Developments in the Emergence Proposal

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In Quantum Gravity, we expect that spacetime itself is emergent

Famous examples:

- AdS/CFT
- BFSS Matrix Theory

Can we extract implications for kinetic terms (spacetime derivatives) of effective theories?

$$\mathcal{L} = \frac{M_p^2}{2} R_{\mu\nu} g^{\mu\nu} - g_{\phi\phi} \partial_\mu \phi \partial^\mu \phi - \frac{1}{4g^2} (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$$

Emergence Proposal: Some Swampland conjectures are manifestations of such emergence

A good toy model of such a connection to keep in mind is the $\mathbb{CP}^N$ model

[D'Adda, Luscher, Di Vecchia '78][Harlow '16]

In the ultraviolet we have N massive scalars z^i which have a non-dynamical collective massless vector mode

$$\mathcal{L} = -\frac{N}{c^2} (D_\mu z_i)^* \left(D^\mu z^i\right) \; . \qquad A_\mu \equiv \frac{1}{2iN} \left(z_i^* \partial_\mu z^i - z^i \partial_\mu z_i^*\right) \; .$$

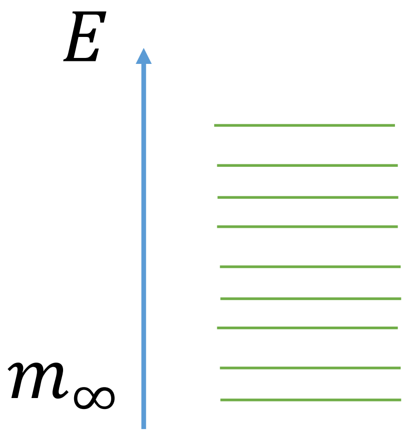
Integrating out the massive degrees of freedom leads, in the infrared, to emergent dynamics for the field

$$\mathcal{L} = -\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} \; , \qquad \frac{1}{g^2} = \frac{N}{12\pi^2} \log\left(\frac{\Lambda_{CP}}{m_z}\right) \; .$$

The Emergence Proposal is the idea that all dynamics for fields arise in the infrared from integrating out towers of states

Weak Gravity Conjecture : $m_\infty \sim g\, M_p$

Distance Conjecture : $m_\infty \sim e^{-\phi}\, M_p$



Spacetime emergence is a duality, and its infrared limit includes a duality between the propagating fields in the action and degrees of freedom in the towers of states associated to it by the Swampland conjectures

Note: the dual tower is not the lightest tower (e.g. graviphoton is not dual to KK modes)

Proposal formulated following a series of investigations [EP ’19]

- Factorisation in AdS/CFT suggests gauge fields are emergent from charged states at high energies

  [Harlow ’16]

- Toy model calculation suggests Swampland conjectures are imposing unification at the Species scale (weak emergence)

  [Heidenreich, Reece, Rudelius ’17; Grimm, EP, Valenzuela ’18; Heidenreich, Reece, Rudelius ’18]

- Proposal that type IIB complex-structure moduli space arises fully from integrating out wrapped D3 branes: So, emergence is an exact Infrared Duality (strong emergence)

  [Grimm, EP, Valenzuela ’18]

[Basile, Blumenhagen, Calderón-Infante, Castellano, Cribiori, Corvilian, Cota, Delgado, EP, Grimm, Gligovic, Herraez, Heidenreich, Ibanez, Lee, Lerche, Li, Marchesano, Melotti, Mininno, Paraskevopoulou, Paoloni, Reece, Rudelius, Ruiz, Uranga, Valenzuela, Weigand, Wiesner, + …]

A good balance between control and complexity is the type II N=2 Calabi-Yau compactification vector multiplets sector

IIB complex-structure + D3 branes = IIA Kahler + D2-D0 branes

For simplicity consider one vector multiplet with scalar component

$$T = t + ib$$

Vector multiplets sector controlled by a prepotential F_0. At large $Re(T)$:

$$F_0 = \underbrace{\mathrm{a}\,\mathrm{T}^3 + \mathrm{b}\,\mathrm{T}^2 + \mathrm{c}\,\mathrm{T} + \mathrm{d}}_{\text{Polynomial / Tree-level}} + \underbrace{\sum_{\gamma \in \mathbb{N}} n_\gamma e^{-2\pi\gamma T}}_{\text{Exponential / Non-perturbative}}$$

A crucial point is that the prepotential does not receive string loop corrections in the supergravity limit (low energy)

This allows for dual interpretations of the exponential terms $\sum_{\gamma \in \mathbb{N}} n_\gamma e^{-2\pi\gamma T}$

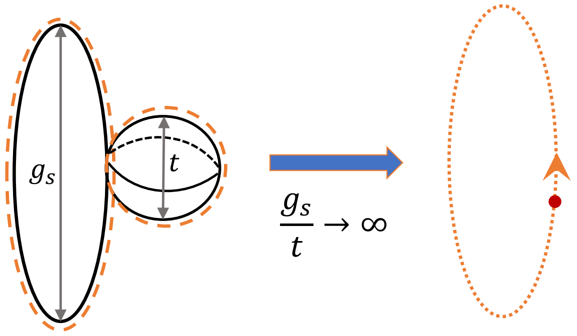
[Gopakumar, Vafa '98] [Dedushenko, Witten '14]

Worldsheet (F1) instantons

D2-D0 particle loops

g_s 0 ⟶ ∞

Membrane particle limit



Euclidean wrapped M2 brane

Euclidean particle worldline

The D2-D0 states can be integrated out in a field-theory (FT) description using a Schwinger integral

$$F_0^{Inst} = \sum_{\beta \in \mathbb{N}, n \in \mathbb{Z}} \int_\epsilon^\infty \frac{ds}{s^3} \alpha_\beta e^{-2\pi s(\beta T + in)}$$

The alternative Euclidean particle worldline description is reached through a Poisson resummation

$$\sum_{k \in \mathbb{Z}} \delta(s-k) = \sum_{n \in \mathbb{Z}} e^{2\pi i n s} \ ,$$

$$F_0^{Inst} = \sum_{\beta \in \mathbb{N}, k \in \mathbb{Z}} \int_\epsilon^\infty \frac{ds}{s^3} \delta(s-k) \alpha_\beta e^{-2\pi s \beta T} = \sum_{\beta \in \mathbb{N},\, k \geq 1} \frac{1}{k^3} \alpha_\beta e^{-2\pi k \beta T}$$

Strong Emergence predicts that also the polynomial (tree-level) prepotential comes from integrating out the states

[Grimm, EP, Valenzuela '18]

Note that ultraviolet physics has been dropped in the calculation

$$F_0^{Inst} = \sum_{\beta\in\mathbb{N}, n\in\mathbb{Z}} \int_{\epsilon}^{\infty} \frac{ds}{s^3}\alpha_\beta e^{-2\pi s(\beta T + in)} = \sum_{\beta\in\mathbb{N}, k\geq 1} \frac{1}{k^3}\alpha_\beta e^{-2\pi k\beta T}$$

Our proposal: The full prepotential comes from including the UV physics

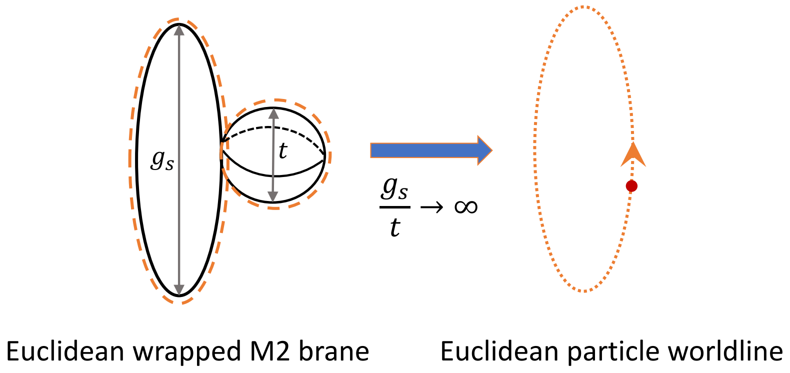
Naively, there are divergences:

$$F_0 = \sum_{\beta\in\mathbb{N}, n\in\mathbb{Z}} \int_{0}^{\infty} \frac{ds}{s^3}\alpha_\beta e^{-2\pi s(\beta T + in)} = \sum_{\beta\in\mathbb{N}, k\geq 0} \frac{1}{k^3}\alpha_\beta e^{-2\pi k\beta T}$$

Inspired by recent attempts to regularize these divergences using zeta-function regularization (but our approach is (probably) different)

[Blumenhagen, Cribiori, Gligovic, Paraskevopoulou ’18]

We note that the reason the ultraviolet physics was dropped is because the states in the tower cannot be treated as particles in those regimes



The divergence can then be understood as an incorrect particle treatment of the degrees of freedom in the ultraviolet

The whole integrating out procedure needs to be modified to account for the ultraviolet physics – we propose that such a modification is possible

Simple example: The resolved conifold (a non-compact setting with a single 2-cycle)

The full prepotential is:

[Gopakumar, Vafa '98]

$$\mathcal{F}_0 = -\frac{(2\pi i)^3}{12}\left[iT^3 + \frac{3}{2}(1+4m)T^2 - \frac{i}{2}T\right] - \zeta(3) + \mathrm{Li}_3\left(e^{-2\pi T}\right) \ ,$$

The instanton contributions are recovered as usual:

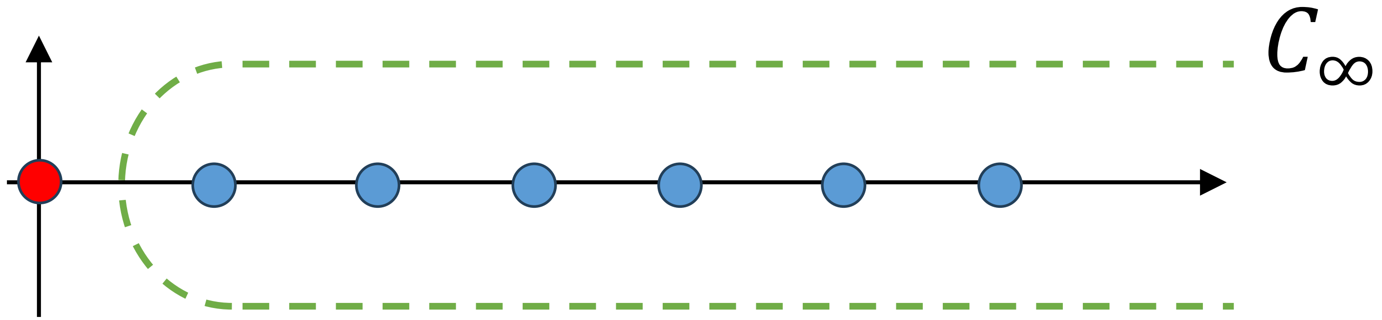
$$\sum_{n\in\mathbb{Z}} \int_{\epsilon}^{\infty} \frac{ds}{s^3} e^{-s2\pi(T+in)} = \sum_{k\geq 1} \frac{1}{k^3} e^{-2\pi k T} = \mathrm{Li}_3\left(e^{-2\pi T}\right) \ .$$

The ultraviolet physics is naively given by:

$$\int_0^{\infty} \frac{ds}{s^3} \delta(s)\, e^{-2\pi s T}$$

It is possible to repackage the instanton integral into a contour integral

$$F_0^{Inst} = \sum_{n \in \mathbb{Z}} \int_\epsilon^\infty \frac{ds}{s^3} e^{-2\pi s(T+in)} = \sum_{k \in \mathbb{Z}} \int_\epsilon^\infty \frac{ds}{s^3} \delta(s-k)\, e^{-2\pi sT} = \oint_{C_\infty} \frac{dz}{z^3} \frac{e^{-2\pi zT}}{1-e^{-2\pi iz}}$$

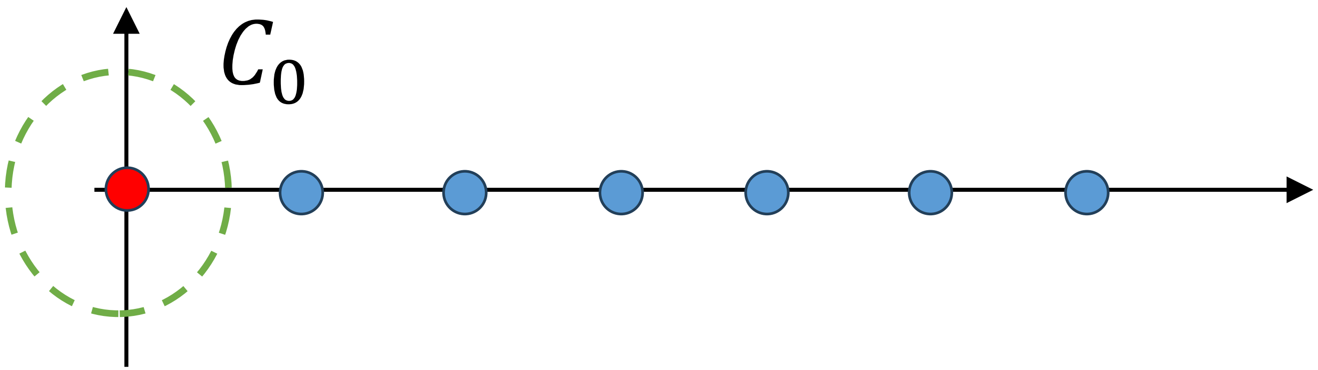


This can be thought of as an analytic continuation of the Schwinger proper time

$$s \to z$$

It is now possible to evaluate the ultraviolet pole at the origin

$$F_0^{Poly} = \frac{1}{2}\oint_{C_0} \frac{dz}{z^3}\frac{e^{-2\pi z T}}{1-e^{-2\pi i z}} = -\frac{(2\pi i)^3}{12}\left[iT^3 + \frac{3}{2}T^2 - \frac{i}{2}T\right]$$



Our interpretation is that indeed the full prepotential arises from integrating out the states, but the ultraviolet contribution needs to evaluated by analytic continuation and a modification around the origin

Can this work for a full compact Calabi-Yau? Yes

However, note that unlike the conifold case, there are multiple D2 branes wrappings that need to be summed

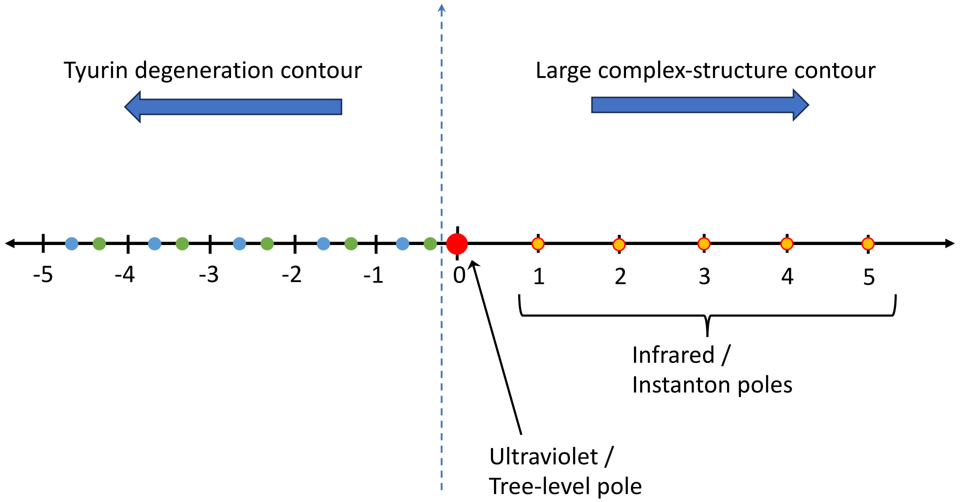
$$F_0^{Inst} = \sum_{\beta \in \mathbb{N}, n \in \mathbb{Z}} \int_{\epsilon}^{\infty} \frac{ds}{s^3} \alpha_\beta e^{-2\pi s(\beta T + in)}$$

The spectrum depends on the the Calabi-Yau in question, and so the integrand should be specific to the Calabi-Yau

We consider the bi-cubic one-parameter Calabi-Yau: $\mathbb{P}^5[3,3]$ [Joshi, Klemm '19]

$$\mathcal{F}_0(T)|^{Poly} = \frac{1}{2}\oint_{C_0}\left[\left(3+i\sqrt{3}\right)\frac{\Gamma\left(\frac{1}{3}+z\right)(-1)^{-z}}{\Gamma\left(\frac{2}{3}-z\right)\Gamma\left(\frac{1}{3}-z\right)^2} + \left(-3+i\sqrt{3}\right)\frac{\Gamma\left(\frac{2}{3}+z\right)(-1)^{-z}}{\Gamma\left(\frac{1}{3}-z\right)\Gamma\left(\frac{2}{3}-z\right)^2} - \frac{(3i+9\sqrt{3})}{2\pi}\frac{\Gamma\left(\frac{1}{3}+z\right)^2}{\Gamma\left(\frac{1}{3}-z\right)^2} + \frac{(9\sqrt{3}-3i)}{2\pi}\frac{\Gamma\left(\frac{2}{3}+z\right)^2}{\Gamma\left(\frac{2}{3}-z\right)^2}\right]\frac{\Gamma(-z)^4\, 3^{6z} e^{-2\pi z T}}{2\pi i z}$$

$$= -\frac{(2\pi i)^3}{4}\left[6iT^3 - T^2 + 9iT\right] + \text{const.}\,.$$



* For Period not Prepotential

Note that the zero pole does not have a large D2 degeneracy factor: only integrating out a few constituents in the ultraviolet

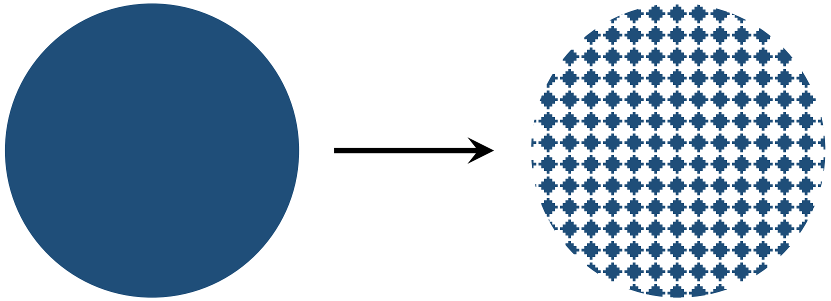
Using the integral as a guide for the ultraviolet physics suggests:

- Full prepotential arises from integrating out the non-perturbative states

- There is a sum over D0 branes, and the polynomial piece comes from the ultraviolet physics of this sum

- There is no sum over D2 degeneracies, only a single contribution from the self-intersection points

- At scales below the D2 size, we should consider them as non-fundamental, but weakly-bound states of a fundamental constituents that we integrate out (3-dimensional theories are free in the UV)



To Appear, w/ Hattab

New proposed picture for Emergence:

- The infrared duality is between light fields and fundamental point-like constituents (likely spacetime quanta)
- The towers of states are capturing the infrared physics of those constituents
- The leading tree-level kinetic terms come from the ultraviolet regime, where the constituents are the weakly-coupled fundamental degrees of freedom
- In general, integrating out those fields cannot be formulated as an equi-dimensional Schwinger integral

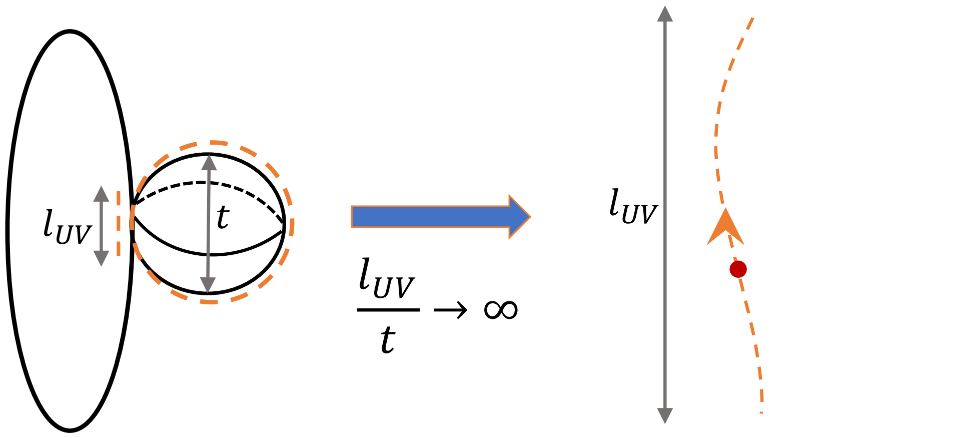
In general, there is no way to integrate out all the degrees of freedom as spacetime particles (standard Schwinger integrals)

However, there do exist regimes where such a treatment can hold

Example: vanishing locus of the resolved conifold $t \to 0$

$$\mathcal{F}_0\big|^{\text{Poly}}_{t=0,m=0} = -\frac{(2\pi i)^3}{12} B_3\left(b\right) \quad ,$$

$$\mathcal{F}_0\big|^{\text{Poly}}_{t=0,m=0} = \frac{1}{2}\sum_{k\neq 0}\frac{1}{k^3}e^{2\pi i k b} \quad .$$



Euclidean unwrapped M2 brane Euclidean particle worldline

Summary

- Interpreted (known) integral representations of CY periods as analytically continued Schwinger integrals

- The tree-level prepotential piece can then be understood as arising from integrating out non-perturbative degrees of freedom

- This interpretation suggests a new perspective on the Emergence Proposal where one does not integrate out the towers of particle states, but resolved them into constituents in the ultraviolet

- There are regimes in moduli space where it is possible to generate the prepotential from integrating out spacetime particles: singular conifold

Thank You