Starobinsky Inflation and the Swampland

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Geometry, Strings and the Swampland Program

Based on [2312.13210, D. Lüst, M. Scalisi, B. Muntz, **JM**]

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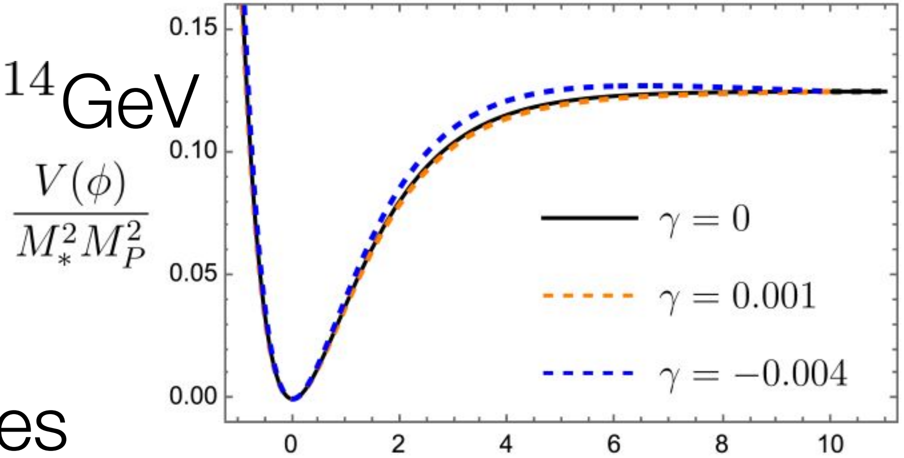

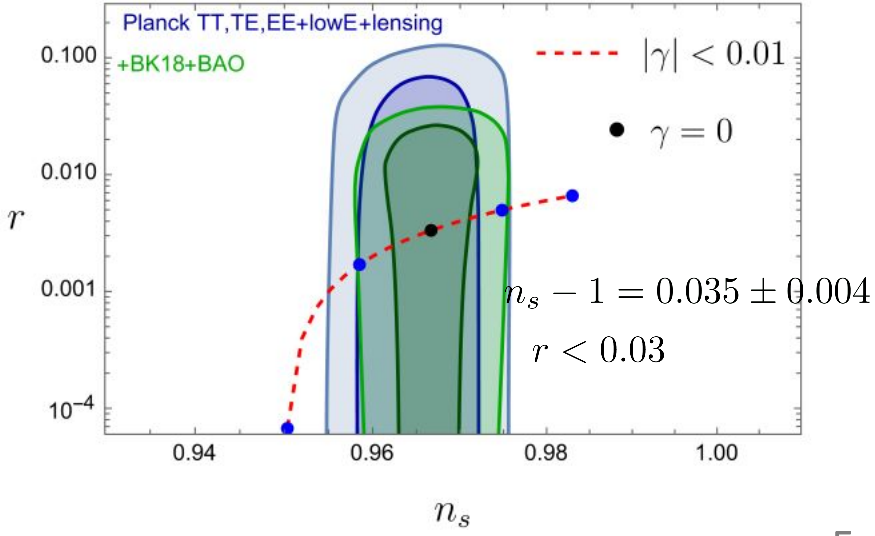

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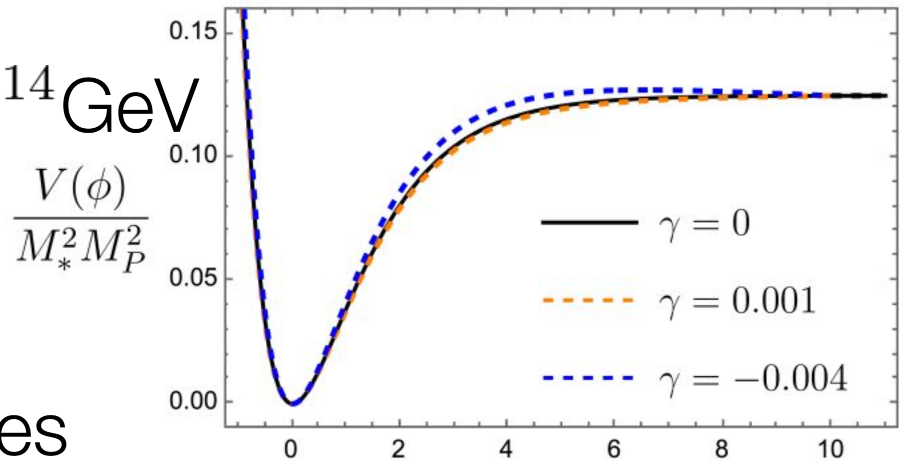
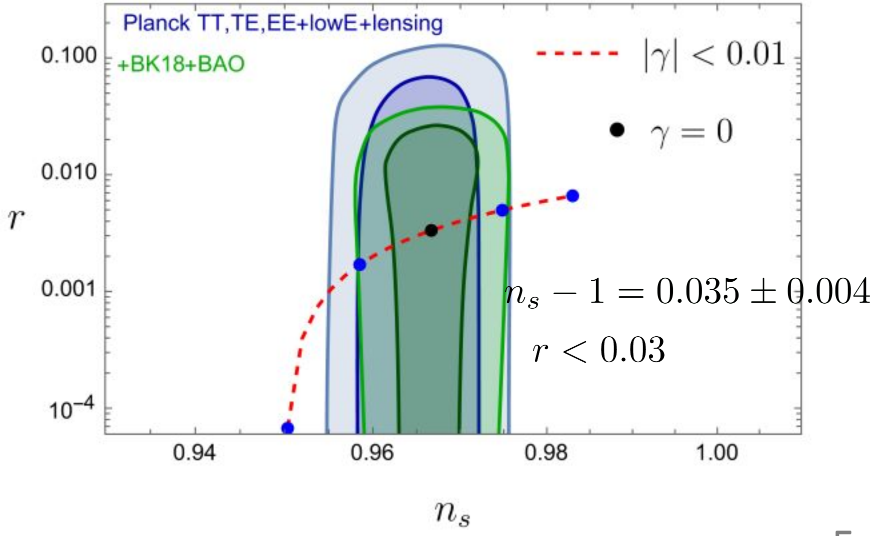
$$\gamma = \frac{\Lambda_s'}{\Lambda_s} \qquad -0.004 \leq \gamma \leq 0.001 \qquad \frac{1}{\sqrt{6}} \leq \left|\frac{\Lambda_s'}{\Lambda_s}\right| \leq \frac{1}{\sqrt{2}}$$





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- Not protected from higher curvature corrections

Conclusions

- The Starobinsky model of inflation can be interpreted as a QG correction to EH gravity.
- In particular, it can be generated by the renormalization effects of a tower of light species.
- Identifying $M \simeq \Lambda_s$, leads to $\Lambda_s \simeq 10^{14}$ GeV, $N_{sp} \simeq 10^{10}$.
- Starobinsky Inflation is spoiled by an exponential scaling $|\gamma| \gtrsim \mathcal{O}(10^{-3})$.
- Top-down arguments can be used to identify $M \simeq \Lambda_s$.