Instituto de Física Teórica presents:

Asymptotic Limits and *curvature*

by Fernando Marchesano

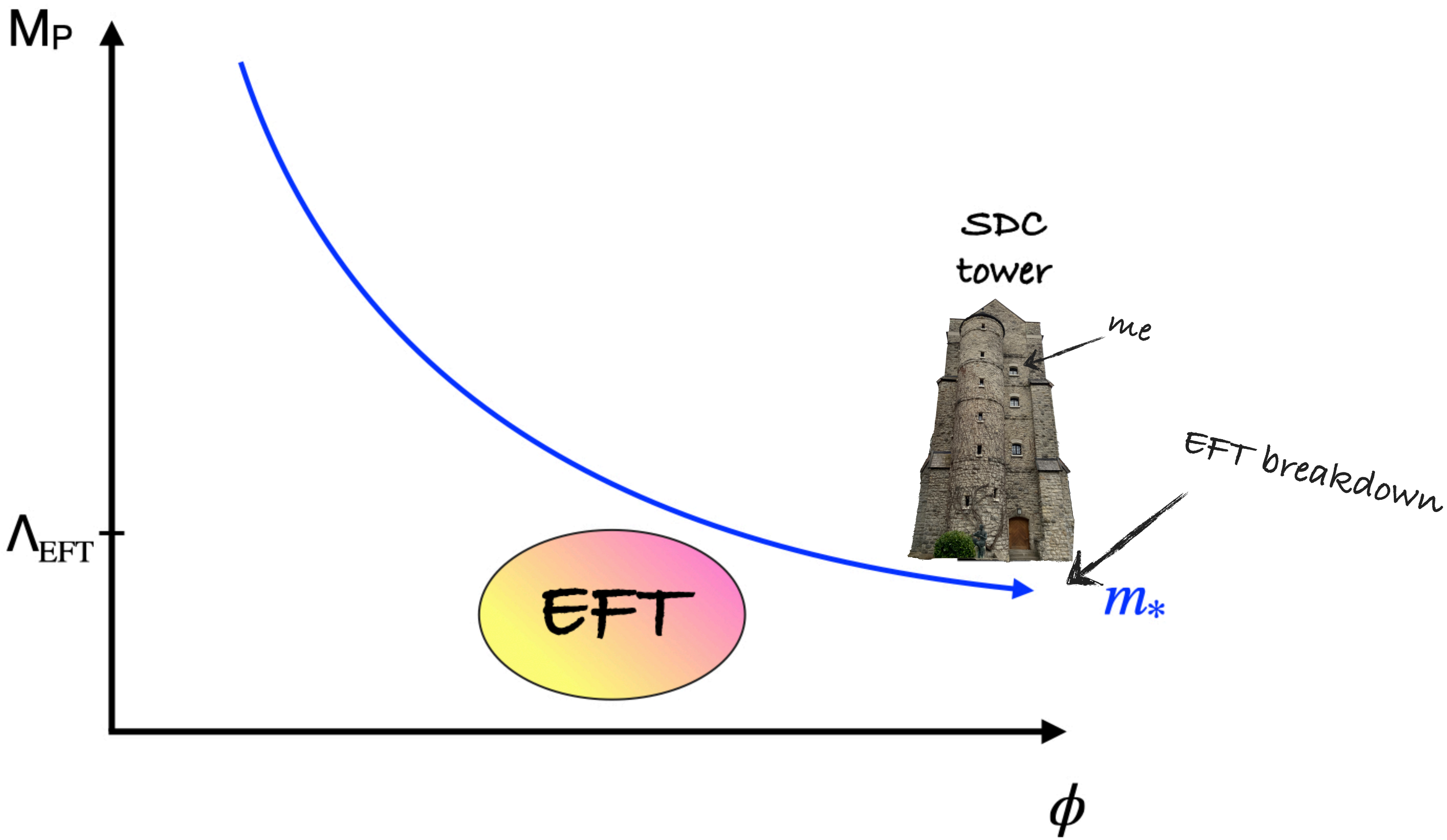
based on 2311.07979

w/Luca Melotti & Lorenzo Paoloni

Swampland Distance Conjecture

Oogurі & Vafa '06

Along infinite distance geodesics there is an infinite tower of states which become exponentially light asymptotically



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Curvature Conjecture

Ooguri & Vafa '06

> The scalar curvature near the points at infinity is non-positive
> (negative if dim $\mathcal{M} > 1$)

Examples: *Greene, Shapere, Vafa, Yau '89*

i) type IIB axio-dilaton

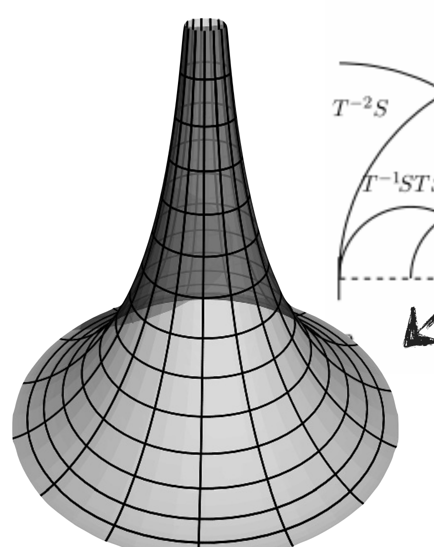
$$\tau = C_0 + i g_s^{-1}$$

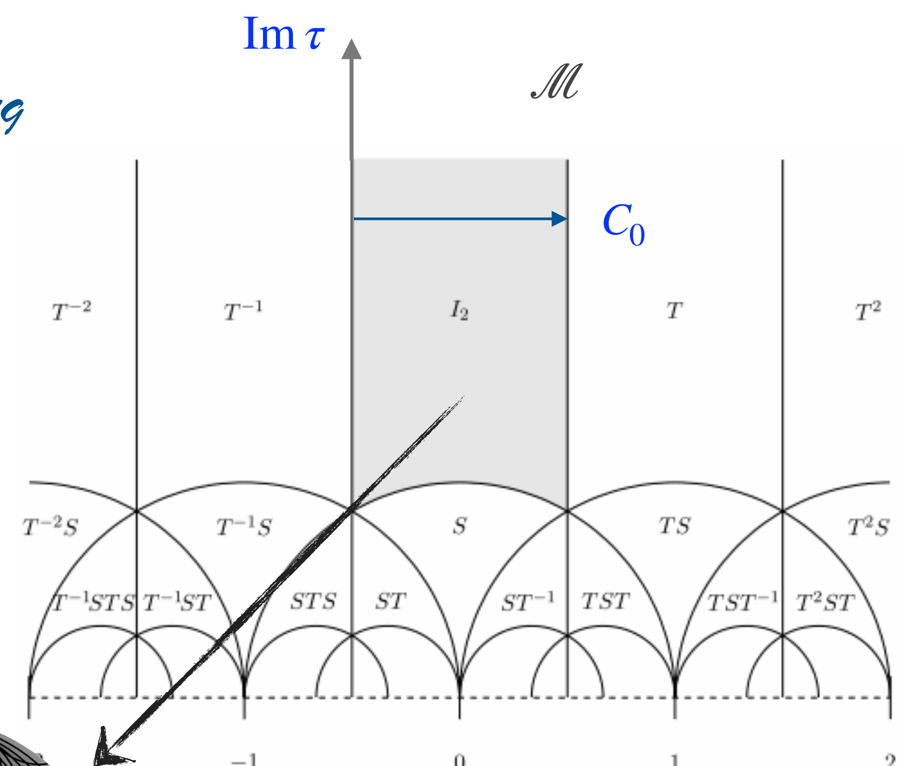
$$K = -\log(-i(\tau - \bar{\tau}))$$

ii) single Kähler modulus

$$T = b + it$$

$$K = -3\log(-i(T - \bar{T}))$$





negative constant curvature

Curvature Conjecture

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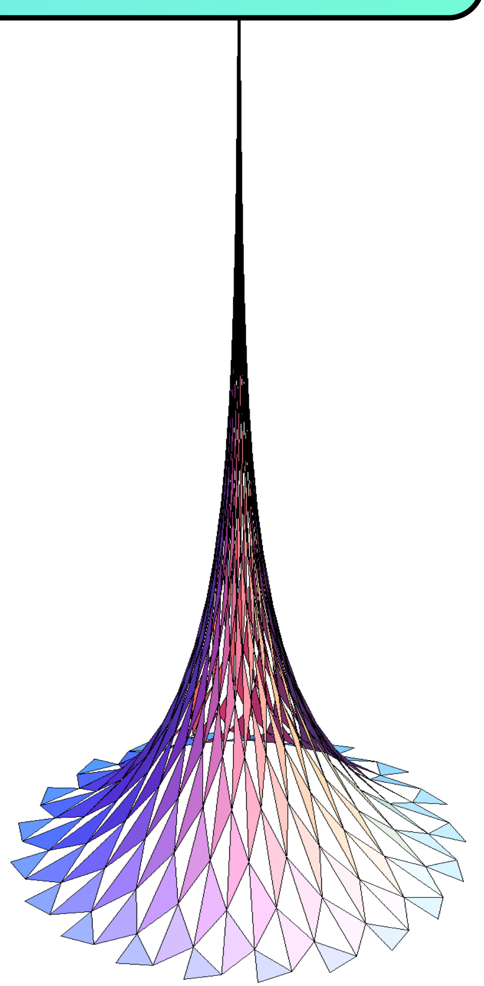
(negative if dim $\mathscr{M} > 1$)

- Further support based on type II string theory compactified on CY manifolds
  $\Longrightarrow$ $4d\ \mathcal{N} = 2$ supergravity theories

- In this context counterexamples were found in CY Vector Multiplet moduli spaces with three moduli

Trenner & Wilson' 09

Asymptotic curvature not only positive but also divergent!



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Asymptotic curvature not only positive but also divergent!

Questions:

When is the asymptotic curvature positive?

When is it divergent?

What is the physical source of the divergence?

The Laboratory: type IIA on a CY, VM

We focus on: Type IIA on a CY$_3$ $\longrightarrow$ $\mathcal{M}_{\mathcal{N}=2} = \mathcal{M}_{\mathrm{HM}} \times \mathcal{M}_{\mathrm{VM}}$

Large volume regime: $K = -\log\left(\int J \wedge J \wedge J + \dots\right)$ $\quad J = t^a \omega_a \implies e^{-K} = \kappa_{abc} t^a t^b t^c$

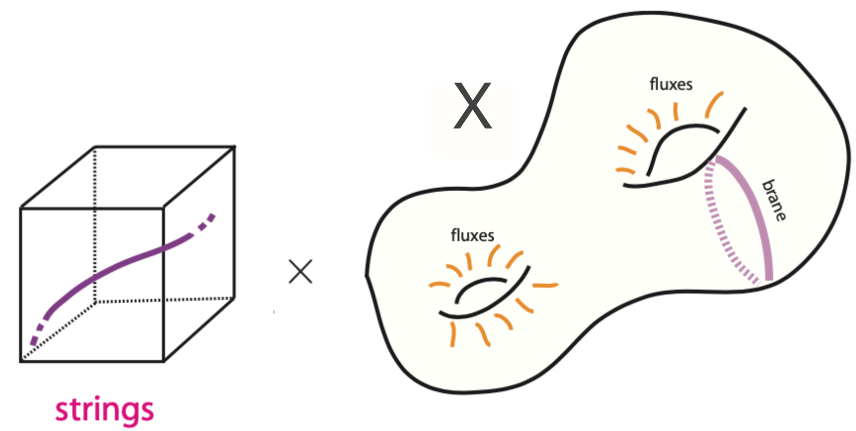
Infinite distance LV limits: physical realisation as backreaction of 4d strings made up from NS5-branes wrapping Nef divisors of X$_6$ (EFT strings) *Lanza et al. '21*

$t^a = t_0^a + e^a \phi,$ with $\phi \to \infty$

$\mathrm{Vol}_X \sim \phi^w \qquad g_s \sim \phi^{w/2} \qquad w = 1,2,3$

SDC tower: D0-branes $m_{\mathrm{D0}} = m_*$

$$\frac{m_*^2}{M_{\mathrm{P}}^2} \sim \phi^{-w} \sim \left(\frac{T_{\mathrm{NS}}}{M_{\mathrm{P}}^2}\right)^w$$



Limits classified in terms of $w = 1,2,3$

Corvilain, Grimm, Valenzuela '18 *Lee, Lerche, Weigand '19*

Type IIA large volume limits

Idea: compute the scalar curvature along EFT string trajectories

Type IIA CY_3 VM sector: EFT strings are type IIA NS5-branes wrapping Nef divisors

$$t^a = e^a \phi \quad \text{with} \quad \phi \to \infty$$

$$g_s^2(\phi) \sim \mathrm{Vol}_X \sim \phi^w \to \infty$$

Large volume and strong 10d coupling

Classification of limits: *Corvilain, Grimm, Valenzuela'18* *Lee, Lerche, Weigand'19*

w	Definition	Dual description
3	$k \neq 0$	M-theory on X
2	$k=0,\ k_a \neq 0$	F-theory on X
1	$k_a=0,\ k_{ab} \neq 0$	Heterotic dual

$$k = \kappa_{abc} e^a e^b e^c$$

$$k_a = \kappa_{abc} e^b e^c$$

$$k_{ab} = \kappa_{abc} e^c$$

Type IIA vs. M-theory description

$$t^a = e^a \phi \quad \text{with} \quad \phi \to \infty$$

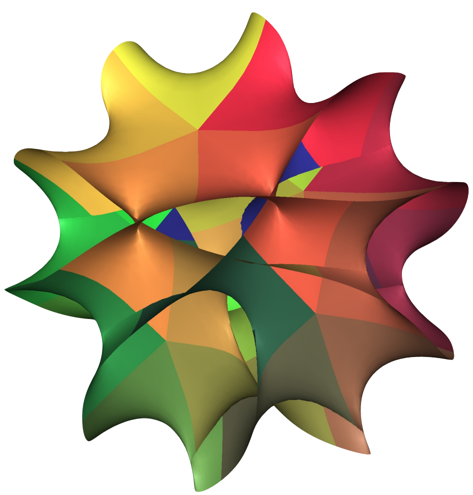
$$g_s^2(\phi) \sim \mathrm{Vol}_X \sim \phi^w \to \infty$$

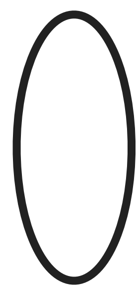
M-theory on $S^1 \times X$

$$M^a = t^a / \mathrm{Vol}_X^{1/3}$$

$$2\pi R_5 = \mathrm{Vol}_X^{1/3}$$

$I_{ab} = 4\mathrm{Vol}_X g_{ab}$ Gauge kinetic matrix J_{ab}





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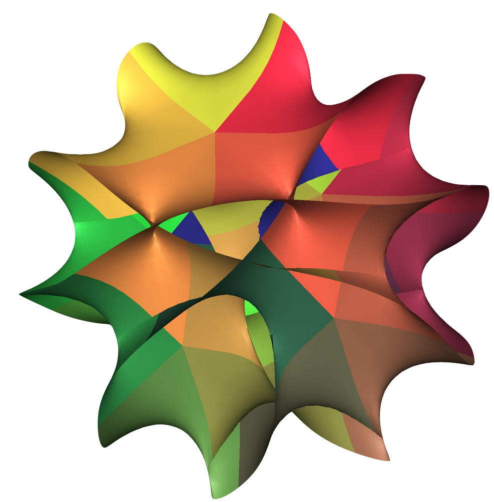

$\phi_4 =$ const.

$\mathrm{Vol_M}$ = const.

Type IIA vs. M-theory description

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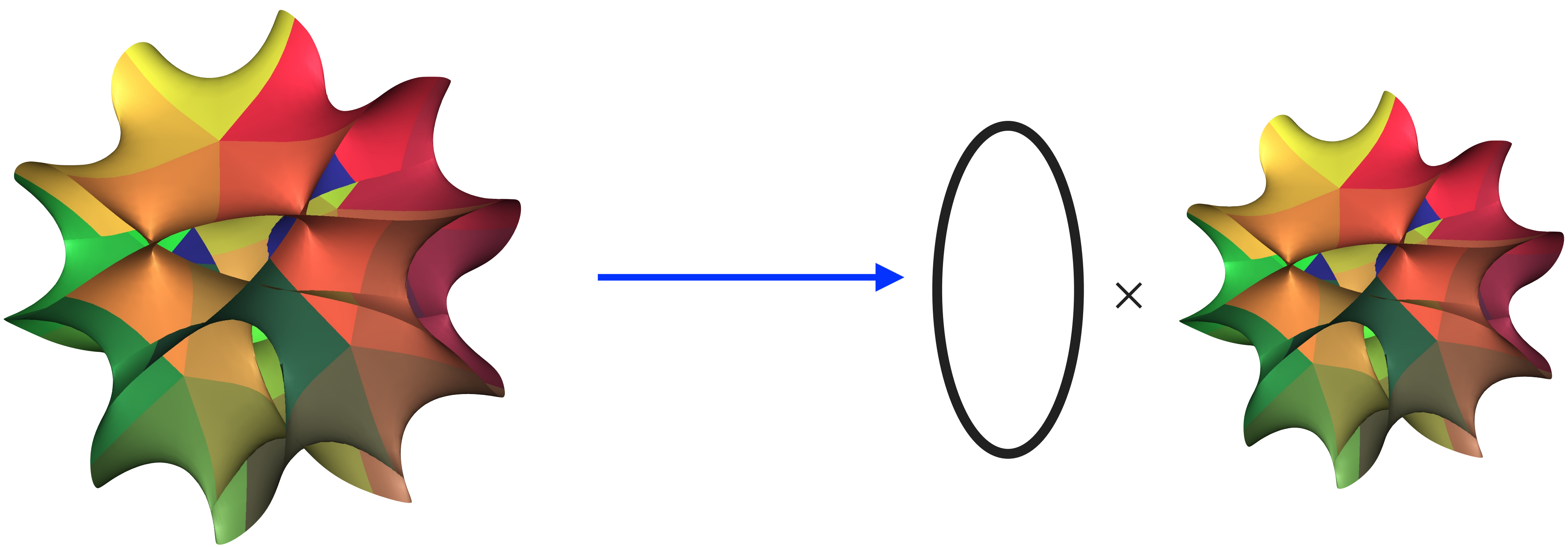

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w=3 limits

$$M^a(\phi) = \left(\frac{6}{k}\right)^{1/3} \left(e^a + \phi^{-1} \left(t_0^a - e^a \frac{k_b t_0^b}{k} \right) + \ldots \right)$$

$k = \kappa_{abc} e^a e^b e^c$

$k_a = \kappa_{abc} e^b e^c$

finite distance trajectory for M-theory on X

w=2,1 limits infinite distance boundary of M-theory on X

The (type IIA) scalar curvature

In special geometry there are simple formulas for the Riemann curvature

Strominger '90

Implementing the large-volume axionic shift symmetries one finds:

$$R_{a\bar{b}c\bar{d}} = g_{ab}g_{cd} + g_{ad}g_{bc} - \frac{g^{ef}\kappa_{ace}\kappa_{bdf}}{64\mathrm{Vol}_X^2}$$

$$\boxed{R_{\mathrm{IIA}}/2 = -n_V(n_V+1) + Y^2}$$

bounded from below

$$Y^2 = \frac{g^{ab}g^{cd}g^{ef}\kappa_{ace}\kappa_{bdf}}{64\mathrm{Vol}_X^2} = \mathrm{Vol}_X I^{ab} I^{cd} I^{ef} \kappa_{ace}\kappa_{bdf}$$

$\sim \phi^w$ $\quad \sim g_\alpha^2$ $\quad$ const.

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$\sim \phi^w$ $\quad\sim g_\alpha^2\quad$ const.

In M-theory variables: $Y^2 = J^{ab}J^{cd}J^{ef}\kappa_{ace}\kappa_{bdf} \simeq \left[g_\alpha^2 g_\beta^2 g_\gamma^2\right]_{5\mathrm{d}}$

$\Longrightarrow$ Smooth function in M-theory moduli space

The (type IIA) scalar curvature

$$\boxed{R_{\mathrm{IIA}}/2 = -\,n_V(n_V+1) + Y^2}$$

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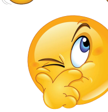
Can only diverge at a boundary of M-theory moduli space (strong gauge coupling)

Types of CY_3 (M-th) Kähler boundaries: *Witten '95*

- Curve collapses to a point → flop transition [no divergence] 
- Divisor collapses to a curve → su(2) enhancement [no divergence] 
- Divisor collapses to a point → non-Lagrangian SCFT [divergence] 
- Infinite distance boundary → weak coupling regime [uncertain] 

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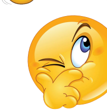
- Infinite distance boundary → weak coupling regime [uncertain] 

Fully collapsing divisors D are non-Nef and generalised del Pezzo

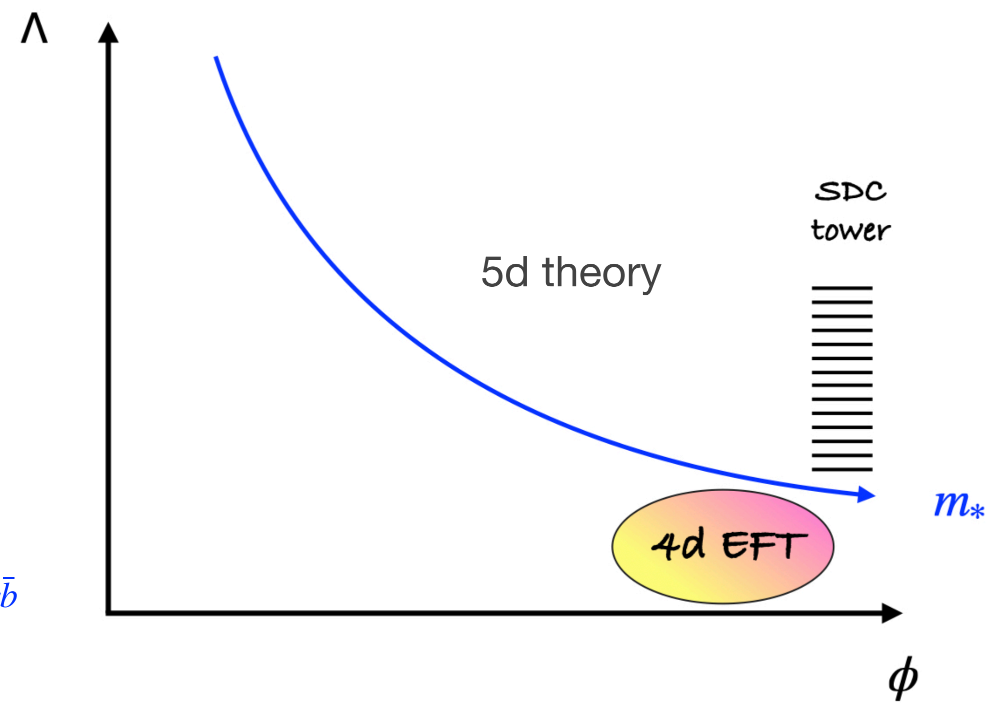
In type IIA variables D does not collapse, but stays of constant volume while $\mathrm{Vol}_X \sim \phi^w$ diverges $\Longrightarrow$ a gauge coupling remains constant along the limit

The 4d EFT viewpoint

A constant gauge coupling along the limit suggests that there is always some interacting gauge theory below the SDC scale, despite the exponential fall-off

Let us reconsider the kinetic terms in units of the 4d cut-off $m_*^2 = M_{\rm P}^2/4{\rm Vol}_X$:

$$M_{\rm P}^2 \int g_{ab}\, dT^a \wedge * d\bar{T}^{\bar{b}} \implies m_*^2 \int I_{ab}\, dT^a \wedge * d\bar{T}^{\bar{b}}$$



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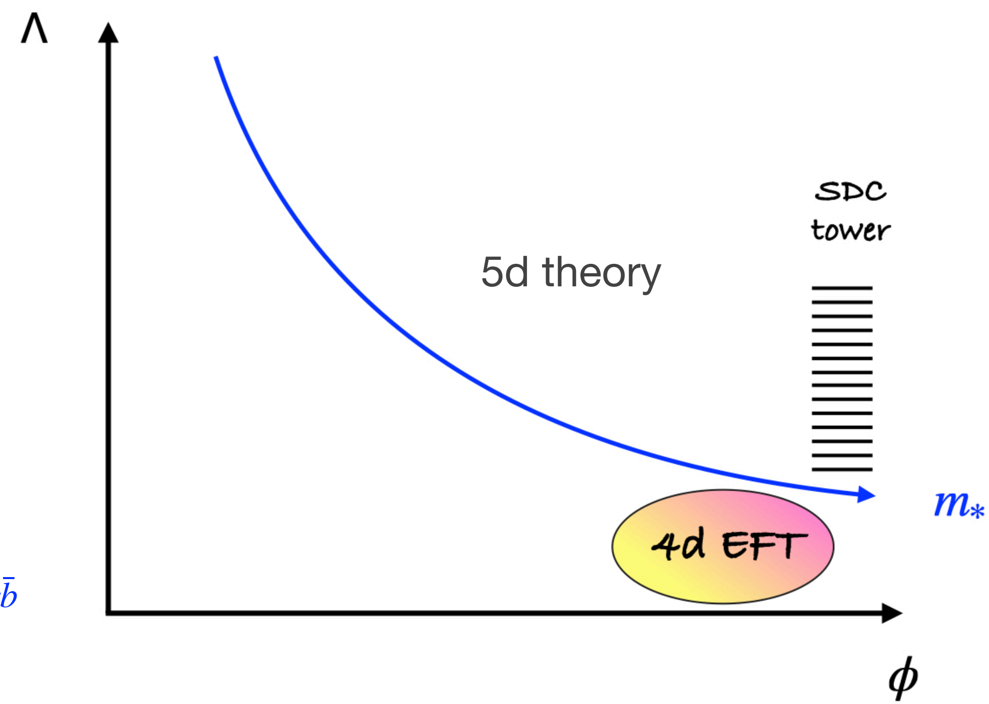

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$\Longrightarrow$ In 4d EFT units, most kinetic terms diverge along the infinite distance limit, but some remain constant when some gauge couplings do as well

Directions that belong to $\ker k_{ab}$, with $k_{ab} \equiv \kappa_{abc} e^c$

$\ker k_{ab} \neq 0 \iff D \subset X_6$
contractible divisor

The 4d EFT viewpoint

Below the SDC scale we recover a 4d N=2 rigid field theory

$$S^{\mathrm{VM}}_{\mathrm{4d,rigid}} = m_*^2 \int I_{\sigma\rho} dT^\sigma \wedge * d\bar{T}^{\bar{\rho}} + \frac{1}{2}\int I_{\sigma\rho} F^\sigma \wedge *_4 F^\rho + R_{\sigma\rho} F^\sigma \wedge F^\rho$$

The dynamical fields are $T^\sigma \in \ker k_{ab}$ $\longleftarrow$ SDC direction ϕ excluded!

Rigid prepotential: $$F_{\mathrm{rigid}} = -\frac{1}{6}\kappa_{\sigma\rho\tau} T^\sigma T^\rho T^\tau + \dots$$

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In terms of the Kähler potential:

$$M_{\mathrm{P}}^{-2} K = -\log\left(\mathscr{K}_0 + \mathscr{K}' + \ldots\right) \simeq -\log \mathscr{K}_0 - \frac{\mathscr{K}'}{\mathscr{K}_0} + \ldots$$

$T^\sigma \in \ker k_{ab}$

$$\Longrightarrow \quad m_*^{-2} K_{\mathrm{rigid}} = -\frac{2}{3}\mathscr{K}'$$

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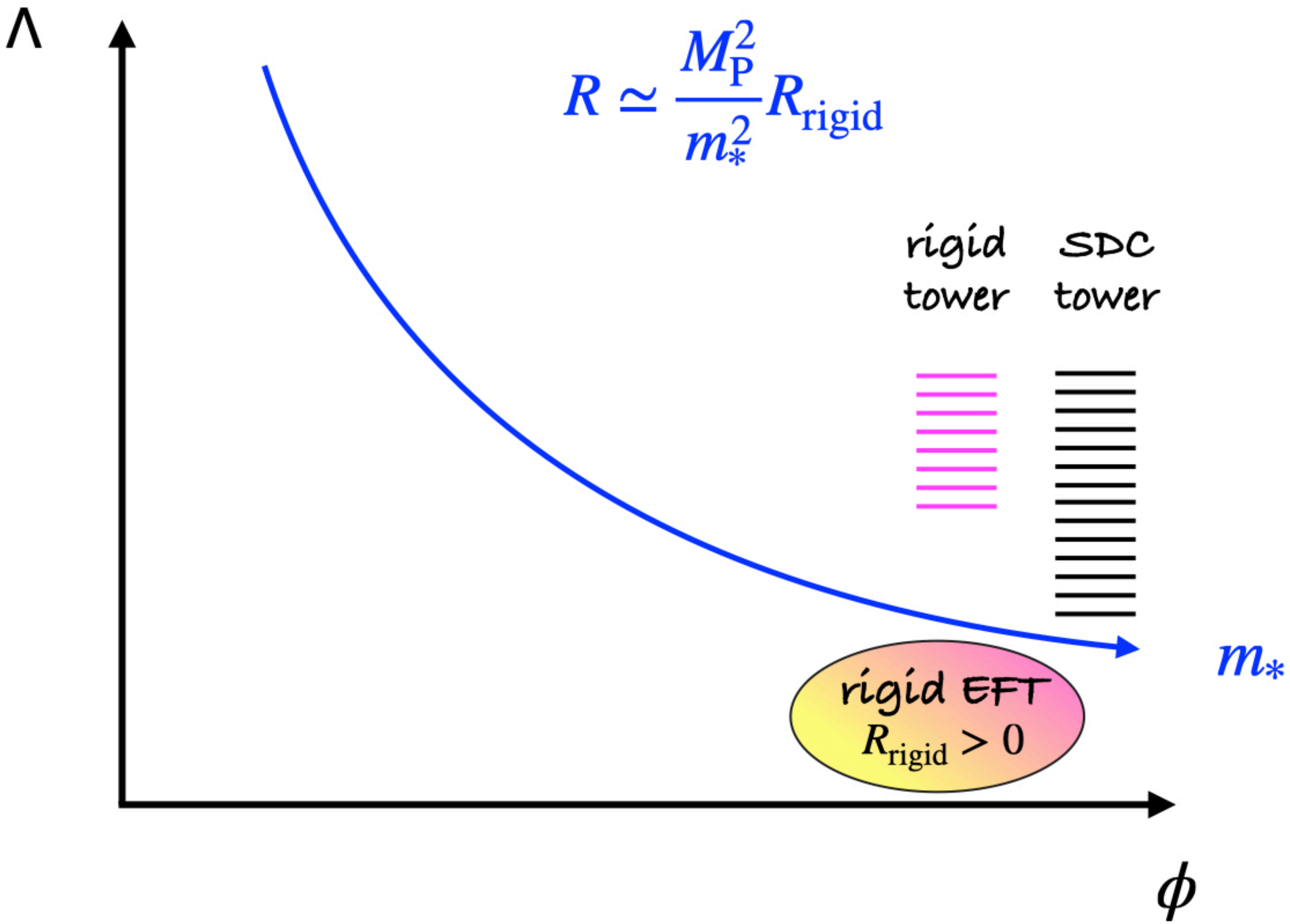
Rigid prepotential: $$F_{\mathrm{rigid}} = -\frac{1}{6}\kappa_{\sigma\rho\tau} T^\sigma T^\rho T^\tau + \dots$$

The curvature of an N=2 rigid theory is always positive:

$$R_{\mathrm{rigid}}/2 = \frac{1}{4} I^{\sigma\rho} I^{\tau\eta} I^{\mu\nu} \kappa_{\sigma\tau\mu}\kappa_{\rho\eta\nu} \simeq \frac{Y^2}{4\mathrm{Vol}_X} = \frac{m_*^2}{M_{\mathrm{P}}^2} Y^2$$

$$\Longrightarrow \quad \boxed{R_{\mathrm{IIA}} \simeq -2n_V(n_V+1) + \frac{M_{\mathrm{P}}^2}{m_*^2} R_{\mathrm{rigid}}}$$

The 4d EFT viewpoint

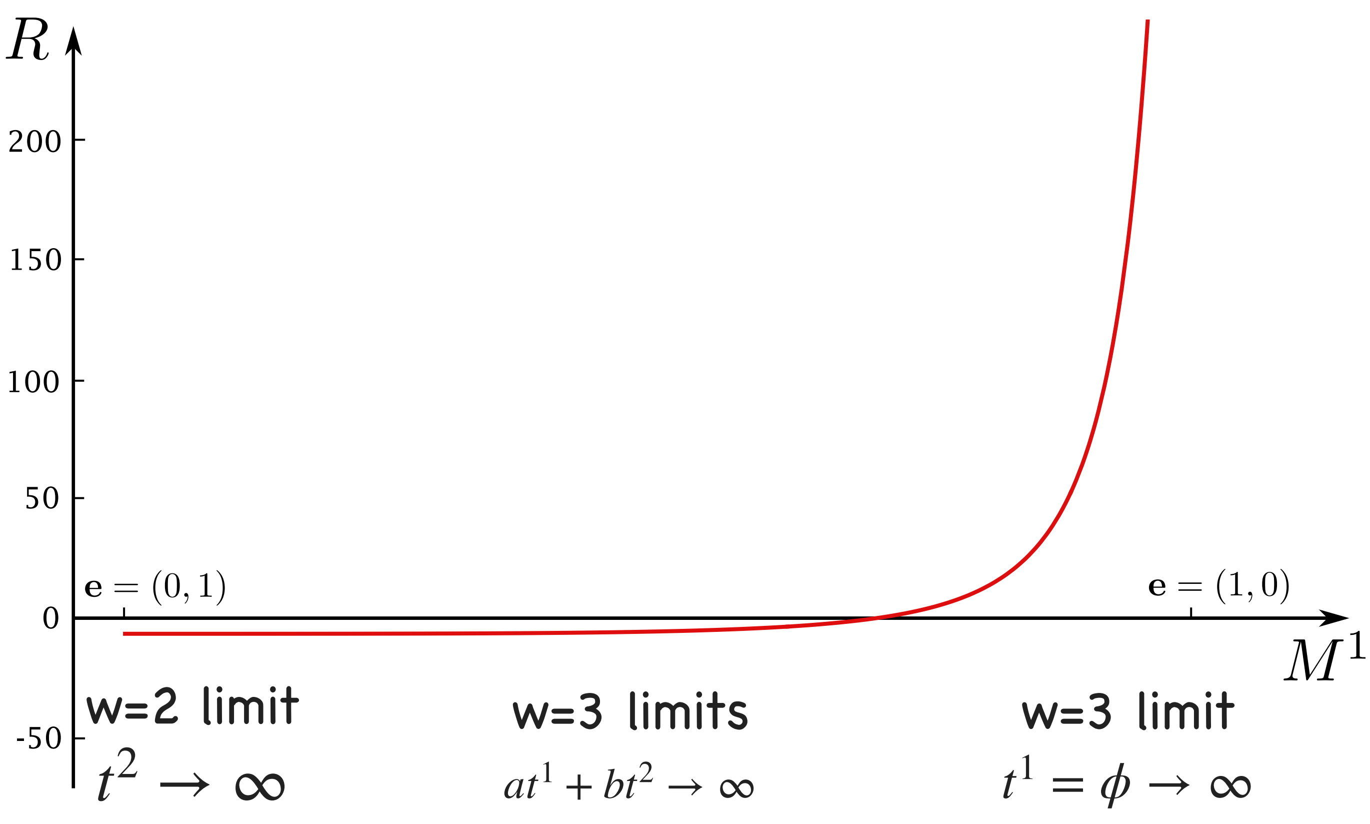


An example

Swiss-cheese CY

Calabi-Yau $X_6 = \mathbb{P}^{(1,1,1,6,9)}[18]$ with two Kähler moduli, smooth fibration with base $\mathbb{P}^2$

$$\kappa_{111} = 9 \qquad \kappa_{112} = 3 \qquad \kappa_{122} = 3 \qquad \kappa_{222} = 0$$



$$R_{\mathrm{IIA}} = \frac{9}{\left(t_0^2\right)^3}\phi^3 + \mathcal{O}(\phi^2)$$

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$\kappa_{111} = 9 \qquad \kappa_{112} = 3 \qquad \kappa_{122} = 3 \qquad \kappa_{222} = 0$

$$k_{ab} = \begin{pmatrix} 9 & 3 \\ 3 & 1 \end{pmatrix}$$

$$\downarrow$$

$$D_B = D_1 - 3D_2 \in \ker k_{ab}$$

$$\downarrow$$

$$\kappa_{BBB} = -9$$

$$\downarrow$$

$$F_{\text{rigid}} = \frac{3}{2}(T^B)^3$$

$$\downarrow$$

$$R_{\text{rigid}} = \frac{3}{2(t^2)^3}$$

$$\downarrow$$

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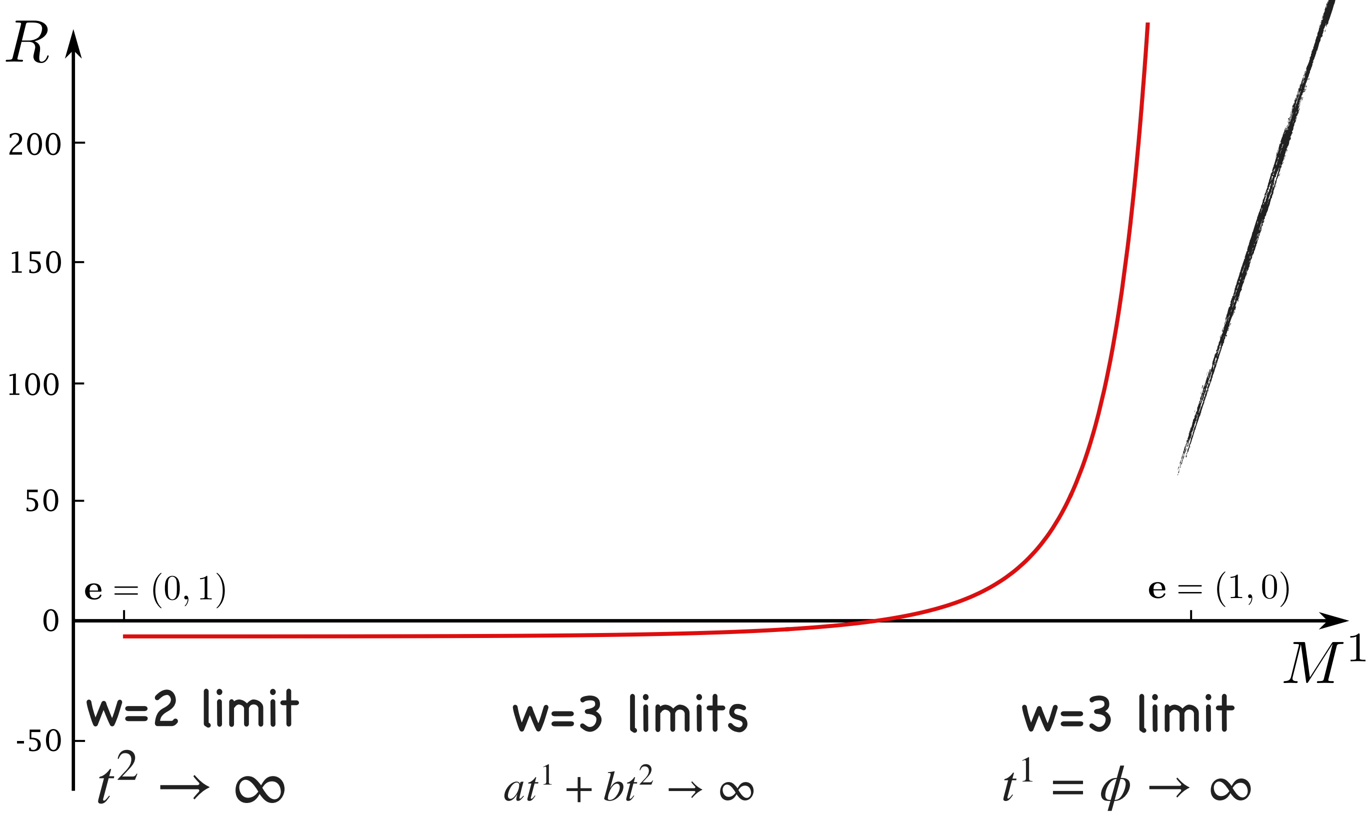


The Curvature Criterion

Along a geodesic trajectory of infinite distance, a moduli space scalar curvature that diverges asymptotically implies the presence of a field theory sector that is decoupled from gravity.

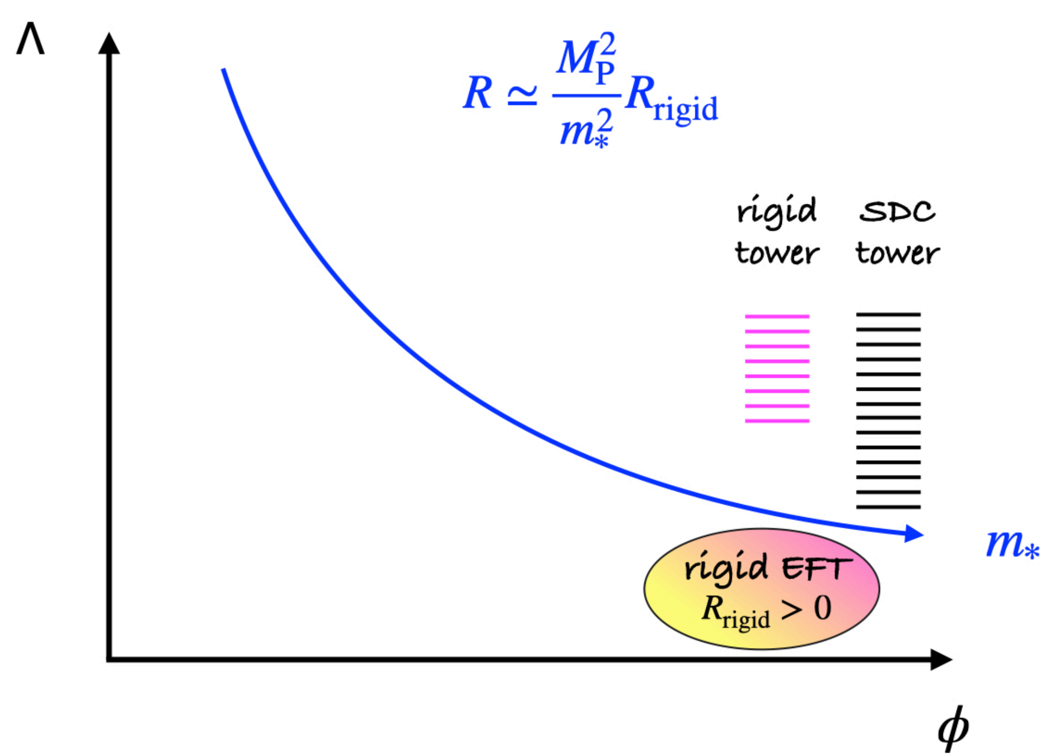
Interpretation: curvature divergence sourced by a gauge theory that dominates over gravity

In our case, the decoupling happens due to a hierarchy of kinetic terms

For the divergence we need $R_{\text{rigid}} \neq 0$

New tower of charged particles

Limits "near" the divergent one have a positive and large asymptotic curvature. Otherwise it is negative.



The Curvature along different limits

Case		$R^{\mathrm{cl}}_{\mathrm{IIA}}(\phi \to \infty)$
w=3	$r = n_V$	$-2n_V^2 + n_V + \mathfrak{C}$
	$r < n_V$	$R^{\mathrm{cl}}_{\mathrm{rigid}} \frac{2}{3} k \phi^3$
w=2	$r = n_V$	$-2n_V^2 + n_V$
	$r < n_V$, smooth	$-2(n_V^2 - 2n_V + 3)$
	$r < n_V$, non-smooth	$\left[-2n_V^2 + 4n_V - 3r\right]^*$
w=1	$r = n_V - 1$	$-2(n_V^2 - 2n_V + 3)$
	$r < n_V - 1$	$R^{\mathrm{cl}}_{\mathrm{rigid}} 2 k_{\Sigma\Lambda} t_0^{\Sigma} t_0^{\Lambda} \phi$

$$n_V = h^{1,1}(X_6)$$

$$\mathfrak{C} = -\frac{k}{6} k^{ab} k^{cd} k^{ef} \kappa_{ace} \kappa_{bdf}$$

$$r = \mathrm{rank}(k_{ab}) \qquad k_a = \kappa_{abc} e^b e^c$$

$$k_{ab} = \kappa_{abc} e^c \qquad k = \kappa_{abc} e^a e^b e^c$$

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$$n_V = h^{1,1}(X_6)$$

$$\mathfrak{C} = -\frac{k}{6} k^{ab} k^{cd} k^{ef} \kappa_{ace} \kappa_{bdf}$$

$R^{\text{cl}}_{\text{rigid}} = 0$, but there can be divergence due to ws instanton corrections

$r = \text{rank}(k_{ab})$ $\quad$ $k_a = \kappa_{abc} e^b e^c$

$k_{ab} = \kappa_{abc} e^c$ $\quad$ $k = \kappa_{abc} e^a e^b e^c$

Aftermath

We have computed the asymptotic behaviour of the scalar curvature for a huge set of cases, using the recent classification of infinite distance limits

Emerging picture: the asymptotic curvature seems to be the result of two competing effects

- Gravitational contribution, that is negative and asymptotically constant along infinite distance limits, as 4d gravity decouples due to the SDC
- Rigid field theory contribution, that is non-negative and diverges if the theory remains dynamical below the SDC scale along the limit

Positive finite curvature is recovered when some gauge interactions are much stronger than that of the graviphoton, but not parametrically

If this is a general feature, it could be used to detect corners of the string theory landscape with non-trivial EFTs

Conclusions

- Swampland criteria have proven to be particularly powerful in asymptotic regions of field space. In this work we have focused on the behaviour of the scalar curvature, for which there is a proposal and counterexamples that challenges it.

- We have analysed the asymptotic behaviour of the scalar curvature in 4d N=2 moduli spaces, which have been recently considered in light of the SDC.

- We have focused on type IIA CY VM sector at large volume, which provide a huge set of limits, recently classified in light of the SDC. In the case the SDC tower always involved D0-branes, so there is an M-theory description.

- The take-home message is that curvature divergences appear when there is a non-trivial EFT below the SDC scale, that decouples from gravity.

- The next step is to test this picture in more general setups, and see if similar lessons can be drawn for other metric invariants or components of the Riemann curvature.


Thank you!