Do black holes know about the emergent string conjecture?

Based on a series of works with Ivano Basile, Niccoló Cribiori and Dieter Lüst.
[2305.10489], *[2311.12113]*, [2401.06851]

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Carmine Montella, 19/03/2024.

The emergent string conjecture

- From a bottom-up point of view, an infinite distance limit in a space of vacua is a factorisation limit, i.e. a $N-$point function can be reduced into $N-$one point functions. *[Stout '21]*

- However, gravity abhors factorisation (due to equivalence principle), thus it must couple to an infinite tower of species. *[Stout '22]* In string theory: tower is asymptotically massless. *(SDC!)*

  - The Emergent String Conjecture expresses the nature of the tower, stating that any infinite distance limit in the space of vacua is either a decompactification limit or a limit in which there is a weakly coupled (critical) string becoming tensionless. *[Lee, Lerche, Weigand '19]*

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The species scale

- The species scale is (an upper bound to) the cut-off of an effective theory of gravity.

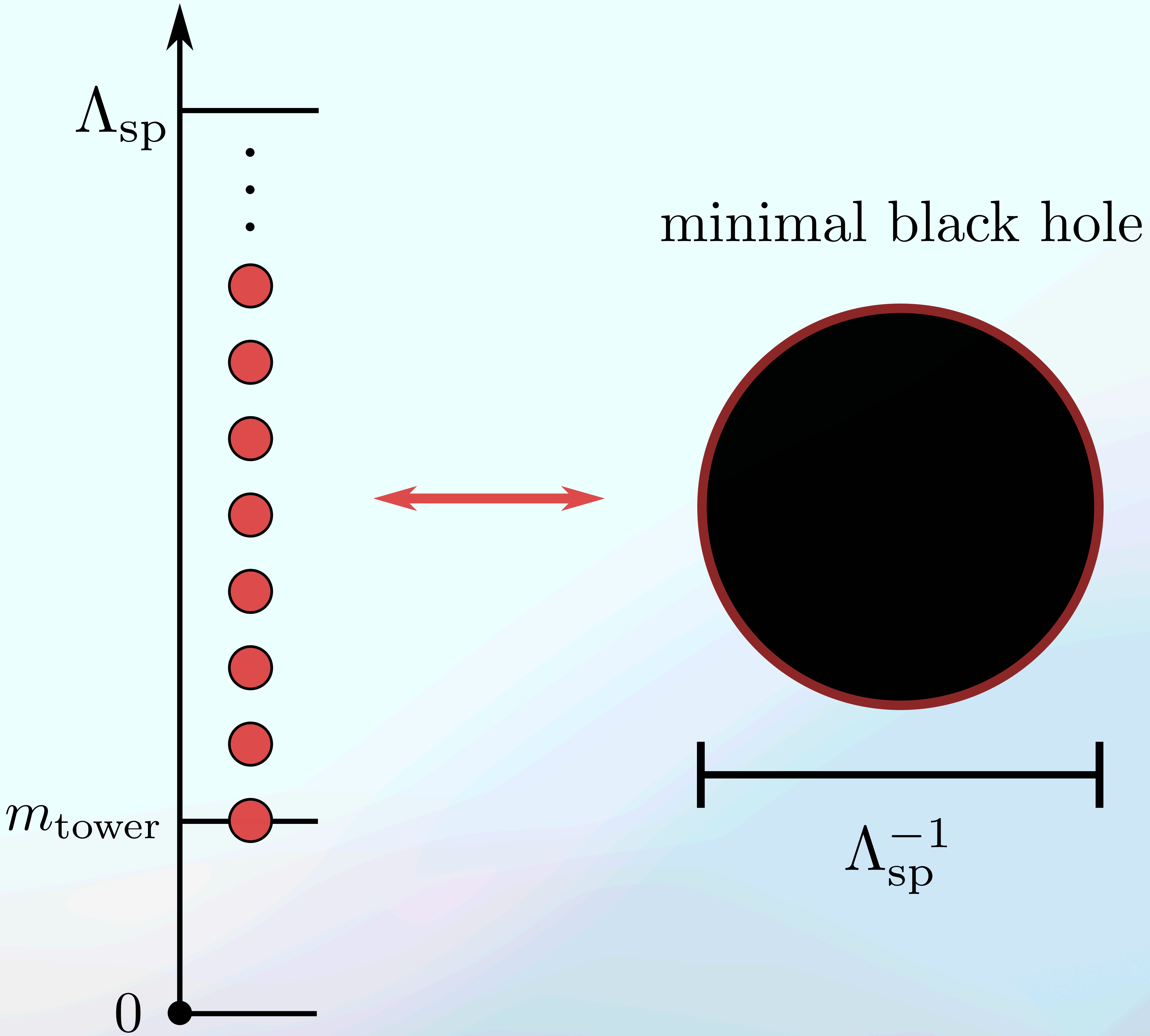
    1. The scale at which perturbative gravity breaks down due to the presence of $N_{sp} \gg 1$ species *[Dvali '09]*:

$$\Lambda_{sp} = \frac{M_{pl,d}}{N_{sp}^{d-2}}$$

2. The cut-off scale Λ_{UV} appears in the higher derivative terms of an effective gravitational action. A modern view in string theory context: *[van de Heisteeg, Vafa, Wiesner, Wu '22-'23]*:

$$S_{\text{EFT}} \sim \frac{M_{\text{pl},d}^{d-2}}{2} \int d^d x \sqrt{-g} \left(R + \sum_n \frac{c_n}{\Lambda_{\text{UV}}^{2n-2}} O_n(g, \text{Riem}, \nabla) \right) .$$

Does the species scale define the smallest black hole?



A depiction of the correspondence or transition between (asymptotically) massless species and minimal black hole

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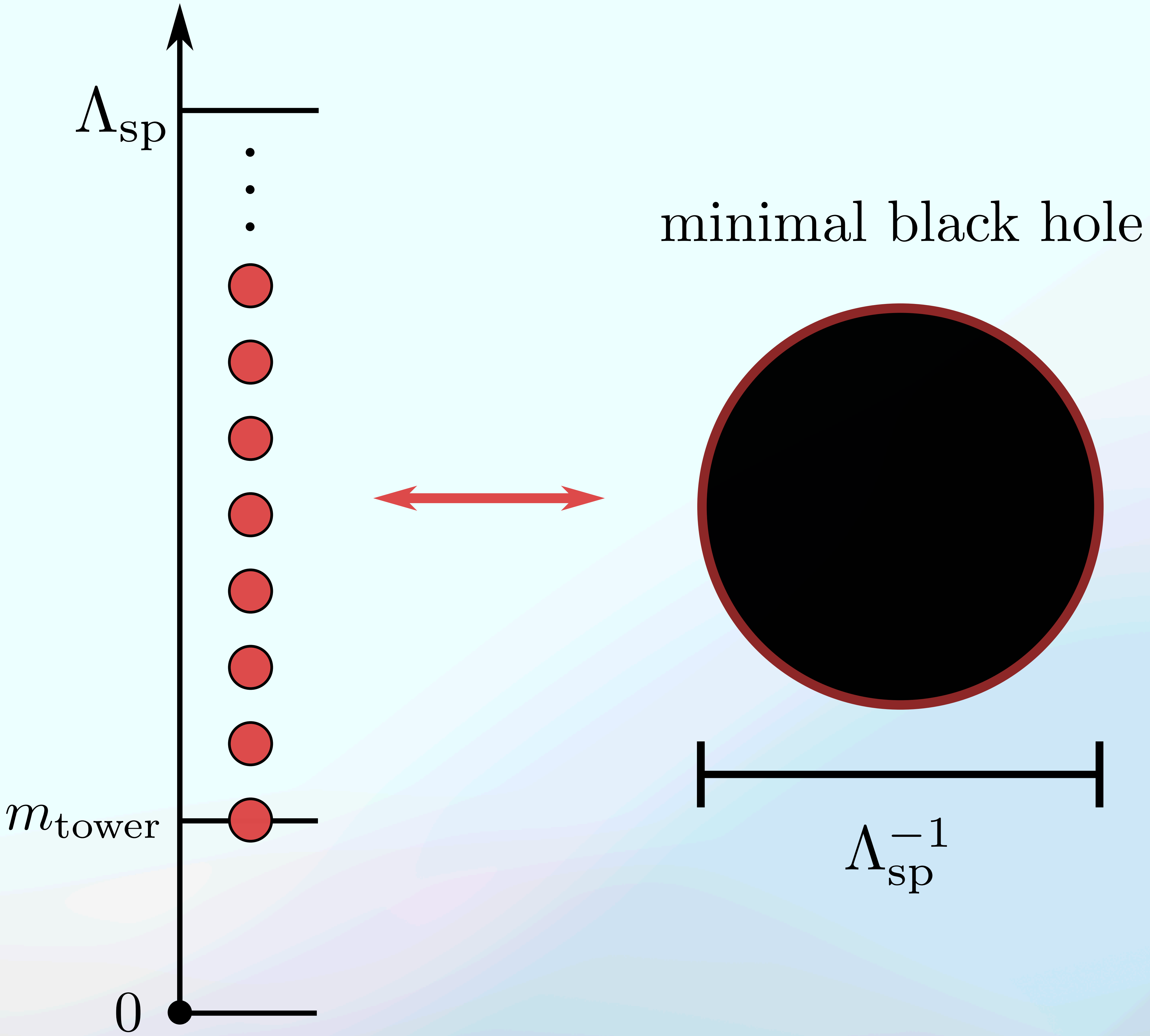

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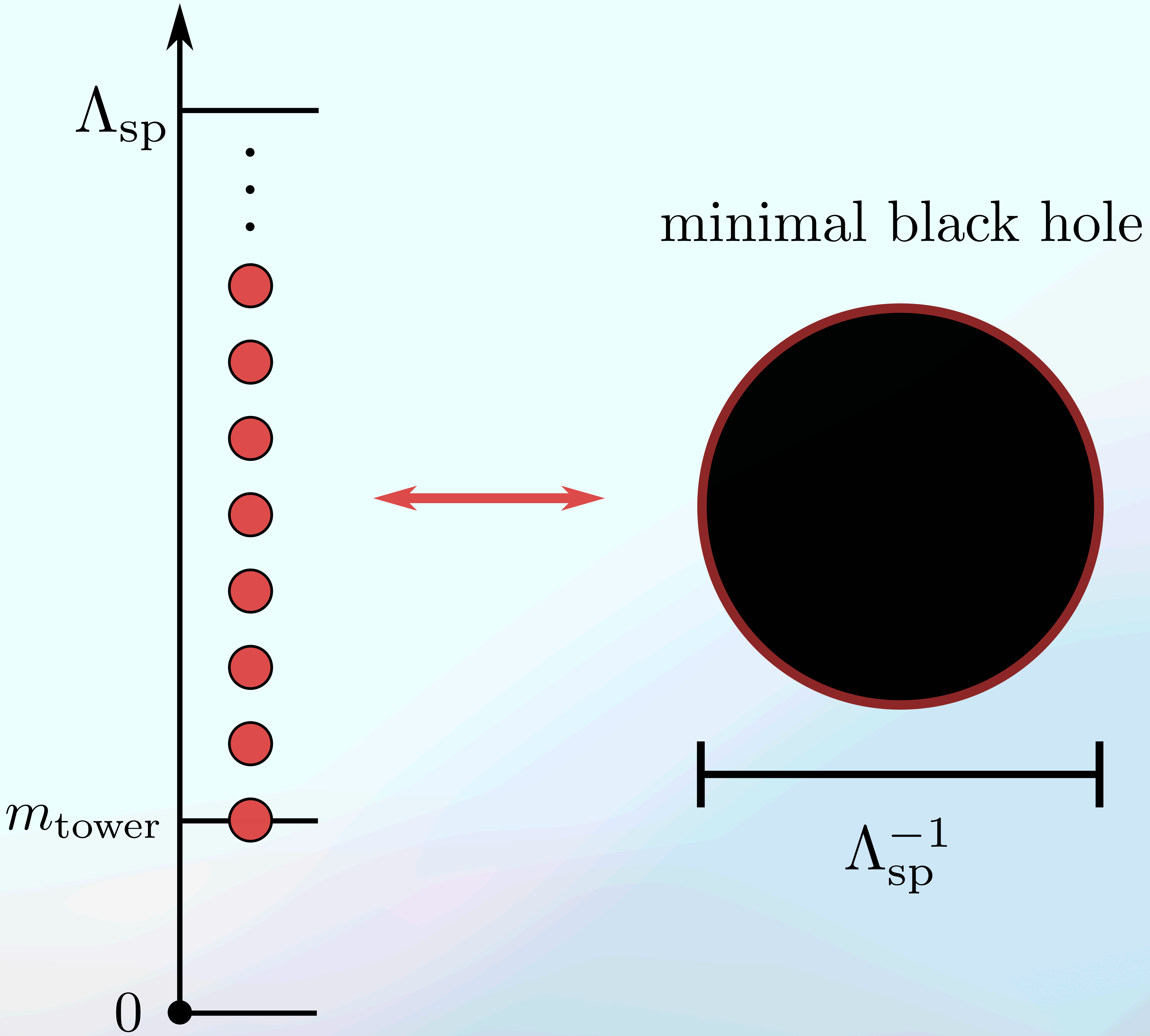

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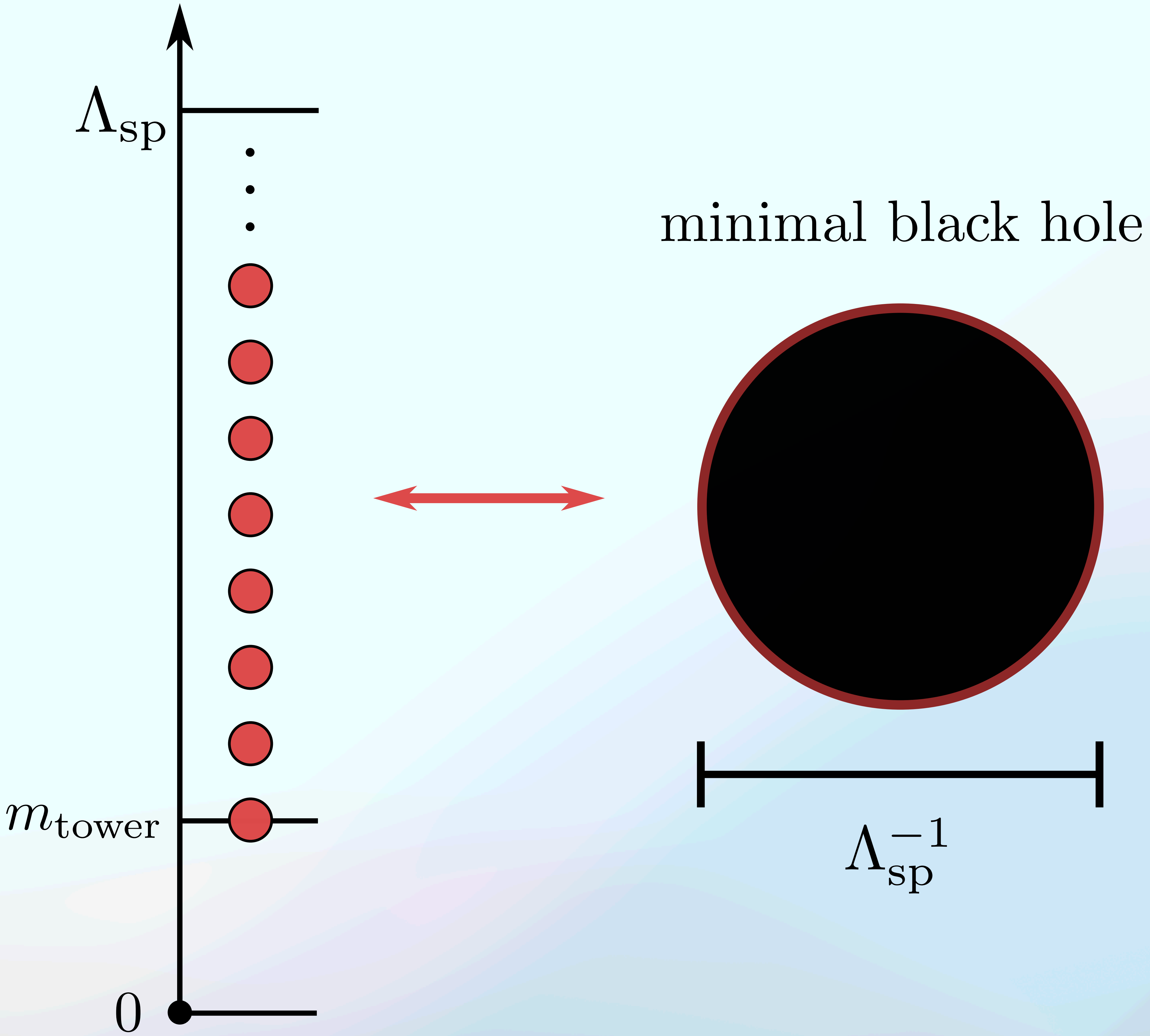

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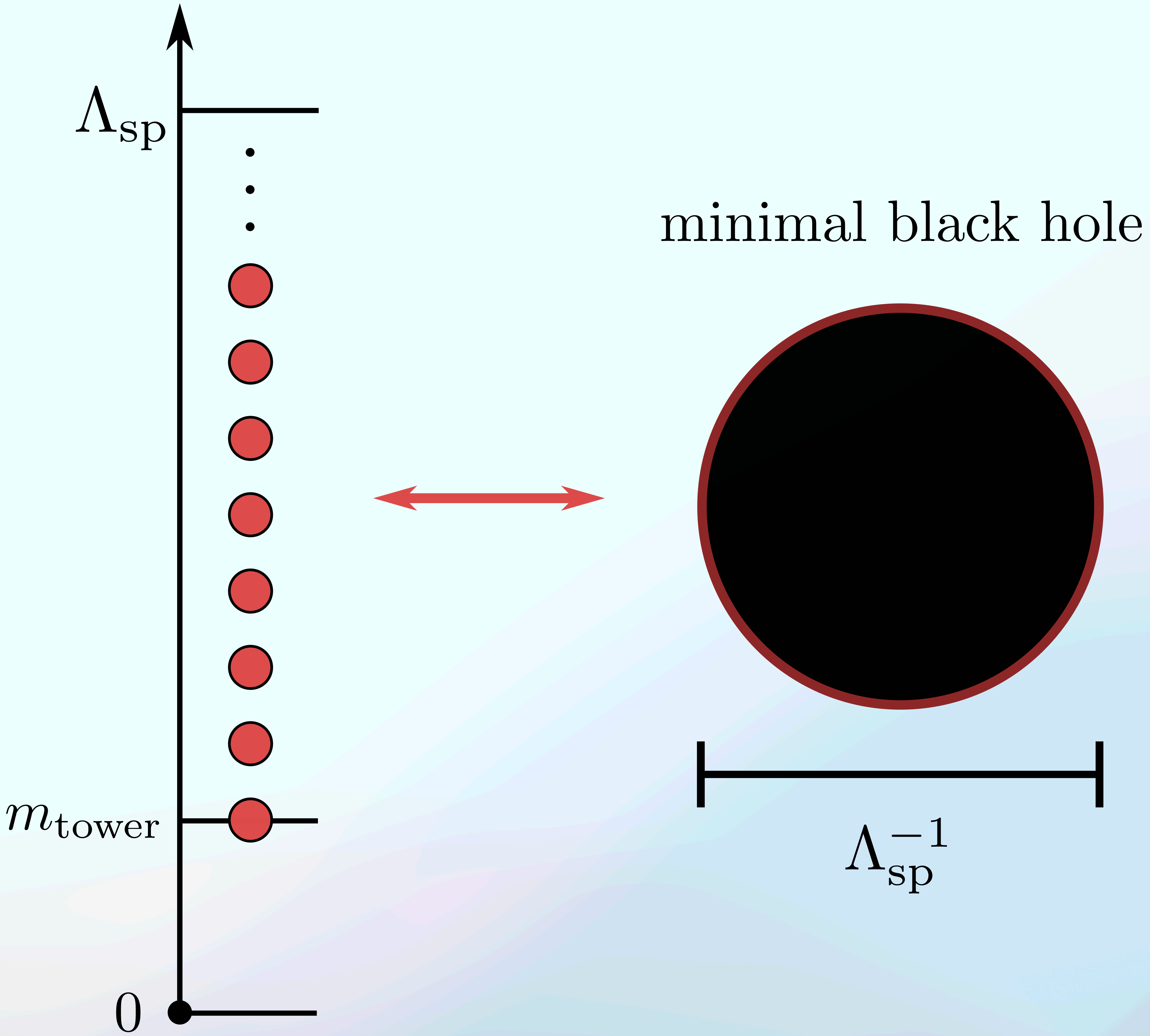

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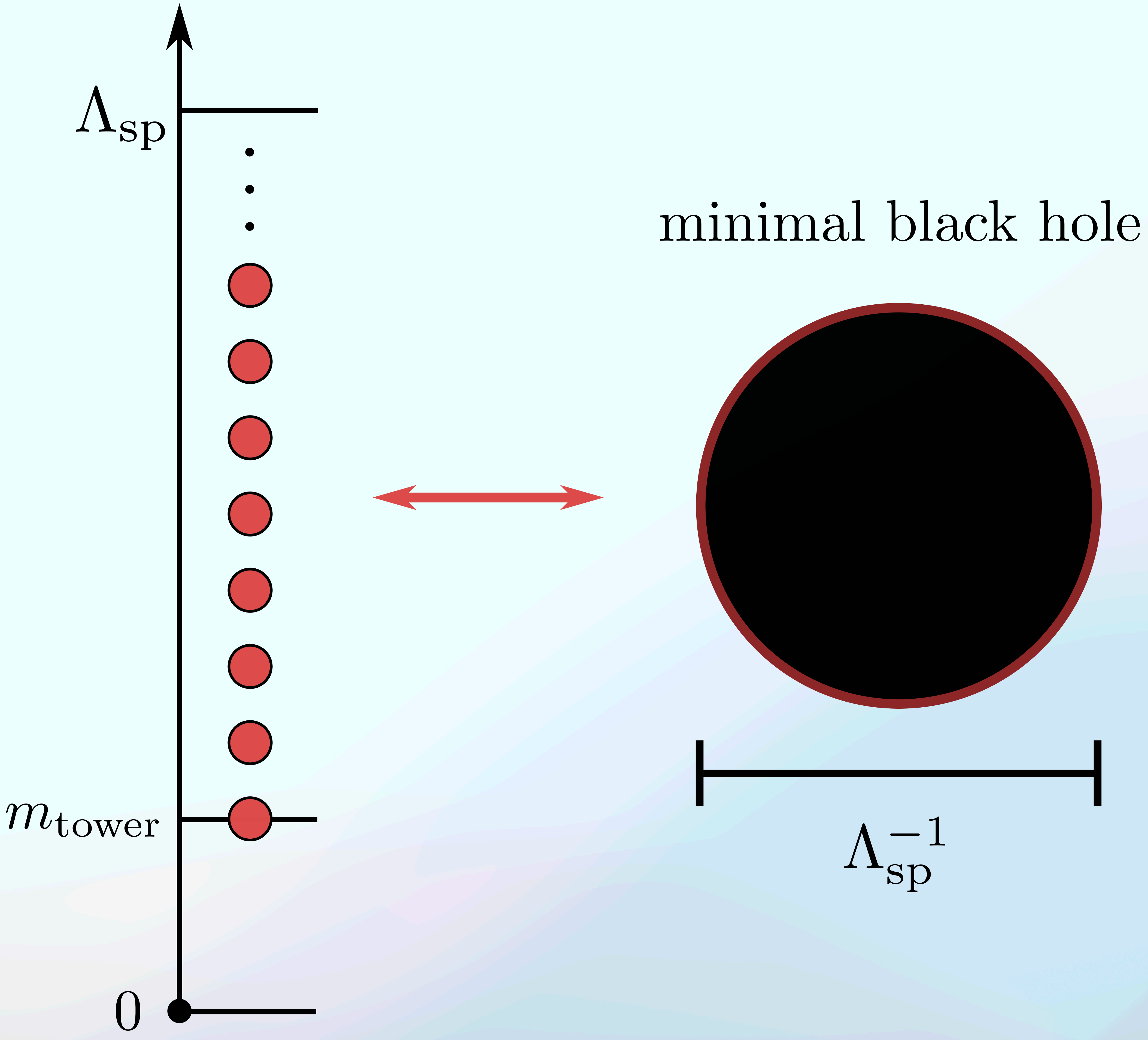

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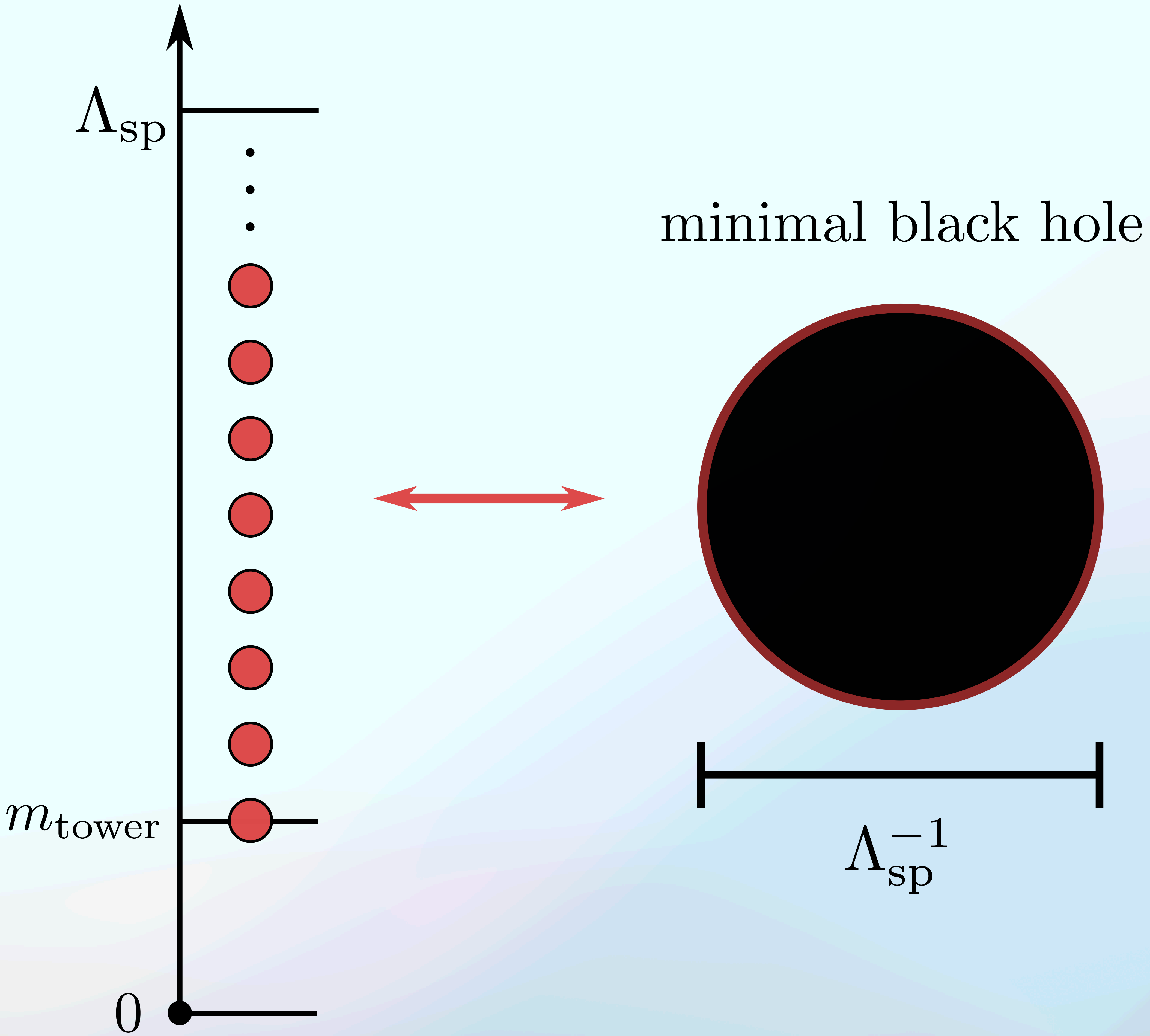
- We give evidence the above correspondence support the ESC!



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Tower of species picture as black hole?

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- We parametrise a general tower as $m_n = m_{\text{tow}}\chi(n)$, with a degeneracy of d_n.

- Fixing the energy E_{sp}, then the micro-canonical entropy can be calculated as
  $S_{sp} = \log D(E_{sp})$, i.e. $Z(q) = \sum_M q^M D(M) = \prod_{n\leq N} \left(1 - q^{\chi(n)}\right)^{-d_n}$

$$S_{sp} \sim N_{\text{sp}} + \sum_{n\leq N} d_n \log \frac{E_{sp}}{N_{sp} m_n} + \text{corr.}$$

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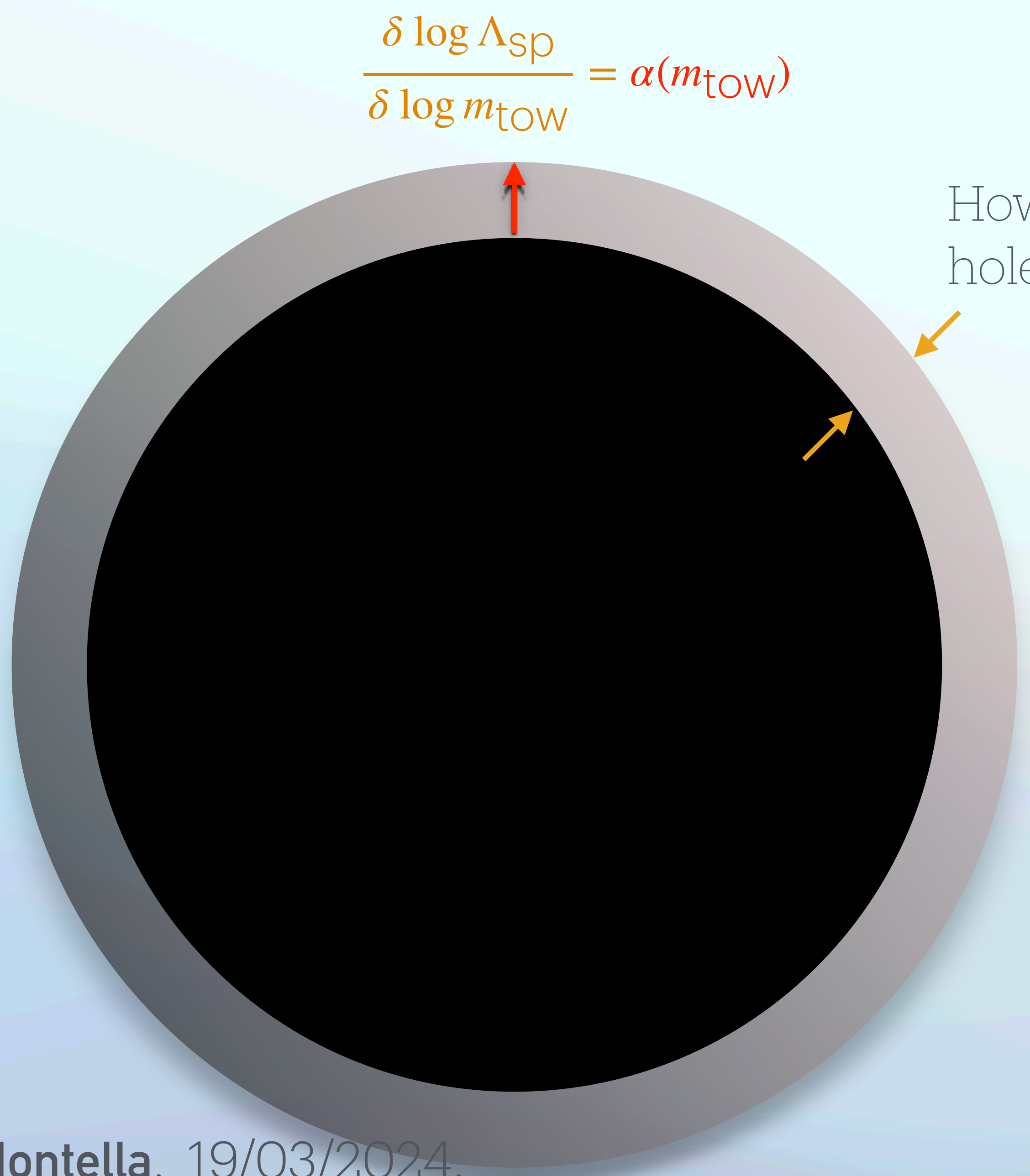
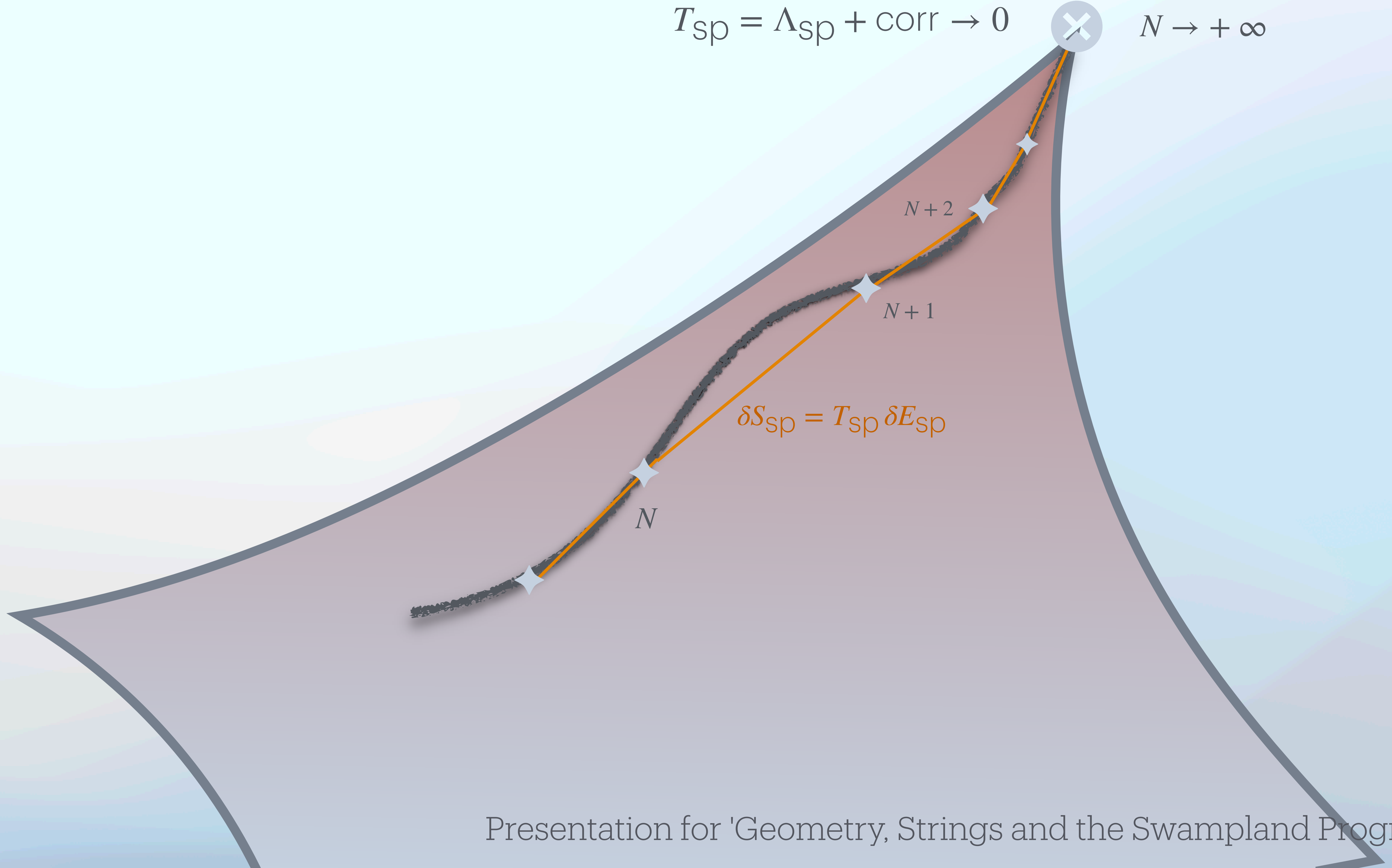
A. The question we ask ourselves is: given leading towers at infinite distance limit, when is it possible to have a correspondence/transition with the smallest black hole in the EFT?

A bottom-up approach

- **In a given EFT, any consistent tower at infinite distance limit must correspond to a minimal black hole, and viceversa. [Basile, Lüst, CM '23]**

- Due to $S_{sp} \sim \Lambda_{sp}^{2-d}$ + corr. for the leading towers, we impose $E_{sp} = \gamma \Lambda_{sp}^{3-d}$ + corr, for any point at infinite distance of the space of vacua.





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- Again, for large N: $\chi(N) = \dfrac{\Gamma(N+\sigma+1)\Gamma(\alpha+1)}{\Gamma(N+1+\alpha)\Gamma(\sigma+1)} \sim N^{\frac{1}{p}}$, now with $p \geq 1$!

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- We can state that at infinite distance limit only two classes of towers can correspond to minimal black holes

$$\Lambda_{\text{sp}} \sim m_{\text{tow}}^{\frac{\hat{p}}{\hat{p}+d-2}} M_{\text{pl,d}}^{\frac{d-2}{d-2+\hat{p}}} \qquad \Lambda_{\text{sp}} \sim m_{\text{tow}}$$

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A. From a bottom-up approach, $\hat{p}\left(\hat{c}(m_n, d_n)\right) \geq 1$ represent a generic parameter defined by the black hole thermodynamics, and different parametrizations of the “microstates".

B. From a top-down perspective it corresponds to the number of extra dimensions! ⟶ $m_{\text{tow}} \equiv m_{\text{KK}}$

- The limit $\hat{p} \to \infty$ is well defined for every thermodynamics quantity, and the black hole thermodynamics returns the string/BH transition ⟶ $m_{\text{tow}} \equiv M_{\text{S}}$

$$T_{sp} \sim M_s \equiv T_{Hag} \qquad M_c \sim M_s^{3-d} = \frac{M_s}{g_s^2}$$

In Planck units.

Thank you for attention!

Have a nice stay!