y}^2 + V(x,y)$

MOTIVATION

Ghosts don’t really want to give us stable systems!

MOTIVATION

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Do we have to kill *all* ghosts?

WHAT IS STABLE?

3 types of stability:

- **Bounded motion/global stability/Lagrange stability**
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- **Island of stability**
  - Eom solutions are stable for certain regions of initial conditions
  - mostly discovered by numerical analyses

OUR MISSION

- Start from Lagrangian $L = \frac{1}{2}\dot{x}^2 - \frac{1}{2}\dot{y}^2 + V(x, y)$
- This PG system has conserved (Lagrangian) energy $E_L = \frac{1}{2}\dot{x}^2 - \frac{1}{2}\dot{y}^2 + V, \dot{E} = 0$
- 2 steps:
  - Generate families of potentials for which the PG system is Liouville integrable
    - Construct V’s such that there exists a second conserved quantity Q
  - pose additional demands on Q so that system is stable
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  - pose additional demands on Q so that system is stable
    - e.g. Q positive definite, strictly positive Hessian

Q takes over the job of E! Thus, we ensure bounded motion.

Thank you for your attention!

More coming soon... :)

BONUS SLIDE!

- Consider $f : \mathbb{R}^n \to \mathbb{R}$, $f \in C^2$, with $\bigtriangledown f\left(x^+\right) = 0, x^+ \in \mathbb{R}$
- Let $\bigtriangledown^2 f\left(x\right)$ be positive definite for all $x \in \mathbb{R}^n$
- Taylor: $f\left(y\right) = f\left(x\right) + \bigtriangledown f\left(x\right)\left(y - x\right) + \frac{1}{2}\bigtriangledown^2 f\left(x\right)(y - x)^2$
- $f\left(y\right) > f\left(x\right) + \bigtriangledown f\left(x\right)\left(y - x\right) \Rightarrow$ strictly convex
- strictly convex with minimum means strictly coercive!
- bounded sublevel sets { $x \in \mathbb{R}^n \mid f\left(x\right) \leqslant c \in \mathbb{R}$ }
- $\Rightarrow f$ has bounded level sets, conserved quantity!
- system's curves are inside level sets, curves are bounded!