Fun in 5d Supergravity

Tom Rudelius

Durham University

Based on 2212.10573 with Naomi Gendler, Ben Heidenreich, Liam McAllister, Jakob Moritz
2108.08309 with Murad Alim and Ben Heidenreich
2206.04063 with Muldrow Etheredge, Ben Heidenreich, Sami Kaya, and Yue Qiu
2309.10024, 2312.00120
(Some figures adapted from talk slides by Naomi and Ben)

Outline

I. 5d Supergravity and Calabi-Yau Geometry
II. Moduli Space Reconstruction
III. Infinite-Distance Limits
IV. Persistence of the “Pattern”
V. Summary

5d Supergravity and Calabi-Yau Geometry

5d supergravity

- 5d supergravity features two multiplets with massless scalar fields
  - Hypermultiplets
  - Vector multiplets, which also feature a vector boson
- In addition, every 5d supergravity theory features a gravity multiplet, which also has a vector boson
- Thus, there are $N = n_v + 1$ vector bosons, and n_v vector multiplet moduli

5d supergravity

- Many properties of a 5d supergravity theory are controlled by a cubic prepotential:

$$\mathcal{F} = \frac{1}{6} C_{IJK} Y^I Y^J Y^K \qquad I = 1, ..., N$$

- Two equivalent ways to construct the vector multiplet moduli space:
    - The $\mathscr{F} = 1$ slice of the space parametrized by the Y^I
    - The projective space parametrized by homogeneous coordinates $Y^I \sim \lambda Y^I$

Gauge and Scalar Couplings

- Helpful to define

$$\mathcal{F}_I = \partial_I \mathcal{F} \qquad\qquad \mathcal{F}_{IJ} = \partial_I \partial_J \mathcal{F}$$

- Gauge kinetic matrix is then

$$a_{IJ} = \mathcal{F}_I \mathcal{F}_J - \mathcal{F}_{IJ}$$

- Metric on moduli space is pullback to $\mathscr{F} = 1$ slice:

$$g_{ij} = \frac{1}{2} a_{IJ} \partial_i Y^I \partial_j Y^J$$

BPS Particles

- In theories with 8+ supercharges, charged particles must satisfy BPS bound:

$$m \geq |\zeta_{q_i}(a_i)|$$

- Central charge ζ depends on moduli a_i, charge q_i of particle
- 4d: ζ complex. 5d: ζ real
- Particles that saturate the BPS bound are called BPS particles

BPS Bound in 5d

- BPS particles saturate the bound:

$$m(q_I) \geq (2\pi^2)^{1/6} |q_I Y^I|$$

- BPS strings saturate the bound:

$$T(\tilde{q}^I) \geq \frac{1}{2}(2\pi^2)^{-1/6} |\tilde{q}^I \mathcal{F}_I|$$

5d Supergravity from M-theory

- One way to construct UV-complete 5d supergravity theories in the Landscape is to compactify M-theory on a Calabi-Yau threefold
- Physical properties of the theory are then identified with geometric properties of the Calabi-Yau threefold

Prepotential from Intersection Numbers

- Prepotential:

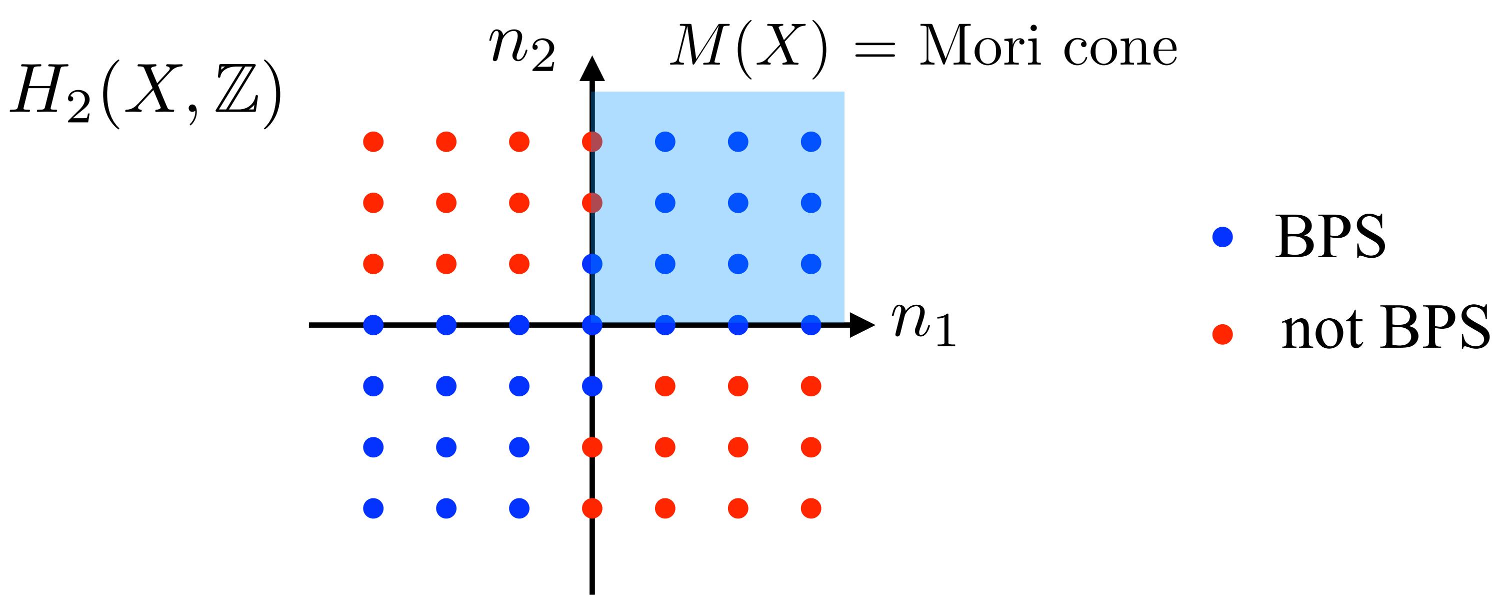
$$\mathcal{F} = \frac{1}{6} C_{IJK} Y^I Y^J Y^K$$

- Integers C_{IJK} are geometrically triple intersection numbers of divisors:

$$C_{IJK} = \int_X D_I \cdot D_J \cdot D_K$$

BPS Particles and Holomorphic Curves

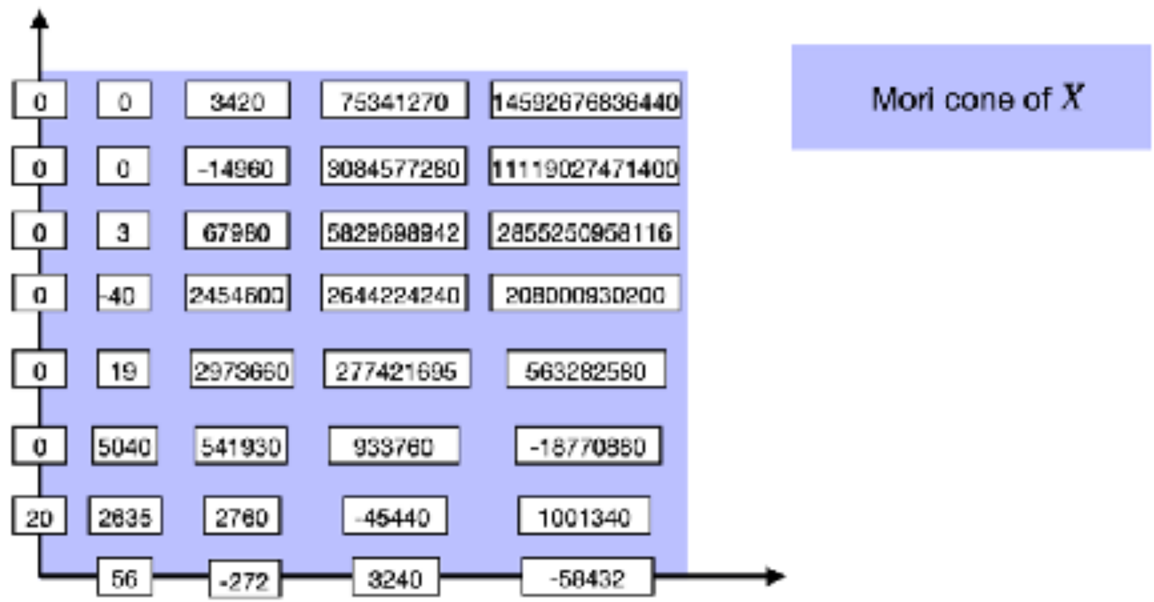
- M2-brane wrapped on holomorphic curve of class $\sum n_i[C_i], n_i \geq 0$ gives BPS particle of charge n_i
  $\Rightarrow H_2(X, \mathbb{Z})$ identified with electric charge lattice



Gopakumar-Vafa Invariants

- BPS states (i.e., holomorphic curves) of each charge are counted by "Gopakumar-Vafa invariants":

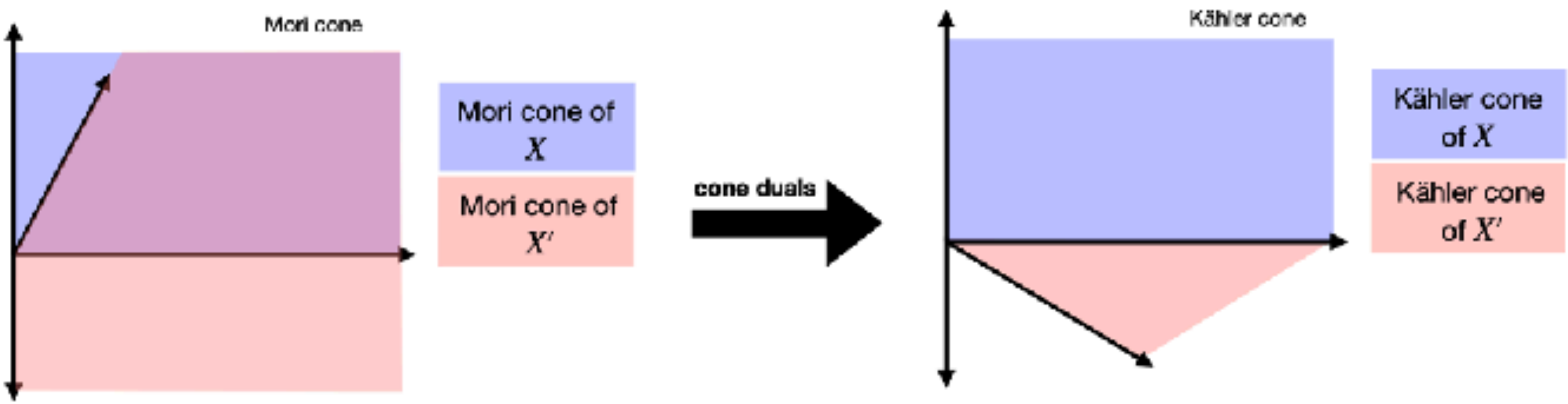
A Calabi-Yau threefold, X, with $h^{1,1} = 2$

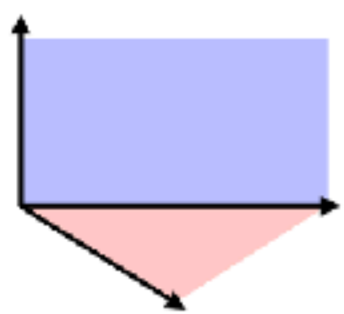


Gopakumar, Vafa, '98

Mori Cones and Flops

- The Mori cone of X is dual to the Kähler cone, which changes upon a "flop" transition to a birationally equivalent CY3 X'
- Physically, the full moduli space of the 5d supergravity contains the union of all the Kähler cones, also known as the extended Kähler cone:





This is the extended Kähler cone $\mathcal{K}$

Effective Curves and Divisors

- Effective curves generate the Mori cone, which is dual to the Kahler cone:

$$M(X) = \mathcal{K}(X)^{\vee}$$

- Similarly, effective divisors generate the effective cone, which is dual to the movable cone

$$\mathrm{Mov}(X) = \mathcal{E}^{\vee}$$

Moduli Space Reconstruction

- In general, it’s difficult to compute the extended Kähler cone of a CY3, and even harder to compute the movable cone
- But, we have constructed an algorithm to do this, with help from GV invariants and supergravity

Alim, Heidenreich, TR ‘21
Gendler, Heidenreich, McAllister, Moritz, TR ‘22

Moduli Space Reconstruction

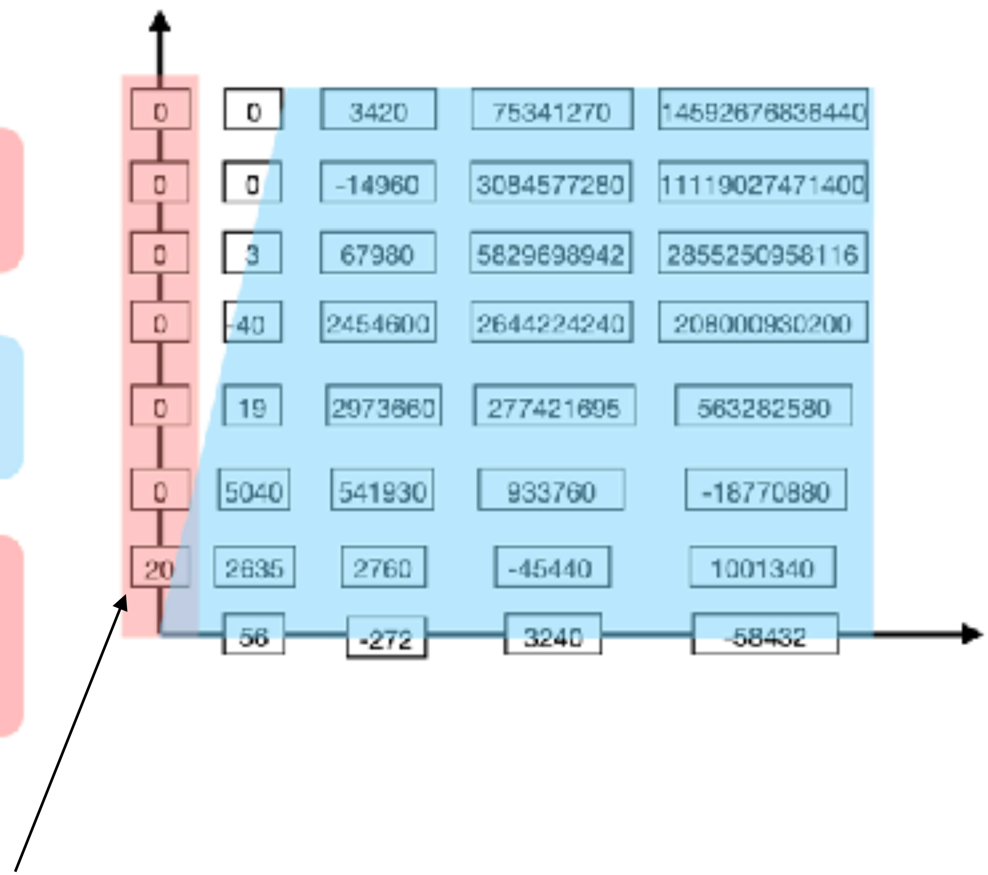
Nops and Flops

A few definitions

Nilpotent curve: a ray with finitely many non-zero GV invariants.

Potent curve: a ray with infinitely many non-zero GV invariants.

Nop curve: nilpotent curve outside the closure of the cone generated by potent curves.

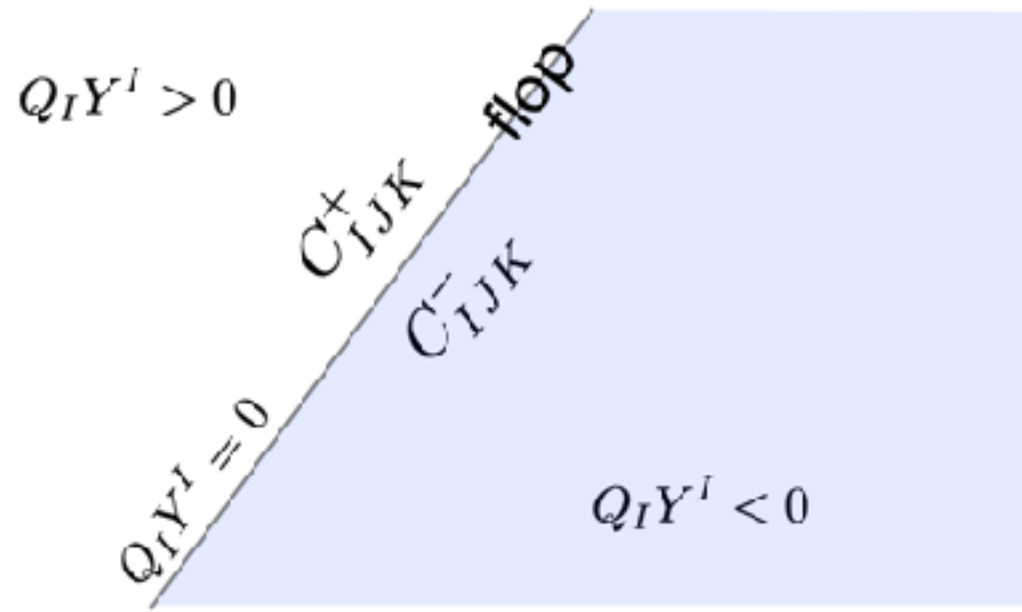


Nop curve ⇔ Floppable Curve

Flops and Intersection Numbers

- At such a flop, N hypers of charge Q_I become massless, triple intersection numbers C_{IJK} shift

$$C^{+}_{IJK} = C^{-}_{IJK} + N Q_I Q_J Q_K$$

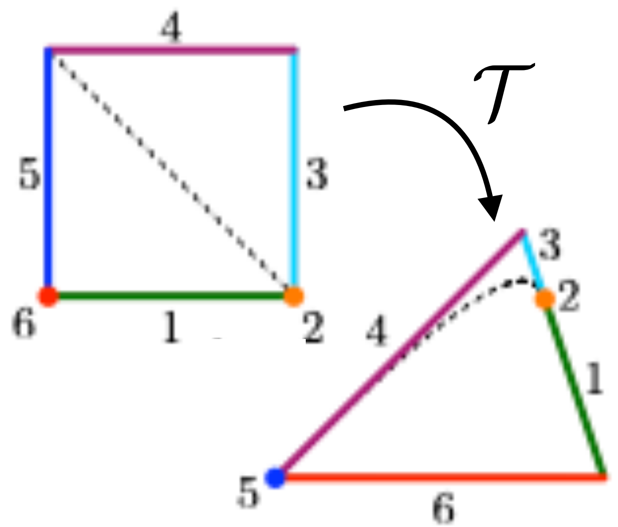


- Repeating this process for all nop curves, find extended Kähler cone, prepotential in each phase

Moduli Space Reconstruction

- Given point in extended Kähler cone Y^I, find point in movable cone via "$\mathscr{T}$ map":

$$\mathcal{T}(Y^I) = \mathcal{F}_I = \frac{1}{2} C_{IJK} Y^J Y^K$$



Testing the Sublattice WGC

- The Sublattice WGC requires an infinite tower of BPS particles everywhere inside the movable cone
- By constructing the movable cone and computing GV invariants, can put this to the test: verified in all 2062 examples studied

Alim, Heidenreich, TR '21
Gendler, Heidenreich, McAllister, Moritz, TR '22

Infinite-Distance Limits

Compactifying 6d String Theory

- Consider a 6d string theory, with 2-form gauge coupling

$$g_2^{(6)} \sim \exp(-\phi)$$

- Compactifying to 5d, this picks up dependence on the radion:

$$g_2 \sim \exp(-\phi - \frac{1}{\sqrt{3}}\rho)$$

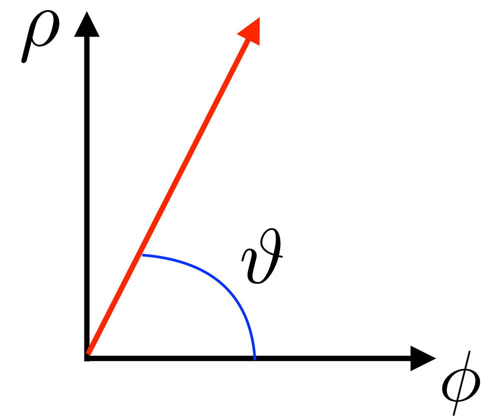
- Also have KK gauge field, with coupling

$$e_{\mathrm{KK}} \sim \exp(-\frac{2}{\sqrt{3}}\rho)$$

Decompactifying 5d String Theory

- Consider the decompactification limit $\rho, \phi \to \infty$, with fixed slope

$$\frac{d\rho}{d\phi} \equiv \tan\vartheta$$



- In this limit, have

$$g_2 \sim \exp(-\lambda||p - p_0||) \qquad e_{\rm KK} \sim \exp(-\alpha||p - p_0||)$$

$$\lambda = \cos\vartheta + \frac{1}{\sqrt{3}}\sin\vartheta \qquad \alpha = \frac{2}{\sqrt{3}}\sin\vartheta$$

Infinite-Distance Limits in 5d SUGRA

- Given prepotential:

$$\mathcal{F} = \frac{1}{6} C_{IJK} Y^I Y^J Y^K$$

- Let's now adopt the perspective that the Y^I are homogenous coordinates, identified under simultaneous rescaling $Y^I \sim \lambda Y^I$
- Consider "straight-line" path in the space of these homogenous coordinates:

$$Y^I = Y_0^I + s Y_1^I \, , s \in [0,1]$$

- Assume $s = 0$ is at infinite distance $\Rightarrow \mathscr{F} \sim s$ or $\mathscr{F} \sim s^2$

Decompactification Limits

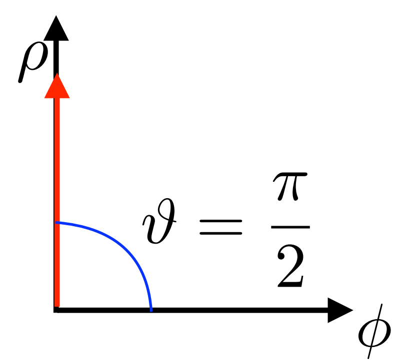
Case 1: $\mathscr{F} \sim s$

- Using formulae for gauge kinetic matrix, metric on moduli space in terms of the prepotential, find

$$e_{\text{KK}} \sim g_{\text{min}} \sim \exp(-\frac{2}{\sqrt{3}}||p - p_0||) \quad \Rightarrow \quad \alpha = \frac{2}{\sqrt{3}}$$

$$g_2 \sim \frac{1}{g_{\text{max}}} \sim \exp(-\frac{1}{\sqrt{3}}||p - p_0||) \quad \Rightarrow \quad \lambda = \frac{1}{\sqrt{3}}$$

- Matches scaling for $\vartheta = \pi/2$ decompactification limit!



Etheredge, Heidenreich, Kaya, Qiu, TR ’22

Emergent String Limits

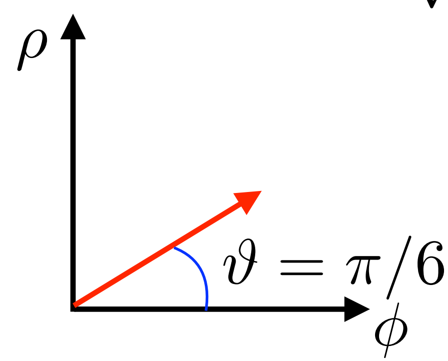
Case 2: $\mathscr{F} \sim s^2$

- Using formulae for gauge kinetic matrix, metric on moduli space in terms of the prepotential, find

$$e_{\mathrm{KK}} \sim g_{\min} \sim \exp(-\frac{1}{\sqrt{3}}||p - p_0||) \quad \Rightarrow \quad \alpha = \frac{1}{\sqrt{3}}$$

$$g_2 \sim \frac{1}{g_{\max}} \sim \exp(-\frac{2}{\sqrt{3}}||p - p_0||) \quad \Rightarrow \quad \lambda = \frac{2}{\sqrt{3}}$$

- Matches scaling for $\vartheta = \pi/6$ emergent string limit!



Etheredge, Heidenreich, Kaya, Qiu, TR ’22

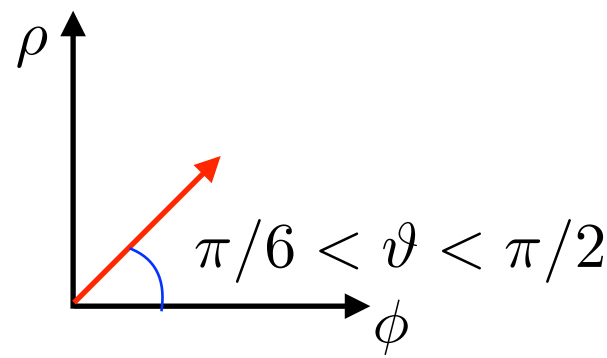
Curved Paths

- What about decompactifcation limits with $\dfrac{\pi}{6} < \vartheta < \dfrac{\pi}{2}$? Need to drop assumption of straight line paths
- Setting

$$Y^1 = 1\,, \quad Y^2 = s\,, \quad Y^3 = s^\beta\,, \quad Y^I = 0\,,\ I > 3$$

- Then for appropriate choices of the prepotential, find expected scaling of α, λ, with

$$\sin\vartheta = \frac{2-\beta}{2\sqrt{\beta^2-\beta+1}}$$



TR ’23

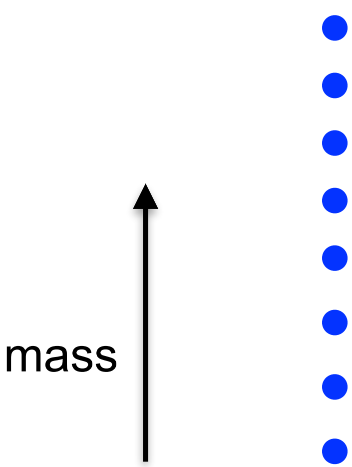
GV Invariants

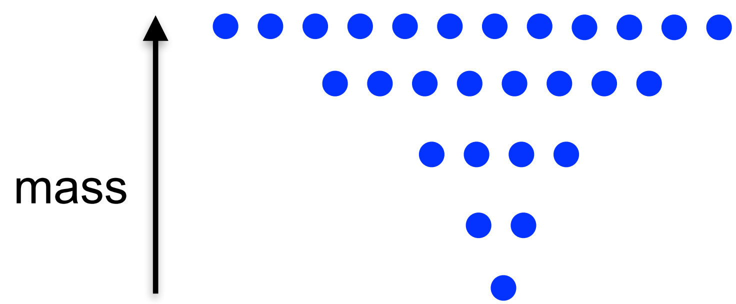
- Match of scaling behavior for gauge couplings relies only on supergravity, no input from string/ M-theory!
- Upon including M-theory, we find further evidence for the Emergent String Conjecture using the spectrum of BPS particles…

Tower Density of States

Decompactification limit $\Rightarrow$ KK tower, tower degeneracies order-one and periodic:

ES limit $\Rightarrow$ KK tower, exponential (Hagedorn) density of states





Examples

Decompactification limit as $Y^2 \to 0$, $(\mathscr{F} \sim Y^2)$:

$q_2 \backslash q_1$	0	1	2	3	4
0	–	40	4	0	0
1	144	496	496	23953120	2388434784
2	164	5616	23100	34528	23100
3	144	44384	602016	2471824	4709216
4	88	279976	10439512	97922024	398659384
5	144	1482384	136431424	2616030416	20133562480
6	164	6751472	1439003864	52447406096	707697743208
7	144	27208608	12779098368	841622542048	18899196173440
8	88	99569856	98370714948	11277044593704	405560003481888

Emergent string limit as $Y^2 \to 0$, $(\mathscr{F} \sim (Y^2)^2)$:

$q_2 \backslash q_1$	0	1	2	3	4
0	–	24	0	0	0
1	396	1152	396	0	0
2	2610	53136	112068	53136	2610
3	35640	2377728	15951564	28024704	15951564
4	605844	103323672	1602730872	6746381496	10576809936
5	12212172	4400303616	132192153792	1084701369600	3472953972948
6	273644244	184590071136	9593083752300	135592015659408	762494479579314

Similar pattern for all 7000+ CICYs!

TR '23, see also Lee, Lerche, Weigand '19, Fierro Cota, Mininno, Weigand, Wiesner '22

Persistence of the Pattern

The Species Scale "Pattern"

- A remarkable pattern observed by Castellano, Ruiz, and Valenzuela in infinite-distance limits

$$\frac{\vec{\nabla} m}{m} \cdot \frac{\vec{\nabla} \Lambda_{\mathrm{QG}}}{\Lambda_{\mathrm{QG}}} = \frac{1}{d-2}$$

- This relation can be proven in asymptotic limits under the ESC and further genericness assumptions (Etheredge, Heidenreich, TR, Ruiz, Valenzuela, to appear)
- However, in the context of 5d supergravity, a version of it can be proven in full generality

5d supergravity

- Recall that 5d supergravity is controlled by a cubic prepotential:

$$\mathcal{F} = \frac{1}{6} C_{IJK} Y^I Y^J Y^K$$

- The vector multiplet moduli space is the $\mathscr{F} = 1$ slice
- Key identity:

$$a^{IJ} = \frac{1}{2} g^{ij} \partial_i Y^I \partial_j Y^J + \frac{1}{3} Y^I Y^J$$

BPS Bound in 5d

- Recall: BPS particles saturate the bound

$$m(q_I) \geq (2\pi^2)^{1/6}|q_I Y^I|$$

- BPS strings saturate the bound

$$T(\tilde{q}^I) \geq \frac{1}{2}(2\pi^2)^{-1/6}|\tilde{q}^I \mathcal{F}_I|$$

The Pattern

- Setting $M_s = \sqrt{2\pi T(\tilde{q}^I)}$, using identity, can prove that for any BPS particle and BPS string,

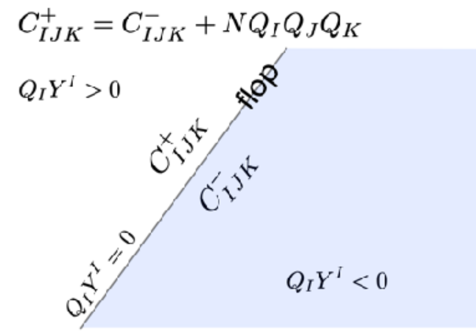
$$g^{ij}\frac{\partial_i m}{m}\frac{\partial_j M_s}{M_s} = \frac{1}{3} - \frac{q_I \tilde{q}^I}{(q_K Y^K)(\tilde{q}^L \mathcal{F}_L)}$$

- If string and particle become light in asymptotic limit, their Dirac pairing vanishes, $q_I \tilde{q}^I = 0$
- Setting $\Lambda_{\mathrm{QG}} = M_s$, we find the pattern:

$$\boxed{\frac{\vec{\nabla} m}{m} \cdot \frac{\vec{\nabla}\Lambda_{\mathrm{QG}}}{\Lambda_{\mathrm{QG}}} = \frac{1}{3}}$$

Persistence of the Pattern

- This result holds not only in the asymptotic limits of moduli space, but also in the interior
- It even carries over into distinct phases, related by flop transitions, where the prepotential and string central charges are modified



- It holds even though $||\vec{\nabla}\log(m)||$ and $||\vec{\nabla}\log(\Lambda_{\mathrm{QG}})||$ vary in the interior of moduli space

Summary

Summary

And now, with that, my talk is done,
I hope that, like me, you too have had fun.
Just one more thing before we're through,
Let us conclude with a brief review.
We saw that 5d supergravity has a moduli space,
Which can be reconstructed using BPS states.
We figured out the locations of the flops,
Using GV invariants to identify the nops.
This let us determine the extended Kahler cone,
And, using the T-map, the effective cone.
Next, we studied gauge couplings of 5d supergravity,
In the limit where a scalar field goes to infinity.
With decompactification limits, we had a perfect match,
Without any top-down input, how neat was that?
Finally, in the end, we saw a pattern that remained,
In the interior of moduli space, not just the asymptotic plane.
The full implications of this, I do not yet understand,
But I know I've had fun hiking with you all through the swampland!

Thank You