KING'S College LONDON

LEVERHULME TRUST

Automorphic Forms and Heterotic EFT

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based on work with J. M. Leedom, A. Westphal and A. Kidambi

Geometry, Strings and the Swampland Program

March 18, 2024

Prologue & Motivation

Heterotic string is a remarkable playground:

- 4d $N = 2$ heterotic - type II duality
- presence of modular symmetries
- control of the $N = 1$ EFT from higher susy subsectors
- outstanding work on vacua and SM

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- control of the $N = 1$ EFT from higher susy subsectors
- outstanding work on vacua and SM $\longrightarrow$ No-go thms for de Sitter vacua

[Green, Martinec, Quigley, Sethi '11][Gautason, Junghans, Zagermann '12]
[Kutasov, Maxfield, Melnikov, Sethi '15][Quigley '15]
[Leedom, NR, Westphal '22]

$\Rightarrow$ Lessons for the Swampland program

Prologue & Motivation

[Leedom, NR, Westphal '22]

No-go thms for de Sitter vacua:

Theorem 1. No dS minima for $F_T = F_S = 0$

Theorem 2. No dS minima for $K = K_{\text{tree}}(S, T)$ and $F_T = 0$ but $F_S \neq 0$

note: racetrack cannot save you

Prologue & Motivation

[Leedom, NR, Westphal '22]

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$$K = -\ln(S + \bar{S}) + \delta K_{np}(S, \bar{S})$$

Prologue & Motivation

[Leedom, NR, Westphal '22]

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metastable dS vacuum

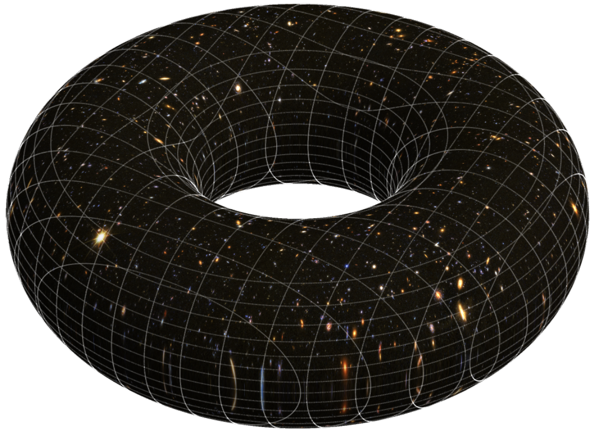
The Setup

SL(2,$\mathbb{Z}$): $ST(U)$ model

Dilaton $S = \frac{1}{g_s^2} + i\theta$

Kahler modulus $T = a + it$

Complex structure modulus $U = \frac{1}{G_{11}}\left(G_{12} + i\sqrt{\det(G)}\right)$



$+ \qquad T \leftrightarrow U$

Sp(4,$\mathbb{Z}$): *STUV* model

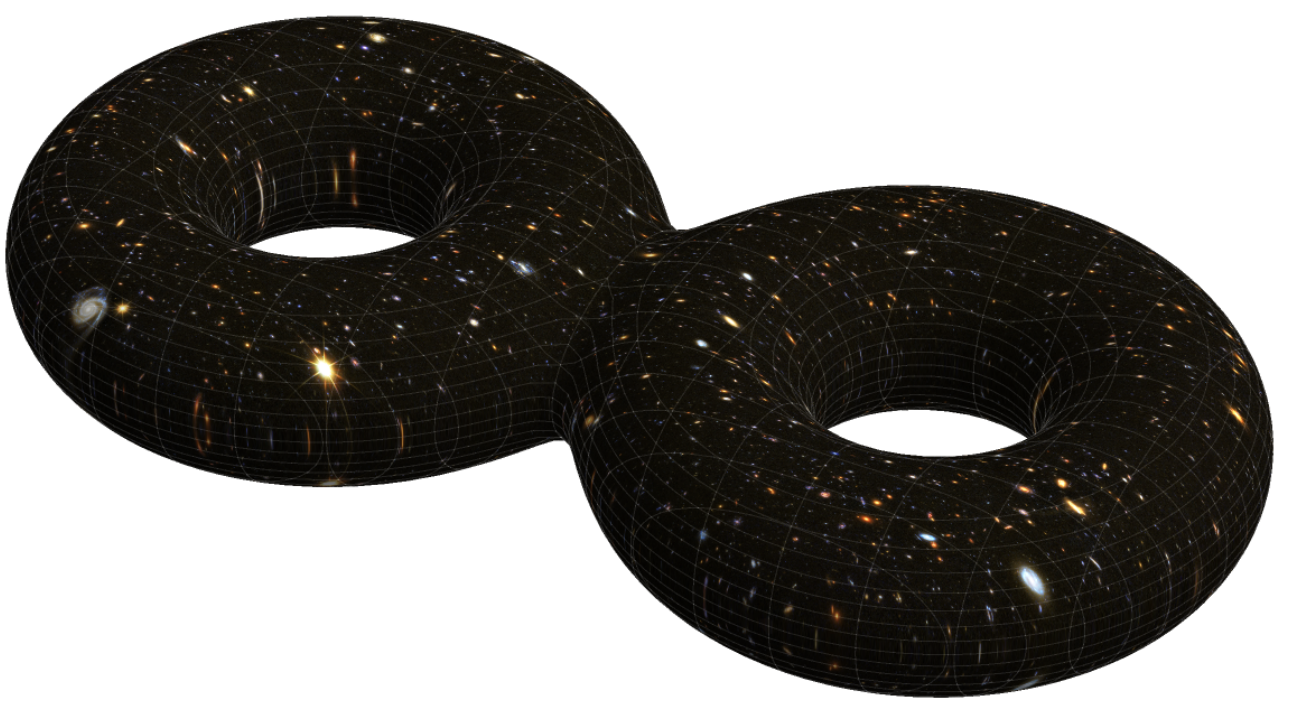
Dilaton $S = \frac{1}{g_s^2} + i\theta$

Kahler modulus $T = a + it + w_1(-w_2 + Uw_1)$

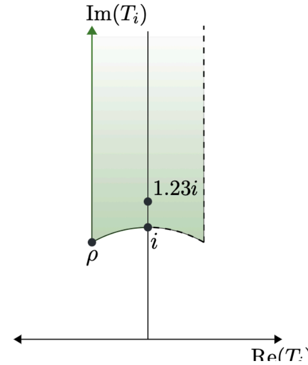
Complex structure modulus $U = \frac{1}{G_{11}}\left(G_{12} + i\sqrt{\det(G)}\right)$

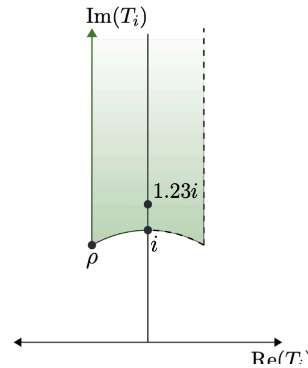
Wilson line $V = -w_2 + Uw_1$

$$\left.\right\} \quad M = \begin{pmatrix} T & \frac{1}{2}V \\ \frac{1}{2}V & U \end{pmatrix}$$



$\mathrm{SL}(2,\mathbb{Z})$

Fundamental domain	 with 2 fixed pts
Modular form of weight k	$f(\gamma \cdot x) = (cx+d)^k f(x)$ $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})$ $ad - bc = 1$
Ring	E_4, E_6, η

	SL(2,ℤ)	Sp(4,ℤ)
Fundamental domain	 with 2 fixed pts	Siegel upper half plane with 6 fixed pts σ_i
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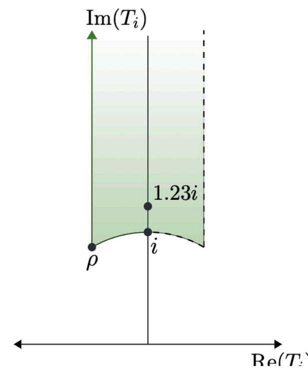
Fundamental domain	 with 2 fixed pts	Siegel upper half plane with 6 fixed pts σ_i
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Cusp form: $g(X) \xrightarrow{X \to \text{cusp}} 0$

The EFT

The EFT: Kahler Potential

$$K_{(2)} = -\ln\left[\det(M - M^{\dagger})\right] = -\ln\left[-(T-\bar{T})(U-\bar{U}) + \frac{1}{4}(V-\bar{V})^2\right]$$

[Lopes Cardoso, Lüst, Mohaupt '94]
[Ferrara, Kounnas, Lüst, Zwirner '91]

under $Sp(4,\mathbb{Z})$:

$$M \to (AM+B)(CM+D)^{-1}$$

$$K_{(2)} \to K_{(2)} + \ln\left[\det(CM^{\dagger}+D)\right] + \ln\left[\det(CM+D)\right]$$

The EFT: superpotential

the superpotential is more involved!

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require **invariance** of $\mathscr{V}$:

defining $G \equiv K_{(2)} + \ln|W_{(2)}|^2 \;\Rightarrow\; \mathscr{V} = e^G\left(G_i G^{i\bar{j}} G_{\bar{j}} - 3\right)$

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$$W_{(2)} \to \det(CM + D)^{-1} W_{(2)}$$

We have a prediction on the form!

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W inherits its automorphic properties from moduli-dependent **threshold corrections**:

- b.c. for the running gauge couplings of the EFT at M_{string}
- moduli dependent = modular forms associated to the symmetry group
- computed in the $N = 2$ subsectors

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flashback to $SL(2,\mathbb{Z})$:

ring E_4, E_6, η

for gaugino condensation $$W \sim \Lambda^3 \sim \mu_0^3 e^{-\frac{f_a}{b_a}}$$

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$$f_a = S$$

tree
level

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$$f_a = S + b_a \ln \eta^6(T)$$

tree level

threshold corrections Δ_a

[Dixon, Kaplunovsky, Louis '91]
[Kaplunovsky, Louis '95]
[Kiritsis, Kounnas '95]

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ring $\mathscr{E}_4$, $\mathscr{E}_6$, χ_{10}, χ_{12}, χ_{35}

$\Rightarrow$ threshold corrections $\quad f_a = S + \Delta_a$

[Mayr, Stieberger '95]
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- without singularities in $\mathscr{F}$: $\Delta_a \sim b_a \ln\left(|\chi_{12}|^2\right)$
- with a singularity in $\mathscr{F}$ for $V \to 0$: $\Delta_a \sim b_a \ln\left(|\chi_{10}|^2\right)$

$$\Rightarrow \quad W_{(2)} \sim \frac{e^{-S/b_a}}{e^{\Delta_a}}$$

The EFT: gaugino condensation

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$$W = \frac{e^{-S/b_a} H(T)}{\eta^6(T)} \qquad H(T) = \left(\frac{E_4(T)}{\eta^8(T)}\right)^n \left(\frac{E_6(T)}{\eta^{12}(T)}\right)^m \mathcal{P}(j(T))$$

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$H(T)$ is the **most general modular function** on $SL(2,\mathbb{Z})$

[Rademacher, Zuckerman '38]

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Theorem:

$$H_{(2)} = \left(\frac{\mathscr{E}_4^3}{\chi_{12}}\right)^n \left(\frac{\mathscr{E}_6^2}{\chi_{12}}\right)^m \left(\frac{\mathscr{E}_4^2\mathscr{E}_6\chi_{10}}{\chi_{12}^2}\right)^\ell \mathscr{P}(j_{(2)})$$

[Kidambi, Leedom, NR, Westphal *WiP*]

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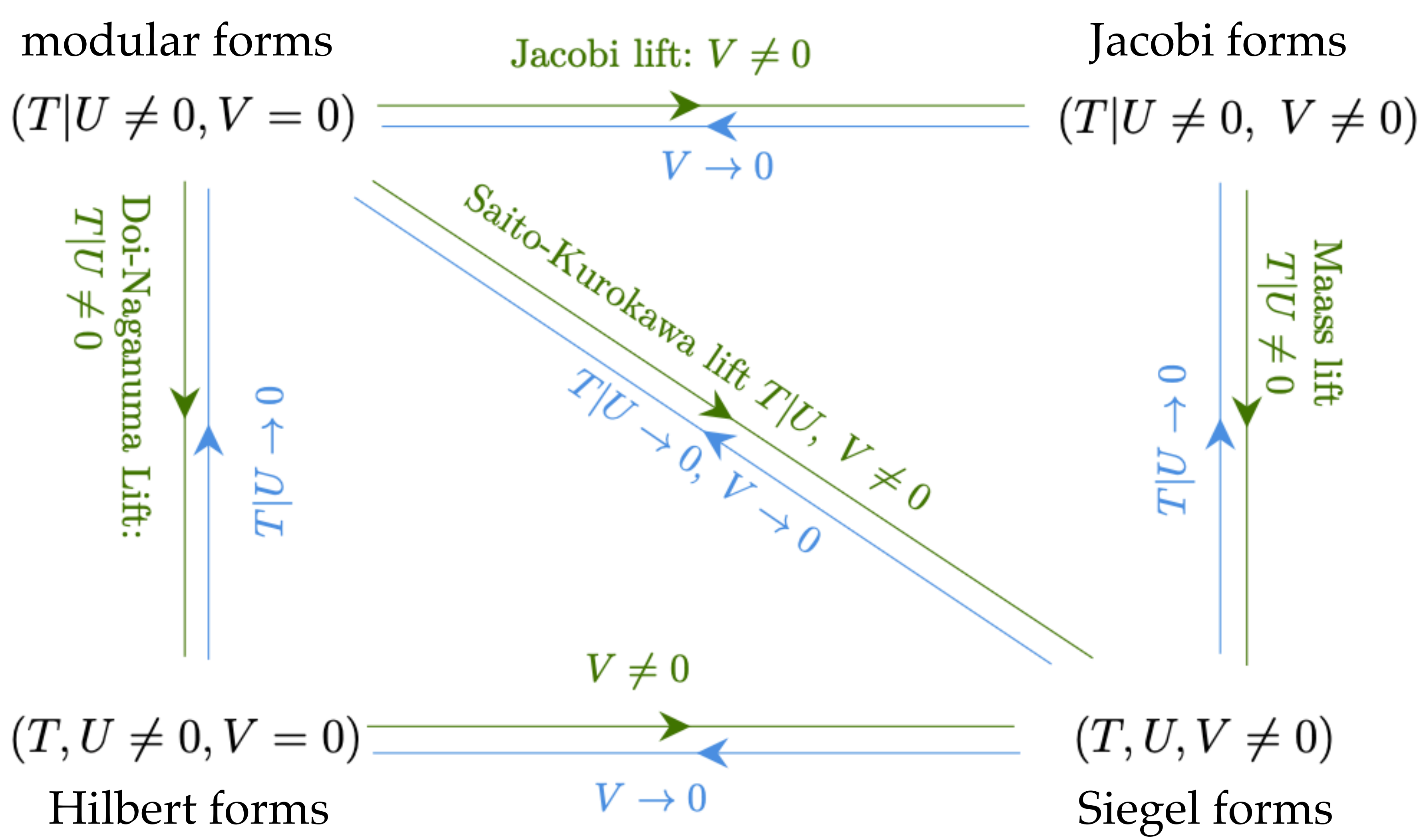

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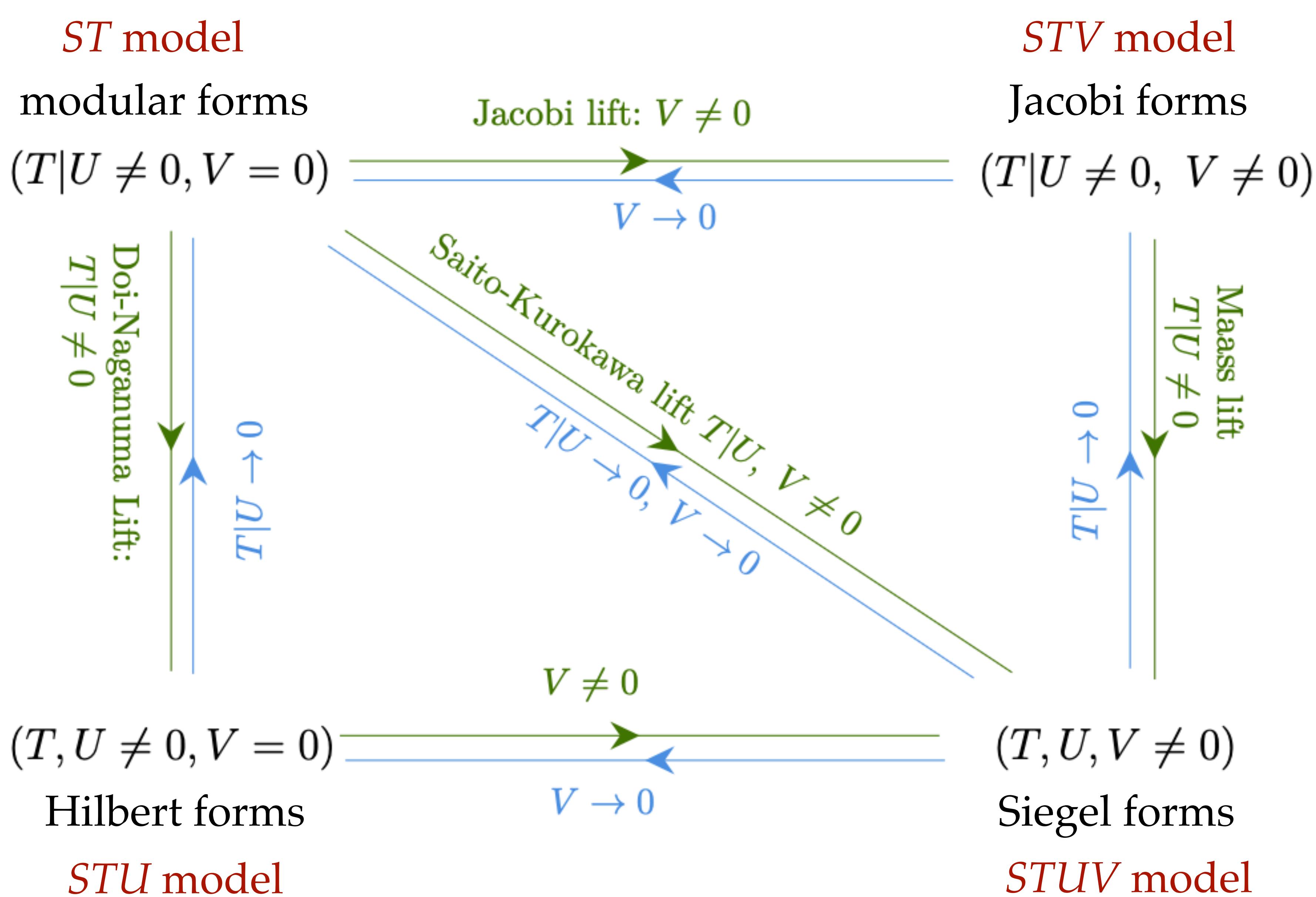

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The check:

1) Expansion in U

$$\mathscr{E}_4 = E_4(T) + 240E_{4,1}(T,V)e^{-2\pi U} + \dots$$

$$\mathscr{E}_6 = E_6(T) - 504E_{6,1}(T,V)e^{-2\pi U} + \dots$$

$$\chi_{10} = \phi_{10,1}(T,V)e^{-2\pi U} + \dots$$

$$\chi_{12} = \eta^{24}(T) + \frac{1}{12}\phi_{12,1}(T,V)e^{-2\pi U} + \dots$$

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$$E_{4,1}(T,V) \to E_4(T)$$

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$$\phi_{10,1}(T,V) = \frac{1}{144}(E_6E_{4,1} - E_4E_{6,1}) \to 0$$

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$$H_{(2)} = \left(\frac{\mathcal{E}_4^3}{\chi_{12}}\right)^n \left(\frac{\mathcal{E}_6^2}{\chi_{12}}\right)^m \left(\frac{\mathcal{E}_4^2\mathcal{E}_6\chi_{10}}{\chi_{12}^2}\right)^\ell \mathcal{P}(j_{(2)}) \quad \longrightarrow \quad H = \left(\frac{E_4}{\eta^8}\right)^n \left(\frac{E_6}{\eta^{12}}\right)^m \mathcal{P}(j)$$

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The Vacua

Gaugino condensation and Vacua

Why care about $H_{(2)}$?

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1) Compute the potential

Gaugino condensation and Vacua

Why care about $H_{(2)}$?

1) Compute the potential

without explicitly computing $\mathscr{V}(S, T, U, V)$, we *prove*:

- all 6 fixed points σ_i are extrema:

  since $\mathscr{V}(S, T, U, V)$ is a Siegel modular function, $\nabla\mathscr{V}(S, T, U, V)|_{\{T,U,V\}=\sigma_i} = 0$

- these extrema are always either Minkowski or AdS minima when $F_S = 0$

[Kidambi, Leedom, NR, Westphal *WiP*]

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[Kidambi, Leedom, NR, Westphal *WiP*]

$\Rightarrow$ can we uplift requiring $F_S \neq 0$?

Gaugino condensation and Vacua

[Leedom, NR, Westphal '22]

Why care about $H_{(2)}$?

2) For $F_S \neq 0$, at the 2 fixed pts:

$$H(T) = \left(\frac{E_4(T)}{\eta^8(T)}\right)^n \left(\frac{E_6(T)}{\eta^{12}(T)}\right)^m \mathscr{P}(j(T))$$

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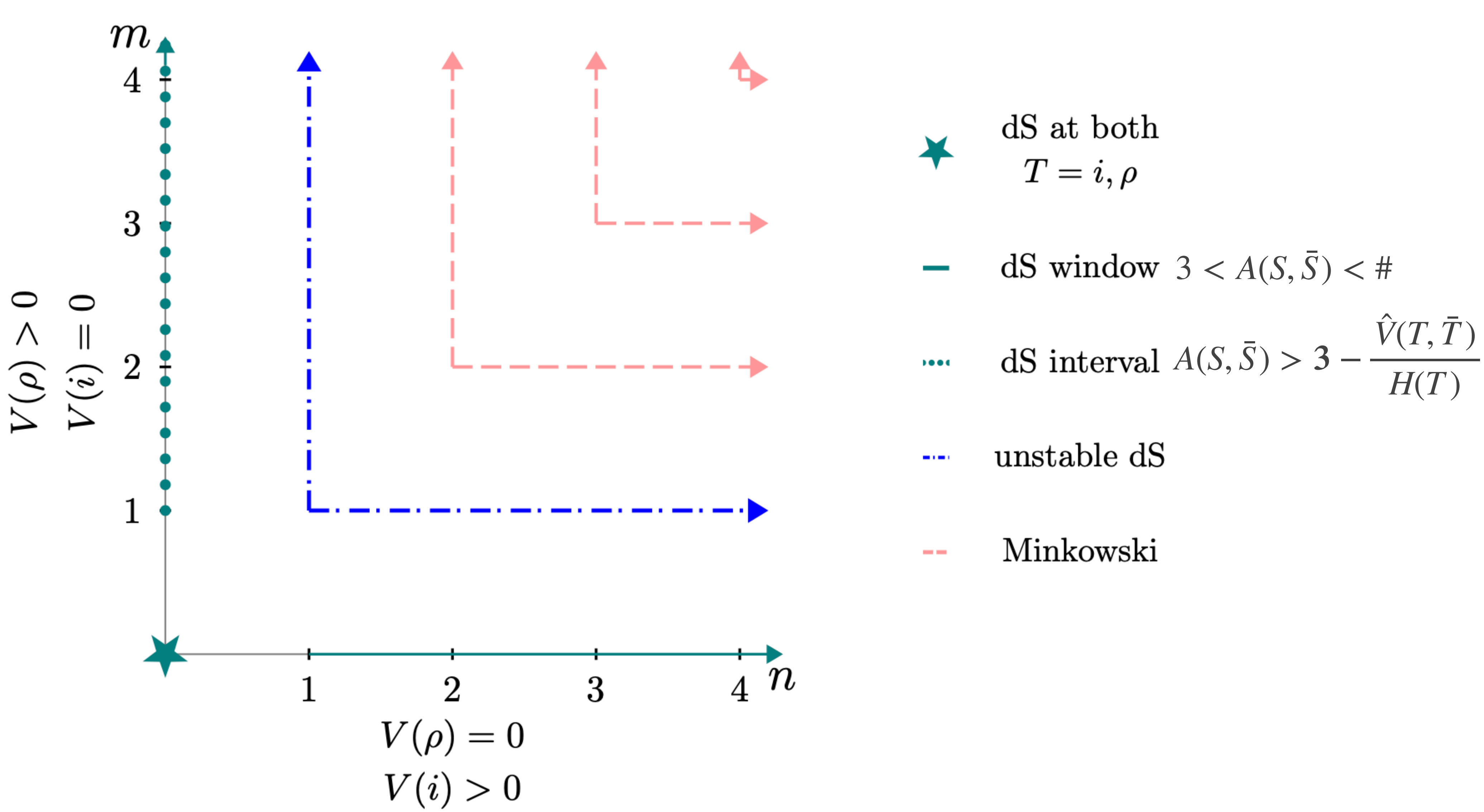

$A(S,\bar{S}) \sim F_S \bar{F}_{\bar{S}}$

Gaugino condensation and Vacua

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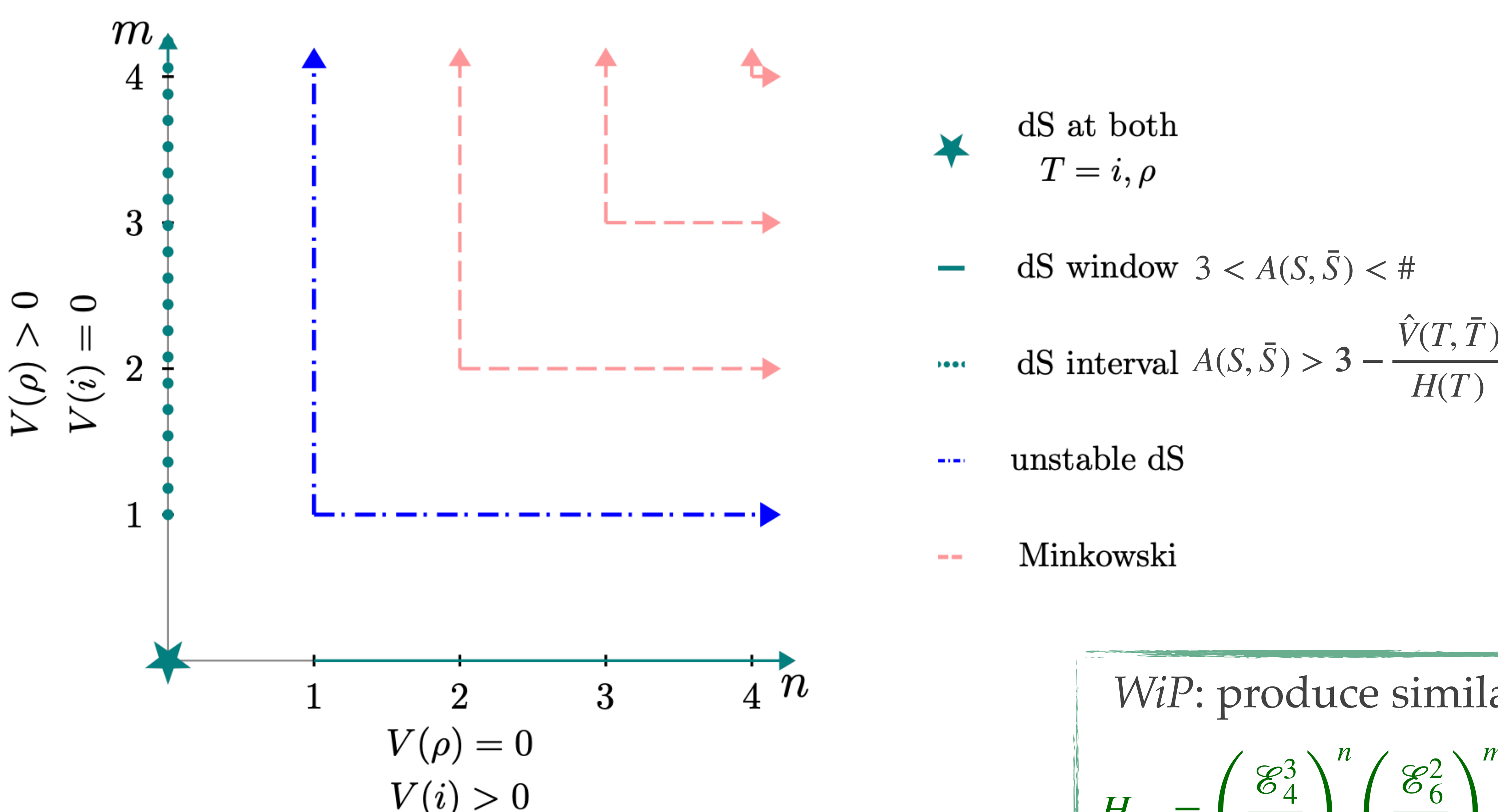

$A(S,\bar{S}) \sim F_S \bar{F}_{\bar{S}}$

WiP: produce similar results for

$$H_{(2)} = \left(\frac{\mathscr{E}_4^3}{\chi_{12}}\right)^{n}\left(\frac{\mathscr{E}_6^2}{\chi_{12}}\right)^{m}\left(\frac{\mathscr{E}_4^2\mathscr{E}_6\chi_{10}}{\chi_{12}^2}\right)^{\ell}\mathscr{P}(j_{(2)})$$

What have we found

- proven new no-go theorems for dS minima

- made considerable progress in the *STUV* setup

What have we found + What has to be done

- proven new no-go theorems for dS minima

- made considerable progress in the $STUV$ setup

  - extend the extrema analysis made for $SL(2,\mathbb{Z})$ to $Sp(4,\mathbb{Z})$

    $\longrightarrow$ new, bigger landscape of heterotic vacua

    $\longrightarrow$ extension of the no-go theorems: new dS vacua

  - relate our findings with the Swampland program

  - understand H and $H_{(2)} \leftrightarrow$ orbifold geometry

What have we found + What has to be done

- proven new no-go theorems for dS minima
- made considerable progress in the *STUV* setup
    - extend the extrema analysis made for $SL(2,\mathbb{Z})$ to $Sp(4,\mathbb{Z})$
        - → new, bigger landscape of heterotic vacua
        - → extension of the no-go theorems: new dS vacua
    - relate our findings with the Swampland program
    - understand H and $H_{(2)} \leftrightarrow$ orbifold geometry

Thank you