# Flux vacua of the mirror octic

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Geometry, Strings, and the Swampland

Ringberg Castle — 19.03.2024

This talk is based on ::

*Flux vacua of the mirror octic*

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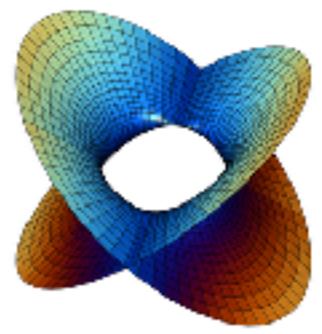
arXiv:2310.06040

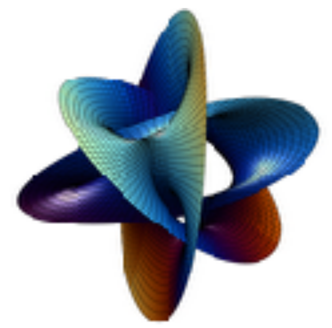
JHEP 01 (2024) 157

Some motivation.

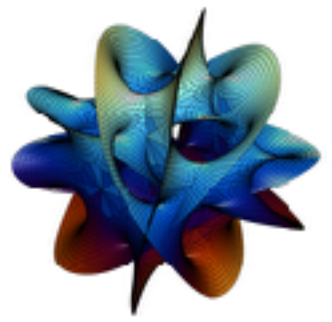
# motivation — compactification



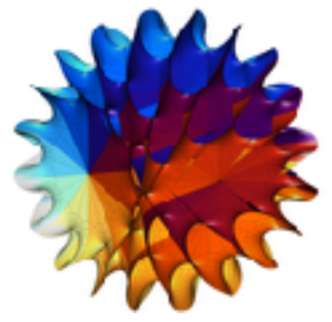
Compactifications of string theory on Calabi-Yau three-folds give rise to four-dimensional effective theories.



These theories often contain massless scalar fields (moduli) that parametrize the compact space.



Deforming the compact space by fluxes can make moduli massive.



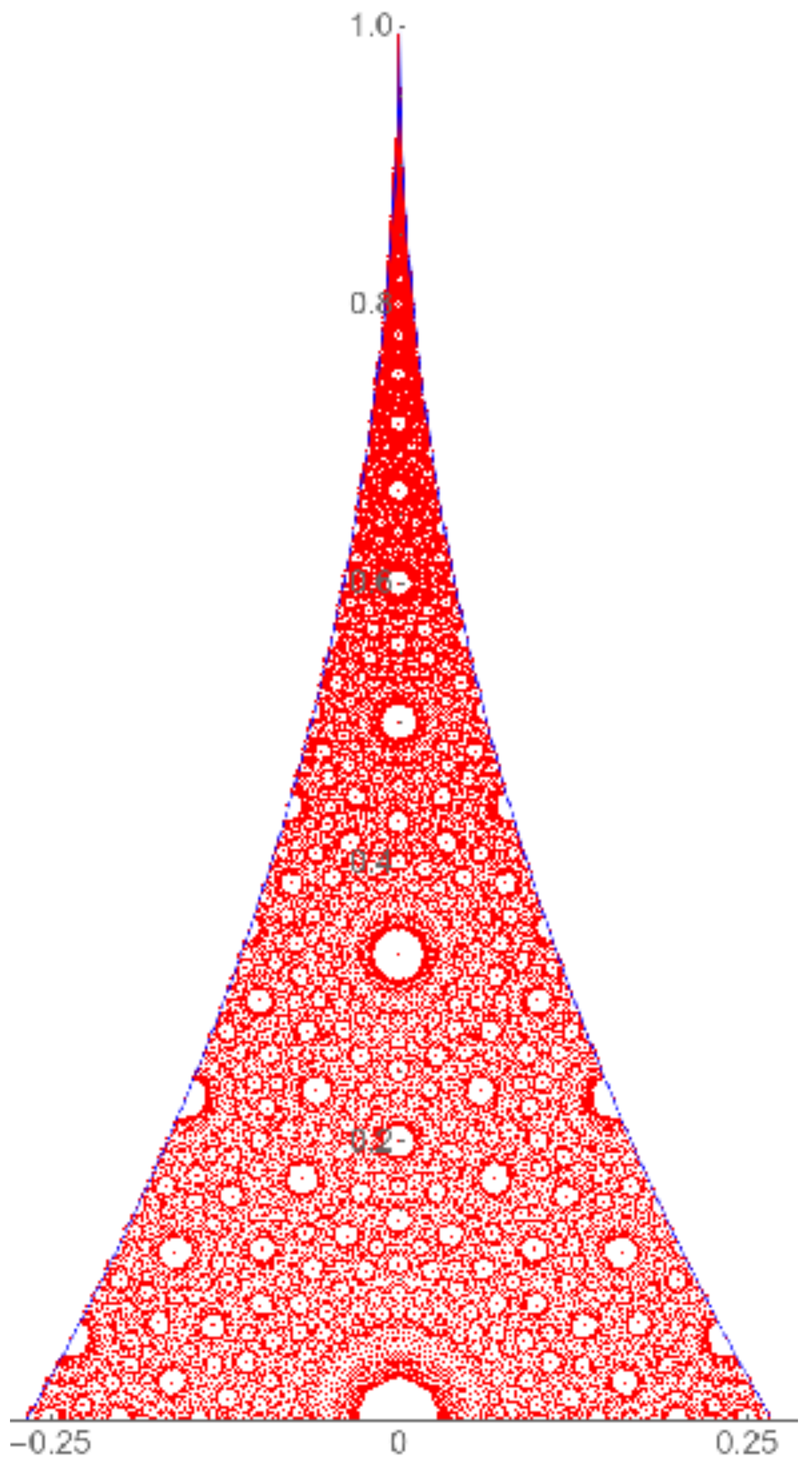
# motivation — type IIB flux vacua

In type IIB orientifold compactifications on Calabi-Yau three-folds $\mathcal{X}$ , fluxes generate mass-terms for the axio-dilaton and complex-structure moduli.

Dasgupta, Rajesh, Sethi — 1999
Giddings, Kachru, Polchinski — 2001

The fluxes are constrained by the tadpole cancellation condition

$$0 < N_{\text{flux}} \leq N_{\text{max}} \,, \qquad\qquad N_{\text{flux}} = \int_{\mathcal{X}} F \wedge H \,.$$



**Questions ::**

- Is the number of flux vacua for a given $N_{\max}$ finite?

  Grimm — 2020
  Bakker, Grimm, Schnell, Tsimerman — 2021

- How many flux vacua exist for a given $N_{\max}$ ?

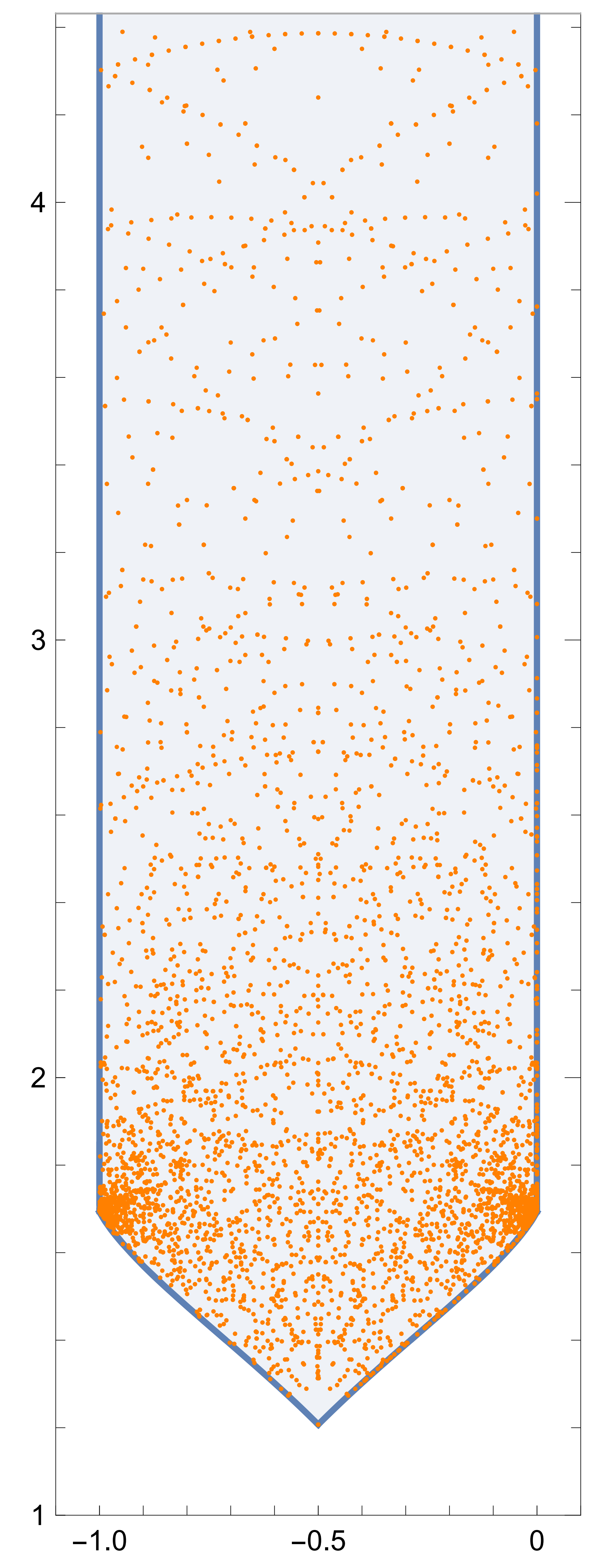
  Ashok, Douglas — 2003
  Denef, Douglas — 2004

- What are general properties of such flux vacua?

  DeWolfe, Giryavets, Kachru, Taylor — 2004
  Conlon, Quevedo — 2004
  Cole, Shiu — 2018
  Dubey, Krippendorf, Schachner — 2023
  ...

**This talk ::**

- Construct all flux vacua with $N_{\max} \leq 10$ for a simple example.



# outline

1. motivation
2. flux vacua
3. the mirror octic
4. results
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The setting.

Consider type IIB orientifold compactifications on Calabi-Yau three-folds $\mathcal{X}$ with O3-/O7-planes.

The four dimensional effective theory contains massless scalar fields (moduli) ::

- 1 axio-dilaton $\tau = c + i\,s\,,$
- $h^{2,1}_{-}$ complex-structure moduli $z^i\,,$
- …

Three-form fluxes $H$ and $F$ along the compact space generate a potential for the moduli ::

$$W = \int_{\mathcal{X}} \Omega \wedge G\,, \qquad \Omega \in H^{3,0}_{-}(\mathcal{X})\,,$$

$$G = F - \tau H\,.$$

Fluxes are constrained by the tadpole cancellation condition

$$0 = N_{\text{flux}} + 2\,N_{\text{D3}} + Q_{\text{D3}}\,.$$

- flux number $N_{\text{flux}} = \int_{\mathcal{X}} F \wedge H$
- number of D3-branes
- contribution of O3-/O7-planes & D7-branes — typically negative

Minima of the scalar potential are determined by vanishing F-terms

$$\begin{aligned} F_\tau &= 0 \\ F_{z^i} &= 0 \end{aligned} \qquad \Longleftrightarrow \qquad G = -i \star G\,.$$

Giddings, Kachru, Polchinski — 2001

Some relations.

For three-forms on $\mathcal{X}$ one can introduce an integral symplectic basis as (with $I = 0, \ldots, h^{2,1}_-$)

$$\{\alpha_I, \beta^I\} \in H^3_-(\mathcal{X}, \mathbb{Z}) \,, \qquad \eta = \int_{\mathcal{X}} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \wedge \, (\alpha, \beta) = \begin{pmatrix} 0 & +\mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix},$$

$$\mathcal{M} = \int_{\mathcal{X}} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \wedge \star (\alpha, \beta) \; .$$

The matrix $\mathcal{M}$ is symplectic, symmetric, and positive-definite. Its eigenvalues come in pairs

$$(\lambda_I, \lambda_I^{-1}) \,, \qquad \lambda_I \geq 1 \,.$$

The fluxes can be expanded in the symplectic basis and combined into vectors as

$$H = h^I \alpha_I + h_I \beta^I \qquad \mathsf{H} = \begin{pmatrix} h^I \\ h_I \end{pmatrix},$$

$$F = f^I \alpha_I + f_I \beta^I \qquad \mathsf{F} = \begin{pmatrix} f^I \\ f_I \end{pmatrix}.$$

The minimum condition can be expressed in the following way

$$G = -i \star G \qquad \longrightarrow \qquad \eta\,\big(\mathsf{F} - \mathsf{H}\,c\big) = -\mathcal{M}\,\mathsf{H}\,s\,.$$

Using this condition one obtains the following relations

$$s = \frac{N_{\text{flux}}}{\mathsf{H}^T\mathcal{M}\,\mathsf{H}}\,, \qquad c = \frac{\mathsf{H}^T\mathcal{M}\,\mathsf{F}}{\mathsf{H}^T\mathcal{M}\,\mathsf{H}}\,, \qquad \mathsf{F}^T\mathcal{M}\,\mathsf{F} = (s^2 + c^2)\big(\mathsf{H}^T\mathcal{M}\,\mathsf{H}\big)\,.$$

The usual bounds on matrix norms imply ($\|\cdot\|^2$ denotes the Euclidean norm)

$$\frac{1}{\lambda_{\text{max}}}\|\mathsf{H}\|^2 \leq \mathsf{H}^T\mathcal{M}\,\mathsf{H} \leq \lambda_{\text{max}}\|\mathsf{H}\|^2\,.$$

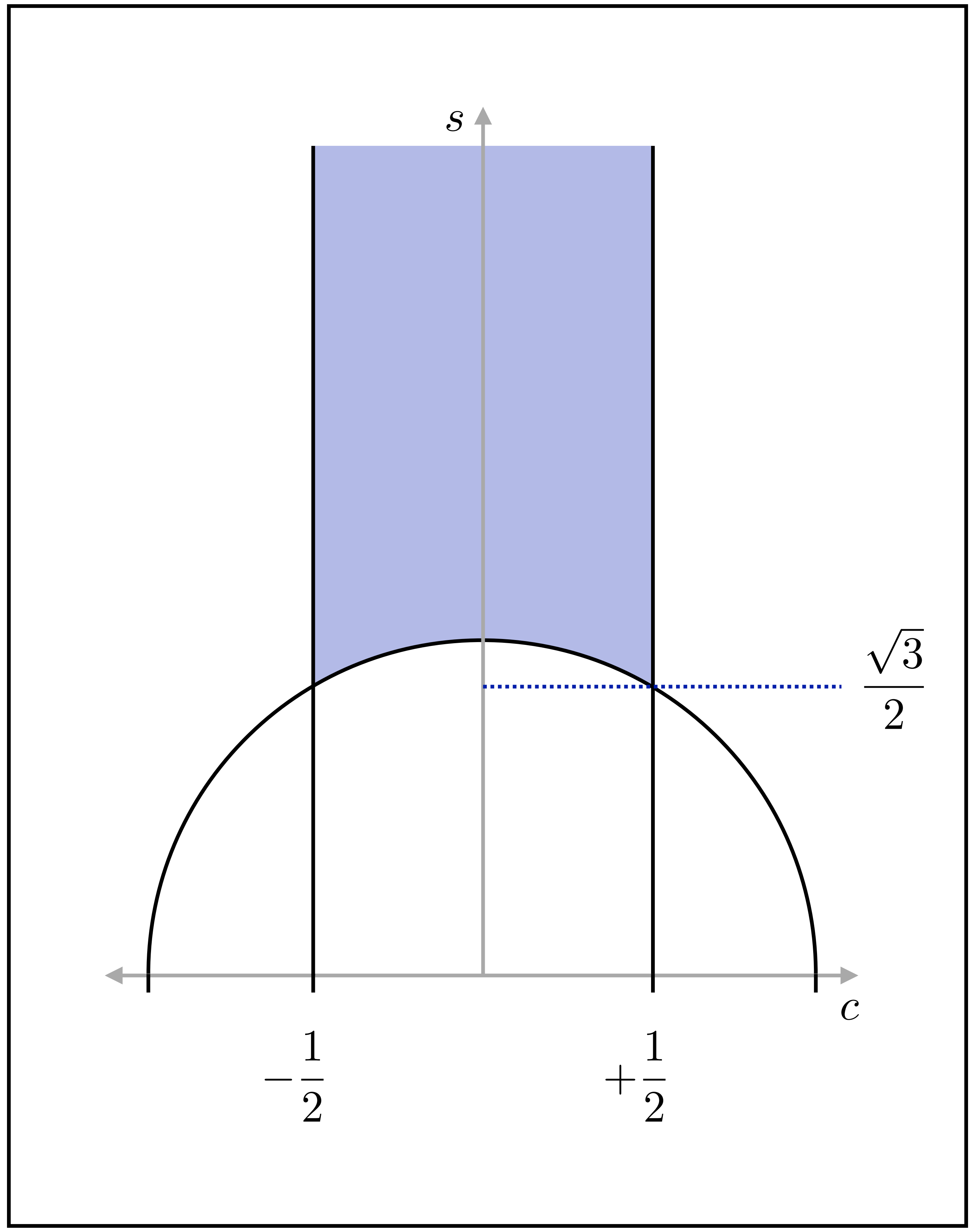
Finite fluxes in finite regions.

Using S-duality, the axio-dilaton can be mapped into the fundamental domain. This implies

$$\frac{\sqrt{3}}{2} \leq s$$

$$\frac{\sqrt{3}}{2} \leq \frac{N_{\text{flux}}}{\mathsf{H}^T \mathcal{M}\, \mathsf{H}}$$

$$\frac{\sqrt{3}}{2} \leq \frac{N_{\text{flux}}}{\mathsf{H}^T \mathcal{M}\, \mathsf{H}} \leq N_{\text{flux}} \frac{\lambda_{\max}}{\|\mathsf{H}\|^2} \longrightarrow \boxed{\|\mathsf{H}\|^2 \leq \frac{2 N_{\text{flux}} \lambda_{\max}}{\sqrt{3}}\,.}$$



With the axio-dilaton in the fundamental domain one also obtains

$$(\mathsf{F}^T\mathcal{M}\,\mathsf{F}) = (c^2+s^2)(\mathsf{H}^T\mathcal{M}\,\mathsf{H})$$

$$(\mathsf{H}^T\mathcal{M}\,\mathsf{H})(\mathsf{F}^T\mathcal{M}\,\mathsf{F}) = (c^2+s^2)(\mathsf{H}^T\mathcal{M}\,\mathsf{H})^2$$

$$(\mathsf{H}^T\mathcal{M}\,\mathsf{H})(\mathsf{F}^T\mathcal{M}\,\mathsf{F}) = (c^2+s^2)\,\frac{N_{\text{flux}}^2}{s^2}$$

$$\frac{\|\mathsf{H}\|^2}{\lambda_{\max}}\,\frac{\|\mathsf{F}\|^2}{\lambda_{\max}} \leq (\mathsf{H}^T\mathcal{M}\,\mathsf{H})(\mathsf{F}^T\mathcal{M}\,\mathsf{F}) = (c^2+s^2)\,\frac{N_{\text{flux}}^2}{s^2} \leq \frac{4}{3}\,N_{\text{flux}}^2$$

$$\boxed{\|\mathsf{F}\|^2 \leq \frac{4N_{\text{flux}}^2\lambda_{\max}^2}{3\,\|\mathsf{H}\|^2}\,.}$$

Summary ::

- For a region of complex-structure moduli space in which the eigenvalues of the Hodge-star operator are bounded,
- and for a given flux number $N_{\text{flux}}$,
- only finitely-many flux choices can lead to a vacuum in that region.

Remark ::

- The bounds can be made slightly stronger.

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1. motivation
2. flux vacua
3. **the mirror octic**
4. results
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The periods.

The octic three-fold is a hyper-surface in weighted projective space $\mathbb{P}_{11114}$ defined by

$$\sum_{i=1}^{4} x_i^8 + 4x_0^2 - 8\psi\, x_0 x_1 x_2 x_3 x_4 = 0\,.$$

The mirror octic has Hodge numbers $(h^{1,1}, h^{2,1}) = (149, 1)\,.$

For the mirror octic an orientifold projection can be chosen such that

$$\begin{aligned} h^{2,1}_- &= 1\,, & h^{2,1}_+ &= 0\,, & N_{\mathrm{O7}} &= 0\,, \\ h^{1,1}_- &= 72\,, & h^{1,1}_+ &= 77\,, & N_{\mathrm{O3}} &= 16\,, \end{aligned} \qquad Q_{\mathrm{D3}} = -8\,.$$

# the mirror octic — topology

The octic three-fold is a hyper-surface in weighted projective space $\mathbb{P}_{11114}$ defined by

$$\sum_{i=1}^{4} x_i^8 + 4x_0^2 - 8\psi\, x_0 x_1 x_2 x_3 x_4 = 0\,.$$

The mirror octic has Hodge numbers $(h^{1,1}, h^{2,1}) = (149, 1)$.

For the mirror octic an orientifold projection can be chosen such that

$$h_-^{2,1} = 1\,, \quad h_+^{2,1} = 0\,, \quad N_{O7} = 0\,,$$
$$h_-^{1,1} = 72\,, \quad h_+^{1,1} = 77\,, \quad N_{O3} = 16\,,$$

$$Q_{D3} = -8\,.$$

# the mirror octic — periods

The periods that determine the holomorphic three-form are solutions to the Picard-Fuchs equation

$$\left[\theta^4 - z(\theta + \tfrac{1}{8})(\theta + \tfrac{3}{8})(\theta + \tfrac{5}{8})(\theta + \tfrac{7}{8})\right]\Pi = 0\,.$$

complex-structure modulus

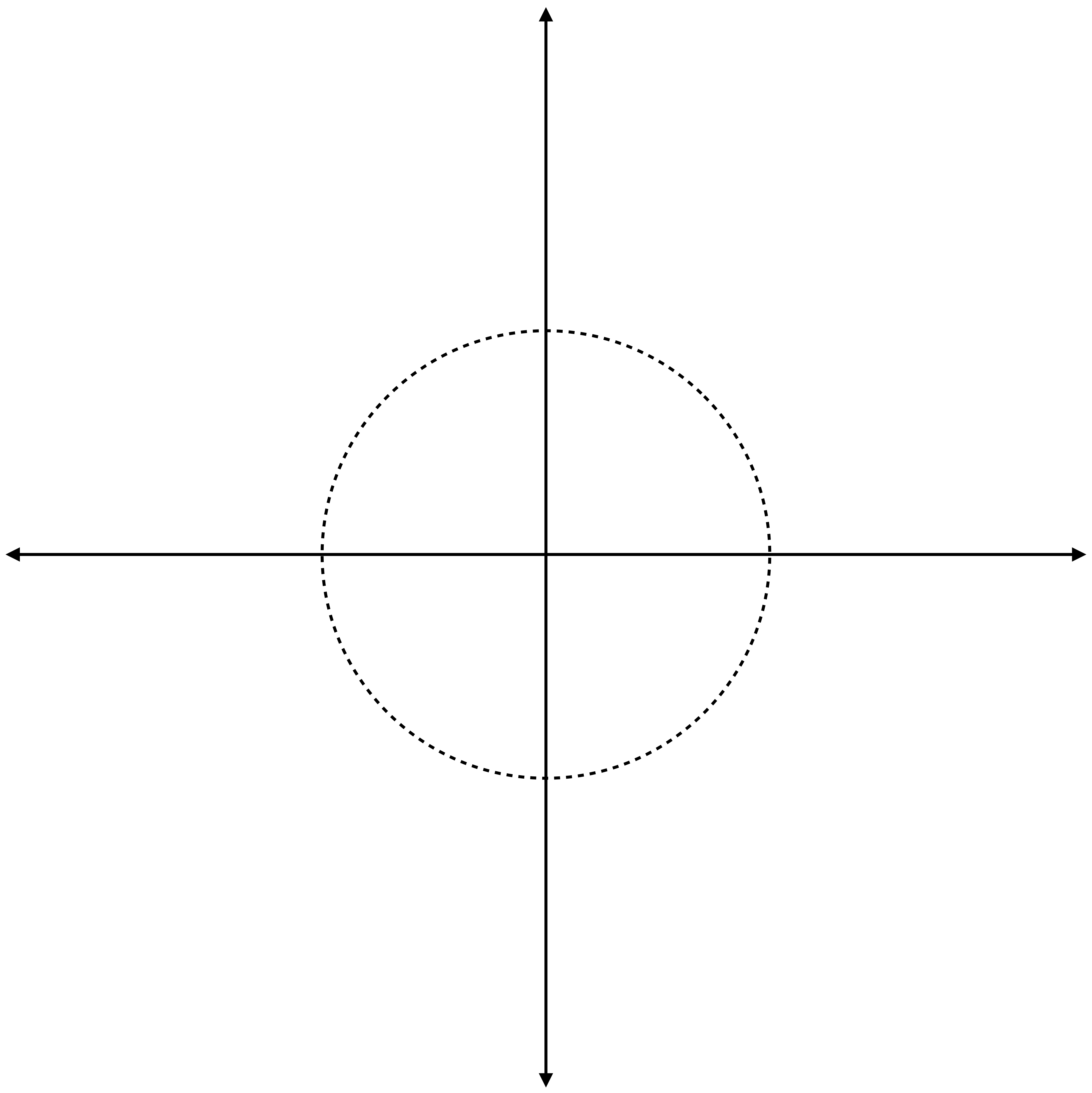
$$\Pi = \begin{pmatrix} \Pi^J \\ \Pi_J \end{pmatrix} \longleftrightarrow \Omega = \Pi^J \alpha_J + \Pi_J \beta^J$$

$$\theta = z\frac{\partial}{\partial z}$$

The solutions are typically expanded in power series that converge for $|z| < 1$.

Morrison — 1991
Font — 1992
Klemm, Theisen — 1992
...
Bastian, van de Heisteeg, Schlechter — 2023

The complex-structure moduli space can be covered by six discs of radius 0.9.



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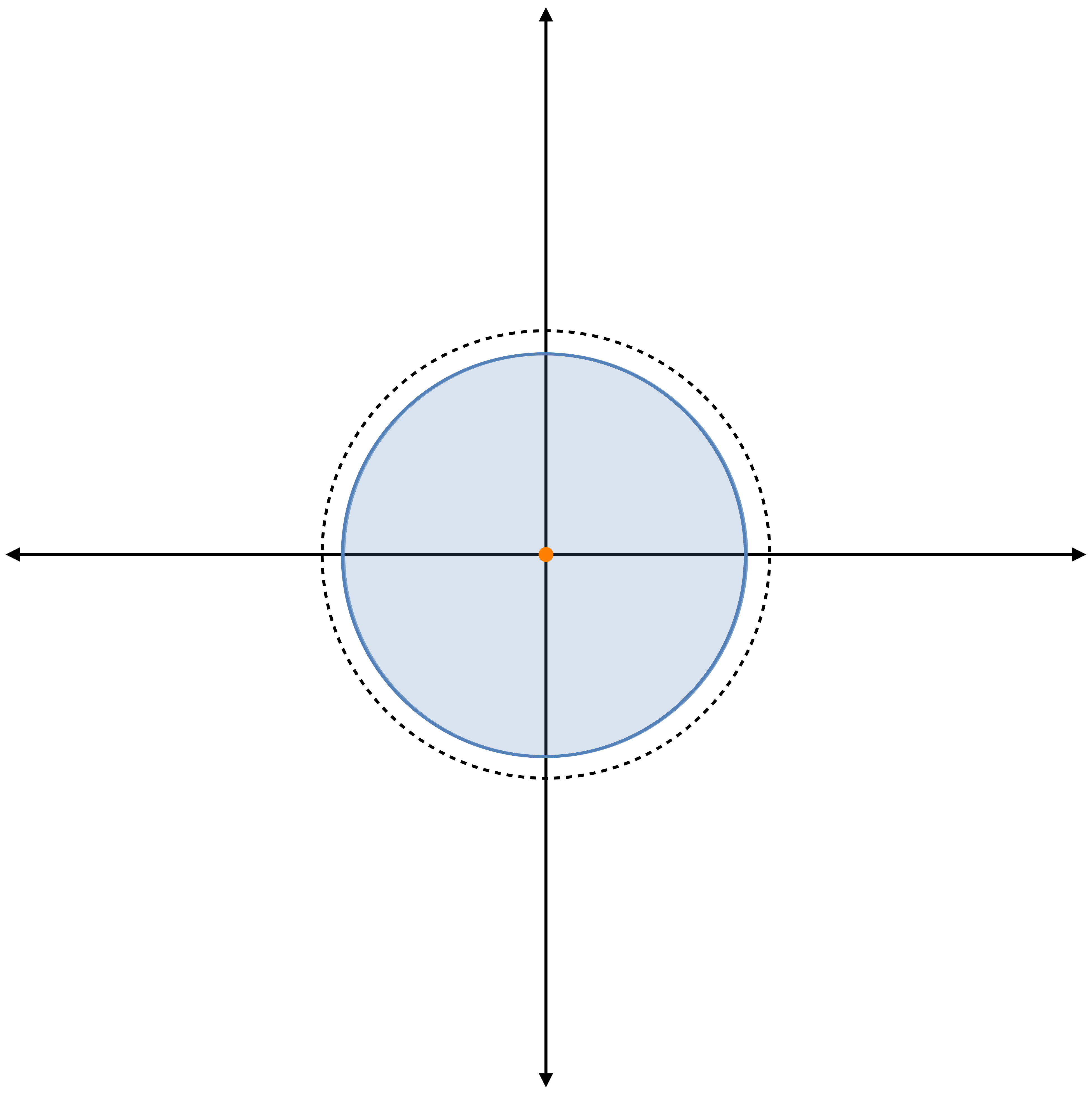

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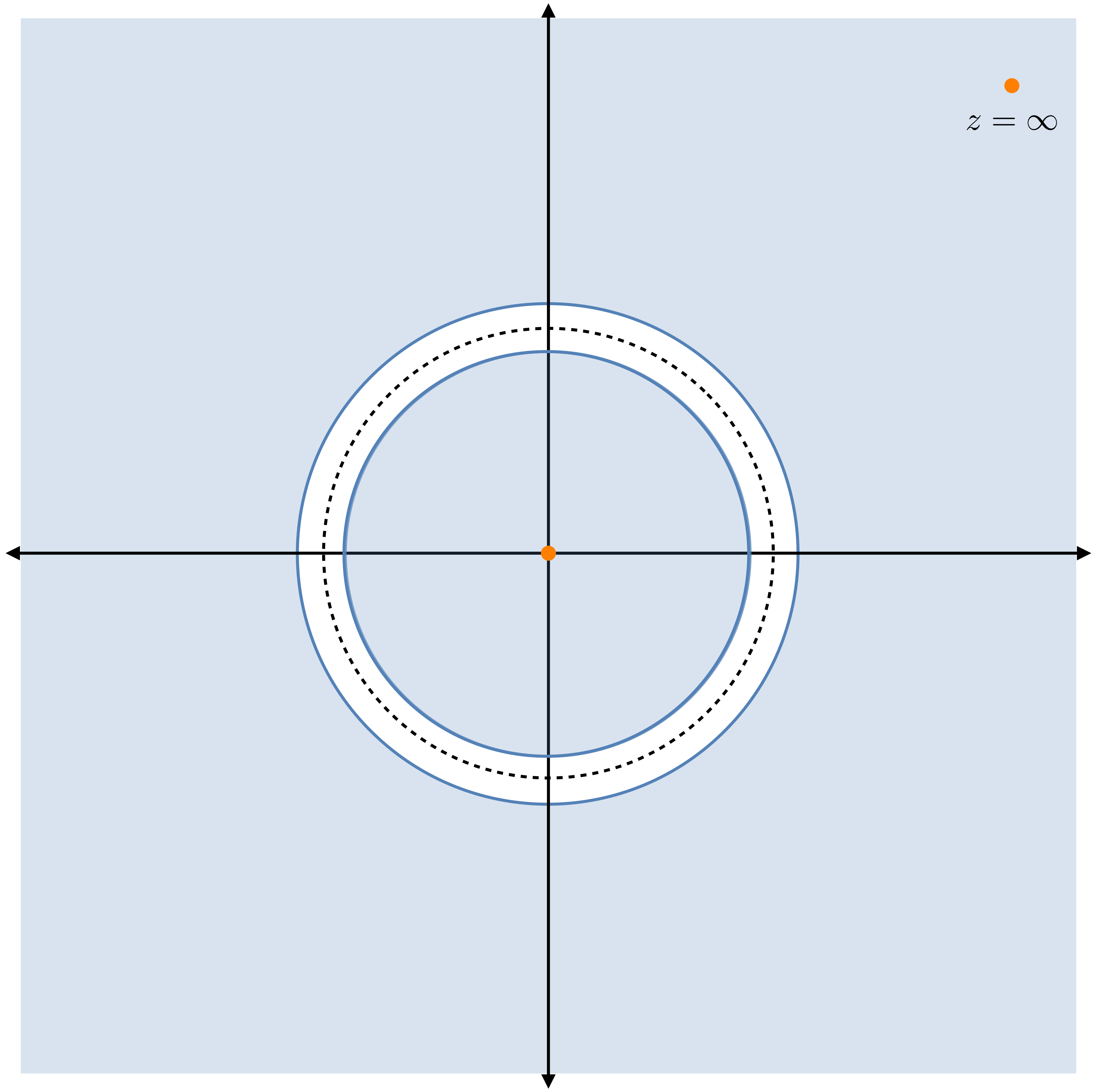

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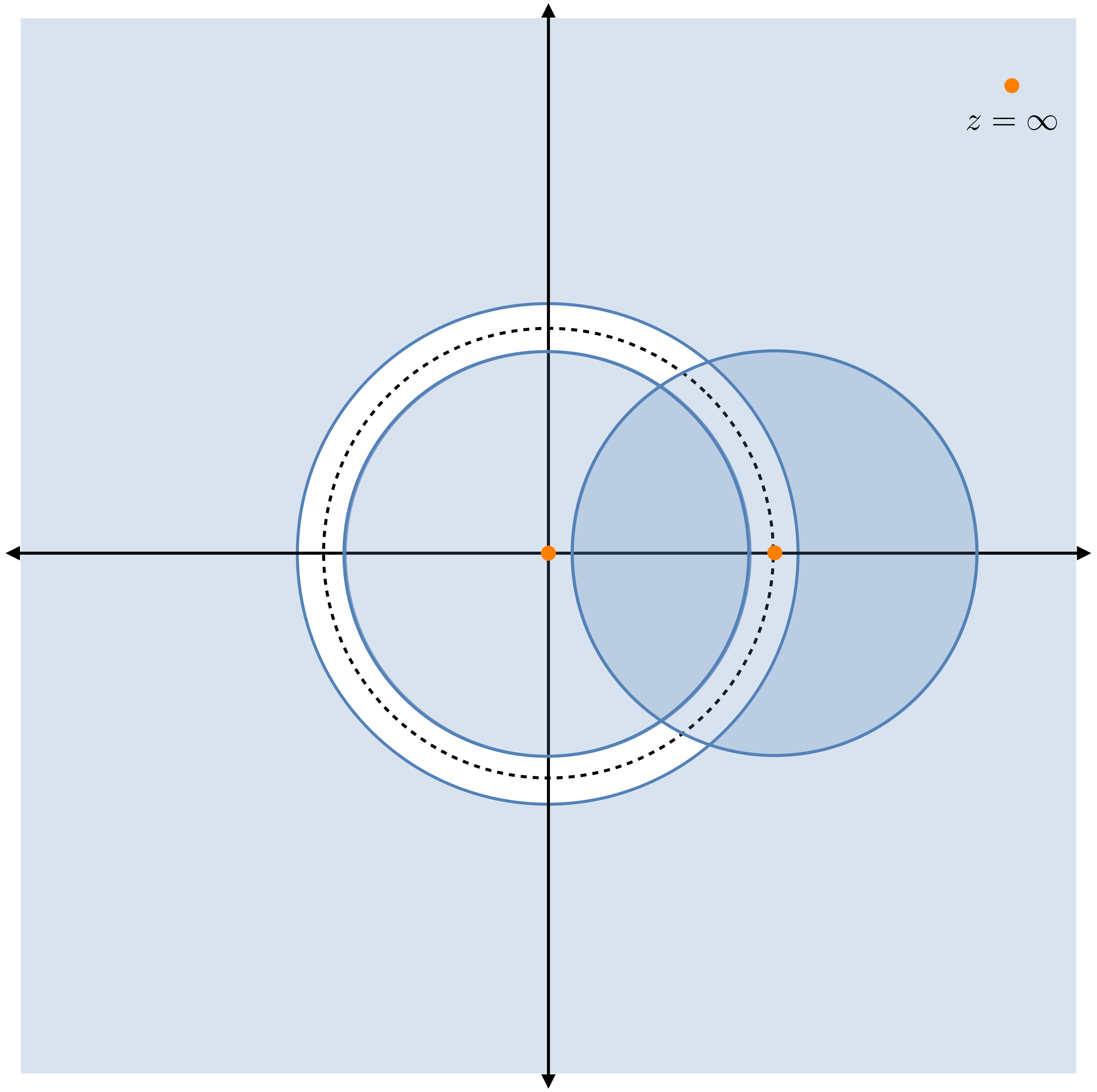

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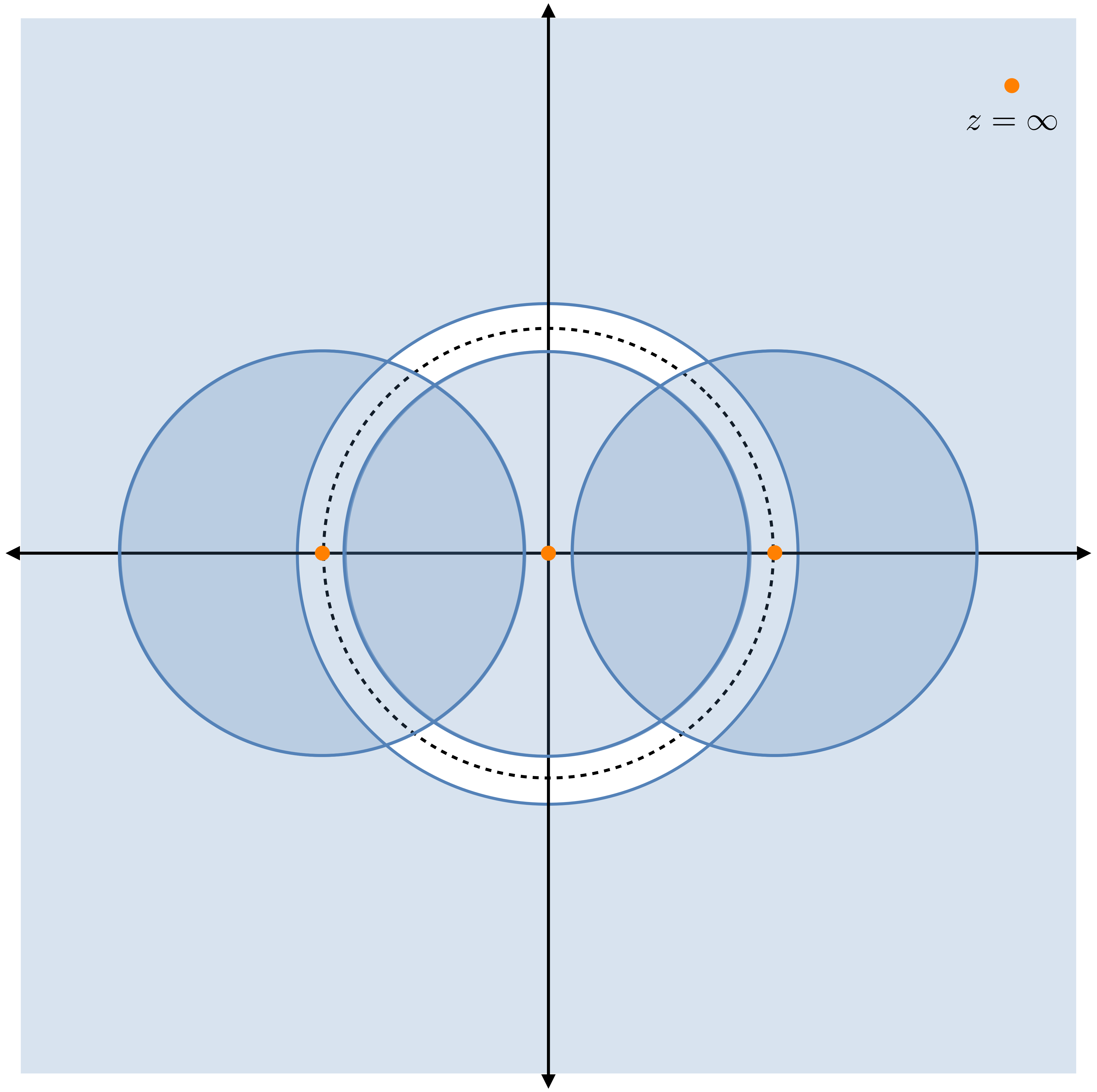
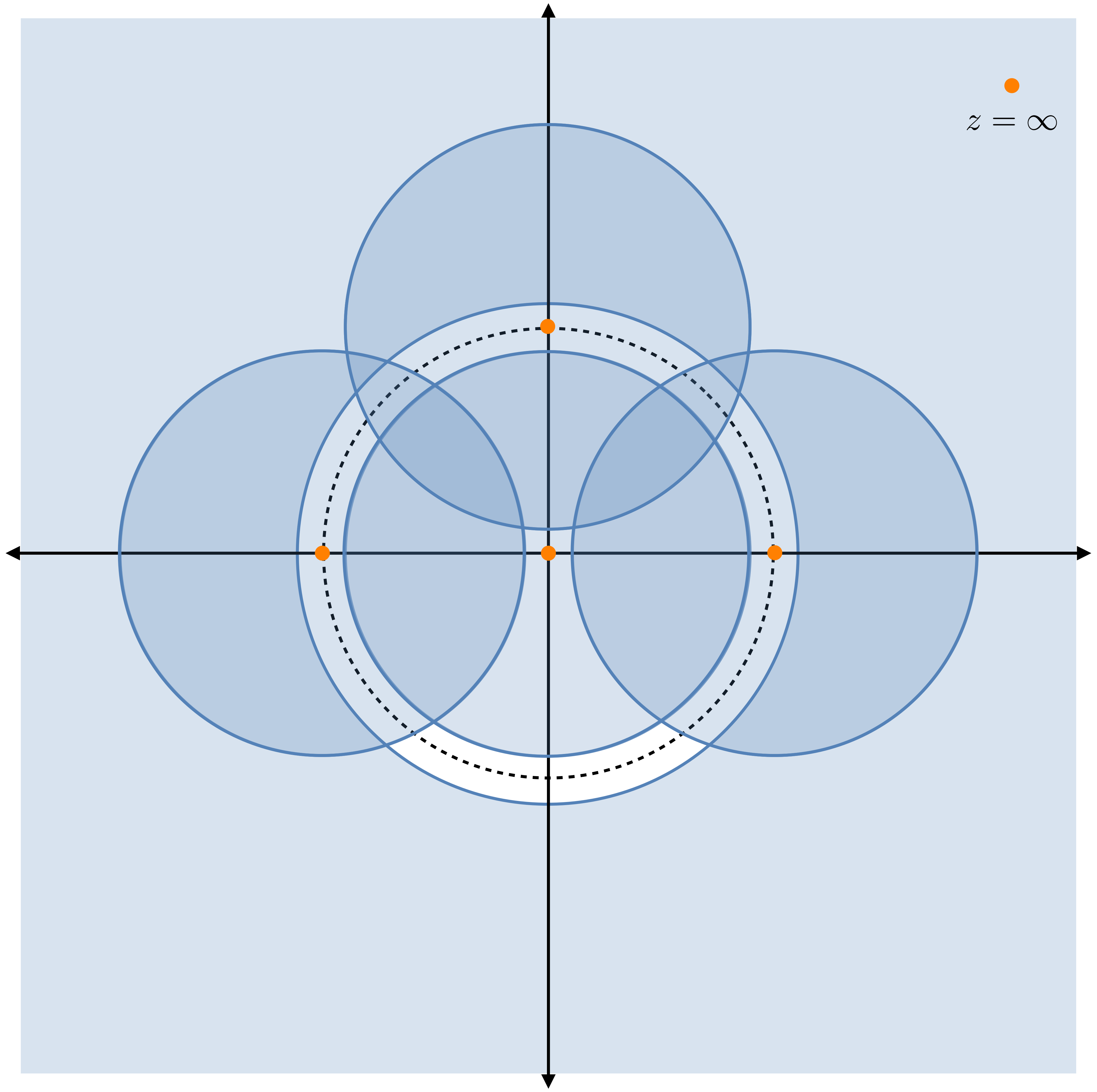
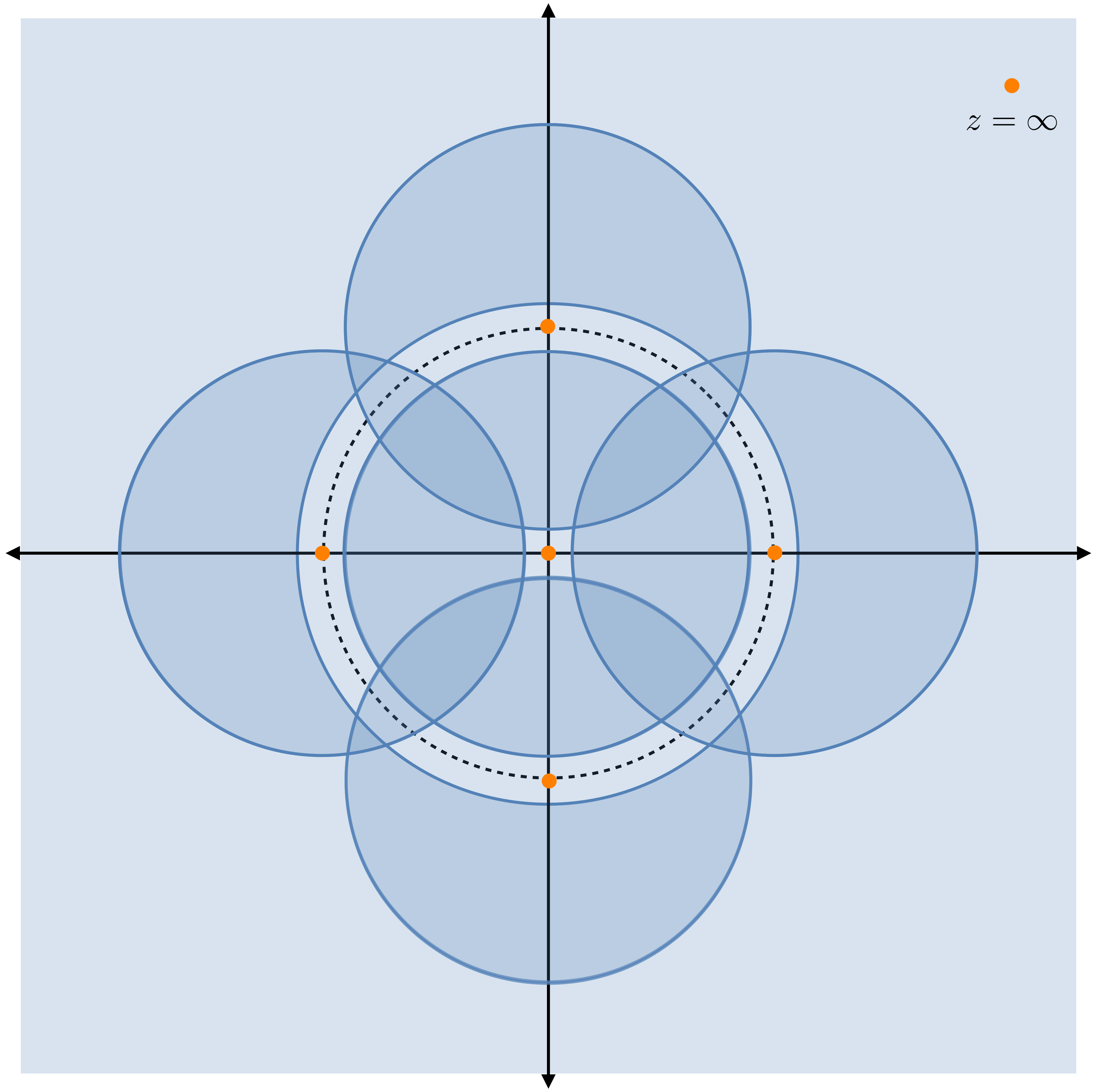


The complex-structure moduli space can be covered by **six discs** of radius 0.9.



# the mirror octic — moduli space I

The complex-structure moduli space can be covered by six discs of radius 0.9.



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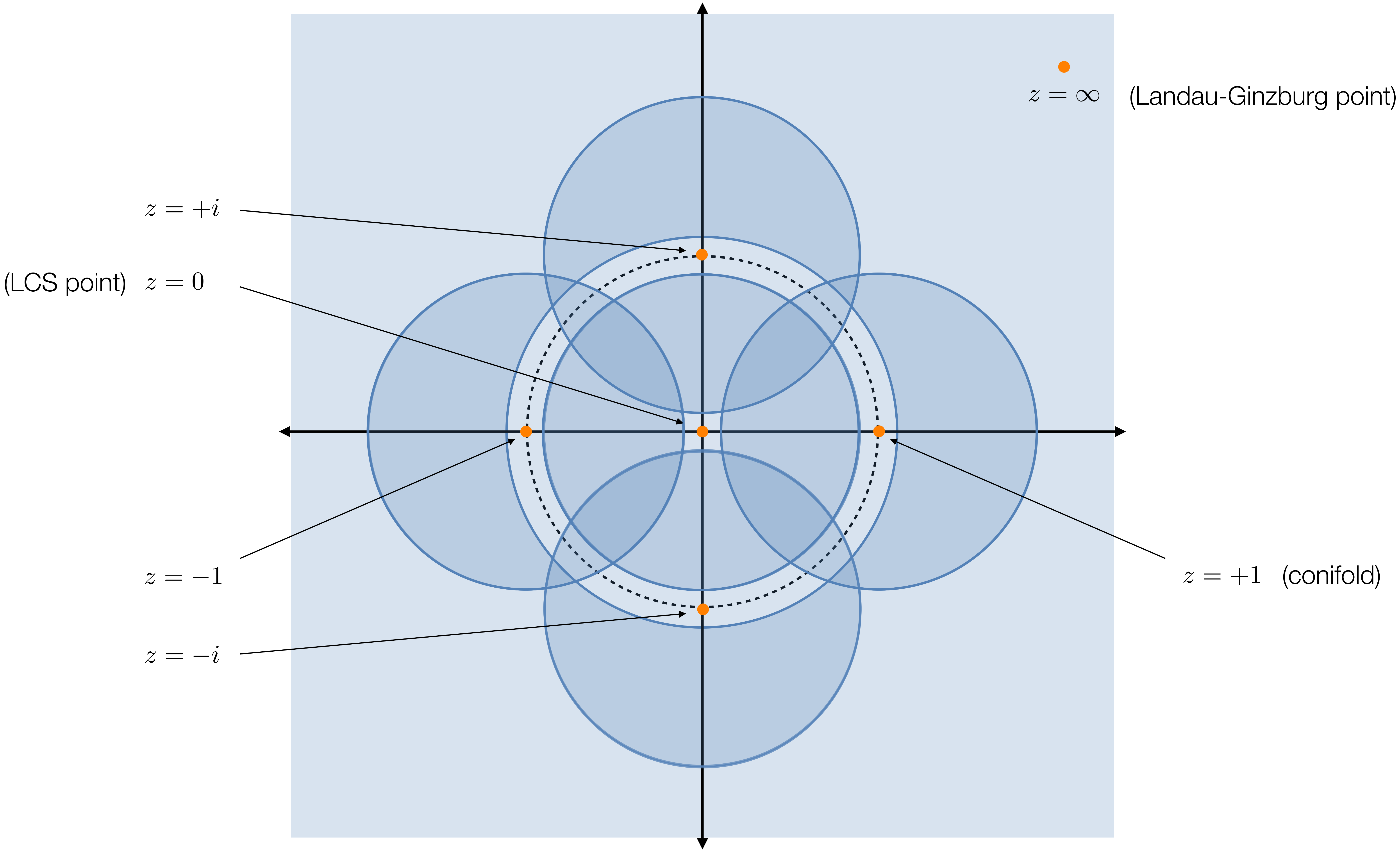
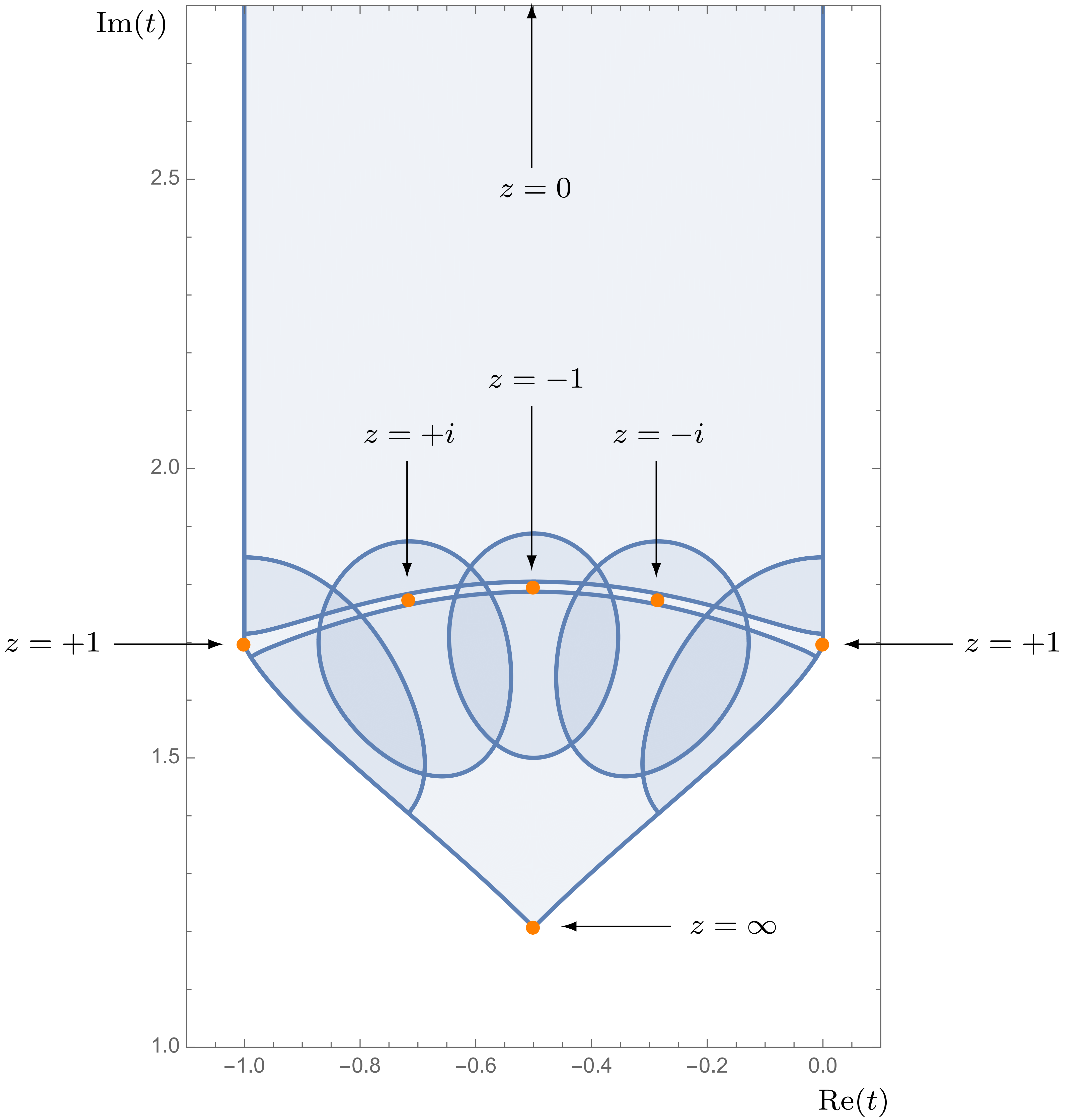


# the mirror octic — moduli space II

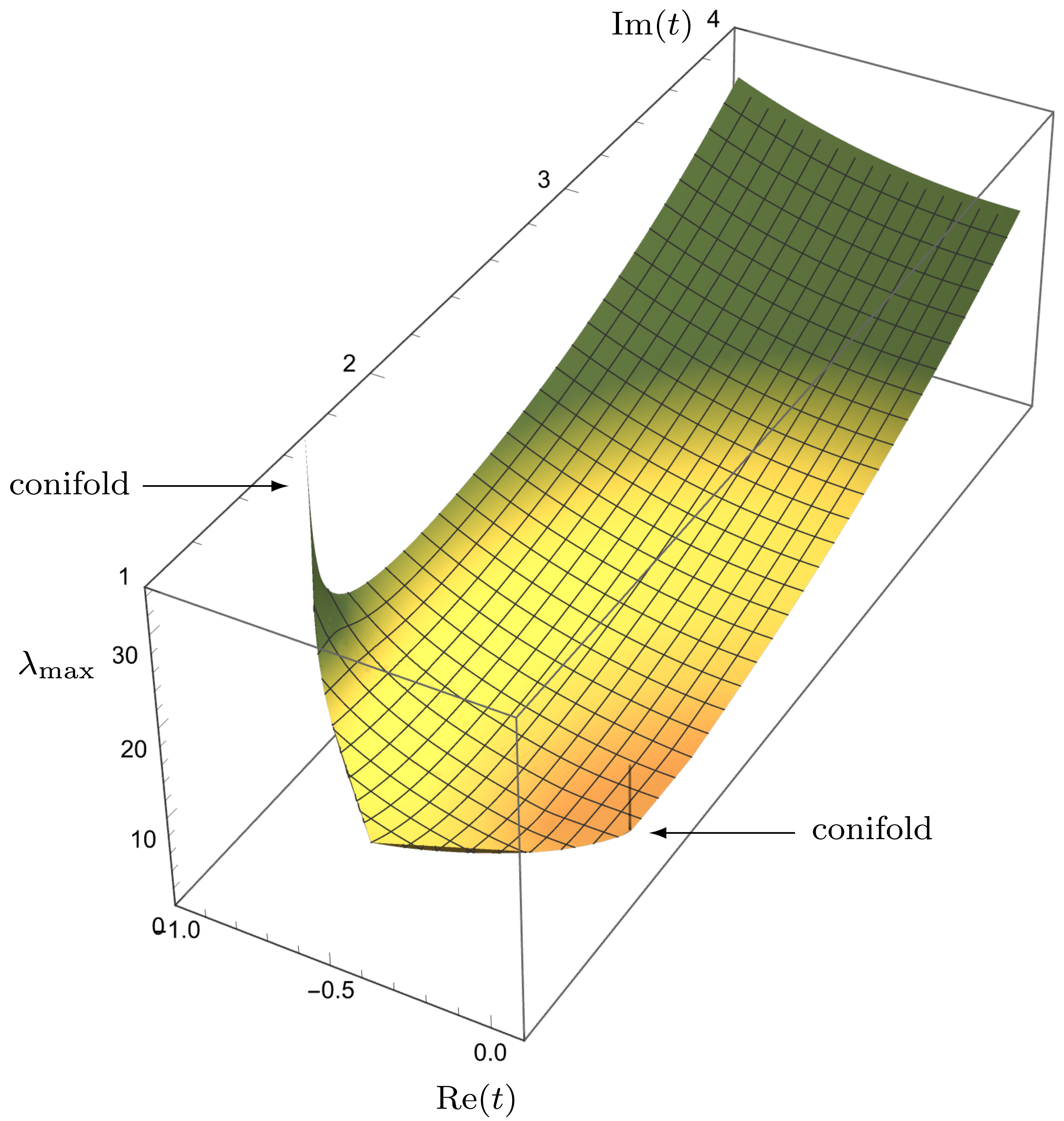
For convenience, the moduli space can be mapped to the Kähler side via the mirror map

$$t(z) = \frac{\Pi^2(z)}{\Pi^1(z)}\,.$$



Strategy flux vacua.

The largest eigenvalue $\lambda_{\max}$ of the Hodge-star operator diverges at the conifold and LCS point.



For each **bulk region** ( $\lambda_{\max}$ is finite ) ::

- Determine periods $\Pi$ up to $\mathcal{O}(100)$.
- Determine relevant flux choices for $N_{\max}$ .
- Try to solve minimum condition.

| point | region | $\lambda_{\max}$ |
|---|---|---|
| $z_{\mathrm{ep}} = 0$ | $10^{-9} \leq \lvert z - z_{\mathrm{ep}} \rvert \leq 0.9$ | 37.98 |
| $z_{\mathrm{ep}} = +1$ | $10^{-9} \leq \lvert z - z_{\mathrm{ep}} \rvert \leq 0.9$ | 3.95 |
| $z_{\mathrm{ep}} = \infty$ | $\lvert z - z_{\mathrm{ep}} \rvert \leq 0.9$ | 8.33 |
| $z_{\mathrm{ep}} = -1$ | $\lvert z - z_{\mathrm{ep}} \rvert \leq 0.9$ | 5.00 |
| $z_{\mathrm{ep}} = +i$ | $\lvert z - z_{\mathrm{ep}} \rvert \leq 0.9$ | 11.24 |
| $z_{\mathrm{ep}} = -i$ | $\lvert z - z_{\mathrm{ep}} \rvert \leq 0.9$ | 3.35 |

For each **boundary region** ( $\lambda_{\max}$ diverges) ::

- Determine periods at leading order.
- Determine relevant flux choices for $N_{\max}$ (partially) analytically.
- Solve minimum condition.

| point | region |
|---|---|
| $z_{\mathrm{ep}} = 0$ | $\lvert z - z_{\mathrm{ep}} \rvert \leq 10^{-9}$ |
| $z_{\mathrm{ep}} = +1$ | $\lvert z - z_{\mathrm{ep}} \rvert \leq 10^{-9}$ |

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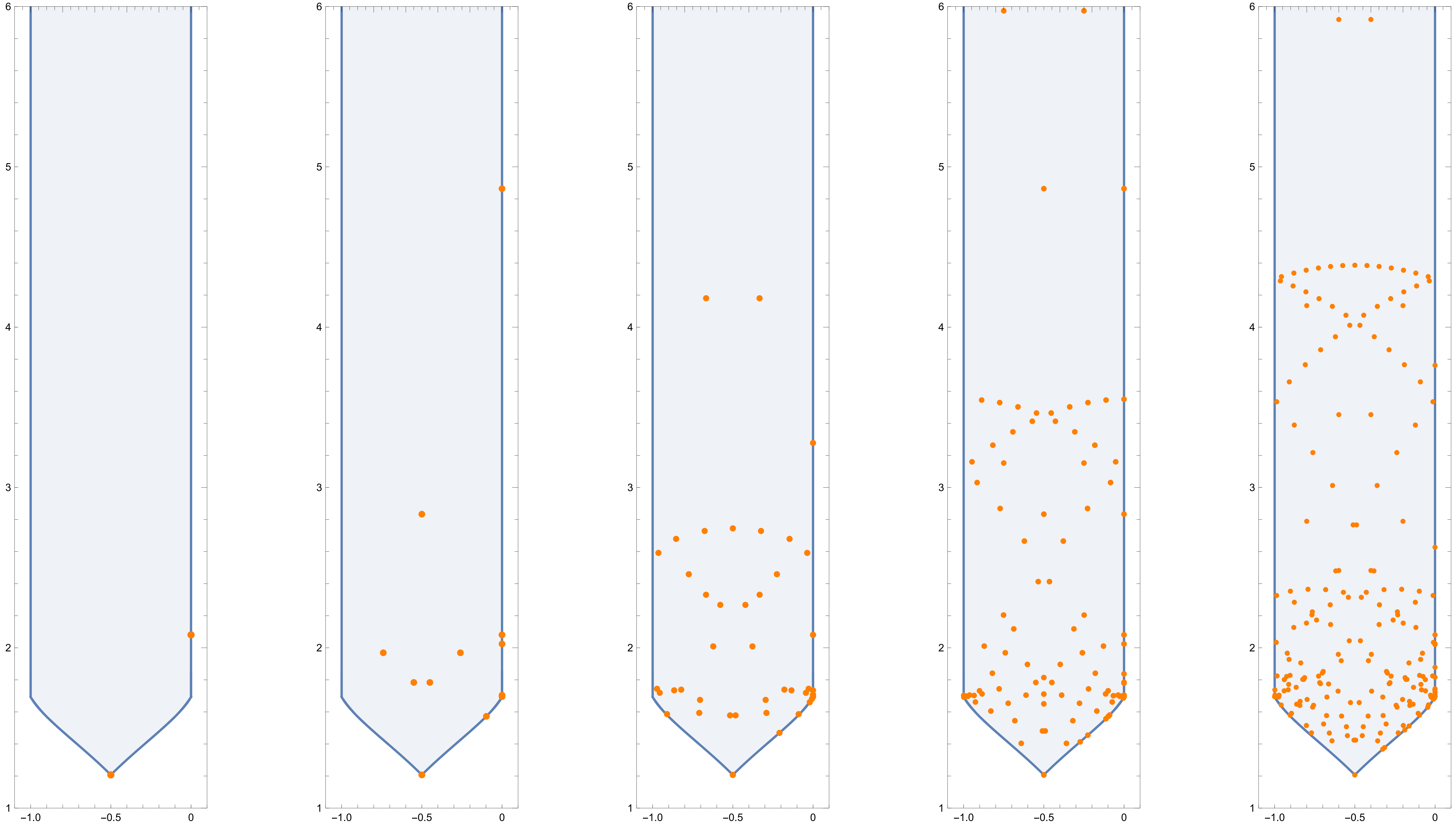
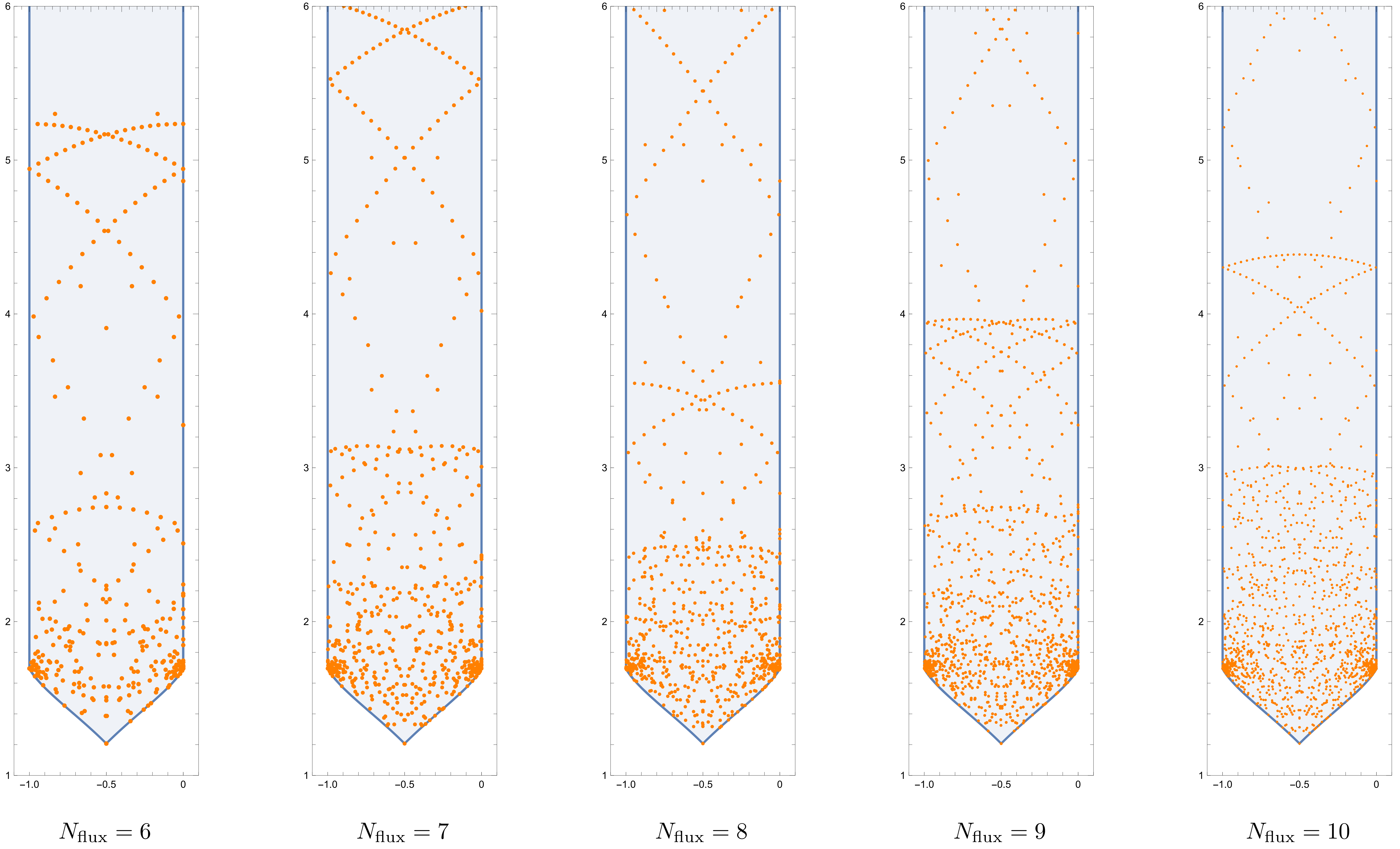
Distribution of vacua.

# results — distribution I

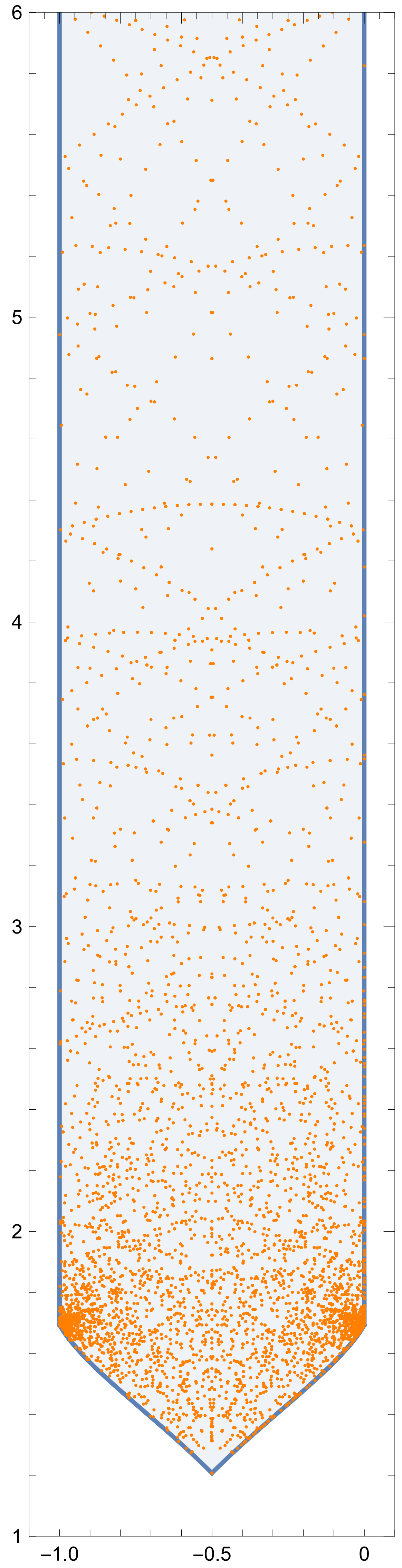



$N_{\text{flux}} = 1$ $N_{\text{flux}} = 2$ $N_{\text{flux}} = 3$ $N_{\text{flux}} = 4$ $N_{\text{flux}} = 5$

# results — distribution II



# results — distribution III



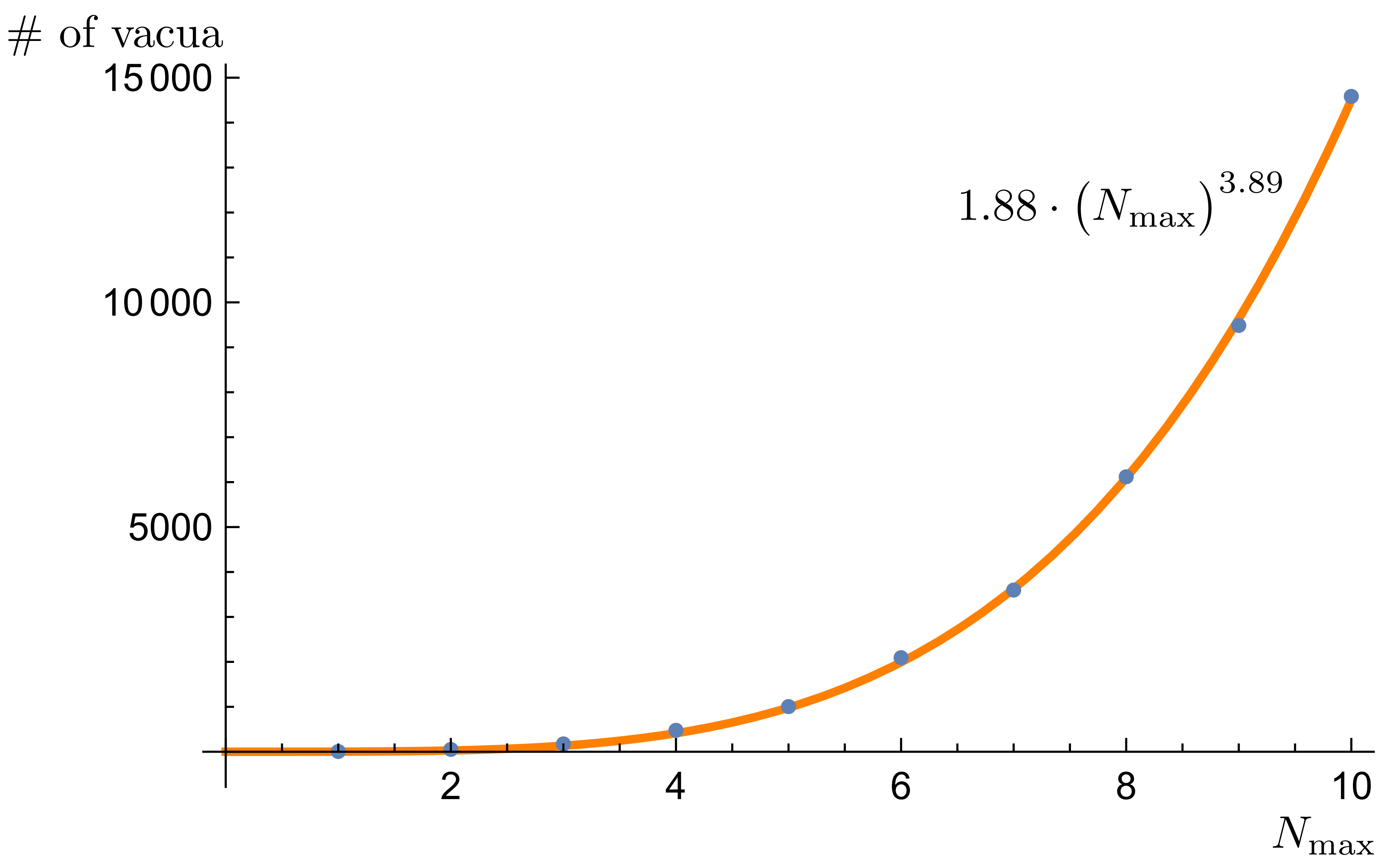

$N_{\text{flux}} \leq 10$

Number of vacua.

# results — number of vacua I

The cumulative number of vacua for a given $N_{\max}$ can be fitted as

$$\mathcal{N} \quad \left(N_{\text{flux}} \leq N_{\max}\right) = 1.88 \cdot \left(N_{\max}\right)^{3.89} .$$



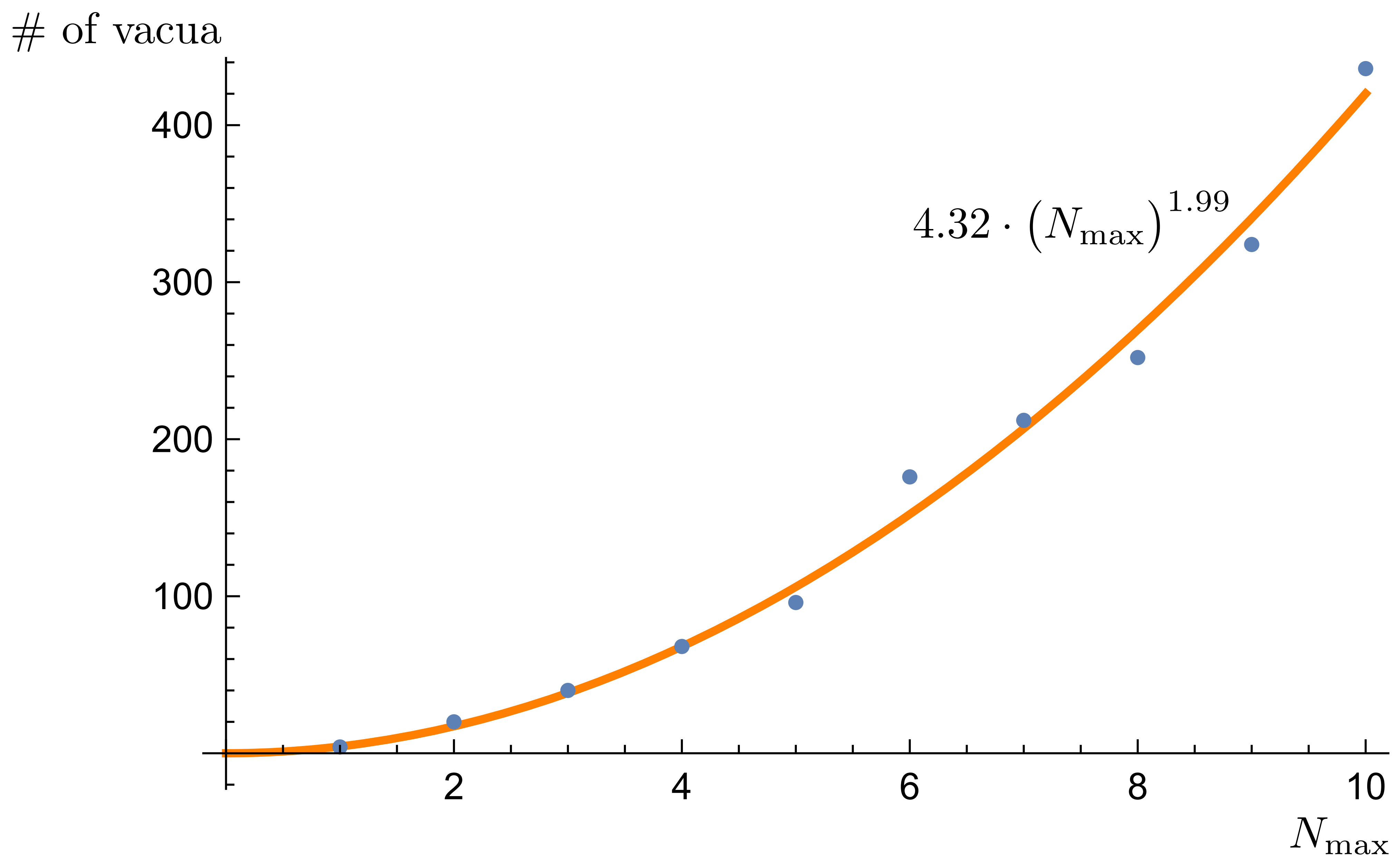

This result agrees approximately with the estimate of Denef/Douglas

$$\mathcal{N}_{\text{est}}\left(N_{\text{flux}} \leq N_{\max}\right) = 1.37 \cdot \left(N_{\max}\right)^{4} .$$

Denef, Douglas — 2004

The cumulative number of vacua with $W_0 = 0$ for a given $N_{\rm max}$ can be fitted as

$$\mathcal{N}\big(N_{\rm flux} \leq N_{\rm max}\big)_{W_0=0} = 4.32 \cdot \big(N_{\rm max}\big)^{1.99} .$$




All these vacua are located at the Landau-Ginzburg point.

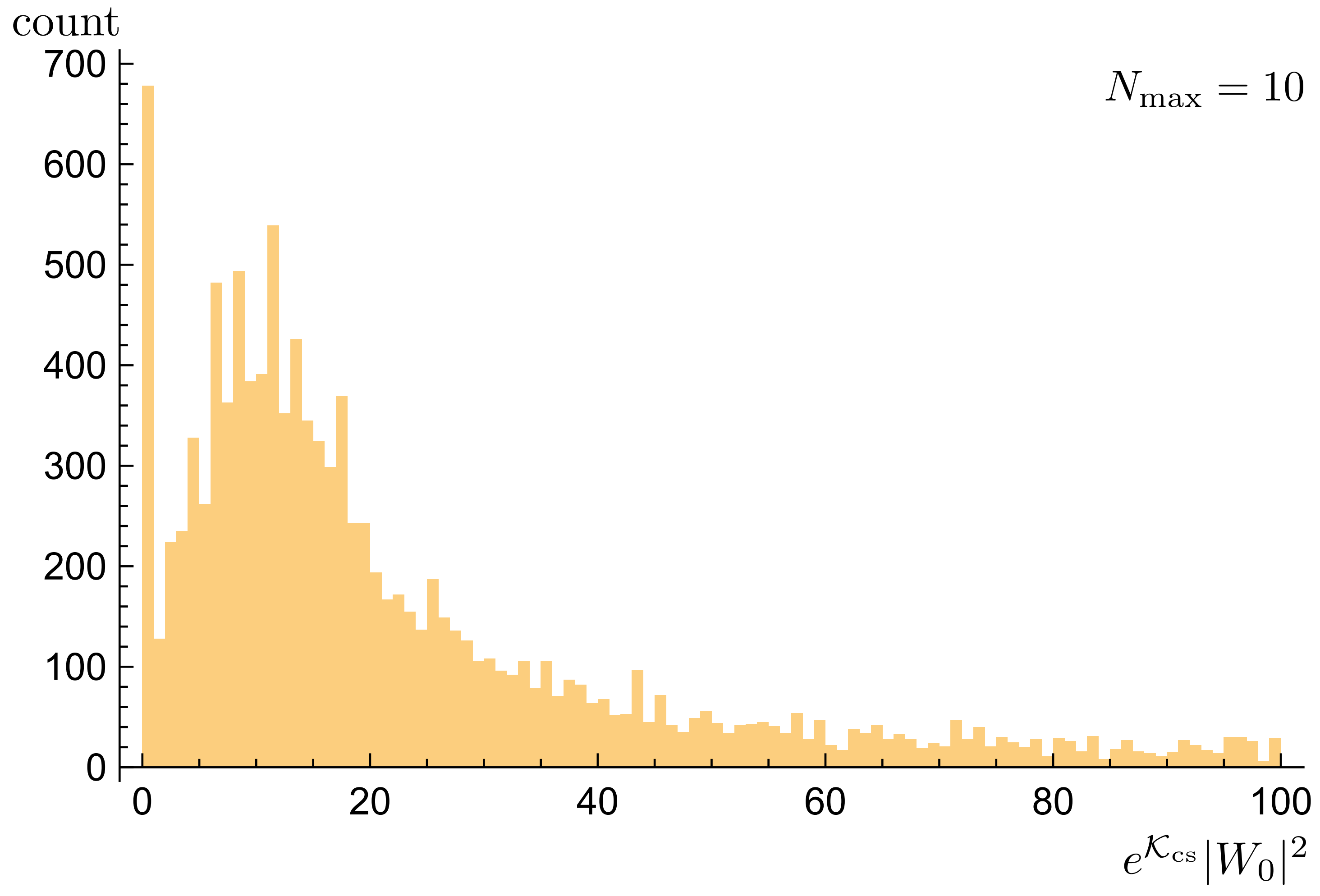
c.f. DeWolfe, Giryavets, Kachru, Taylor — 2004

Distributions.

# results — distributions

Relevant for moduli stabilization via KKLT and LVS is the quantity

$$e^{\mathcal{K}_{\rm cs}}|W_0|^2\,.$$

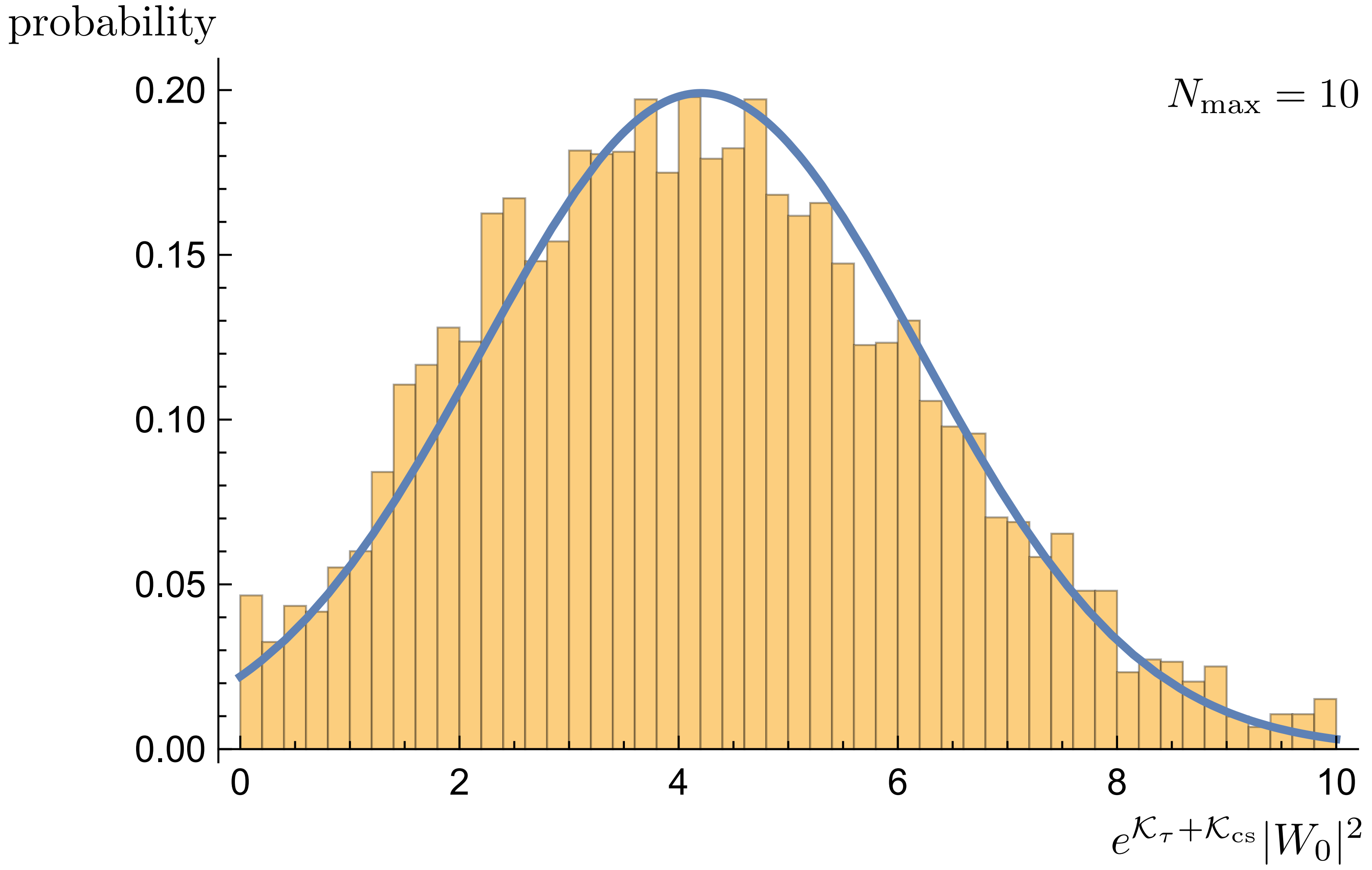



The gravitino-mass is determined by the expression

$$e^{\mathcal{K}_\tau+\mathcal{K}_{\rm cs}}|W_0|^2\,.$$

Excluding $W_0=0$, it follows a normal distribution with mean and standard deviation

$$(\mu,\sigma)=(0.427\,N_{\rm max}\,,\,0.204\,N_{\rm max})\,.$$




c.f. Ebelt, Krippendorf, Schachner — 2023

Sampling vs. complete scan.

In a sampling approach, one often choses flux quanta from a box of length $L_{\rm box}$ as

$$h^I,\, h_I,\, f^I,\, f_I \in [-L_{\rm box}, +L_{\rm box}]\,.$$

The ratio $r(L_{\rm box}, N_{\rm max})$ of vacua possibly-captured in this way can be defined as

$$\left.\frac{\#\text{ of vacua for box of size } L_{\rm box}}{\#\text{ of all flux vacua}}\right|_{N_{\rm flux}\leq N_{\rm max}}.$$

| $N_{\rm max}$ \ $L_{\rm box}$ | 1 | 2 | 3 | 4 | 5 | 10 | 15 | 25 | 35 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.667 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 0.173 | 0.808 | 0.962 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | 0.051 | 0.475 | 0.814 | 0.898 | 0.960 | 1 | 1 | 1 | 1 |
| 4 | 0.019 | 0.344 | 0.629 | 0.838 | 0.902 | 0.996 | 1 | 1 | 1 |
| 5 | 0.009 | 0.211 | 0.483 | 0.691 | 0.826 | 0.982 | 1 | 1 | 1 |
| 6 | 0.004 | 0.137 | 0.391 | 0.604 | 0.744 | 0.968 | 0.998 | 1 | 1 |
| 7 | 0.002 | 0.087 | 0.305 | 0.503 | 0.659 | 0.942 | 0.991 | 1 | 1 |
| 8 | 0.001 | 0.060 | 0.239 | 0.443 | 0.588 | 0.917 | 0.983 | 1 | 1 |
| 9 | 0.001 | 0.040 | 0.199 | 0.378 | 0.526 | 0.886 | 0.970 | 0.999 | 1 |
| 10 | 0.001 | 0.026 | 0.156 | 0.328 | 0.474 | 0.860 | 0.958 | 0.997 | 1 |

The ratio $r(L_{\rm box}, N_{\rm max})$.

Summary ::

- We developed a strategy to determine all flux vacua for type IIB orientifold compactifictions.
- We applied this strategy to the mirror octic for $N_{\text{flux}} \leq 10\,.$
- We analysed properties of the dataset.

Remarks ::

- The main technical difficulty is the required computing time.
- The dataset (together with a Mathematica notebook) is freely available.