Conference: Geometry, Strings and the Swampland Program, Ringberg Castle, March 18, 2024

Navigating the Landscape (and the swampland)

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Bibliography

Ongoing work with:

Ahmad Ghodsi, Francesco Nitti, Christopher Rosen, to appear soon

Ahmad Ghodsi, Francesco Nitti

Published in ArXiv:2309.04880

Ahmad Ghodsi, Francesco Nitti, Valentin Noury

Published in ArXiv:2209.12094

and a whole line of previous papers.

Introduction

• There are a lot of problems in ST and QFT that use the same technical setup.

♠ ST compactifications on products of Einstein manifolds.

♠ Studies of dynamical cobordisms in the swampland program.

♠ RG flows of holographic QFTs on flat or curved manifolds (like Spheres or AdS)

♠ The physics of defects in holographic QFTs

♠ Euclidean Wormholes and their puzzles .

The setup

- The study of classical solutions in effective gravitational theories

$$S_{grav} = M_p^{D-2} \int d^D x \sqrt{g} \Big[R - \frac{1}{2} G_{IJ} \partial\phi^I \partial\phi^J - V(\phi) -$$

$$- \frac{Z_{ab}}{4} F_p^a \cdot F_p^b + \text{anomaly} \quad \text{terms} + \cdots \Big]$$

- The generic metric ansatz (the conical ansatz) is

$$ds^2 = du^2 + \sum_{i=1}^{N} \; e^{2A_i(u)} ds_i^2 \quad , \quad ds_i^2 = \zeta^i_{\mu_i \nu_i} dx^{\mu_i} dx^{\nu_i}$$

where ds_i^2 is the metric of M_i that is an Einstein manifold

$$R_{\mu_i \nu_i} = M_p^2 \; \kappa_i \zeta^i_{\mu_i \nu_i}$$

- κ_i can be scaled to $\pm 1, 0$ by shifting the $A_i(u)$.

- This ansatz is accompanied with

$$\phi^I(u) \quad , \quad F_p^a \sim \sum_{i=1}^{N} \alpha_i^a(u) \cdot \; \text{Volume}_i$$

• The equations boil down to non-linear ODEs for the functions $A_i(u)$, $\phi^I(u)$, $\alpha_i^a(u)$.

• The "parameters" that enter in these equations are κ_i and $V(\phi^I), Z^{ab}(\phi^I)$, that we shall call "the bulk data".

• The equations can be interpreted in terms of dimensional reduction: if we consider as "internal", a subset of the manifolds M_i, then the associated A_i and a_i^a become scalar fields in the lower dimensional theory.

• The kinetic metric of the A_i originates in the Ricci scalar, the metric of the axions a_i^a originates from Z^{ab} and extra terms in the scalar potential appear, originating from the curvatures κ_i of the internal manifolds.

• The simplest solutions of these equations are the critical solutions:

♠ Constant scalars sitting at the extremal points of the scalar potential.

♠ Scalars running off to the boundaries of scalar field space (which have an interpretation in the higher-dimensional incarnations of the theory).

• All such solutions have some scaling symmetries that are encoded in the geometry

♠ Apart from the bulk data, the solutions depend on integration constants.

♠ Such integration constants are fixed by a combination of boundary conditions and regularity of bulk solutions.

• The interpretation of boundary conditions depend on the context (asymptotically, flat, AdS, $\cdots$.)

• There are many possible boundaries in the ansatz:

♠ Boundaries originating in the Einstein manifolds (along u):

1. Space-like boundaries if for example M_i=Anti de Sitter.

2. Time like boundaries if for example M_i=de Sitter.

3. Light-like boundaries when for example M_i=Minkowski.

♠ Boundaries transverse to u:

These are space-like (AdS-type) boundaries.

The related cosmological ansatz

- In the previous ansatz the "radial" coordinate u was space-like.

- In asymptotically AdS cases it is the holographic coordinate.

- There is a closely related "cosmological ansatz"

$$ds^2 = -dt^2 + \sum_{i=1}^{N} e^{2A_i(t)} ds_i^2 \quad , \quad ds_i^2 = \zeta^i_{\mu_i \nu_i} dx^{\mu_i} dx^{\nu_i}$$

and all other fields are functions of t.

- The equations of this ansatz are very closely related to those of the space-like ansatz:

$$u \to t \quad , \quad V \to -V \quad , \quad \kappa_i \to -\kappa_i$$

- Therefore complete knowledge of the spacelike ansatz provides complete knowledge on the time-like ansatz (and vice versa)

- The regularity conditions change when $u \Leftrightarrow t$.

Navigating the landscape

• As mentioned the simplest solutions are the critical solutions.

• They include solutions that run off to the boundaries of field space.

• Such boundaries correspond to:

♠ Decompactification limits when M_i are driven to infinite size.

♠ Conifold limits when M_i are driven to zero size (in a regular way).

• **It can be shown that these critical solutions are the end-points of the general solutions**.

• Therefore the classification problem of ALL solutions translates to:

♠ When are there solutions between two given critical solutions?

♠ What parameters control such solutions?

• It turns out that these are non-trivial questions.

A simple landscape

• We assume a single scalar plus gravity: (Einstein-dilaton theory)

$$S_{Bulk} = M_P^{d-1} \int du\, d^d x \sqrt{-g} \Big(R - \frac{1}{2} g^{ab} \partial_a \Phi \partial_b \Phi - V(\Phi) \Big) .$$

and a conical space with a single slice geometry (a constant curvature manifold)

$$ds^2 = du^2 + e^{2A(u)} \zeta_{\mu\nu} dx^\mu dx^\nu , \qquad \Phi = \Phi(u)$$

• The slice is a manifold M_ζ whose metric ζ is any (constant) negative curvature Einstein metric.

$$R^{(\zeta)}_{\mu\nu} = \kappa \zeta_{\mu\nu} \, , \qquad R^{(\zeta)} = d\kappa \, , \qquad \kappa = -\frac{(d-1)}{\alpha^2} \, .$$

• κ can be rescaled by a shift in $A(u)$.

• The solution is characterized by the scalar field profile $\Phi(u)$ and by the scale factor $A(u)$, which are related via the bulk Einstein equations.

• For all constant curvature metrics, the equations are the same!

• M_ζ can be any of M_d, AdS_d, S^d, dS_d etc.

• The equations of motion read:

$$2(d-1)\ddot{A}+\dot{\Phi}^2+\frac{2}{d}e^{-2A}R^{(\zeta)}=0\,,$$

$$d(d-1)\dot{A}^2-\frac{1}{2}\dot{\Phi}^2+V(\Phi)-e^{-2A}R^{(\zeta)}=0\,,$$

$$\ddot{\Phi}+d\dot{A}\dot{\Phi}-V'=0\,.$$

• They have three integration constants: One of them is not important as it is a shift in u.

The first order formalism

• We define the first order superpotential "superpotentials" (no supersymmetry)

$$\dot{A} \equiv -\frac{1}{2(d-1)} W(\Phi)\,,$$

$$\dot{\Phi} \equiv S(\Phi) \quad , \quad T(\Phi) \equiv R_\zeta e^{-2A}$$

• The gravitational equation transform into an equation with respect to Φ:

$$dS^3 S'' - \frac{d}{2} S^4 - S^2 (S')^2 = \frac{d}{d-1} S^2 V - (d+2) S S' V' + d S^2 V'' - (V')^2\,.$$

and the rest can be computed from:

$$W = \frac{2(d-1)}{d}\Big(S' - \frac{V'}{S}\Big) \quad , \quad T = \frac{d}{2} S (W' - S)\,,$$

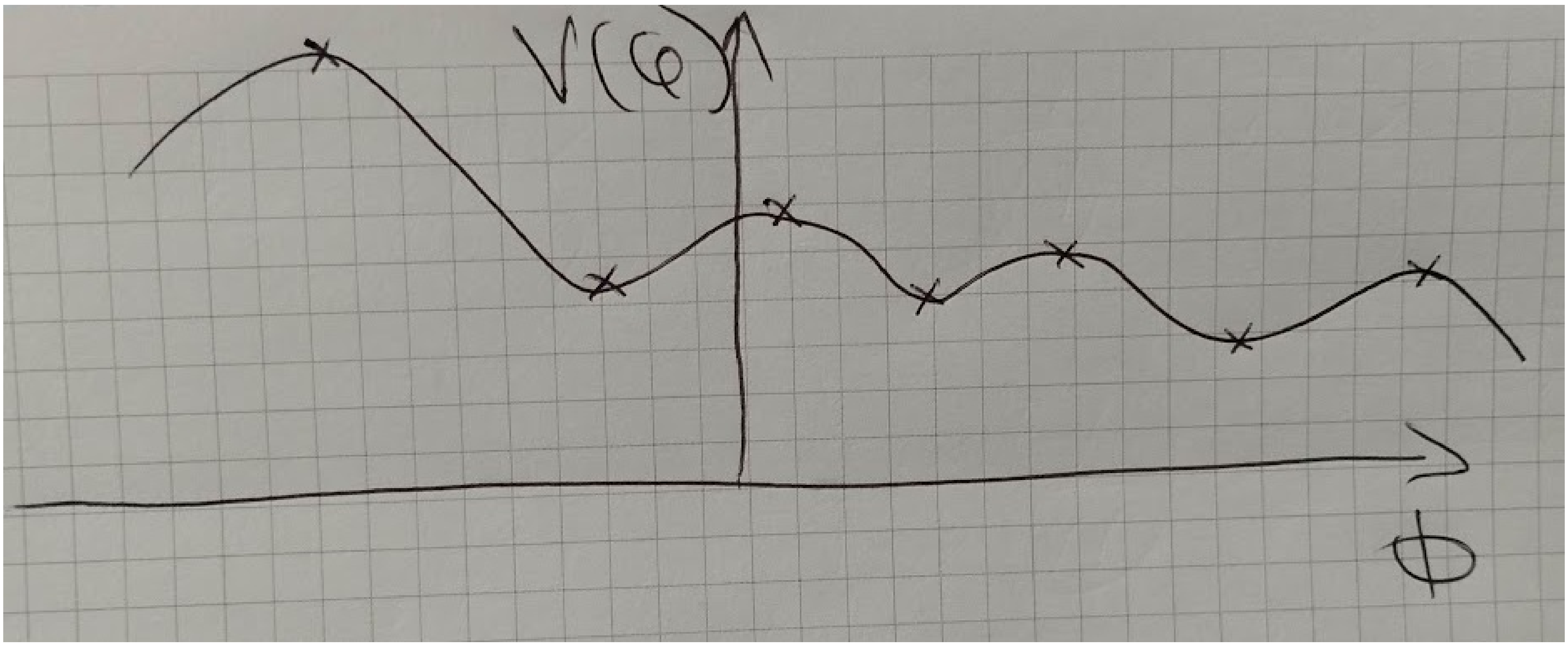
• The initial definitions determine the "trivial integration constants".

• The non-trivial integration constants are hidden in the second order equation for S.

Asymptotically AdS solutions and holography

• The solutions in this case correspond to ground states of d-dimensional holographic theories on the manifold M_ζ.

• When the scalar is constant, it can sit at an extremum of the potential. This is dual to a holographic CFT_d on M_ζ.



- All other solutions have the scalar rolling.

- In holography they have the interpretation of RG flows.

- Where do these flows start an end?

The end-points of non-trivial flows

• A non-trivial flow has two arbitrary parameters:

♠ They are leading boundary conditions on the gravitational side.

♠ They are coupling constants on the QFT side.

• The boundary condition of the scalar corresponds to the dimensionful (relevant) coupling, g perturbing the CFT$_{UV}$.

• If M_ζ has curvature R_M This is another dimensionful coupling of the holographic QFT.

♠ Overall there is a single dimensionless coupling: the dimensionless curvature.

$$\mathcal{R} \equiv \frac{R_H}{g^{\frac{2}{\Delta}}}$$

♠ The physics depends non-trvially on $\mathcal{R}$.

Flat slices: $\mathcal{R} = 0$

• It can be shown that Regular solutions START AND END at extrema of the potential.

• This is the holographic version that a QFT is a flow $CFT_{UV} \to CFT_{IR}$.

• Near a maximum of the potential, there are two branches of solutions known as the $-$ and the $+$ branch.

$$\ell \, W_{\pm} = 2(d-1) - \frac{\Delta_{\pm}}{2}(\phi - \phi_0)^2 + \cdots$$

$$\Delta_{\pm} \equiv \frac{d}{2} \pm \sqrt{\frac{d^2}{4} - \frac{d(d-1)}{2}\frac{V''}{V}}$$

♠ The $-$ branch contains the generic solutions that contain two integration constants. It correspond to the CFT perturbed by a relevant operator with non-zero coupling.

♠ The $+$ branch contains only the special solution for which the leading integration constant vanishes (relevant vev-driven flow).

• For both types of solutions above, the metric has an AdS boundary at the maximum.

• We denote these asymptotics as $Max_{\pm}$.

• Near a minimum of the potential we also have the $+$ and $-$ branches of solutions.

♠ The $-$ branch contains the generic solution (two integration constants). The geometry there is the center of AdS.

♠ The $+$ branch contains the special solution (one integration constant). The bulk metric has an AdS BOUNDARY in this case

• We denote these asymptotics as $Min_{\pm}$.

• The Min_{-} asymptotics describe the IR end-point of a flow.

• The Min_{+} asymptotics describes a UV fixed-point perturbed by the vev of an irrelevant operator.

• $Max_\pm$ and Min_+ are associated to AdS boundaries and therefore to QFT UV fixed points.

• Min_-, to a shrinking slice geometry and therefore to an IR Fixed point.

• The $+$ branch solutions, as they contain less integration constants, exist only in fine-tuned cases.

QFTs on $\mathcal{R} > 0$

• This case includes S^d and dS^d as slices.

• Around Maxima of the potential we still have the $Max_\pm$ solutions with a similar interpretation as in the flat case.

• But at minima the Min_- does not exist! Only the Min_+ exists but this has an AdS boundary.

♠ However here the flows can end at any point, not only at extrema of the potential.

♠ At such a point the Manifold M_ζ shrinks, REGULARLY, to zero size.

♠ This reflects the existence of a non-trivial gap in the dual QFT.

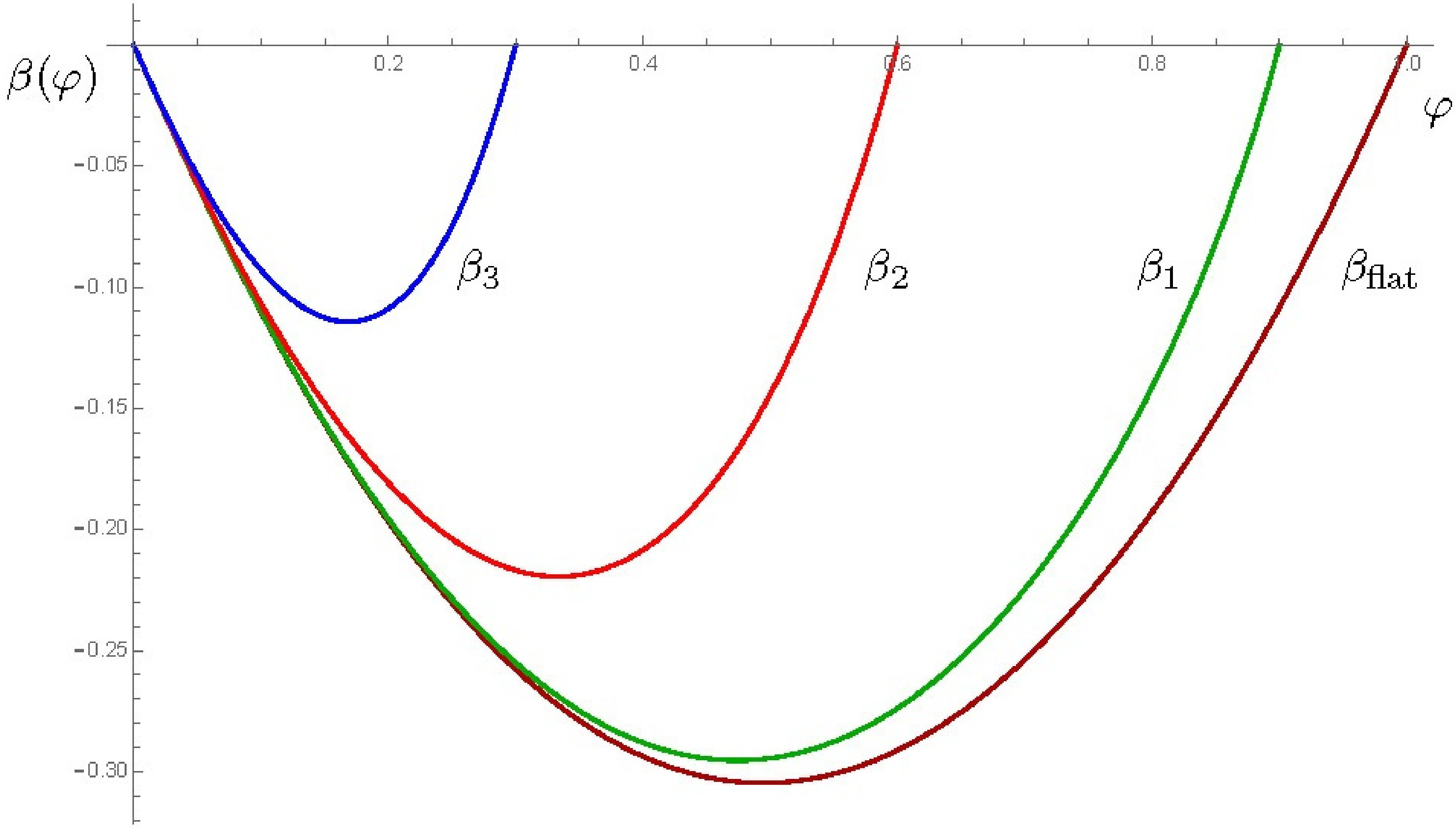


Figure 17: Solutions for $\beta(\varphi) = -2(d-1)\frac{S(\varphi)}{W(\varphi)}$ with $\mathcal{R} \geq 0$ and for the potential (5.1) with $\Delta_- = 1.2$. The five solutions $\beta_i(\varphi)$ with $i = 1, \ldots, 5$ differ in the value of their IR endpoint φ_0.

QFTs on $\mathcal{R} < 0$

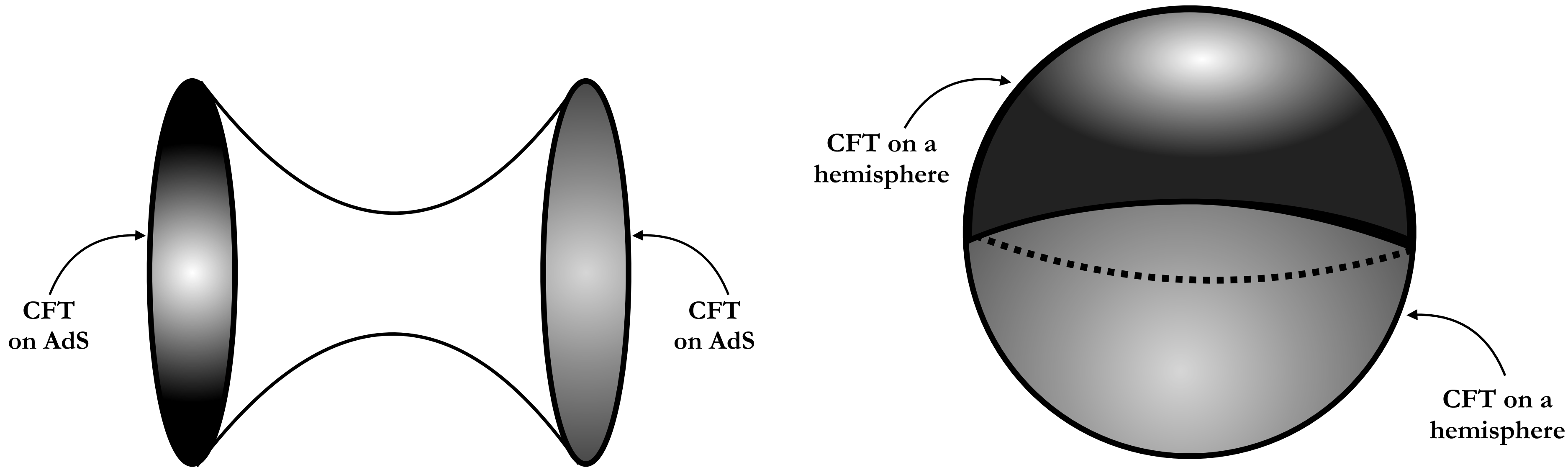
• This includes slices that are AdS or other manifolds with constant negative curvature.

• Like $\mathcal{R} > 0$ Min_- asymptotics do not exist. Only $Max_\pm$ and Min_+ which are all UV Fixed points (AdS boundaries).

• Unlike $\mathcal{R} > 0$, the flows cannot end anywhere else, because a shrinking negative curvature space always has a curvature singularity.

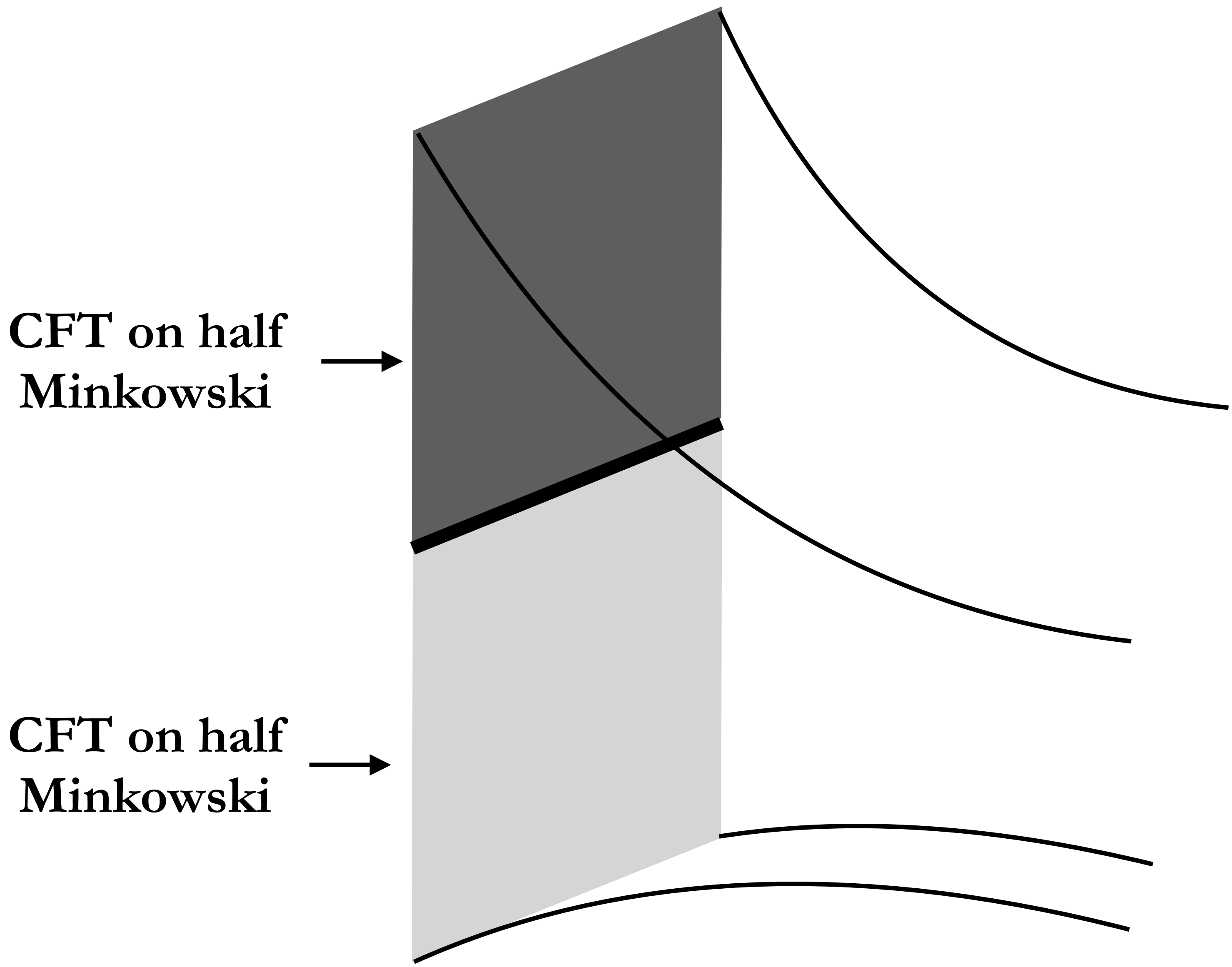
• Therefore in this case, flows have only UV asymptotics=AdS boundaries.

• This is compatible with the flows, because in all other cases ($\mathcal{R} \geq 0$) the scale factor was monotonic. But for $\mathcal{R} < 0$ it is not!



• When the slices have finite volume, this solution is a wormhole with two boundaries.

• When the slices are full AdS_d spaces, then the dual describes two CFTs interacting via their common boundary


CFT on half Minkowski
CFT on half Minkowski

The Classification of complete flows: $\mathcal{R} = 0$

♠ All flows start and end at extrema of the potential.. They have a single AdS boundary.

• (Max_-, Min_-). This is the generic relevant flow driven by a relevant operator.

• (Max_+, Min_-). This is a flow driven by the vev of a relevant operator.

• (Min_+, Min_-). This is a flow driven by the vev of an irrelevant operator.

The Classification of complete flows: $\mathcal{R} > 0$

- In this case, although flows can start at extrema of the potential, (both maxima as $Max_{\pm}$ and minima as Min_{+}), they always end at intermediate points, not at extrema.

- The end is always an IR end-point where the slice volume vanishes.

The Classification of complete flows: $\mathcal{R} < 0$

- This is the most complex case.

- All regular flows must start and end at extrema of the potential.

- The asymptotic solution Min_- does not exist because $\mathcal{R} \neq 0$,

- We have in total the following $3 \times 3 = 9$ options,

$$(Max_- \quad , \quad Max_+ \quad , \quad Min_+) \quad \otimes \quad (Max_- \quad , \quad Max_+ \quad , \quad Min_+)$$

all of them having two AdS boundaries.

- (Max_-, Max_-).

- (Max_-, Max_+) and its reverse (Max_+, Max_-).

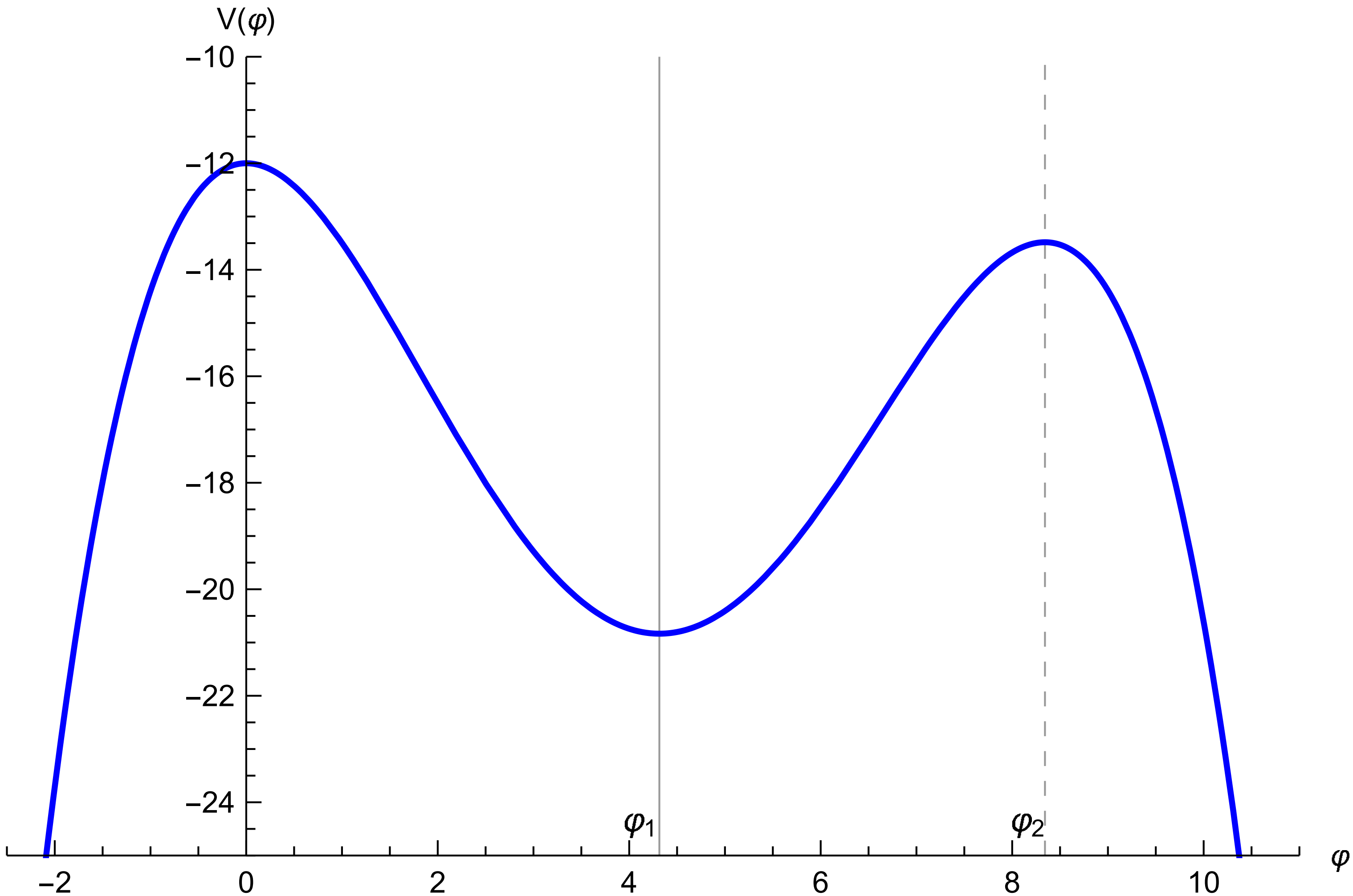
- (Max_-, Min_+) and its reverse, (Min_+, Max_-).

- (Max_+, Min_+) and its reverse, (Min_+, Max_+)

- (Min_+, Min_+) and (Max_+, Max_+).

A concrete example

- We pick $d = 4$ and a generic quartic potential



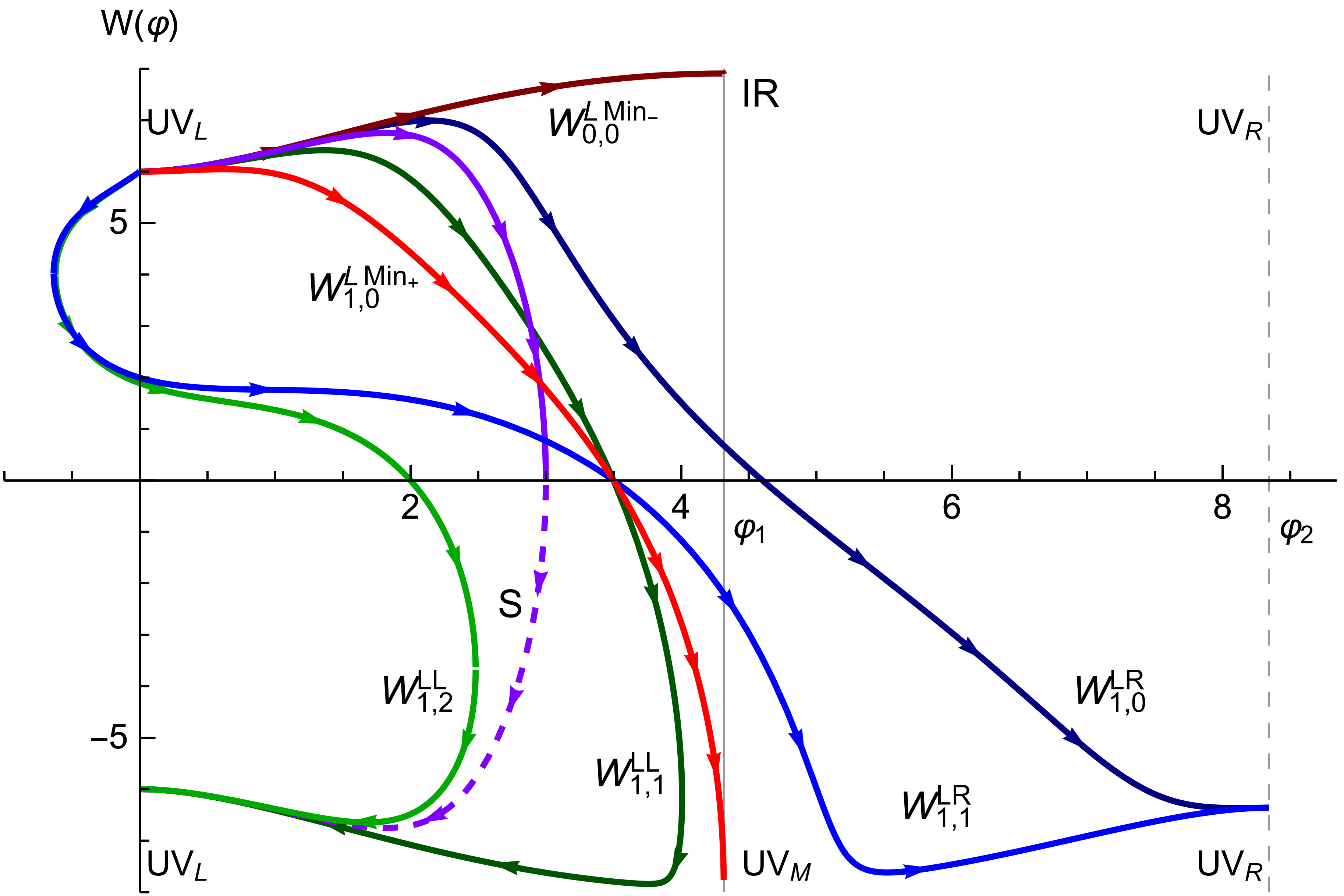
• "Technical" definitions:

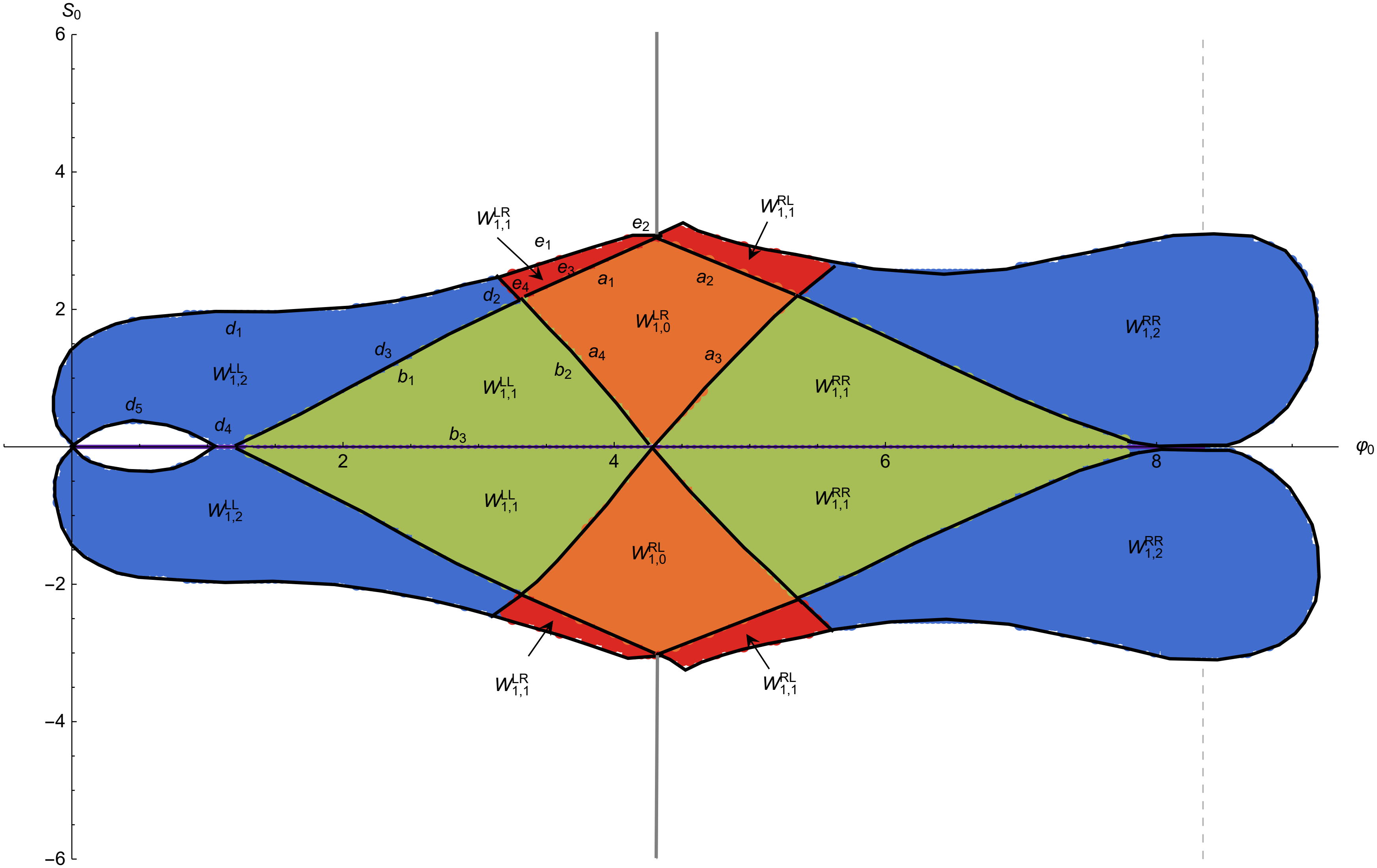
♠ A-bounce is a point where $\dot{A} = 0 \to W = 0$. It always exists when the slice curvature is negative.

• We denote the position of an A-bounce by Φ_0.

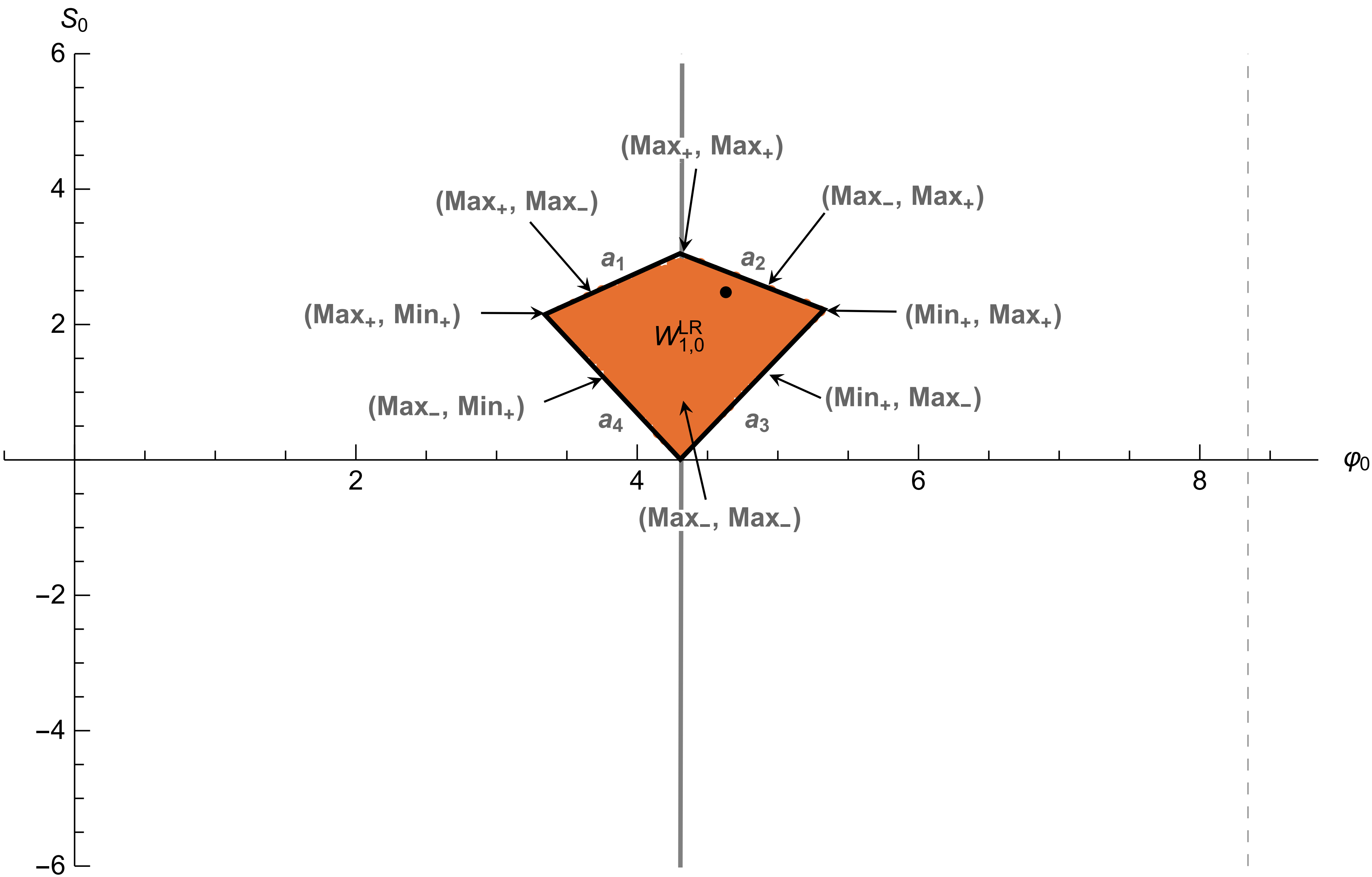
♠ Φ-bounce is a point where $\dot{\Phi} = 0 \to S = 0$.

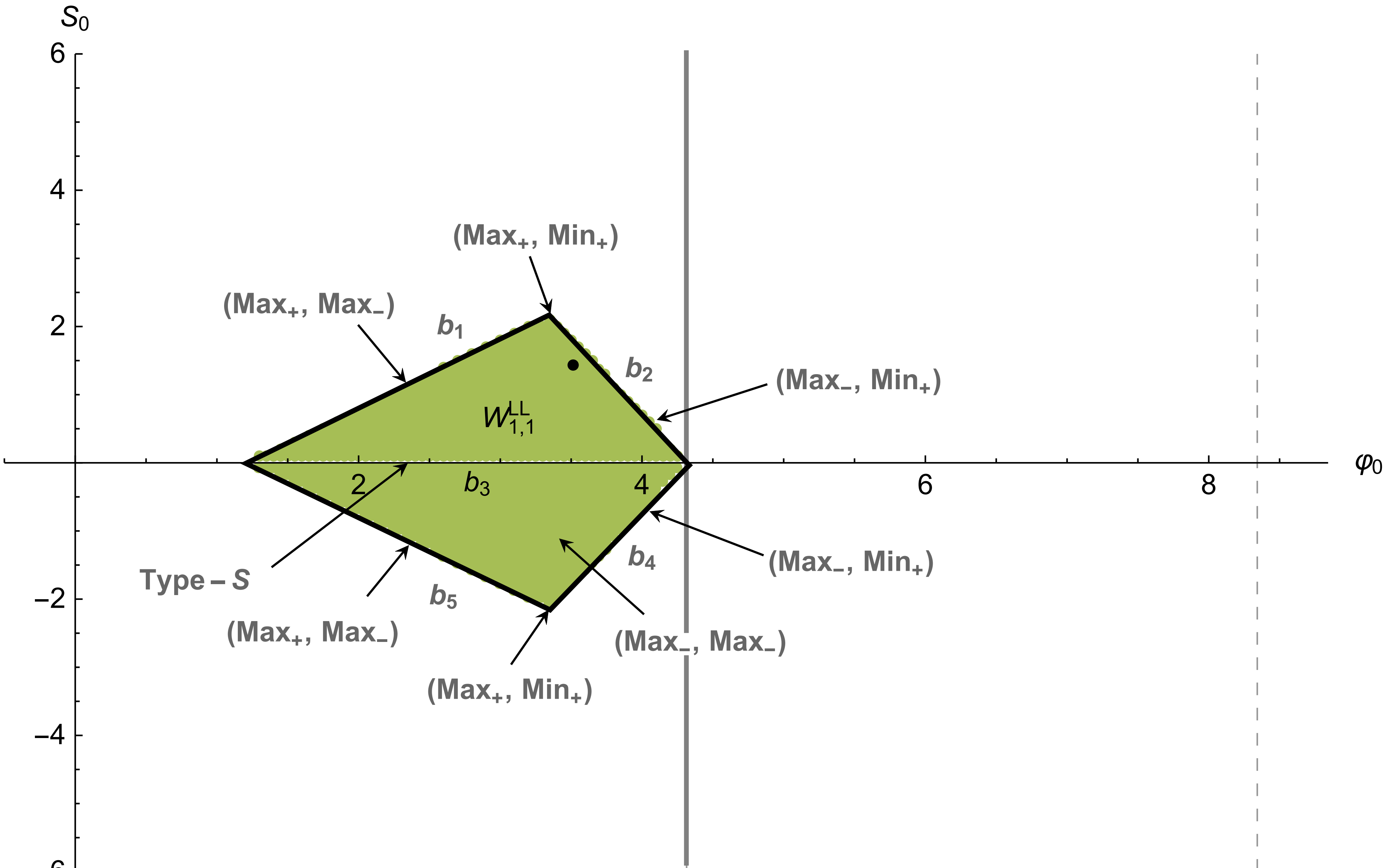
The space of solutions




S_0
6
4
2
−2
−4
−6
φ_0
2
4
6
8
$W^{LR}_{1,1}$
$W^{RL}_{1,1}$
e_1
e_2
e_3
e_4
a_1
a_2
a_3
a_4
d_1
d_2
d_3
d_4
d_5
b_1
b_2
b_3
$W^{LR}_{1,0}$
$W^{RL}_{1,0}$
$W^{LL}_{1,2}$
$W^{LL}_{1,1}$
$W^{RR}_{1,1}$
$W^{RR}_{1,2}$
$W^{LR}_{1,1}$
$W^{RL}_{1,1}$

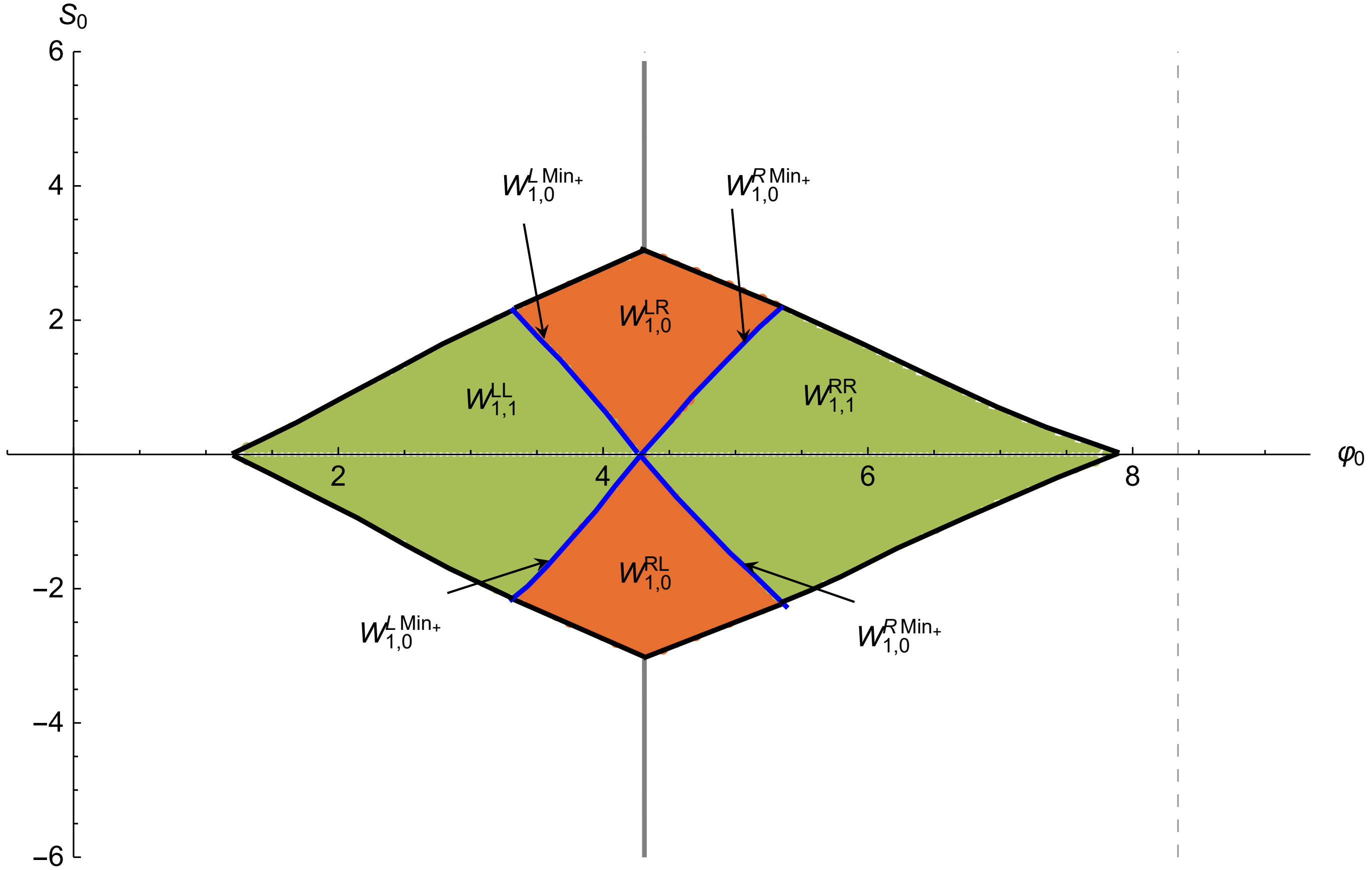
The region boundaries and tuned flows

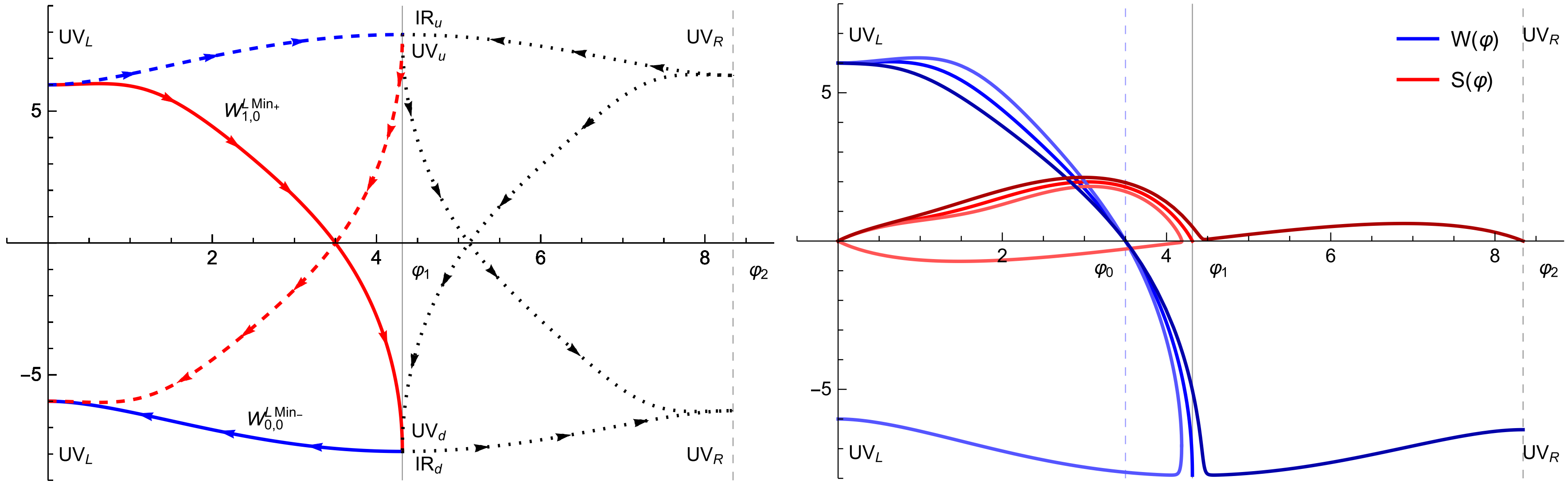




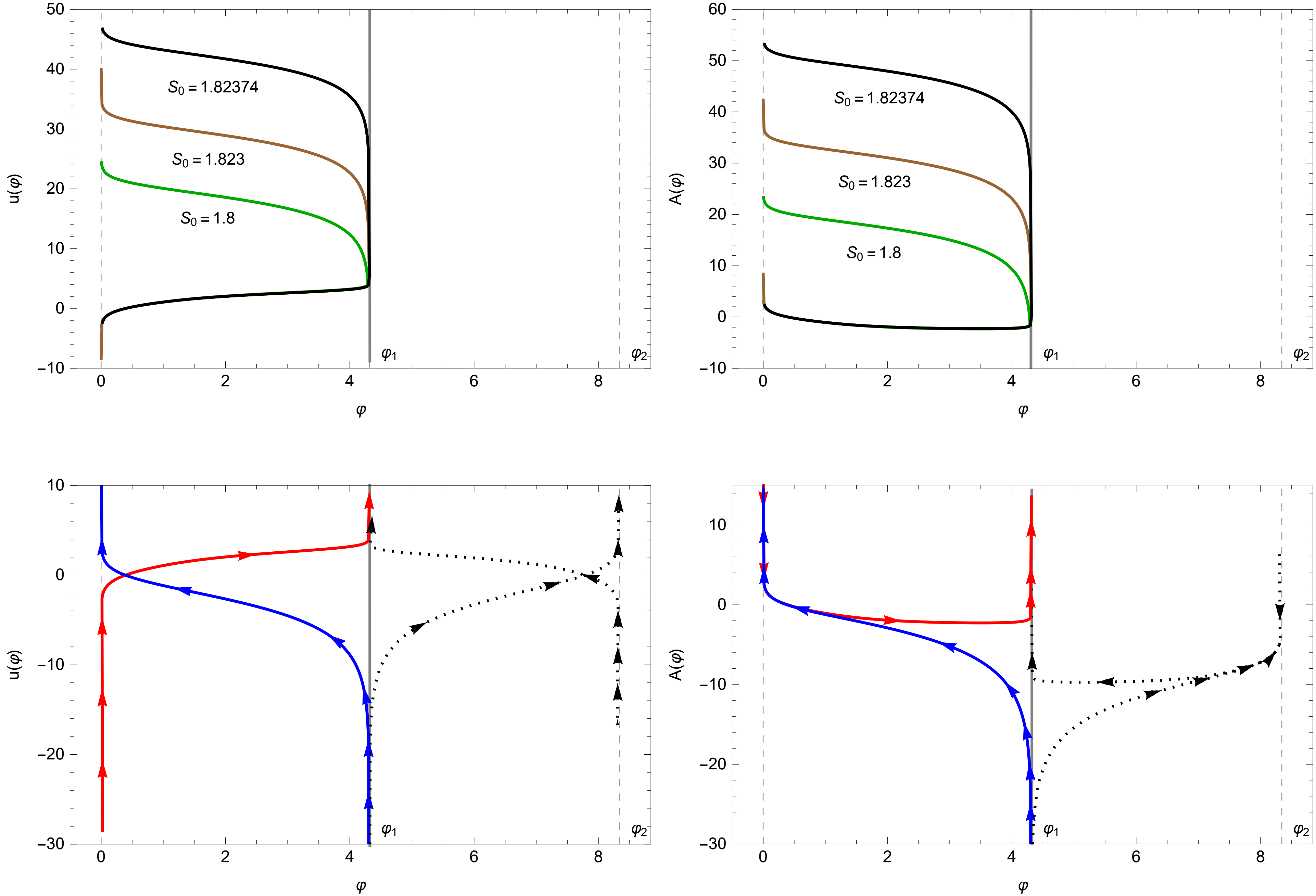
S_0
6
4
2
−2
−4
−6
2
4
6
8
φ_0
(Max+, Min+)
(Max+, Max−)
b_1
b_2
(Max−, Min+)
$W^{LL}_{1,1}$
b_3
(Max−, Min+)
b_4
Type − S
b_5
(Max+, Max−)
(Max−, Max−)
(Max+, Min+)

Flow fragmentation, walking and emergent boundaries





(a): An example of an RG flow between a maximum and a minimum. For the solid curves, (Max_-, Min_+) is a flow between a UV fixed point at maximum $\Phi = 0$ and another UV fixed point at the minimum $\Phi = \Phi_1$. For the (Max_-, Min_-) part of the solution, the minimum is an IR fixed point. The dashed curves show the flipped image of the solid curves. The black dotted curves are other possible RG flows with the same UV fixed points. (b): At a fixed Φ_0 when the value of S_0 is exactly on the border of type $W^{LR}_{1,0}$ and type $W^{LL}_{1,1}$, we have the $W^{LMin_+}_{1,0}$ branch solution (the middle flow). If we increase or decrease the value of S_0 we have the $W^{LR}_{1,0}$ or $W^{LL}_{1,1}$ solutions respectively.



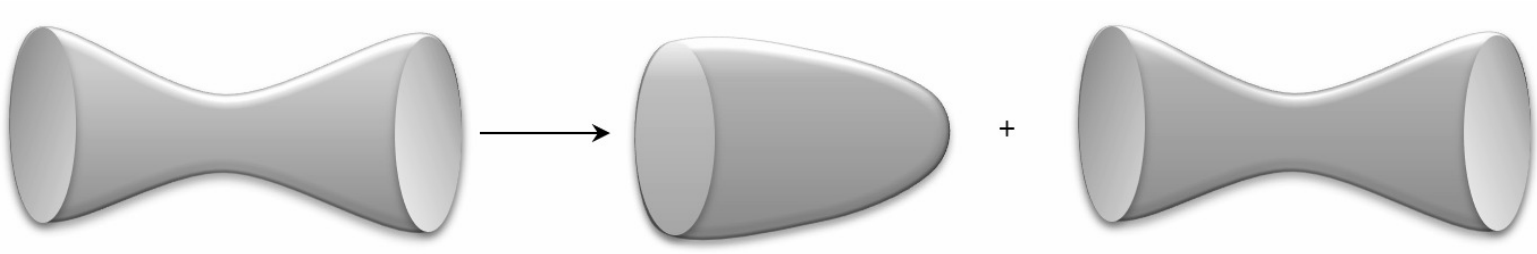
The behavior of the holographic coordinate and scale factor in terms of Φ for the $W_{1,0}^{LMin_+}$ and $W_{0,0}^{LMin_-}$ RG flows. The red curve belongs to $W_{1,0}^{LMin_+}$ and the blue to $W_{0,0}^{LMin_-}$.

• In this limiting region we have an explicit example of solution fragmentation.

• There are two phenomena visible in this example.

♠ Walking. This the phenomenon when an intermediate AdS region appears between the UV and IR, or between UV and UV as is the case here

♠ The emergence of a new boundary.



$$(Max_-, Max_-) \quad \to \quad (Max_-, Min_-) \quad \oplus \quad (Min+, Max_-)$$

• Such flows can be rotated into cosmological solutions with a cosmological bounce, no singularity and "inflation" at the place of big bag.

Open Ends

• There are also flows that end up at the boundaries of the scalar space: $\Phi \to \pm\infty$. They can be treated with similar tools, and the information is known.

• These cases are interesting holographically as they contain confining holographic theories.

• They are also interesting for swamplanders.

• Analysis involving possitive potentials has already been done but there is more to be done in this direction.

• Hopefully, eventually we will have enough tools to intuitively understand the structure of the space of solutions, and eventually cosmology

THANK YOU!

QFT on AdS

- This problem was first seriously adressed by Callan and Wilczek in 1990.
- Their interest was in IR physics.
- Their motivation were the IR divergences that plagued QCD perturbation theory and which made perturbative calculations hard to control.
- The important property of AdS space for this purpose was that even massless fields, had propagators that vanished exponentially as large distances, like massive fields in flat space.
- The reason is that the Laplacian and other relevant operators have a gap in AdS.
- On the other hand, unlike the sphere, AdS has infinite volume.
- Critical systems are described by mean field theory above the upper critical dimension. But AdS acts as an infinite-dimensional space. Therefore critical fluctuations should be weak in any dimension.

• Generically speaking, AdS is expected to "quench" strong IR physics.

• An extra ingredient is that the QFT on AdS must realize the AdS symmetry that is like conformal invariance in one-less dimension.

Callan+Wilczek

• The structure of instantons is also expected to be different:

♠ In flat space, in QCD we expect to have an instanton liquid rather than a (dilute) instanton gas.

Witten

♠ Above the deconfinement phase transition, we expect an instanton gas instead.

• In AdS an instanton gas is generically expected.

Callan+Wilczek

• Chiral invariance for fermions is broken by boundary conditions in AdS.

• An important ingredient for QFT in AdS: boundary conditions.

A confining gauge theory on AdS_4

• There are two types of boundary conditions: electric (Dirichlet) and magnetic (Neumann)

Aharony+Marolf+Rangamani

♠ With electric: gluons are allowed in AdS, they are gapped, and there is an $SU(N)$ global symmetry at weak coupling. Only boundary currents possible.

♠ With magnetic: electric charges are not allowed in bulk, there are O(1) degrees of freedom, and there is confinement (imposed by the bcs).

• There are also many other boundary conditions associated to subgroups.

• For asymptotically free gauge theories with Dirichlet boundary conditions a confinement/deconfinement phase transition is expected

Aharony+Berkooz+Tong+Yankielowicz

♠ $\Lambda L_{Ads} \gg 1$ Confining phase.

♠ $\Lambda L_{AdS} \ll 1$ Deconfined phase.

• With magnetic boundary conditions one expects confinement at all scales, and a free energy of $O(1)$. This is a kind of trivial confinement as no electric charges are allowed in the bulk.

• So far, the only clear criterion for confinement is the order of magnitude of the free energy: $O(1)$ or $O(N^2)$ when $N \to \infty$.

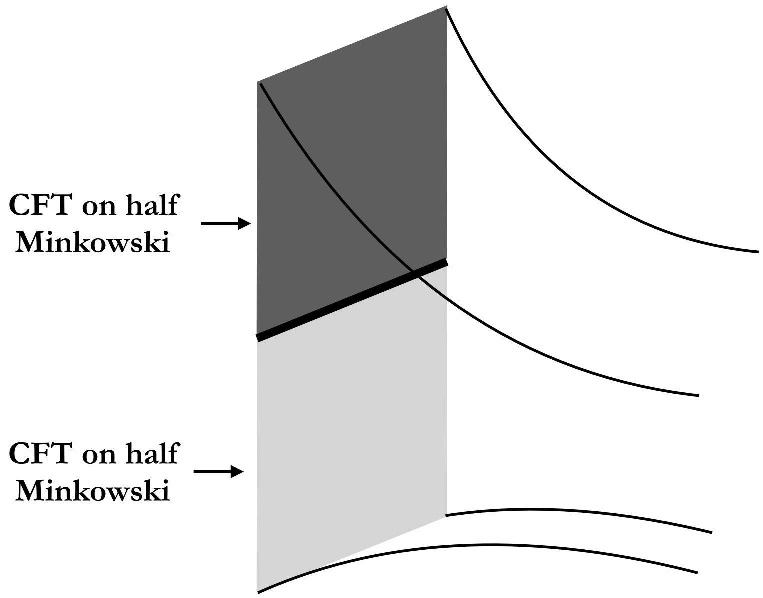
• Wilson loops do not provide an easy criterion for confinement, as for large Wilson loops, the area and the perimeter scale the same way, in global coordinates.

• It is possible that subleading differences may tell the difference.

• But in Poincaré coordinates there are two classes of loops with different behavior for length and area.

• However QFT on AdS in different coordinates gives rise to a different quantum theory.

(Holographic) Interfaces



• We may do a conformal transformation on each of the pieces to map it to AdS in Poincaré coordinates with the boundary at the interface.

• Clearly the two boundaries touch on the interface.

• If the interface is conformal, we expect a O(d,1) symmetry. This will be realized geometrically in the holographic solution

The holographic picture

• A natural metric anzatz for the ground state of a QFT_d on AdS_d is

$$ds^2 = du^2 + e^{2A(u)}\zeta_{\mu\nu}dx^\mu dx^\nu \tag{1}$$

where $\zeta_{\mu\nu}$ the unit radius AdS_d metric.

• The asymptotics of e^A near the boundary $u \to -\infty$ control the source for the radius of the AdS_d slice metric.

• Any QFT on AdS_d has the symmetry of AdS_d: O(1,d).

• If the theory is also scale invariant the symmetry enhances to O(1,d+1) and in this case the bulk solution is global AdS_{d+1}, sliced with AdS_d slices and

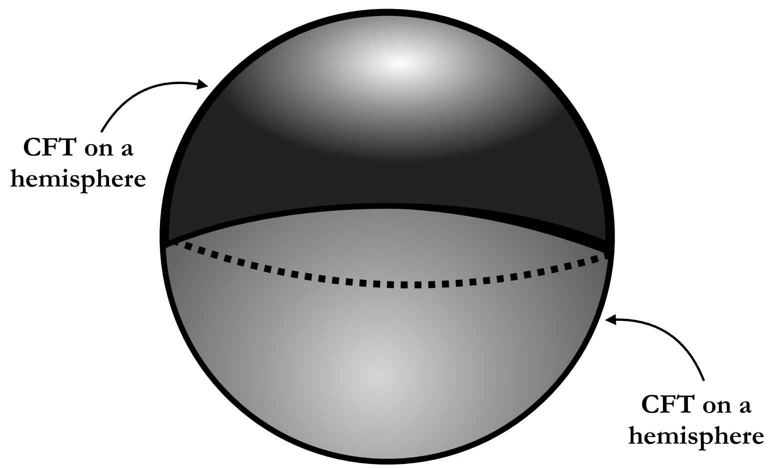
$$e^A = \cosh\frac{u}{\ell}$$

and $-\infty < u < +\infty$. This is a non-monotonic scale factor.

• The metric has two (apparent) AdS boundaries. One, B_+, at $u = -\infty$ and another B_- at $u = +\infty$.

• The metric is locally AdS, and can be mapped to global AdS by a (large) diffeomorphism.

• In the Euclidean case, the two boundaries are isomorphic to $B_\pm^d$ and they intersect at the equator forming the single boundary S^{d-1} of AdS$_d$.



• Similar remarks hold for the Minkowski signature AdS space.

• Unlike the case of non-negative curvature slices, in the case of AdS slices, the negative curvature of the slice is responsible for the scale factor e^A NOT being monotonic.

• In the bulk AdS case, corresponding to a CFT on AdS$_d$, the gravitational solution is interpreted as two copies of the (same) CFT:

one on $B^+ \sim AdS_d$ and the other on $B_- \sim AdS_d$.

• The boundaries of B_+ and B_- are common and are isomorphic to the equator of the S^{d-1}.

• Therefore the interpretation is of two copies of a CFT on AdS_d with common boundary and transparent boundary conditions on the common boundary

• However, we may turn on more fields and in general the two UV CFTs can be different.

• In Poincaré coordinates for the slices this configuration corresponds to two theories on R^d_+ with an interface between them.

Wormholes versus interfaces

• The general case with scalar operators (and RG flows) turned on and with the asymptotic metric a general negative constant curvature metric $\zeta_{\mu\nu}$ is still described by the ansatz

$$ds^2 = du^2 + e^{2A(u)}\zeta_{\mu\nu}dx^\mu dx^\nu \tag{2}$$

with

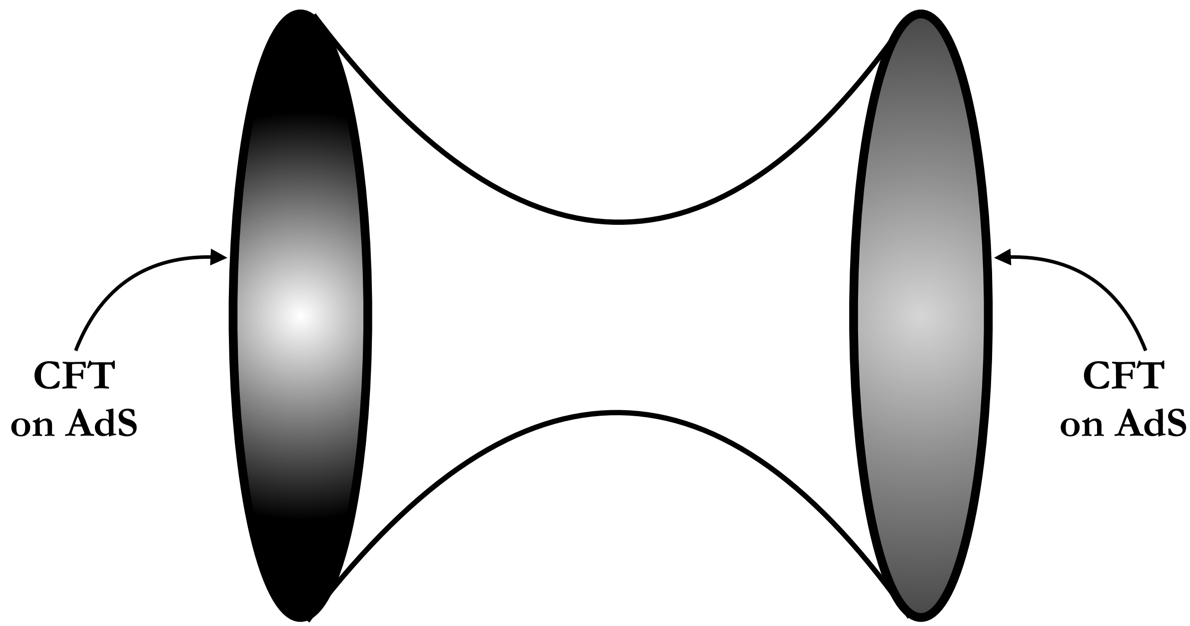
$$R(\zeta)_{\mu\nu} = -\frac{d-1}{L^2}\zeta_{\mu\nu}$$

while other fields, (like scalars) can change continuously between $-\infty < u < +\infty$.

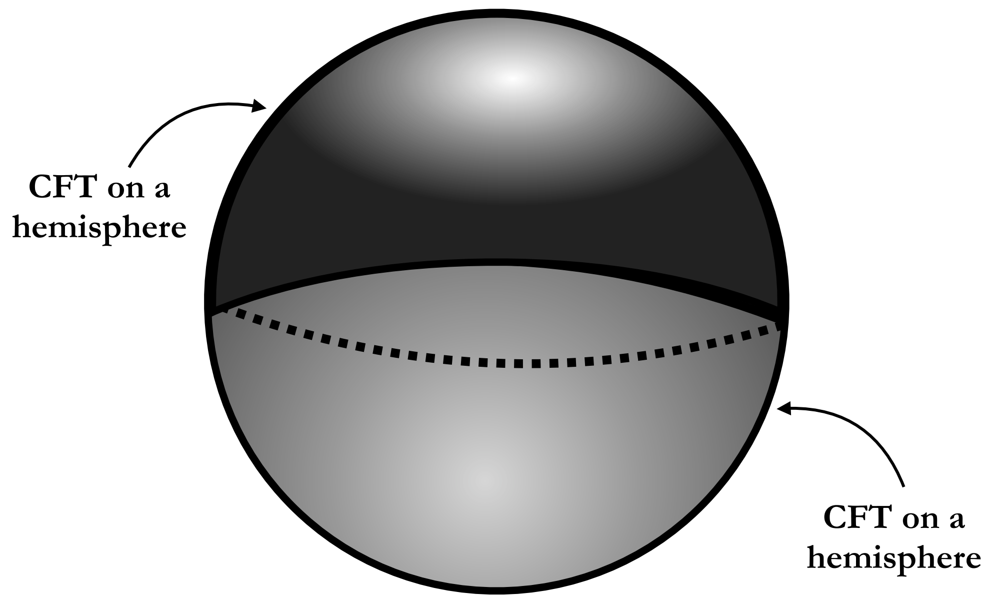
• Such a (regular) solution to the gravitational equations has always two boundaries at $B_\pm$ at $u = \pm\infty$.

• The interpretation of the solution depends however on the nature of the negative curvature Einstein manifold M_ζ with metric ζ.

• If the negative curvature manifold M_ζ is compact ($g > 2$ Riemann surface in $d = 2$ or Schottky manifolds in $d > 2$) then the solution describes a wormhole with negative curvature slices.



• When the slices are full AdS$_d$ spaces in global coordinates then the dual describes two CFTs interacting via their common boundary



• In the conformal case $e^A = \cosh \frac{u-u_0}{\ell}$ and

$$\frac{R_L}{R_R} = e^{\frac{2u_0}{\ell}}$$

• When the slices are AdS in Poincaré coordinates the solutions describes a (Janus) interface between two CFTs.

Proximity in QFT

• The notion of "proximity" in Quantum Field Theory is an intuitive notion.

• One possible definition of the notion of proximity among CFTs is : can QFT_1 and QFT_2 live in the same Hilbert space?

• If there is flow connecting CFT_1 to CFT_2 we can claim that the two theories can live in the same Hilbert space.

• Another was formulated by van Raamsdonk: the CFT masquerade, mostly relevant for CFT duals.

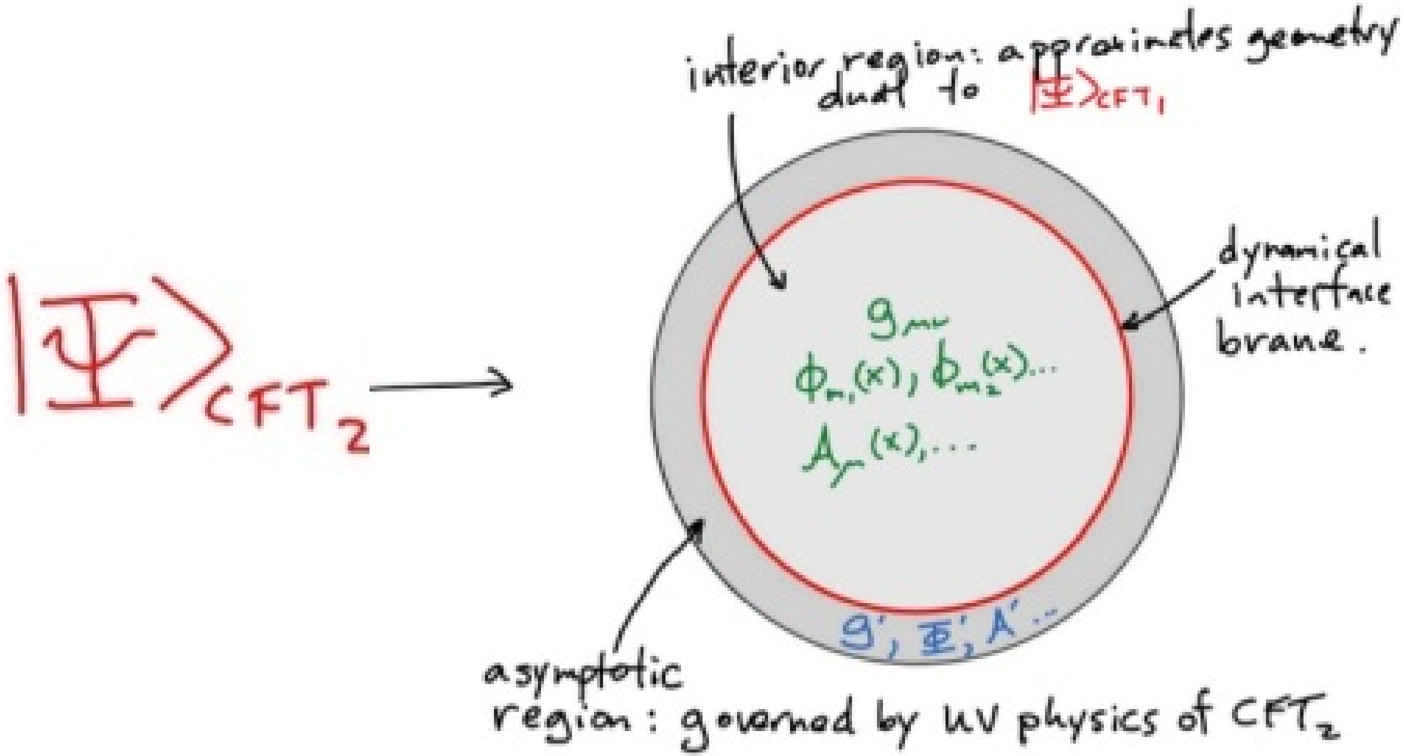
"When the states of CFT_1 can be approximated by CFT_2?"

or

"Can a suitably chosen state of CFT_1, faithfully encode the space-time dual to a state of CFT_2?"

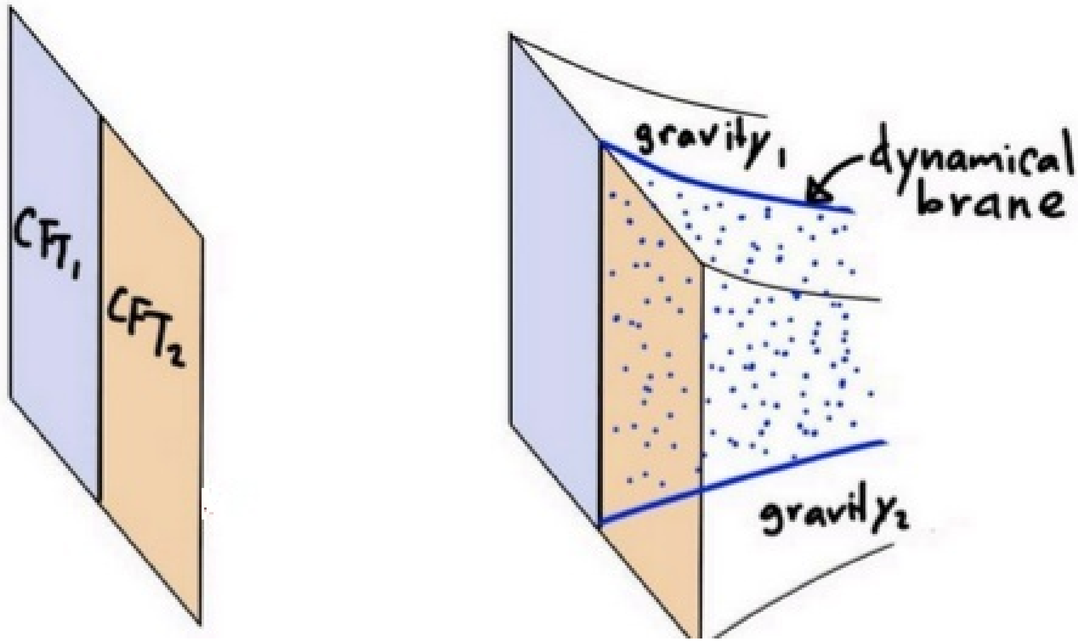
or

”Can two theories with different operator spectra describe the same bulk geometry?”



• Van Raamsdonk gave simple solvable examples where the two CFTs are interfaced by a bulk brane.

• This notion is very close to the RG connection, as a continuous version of this setup is a holographic RG flow.



- Another example is theories that can share an inteface.

- They may be generating a bulk brane or

Takayanagi

- They may be like Janus interface geometries.

Bak+Gutperle+Hirano, + many others

(Holographic) Conformal Defects

• Consider a D-dimensional flat-space QFT, and a $d < D$-dimensional localized (flat-space, non-dynamical) defect.

• This provides a transverse $O(D-d)$ symmetry in the theory.

• Consider also the possibility that the defect is conformal: The associated symmetry is $O(d+1,1)$ and commutes with $O(D-d)$.

• If there is a holographic realization of this, then the geometry should realize the $O(d+1,1) \times O(D-d)$ symmetry. It should therefore contain an $\text{AdS}_{d+1} \times S^{D-d-1}$ manifold.

• The ground state of such holographic conformal defects, will be described by a conifold metric with $\text{AdS}_{d+1} \times S^{D-d-1}$ slices.

• The boundary of such solutions has several components:

♠ One is the boundary of the total space, and this is conformal to $\mathrm{AdS}_{d+1}\times \mathrm{S}^{D-d-1}$, which is also conformal to flat space, $\mathbb{R}^d$.

♠ There is another piece of the boundary, namely the union of the boundaries of the AdS_{d+1} slices. Insertions on that boundary correspond to defect operators.

• Conifold solutions over $\mathrm{AdS}_d\times \mathrm{S}^n$ corresponding to conformal defects of flat space holographic CFTs have been thoroughly studied.

Ghodsi+Kiritsis+Nitti

They have two possible interpretations:

♠ As a holographic CFT$_{d+n}$ on AdS$_d \times$S^n.

♠ As a (d-1)-dimensional defect in a $D = d + n$-dimensional CFT.

• This dual interpretation is compatible as the transverse radial distance to the defect can act as a RG scale.

• In the same vain, $\mathbb{R}^{d+n}$ is conformal to AdS$_d \times$S^n

• Unlike the case of interfaces, the scale factors are always monotonic.

• The conformal interface corresponds to $d = D - 1$ and the remaining symmetry is realized by AdS$_D$. Also S^0 has two points and corresponds to the two sides of the interface.

The AdS-sliced RG flows

• We assume an Einstein-dilaton theory in order to simplify our explorative task.

$$S_{Bulk} = M_P^{d-1} \int du\, d^d x \sqrt{-g} \left(R - \frac{1}{2} g^{ab} \partial_a \Phi \partial_b \Phi - V(\Phi) \right).$$

$$ds^2 = du^2 + e^{2A(u)} \zeta_{\mu\nu} dx^\mu dx^\nu, \qquad \Phi = \Phi(u)$$

• The slice is a manifold M_ζ whose metric ζ is any (constant) negative curvature Einstein metric.

$$R^{(\zeta)}_{\mu\nu} = \kappa \zeta_{\mu\nu}, \qquad R^{(\zeta)} = d\kappa, \qquad \kappa = -\frac{(d-1)}{\alpha^2}.$$

• κ can be rescaled by a shift in $A(u)$.

• The solution is characterized by the scalar field profile $\Phi(u)$ and by the scale factor $A(u)$, which are related via the bulk Einstein equations.

• Note that for all constant curvature metrics, the equations are the same!

• We shall systematically study the solutions to these equations for $R^{(\zeta)} \sim \kappa < 0$.

• The regular solutions have generically two boundaries $B_\pm$ at $u = \pm\infty$.

• They are both conformal to M_ζ.

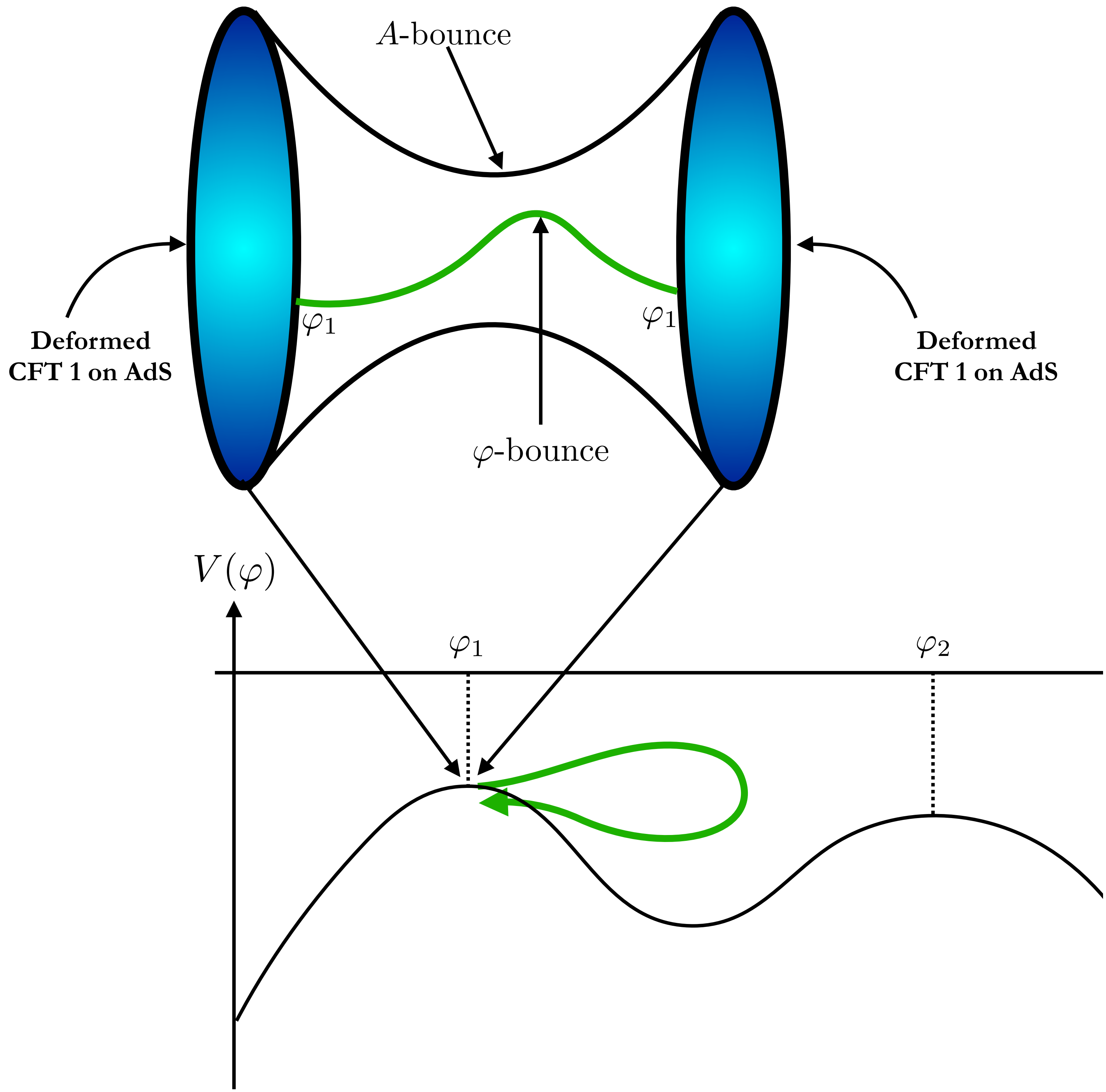
• The end-points are at (finite) extrema $\Phi_\pm$ of the bulk potential $V(\Phi)$.

• The associated CFTs are CFT_+ and CFT_-

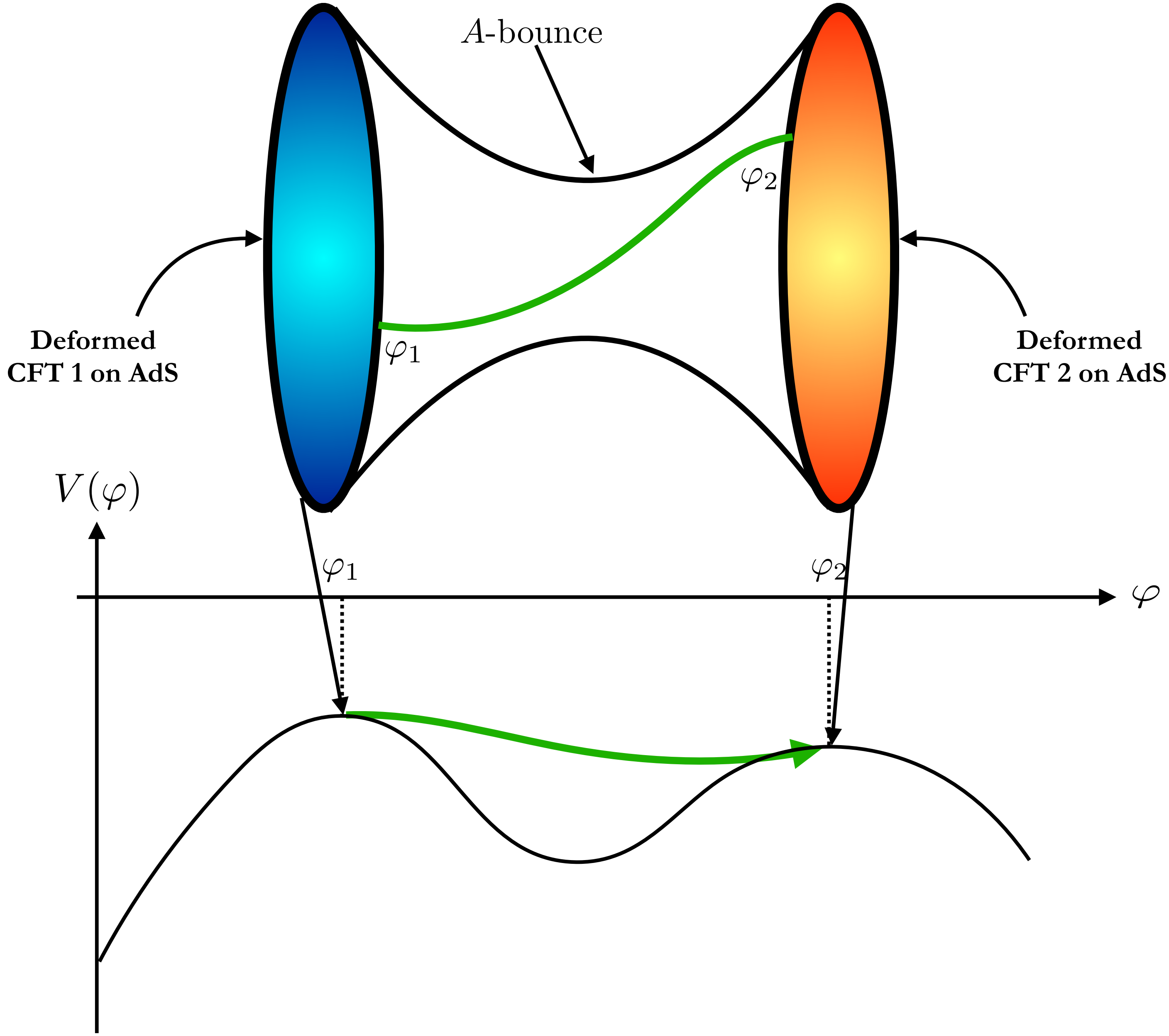
• Every solution (A, Φ) to these equations corresponds to:

♠ A wormhole solution if M_ζ is compact. It connects CFT_+ at B_+ to CFT_- at B_-.

♠ An interface solution if M_ζ is non-compact. The interface $B_+ \cup B_-$ is between CFT_+ and CFT_-.



A-bounce
Deformed
CFT 1 on AdS
φ_1
φ_1
Deformed
CFT 1 on AdS
φ-bounce
$V(\varphi)$
φ_1
φ_2


A-bounce
Deformed
CFT 1 on AdS
Deformed
CFT 2 on AdS
φ_1
φ_2
$V(\varphi)$
φ_1
φ_2
φ

The first order formalism

- We define the "superpotentials" (no supersymmetry)

$$\dot{A} \equiv -\frac{1}{2(d-1)} W(\Phi)\,,$$

$$\dot{\Phi} \equiv S(\Phi)\,,$$

Asymptotics near potential extrema

• Regular solutions START AND END (generically) at extrema of the potential.

• Near a maximum of the potential, there are two branches of solutions known as the $-$ and the $+$ branch.

$$\ell\, W_{\pm} = 2(d-1) - \frac{\Delta_{\pm}}{2}(\phi - \phi_0)^2 + \cdots$$

♠ The $-$ branch contains the generic solutions that contain both source and vev.

♠ The $+$ branch contains only the special solution for which the source vanishes (relevant vev-driven flow).

• For both types of solutions above, the metric has an AdS boundary at the maximum.

• We denote these asymptotics as $Max_{\pm}$.

• Near a minimum of the potential we also have the $+$ and $-$ branches of solutions.

♠ The $-$ branch contains the generic solution.

• It does not exist for non-zero slice curvature. It exists only for flat slices and in that case it describes the IR-end of an RG flow.

♠ The $+$ branch contains the special solution. The bulk metric has an AdS BOUNDARY in this case

• The solution describes a UV fixed-point perturbed by the vev of an irrelevant operator.

• In principle, it can exist for both flat and curved slices.

• We denote these asymptotics as $Min_{\pm}$.

• $Max_{\pm}$ and Min_{+} are associated to AdS boundaries and therefore to QFT UV fixed points.

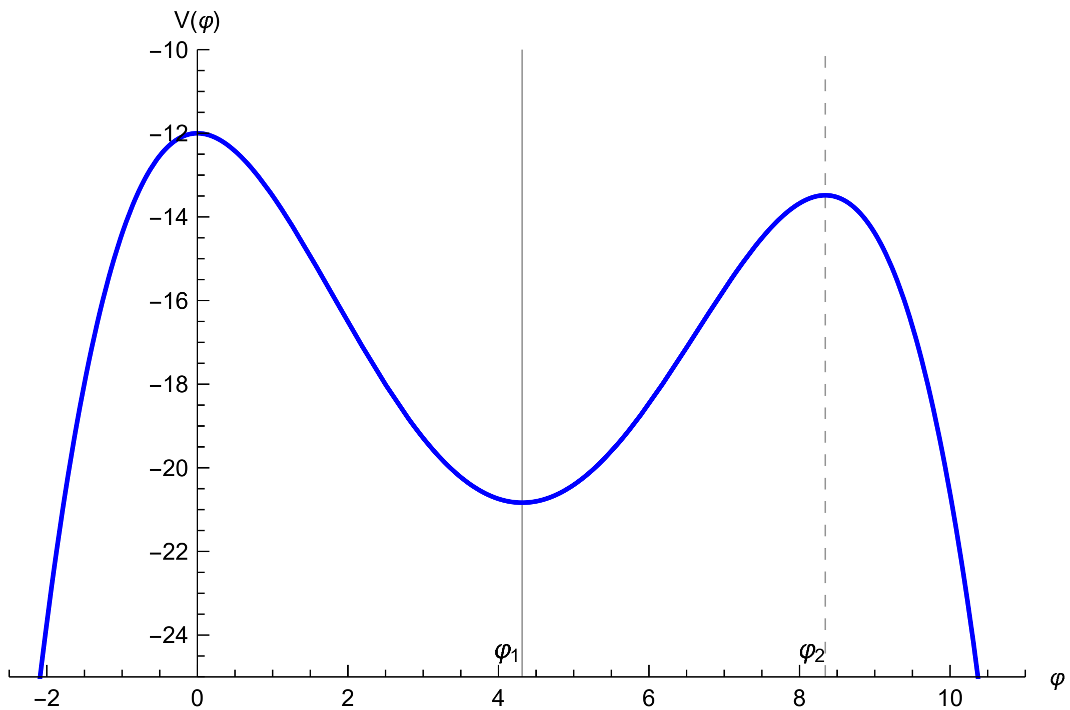
• Min_{-}, to a shrinking slice geometry and therefore to an IR Fixed point.

• The $+$ branch solutions, as they contain less integration constants, exist only in fine-tuned cases.

• The Min_{-} solution does not exist, when the (dimensionless) curvature of the slice $\mathcal{R} \neq 0$.

Classifying the solutions, Part I

- We pick $d = 4$ and a generic quartic potential



- The left maximum is at $\Phi = 0$.

- The right maximum is at $\Phi_2 = 8.34$.

- The minimum is located at $\Phi_1 = 4.31$.

• "Technical" definitions:

♠ A-bounce is a point where $\dot{A} = 0 \to W = 0$. It always exists when the slice curvature is negative.

• We denote the position of an A-bounce by Φ_0.

♠ Φ-bounce is a point where $\dot{\Phi} = 0 \to S = 0$.

• We always start our solution at an A-bounce at $\Phi = \Phi_0$ $(W(\Phi_0) = 0)$and we solve the first order equations

$$\frac{d}{2(d-1)}W^2 + (d-1)S^2 - dSW' + 2V = 0\,,$$

$$SS' - \frac{d}{2(d-1)}SW - V' = 0\,.$$

• We only need an extra "initial" condition: $S_0 \equiv \dot{\Phi}|_{\Phi=\Phi_0} \equiv S(\Phi_0)$.

• The two parameters $(\Phi_0, S_0) \in R^2$ are the complete initial data of the first order system.

• For each pair (Φ_0, S_0) there is a unique solution.

Conformal Theories on AdS

• The prime example, N=4 SYM was analyzed in some detail.

Gaiotto+Witten,Aharony+Marolf+Rangamani, Aharony+Berdichevsky+Berkooz+Shamir

• Boundary conditions on R^4_+ that preserve supersymmetry have been classified, and there are many.

Gaiotto+Witten

• Upon a conformal transformation the theory can be put on AdS_4 in Poincaré coordinates.

• Dirichlet bc generically involve non-trivial vevs for three of the six scalars.

• At weak coupling the theory is generically non-confining.

• But at strong coupling some boundary conditions induce confinement.

• For example, using S-duality, the $g >> 1$ theory with a Higgs condensate is mapped to a $g << 1$ theory with a magnetic condensate that should be confining.

• In particular, S-duality interchanges (among others) Dirichlet and Neumann bc.

• With Neumann bc no order parameter exists that distinguishes a confining from a non-confining phase.

• Therefore, no sharp transition is expected in accordance with the large susy.

The bulk Einstein Equations

• The solution is characterized by the scalar field profile $\Phi(u)$ and by the scale factor $A(u)$, which are related via the bulk Einstein equations.

$$2(d-1)\ddot{A} + \dot{\Phi}^2 + \frac{2}{d}e^{-2A}R^{(\zeta)} = 0$$

$$d(d-1)\dot{A}^2 - \frac{1}{2}\dot{\Phi}^2 + V - e^{-2A}R^{(\zeta)} = 0$$

$$\ddot{\Phi} + d\dot{A}\dot{\Phi} - V' = 0\,,$$

The first order formalism

- We define the "superpotentials" (no supersymmetry)

$$\dot{A} \equiv -\frac{1}{2(d-1)}W(\Phi)\,, \quad \dot{\Phi} \equiv S(\Phi)\,, \quad R^{(\zeta)}e^{-2A(u)} \equiv T(\Phi)\,.$$

- The equations of motion become

$$\frac{d}{2(d-1)}W^2 + (d-1)S^2 - dSW' + 2V = 0\,,$$

$$SS' - \frac{d}{2(d-1)}SW - V' = 0\,.$$

- Once a solution is found we can evaluate

$$T(\Phi) = \frac{d}{4(d-1)}W^2(\Phi) - \frac{S(\Phi)^2}{2} + V(\Phi)$$

The bulk integration constants vs QFT parameters

• When $R^{\zeta} > 0$ the flows describe spaces with a single boundary dual to a single QFT with a relevant coupling.

• The bulk equations have three (dimensionless) integration constants.

• One corresponds to the dimensionless curvature $\mathcal{R}$.

• The second corresponds to the (dimensionless) scalar vev. It must be tuned for regularity.

• The third is not physical as it can be removed by a radial translation.

♠ In the first order formalism the (W,S) equations have two integration constants: one is $\mathcal{R}$, and the second is the scalar vev. The scalar vev is tuned in terms of $\mathcal{R}$ regularity.

• Then T is determined uniquely and from it we determine $A(\Phi)$.

• The first order equation for Φ has one more integration constants.

• This integration constant is trivial and is not a parameter of the dual theory (it is the relevant scale).

• In total, in both cases there is a free arbitrary constant $\mathcal{R}$ and the second (vev) is a function of $\mathcal{R}$.

The bulk integration constants again

• The number of integration constants in the bulk equation is the same(3).

• Here, there is no regularity condition. The solutions are generically regular. Therefore, the scalar vev is an independent parameter and does not depend on $\mathcal{R}$.

• One constant is always redundant as usual.

• All parameters at the second boundary are determined from the solution, evolved from the first boundary.

• Overall our two-boundary solutions depend on two dimensionless independent parameters.

• This is one less from the three we would expect in the general case: $\mathcal{R}_{i,f}$ and ξ.

♠ We shall recover the extra missing parameter by generalizing our solutions later.

Classification of complete flows

♠ $\mathcal{R} = 0$. All flows start and end at extrema of the potential.. They have a single AdS boundary.

• (Max_-, Min_-). This is the generic relevant flow driven by a relevant operator.

• (Max_+, Min_-). This is a flow driven by the vev of a relevant operator.

• (Min_+, Min_-). This is a flow driven by the vev of an irrelevant operator.

♠ $\mathcal{R} > 0$.

• In this case, although flows can start at extrema of the potential, (both maxima as $Max_\pm$ and minima as Min_+), they always end at intermediate points, not at extrema.

• The end is always an IR end-point where the slice volume vanishes.

♠ $\mathcal{R} < 0$.

• It is not possible for a flow to be regular and end at intermediate points (non-extrema of the potential), (there is no slicing of flat space with AdS slices).

• Therefore, all regular flows must start and end at extrema of the potential.

• As the asymptotic solution Min_- does not exist when $\mathcal{R} \neq 0$, we have in total the following $3 \times 3 = 9$ options,

$$(Max_- \quad , \quad Max_+ \quad , \quad Min_+) \quad \otimes \quad (Max_- \quad , \quad Max_+ \quad , \quad Min_+)$$

all of them having two AdS boundaries.

• (Max_-, Max_-), $(Max_+ Max_+)$.

• (Max_-, Max_+) and its reverse (Max_+, Max_-).

• (Max_-, Min_+) and its reverse, (Min_+, Max_-).

• (Max_+, Min_+) and its reverse, (Min_+, Max_+)

• (Min_+, Min_+).

• As mentioned the Max_+ and Min_+ asymptotics are fine-tuned (they have half the adjustable integration constants).

• Therefore the generic solutions will be of the (Max_-, Max_-) type.

• Single fine-tuning of the potential or the integration constants is needed for the (Max_-, Max_+) and (Max_-, Min_+) solutions to exist.

• Double fine-tuning is needed for (Max_+, Max_+), (Max_+, Min_+) and (Min_+, Min_+) to exist.

• We shall find examples of all types fine-tuned or not except the (Min_+, Min_+) solutions.

• The reason is that we have a potential with only one minimum.

Classifying the solutions, II

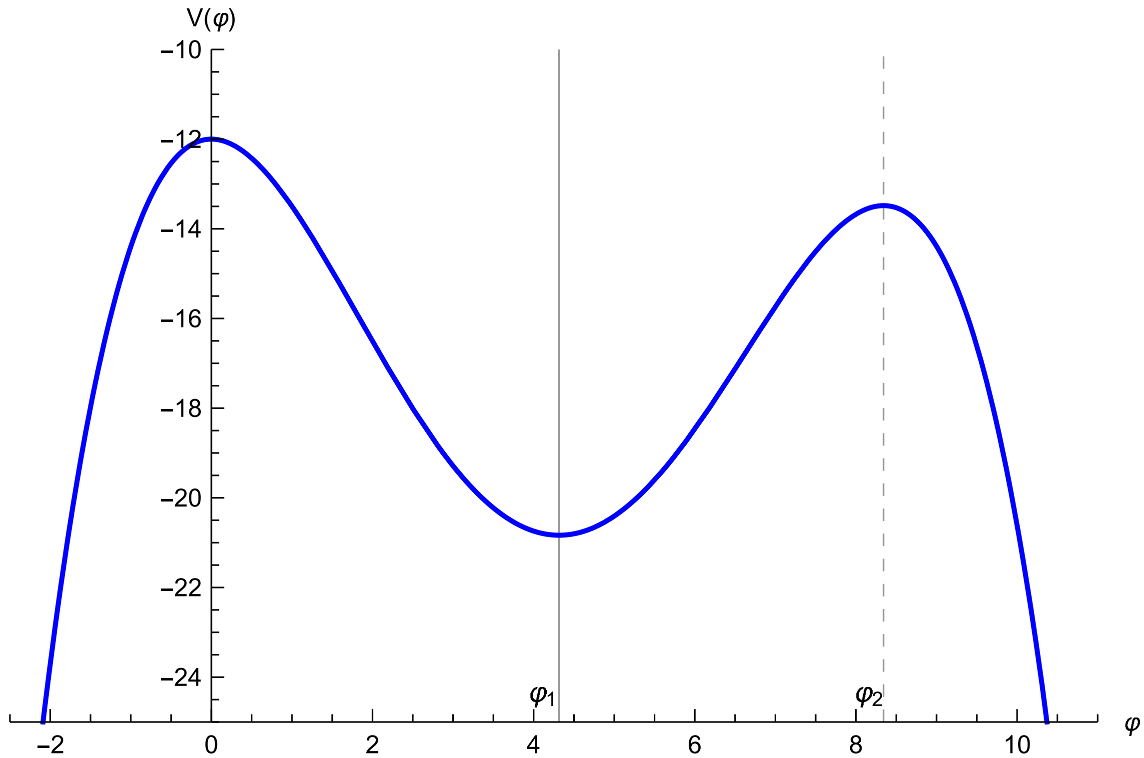
- We picked $d = 4$ and a generic quartic potential that we parametrized as

$$V(\Phi) = -\frac{12}{\ell_L^2} + \frac{\Delta_L(\Delta_L - 4)}{2\ell_L^2}\Phi^2 - \frac{(\Phi_1 + \Phi_2)\Delta_L(\Delta_L - 4)}{3\ell_L^2\Phi_1\Phi_2}\Phi^3 + \frac{\Delta_L(\Delta_L - 4)}{4\ell_L^2\Phi_1\Phi_2}\Phi^4,$$

where Φ_1 and Φ_2 are defined as

$$\Phi_1 = \frac{12\ell_R^2\sqrt{\ell_R^2 - \ell_L^2}\,\Delta_L(\Delta_L - 4)}{\sqrt{\ell_R^2\Delta_L(\Delta_L - 4) - \ell_L^2\Delta_R(\Delta_R - 4)}\left(\ell_R^2\Delta_L(\Delta_L - 4) + \ell_L^2\Delta_R(\Delta_R - 4)\right)}$$

$$\Phi_2 = \frac{12\sqrt{\ell_R^2 - \ell_L^2}}{\sqrt{\ell_R^2\Delta_L(\Delta_L - 4) - \ell_L^2\Delta_R(\Delta_R - 4)}}.$$



• The left maximum is at $\Phi = 0$. The AdS length is $\ell_L = 1$ and the scaling dimension $\Delta_L = 1.6$.

• The right maximum is at $\Phi = 8.34$. The AdS length is $\ell_R = 0.94$ and the scaling dimension $\Delta_R = 1.1$.

• The minimum is located at $\Phi_1 = 4.31$. It has $\Delta_+^{min} = 4.37$.

• "Technical" definitions:

♠ A-bounce is a point where $\dot{A} = 0 \to W = 0$. It always exists when the slice curvature is negative.

• Our solutions will have a single A-bounce. We shall denote its position by Φ_0.

♠ Φ-bounce is a point where $\dot{\Phi} = 0 \to S = 0$. It is a point where the first order equations break down but the second order equations do not.

♠ An IR-bounce is a point where both $\dot{A} = \dot{\Phi} = 0$.

• All bounces are defined AWAY from extremal points of V.

• We always start our solution at the (unique) A-bounce at $\Phi = \Phi_0$ and we solve the first order equations

$$\frac{d}{2(d-1)}W^2 + (d-1)S^2 - dSW' + 2V = 0\,,$$

$$SS' - \frac{d}{2(d-1)}SW - V' = 0\,.$$

• We only need an extra "initial" condition: $S_0 \equiv \dot{\Phi}|_{\Phi=\Phi_0} \equiv S(\Phi_0)$.

• The two parameters $(\Phi_0, S_0) \in R^2$ are the complete initial data of the first order system.

• For each pair (Φ_0, S_0) there is a unique solution.

• We then start solving the equations to the left and right of Φ_0 until we reach an AdS boundary on each side. Then our solution (W, S) is complete.

• We then solve the equations for Φ, A.

$$R^{(\zeta)}e^{-2A(u)} = \frac{d}{4(d-1)}W^2(\Phi) - \frac{S(\Phi)^2}{2} + V(\Phi) \quad , \quad \dot{\Phi} = S$$

The QFT couplings

• At each boundary, initial or final the metric asymptotes to M_ζ and the only parameter (source) is its curvature, $R_{i,f}$.

• The scalar will also have sources at the two boundaries:

$$\Phi(u) \to \Phi_-^{(i)} \quad , \quad u \to -\infty,$$

$$\Phi(u) \to \Phi_-^{(f)} \quad , \quad u \to +\infty,$$

• Therefore, we have four dimensionful couplings: $R_{i,f}$, $\Phi_-^{(i,f)}$.

• As the overall scale is irrelevant, the pair of theories is characterized by three dimensionless numbers which we take to be:

$$\mathcal{R}_i = \frac{R_i^{UV}}{\left(\Phi_-^{(i)}\right)^{2/\Delta_-^i}}, \quad \mathcal{R}_f = \frac{R_f^{UV}}{\left(\Phi_-^{(f)}\right)^{2/\Delta_-^f}}, \quad \xi = \frac{\left(\Phi_-^{(i)}\right)^{1/\Delta_-^i}}{\left(\Phi_-^{(f)}\right)^{1/\Delta_-^f}}$$

Three parameter solutions

- So far our ansatz missed one dimensionless parameter

- To recover it we modify it to:

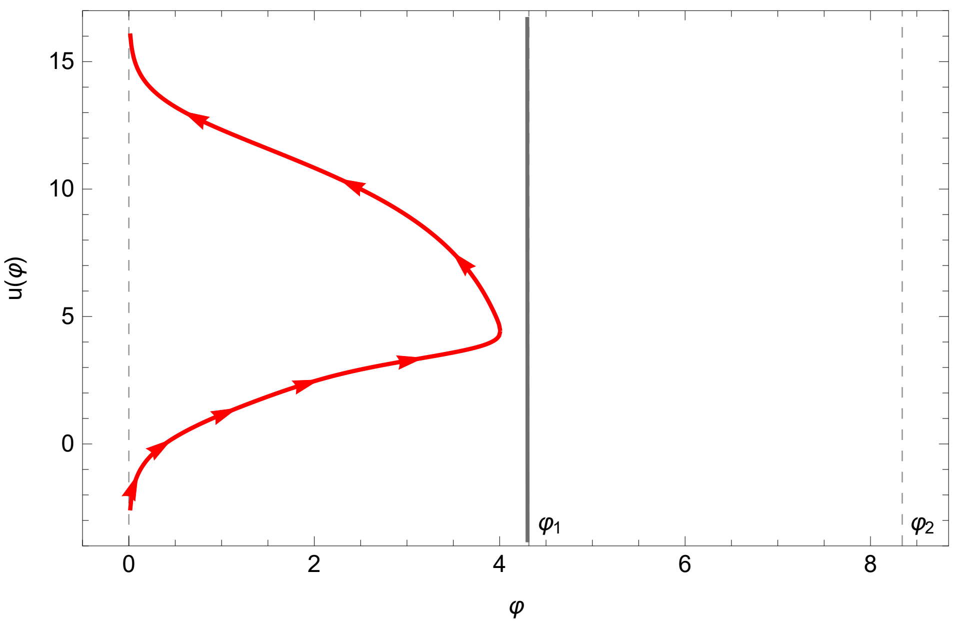
$$A = \begin{cases} \bar{A}(u) & u < u_* \\ \\ \bar{A}(\tilde{u} - \delta) & u_* + \delta < \tilde{u} < +\infty \end{cases} ,$$

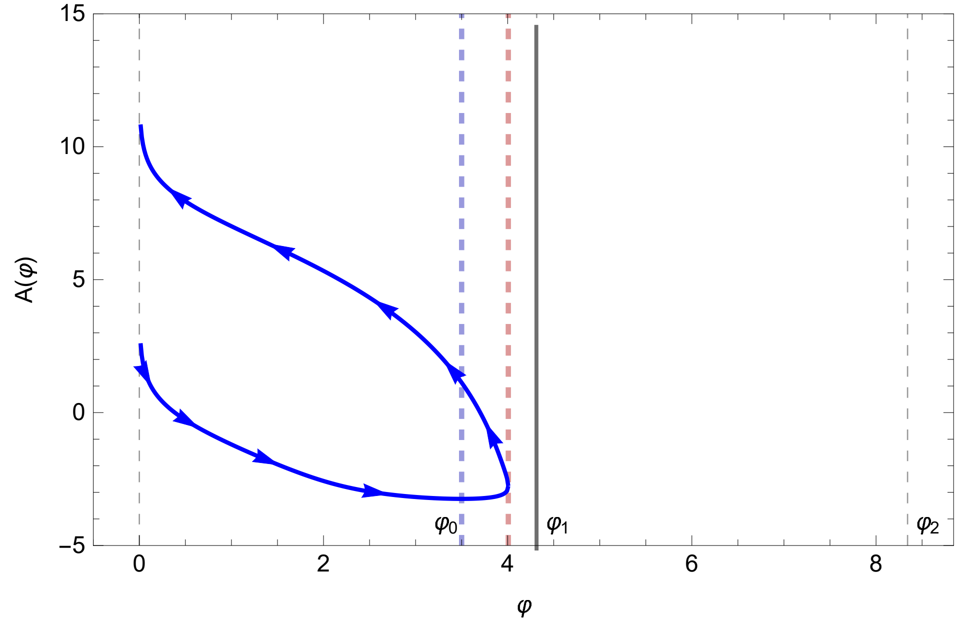
$$\Phi = \begin{cases} \bar{\Phi}(u) & u < u_* \\ \\ \bar{\Phi}(\tilde{u} - \delta) & u_* + \delta < \tilde{u} < +\infty \end{cases} ,$$

- This satisfies the Israel conditions at $u = u_*$ and A, Φ and their derivatives are continuous.

$$R_i^{UV} = \bar{R}_i^{UV}, \quad \Phi_-^i = \bar{\Phi}_-^i, \quad R_f^{UV} = e^{2\delta/\ell} \bar{R}_f^{UV}, \quad \Phi_-^f = e^{\delta \Delta_-^f/\ell} \bar{\Phi}_-^f$$

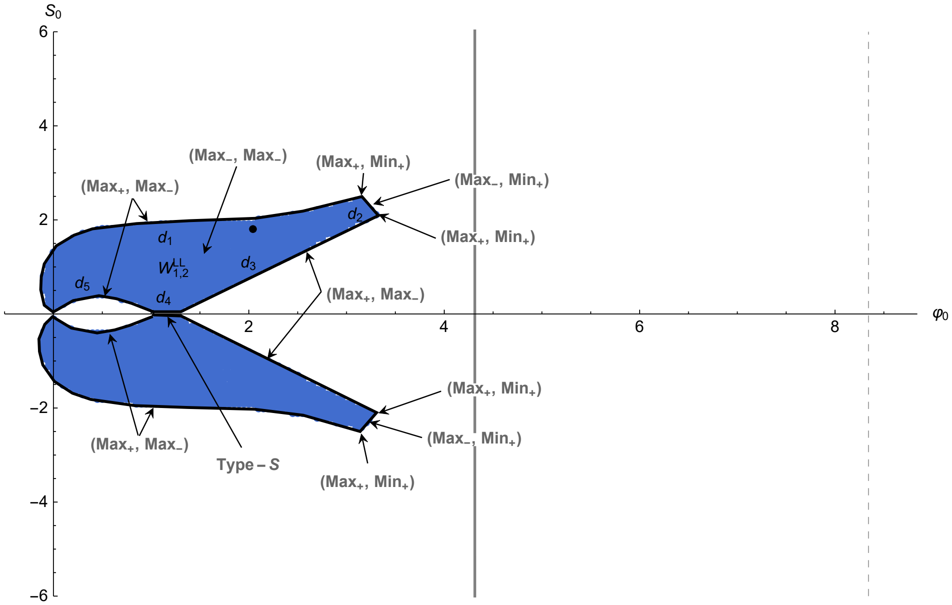
$W_{1,1}^{LL}$

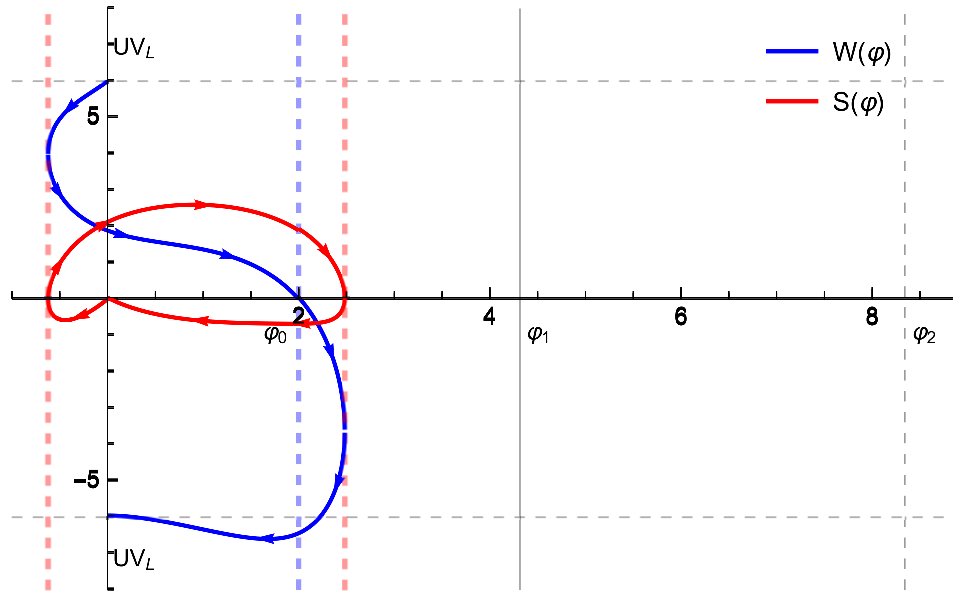




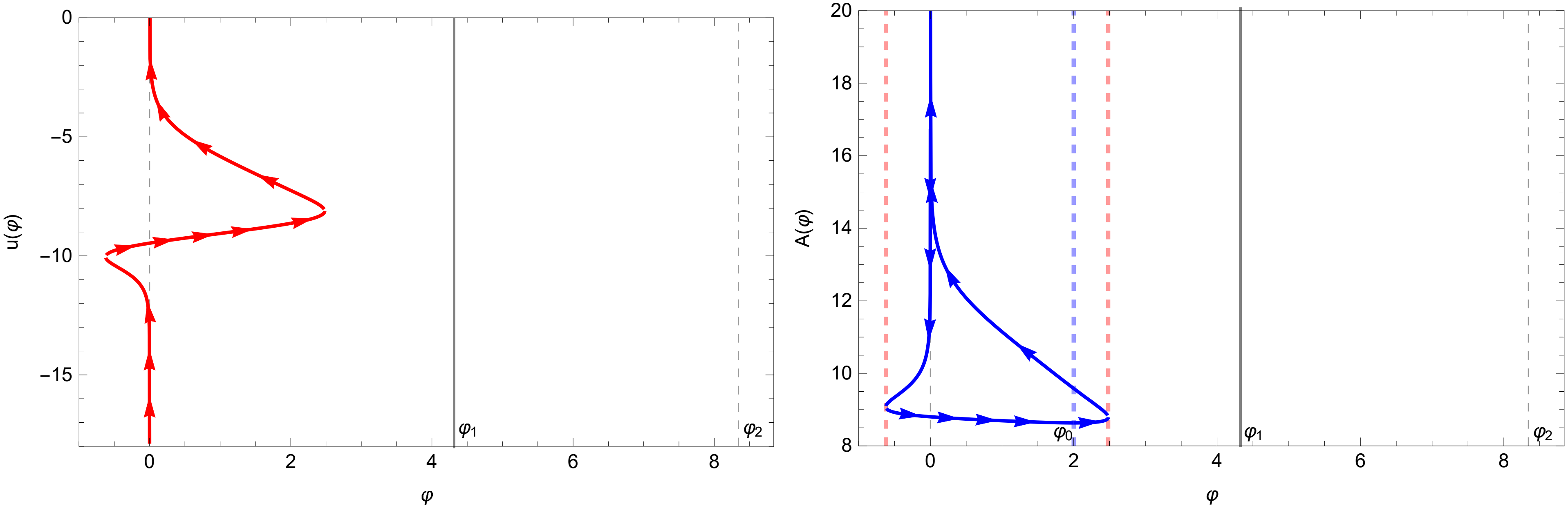
(a): The holographic coordinate at top UV_L tends to $-\infty$ and at bottom UV_L to $+\infty$.
(b): The scale factor has an A-bounce at $\Phi_0 = 3.5$ (blue dashed line) and a Φ-bounce at $\Phi = 4.0$ (red dashed line).

$W^{LL}_{1,2}$



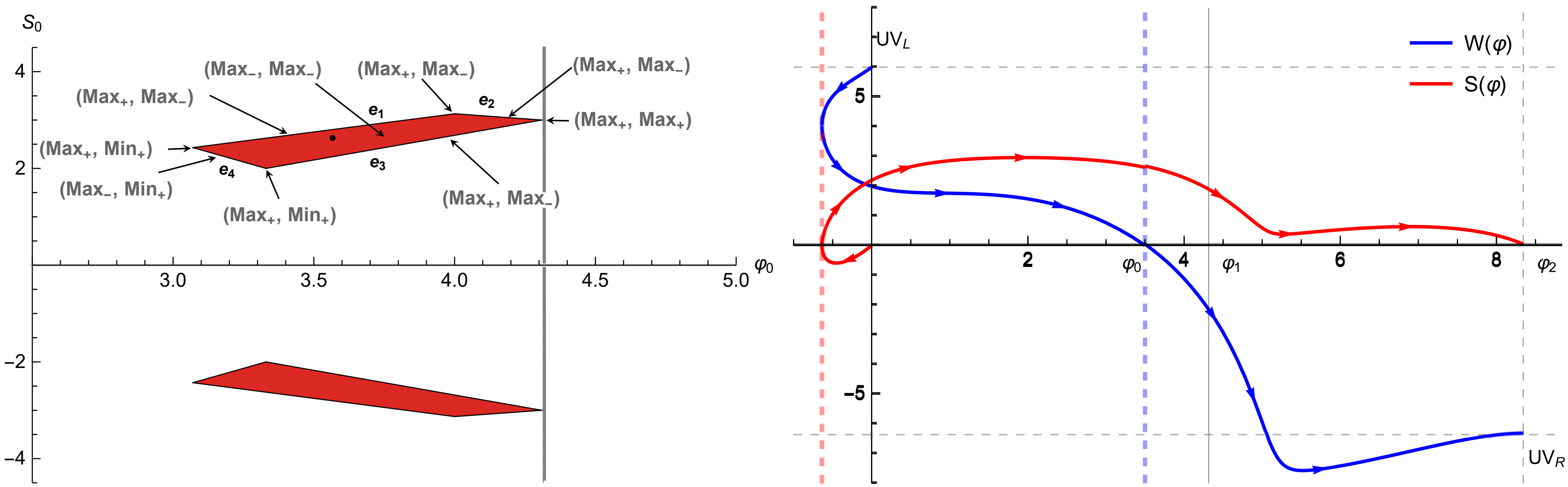


(a): The space of the $W^{LL}_{1,2}$ solutions is the upper blue region. The black dot represents the specific solutions of the diagram (b). The lower blue region corresponds to the solutions with an extra Φ-bounce near the bottom UV_L. (b): The blue and red curves for W, S, describe an RG flow that connects the UV_L fixed point to itself but after two Φ-bounces. The location of the Φ-bounces are indicated by red dashed lines.

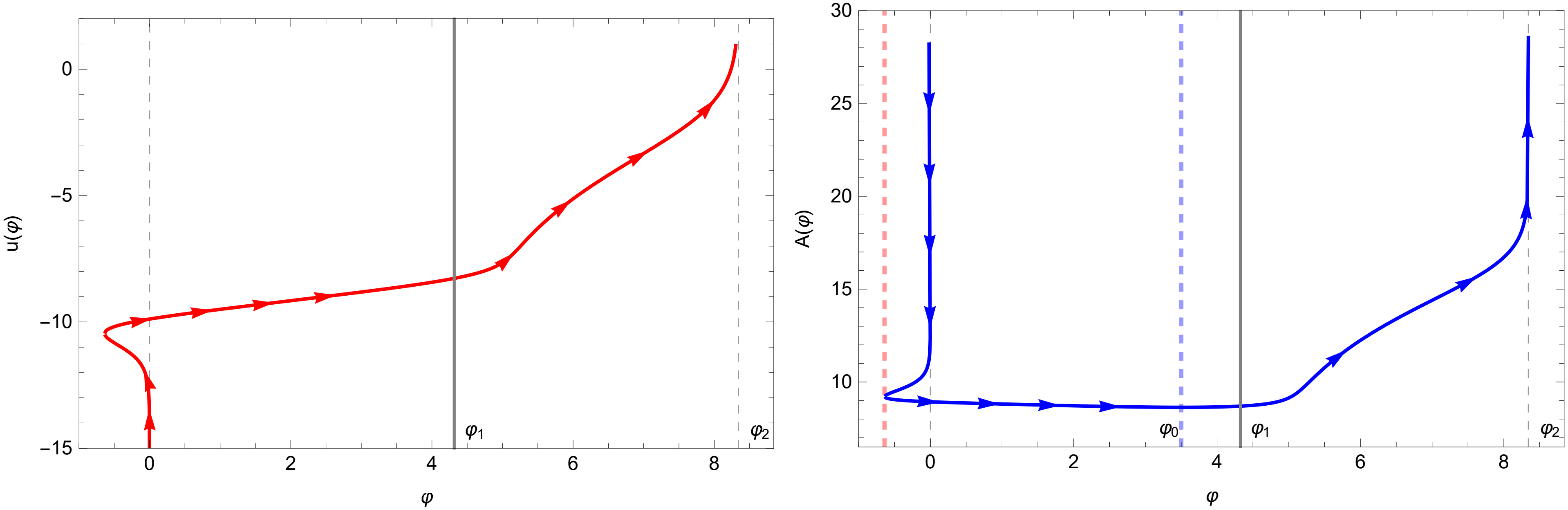


(a): The holographic coordinate at top UV_L boundary tends to $-\infty$ and for bottom UV_L to $+\infty$. (b): The scale factor has an A-bounce at $\Phi = 2.0$, the blue dashed line. The first Φ-bounce on the left occurs at $\Phi = -0.62$ and the second one at $\Phi = 2.48$, the red dashed lines.

$W_{1,1}^{LR}$

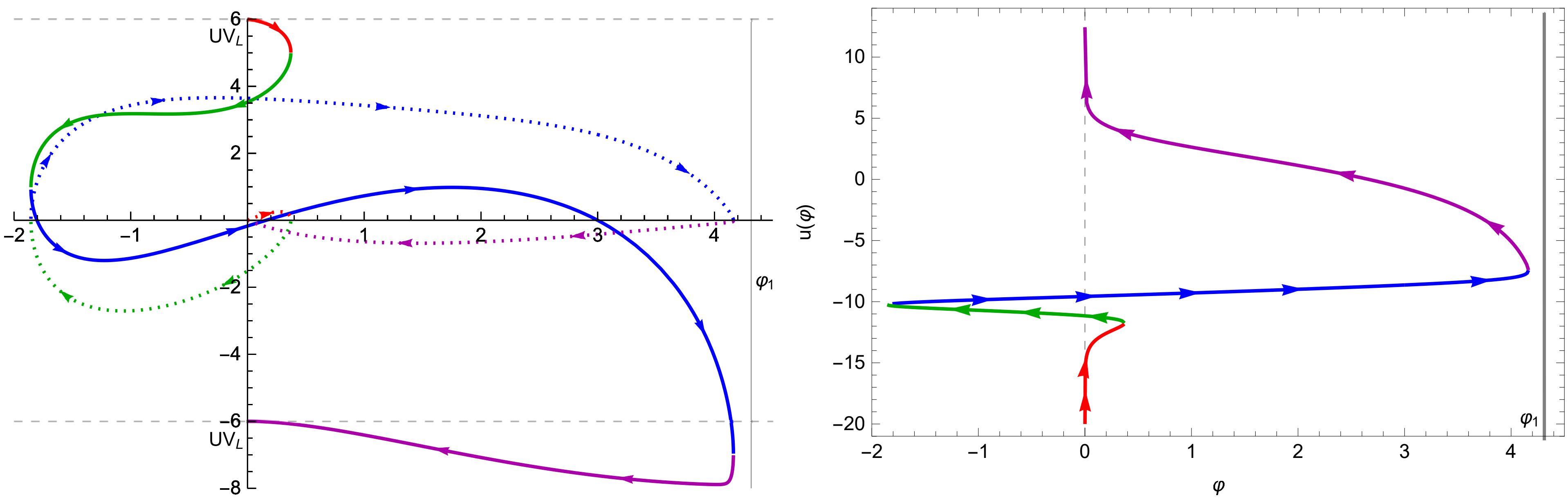


(a): A zoomed picture of the space of the $W_{1,1}^{LR}$ solutions. The black dot represents the RG flow in the diagram (b). (b): The RG flows of type $W_{1,1}^{LR}$ are between the UV_L boundary and UV_R. There is a Φ-bounce at $\Phi < 0$, the red dashed line. Notice that the red region at $S_0 < 0$ in figure (a) is the space of solutions with an extra Φ-bounce near UV_L but at $W < 0$.

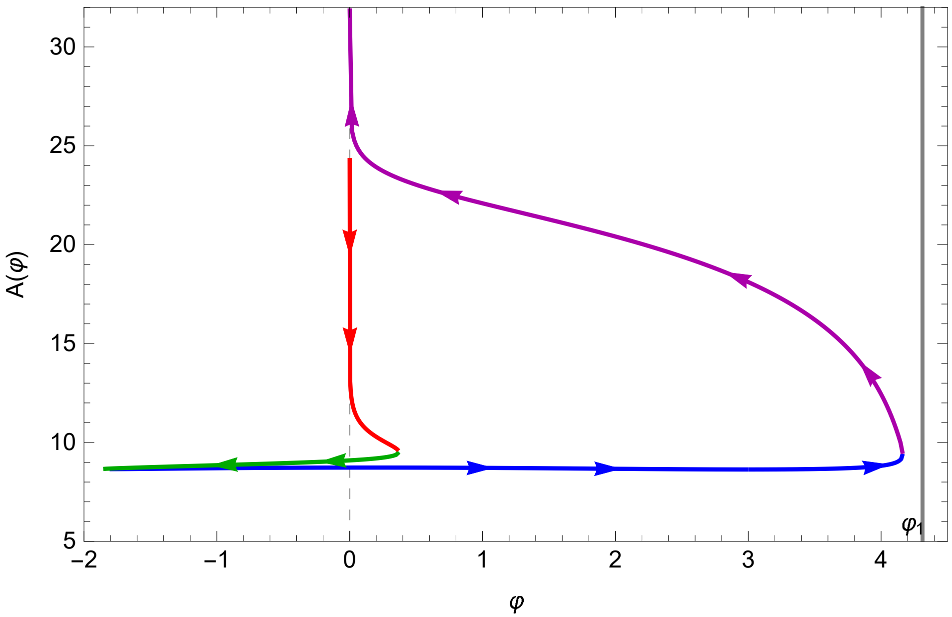


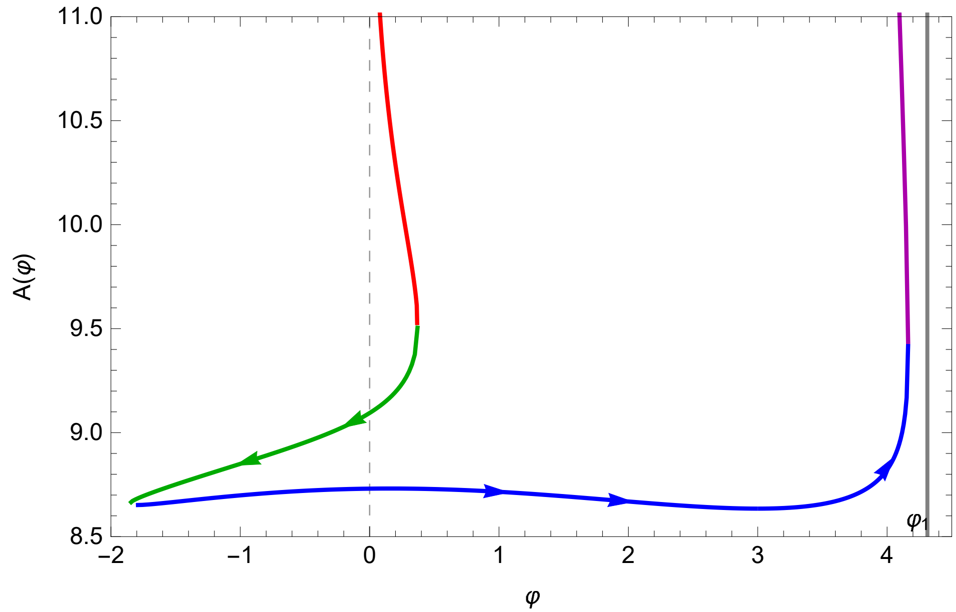
(a): The holographic coordinate at UV_L boundary tends to $-\infty$ and at UV_R to $+\infty$. (b): The scale factor has an A-bounce at $\Phi_0 = 3.5$, the blue dashed line. A Φ-bounce occurs at $\Phi = -0.64$, the red dashed line.

A (3,3) (A-bounce,Φ-bounce) solution

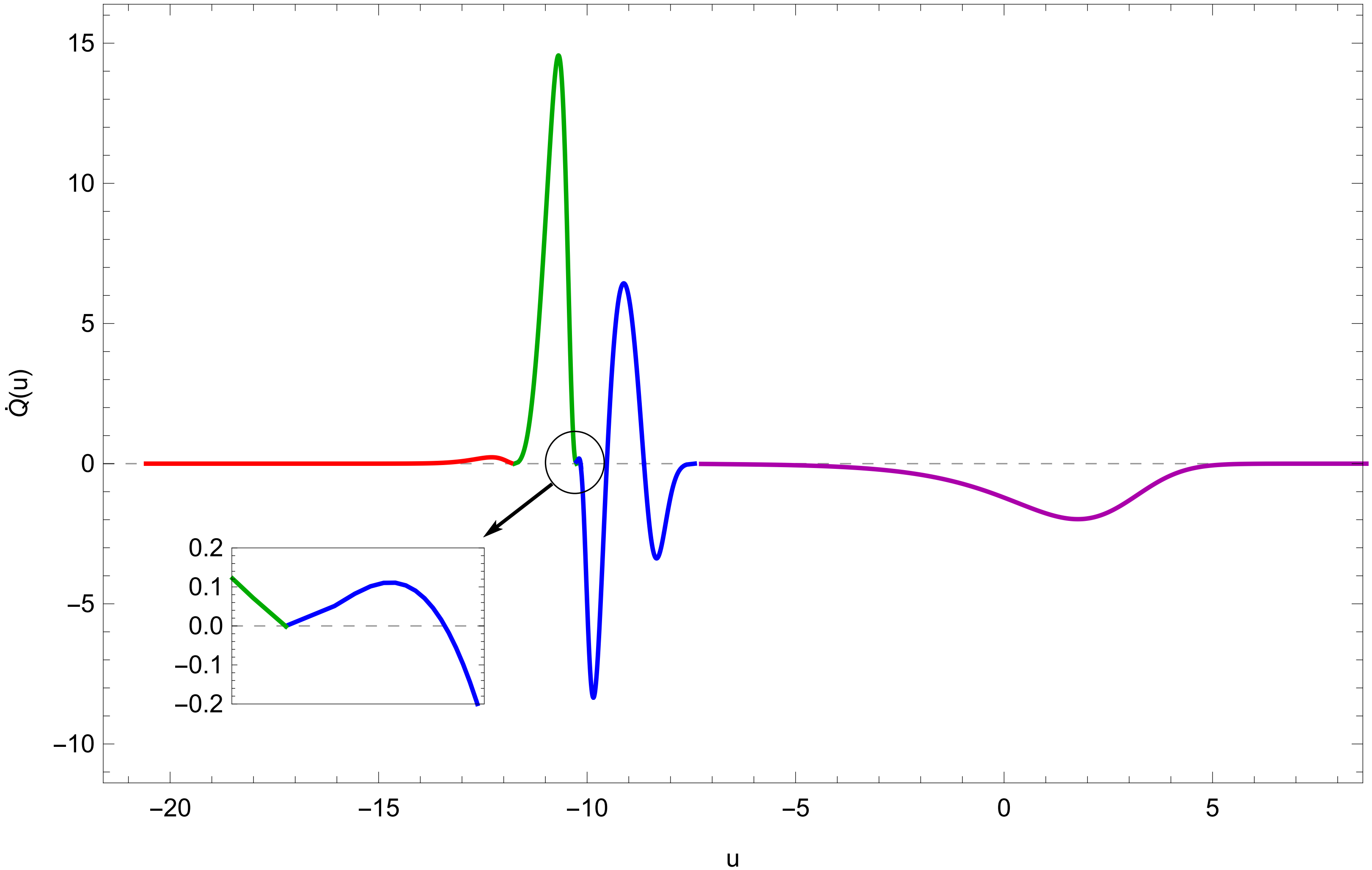


(a): An example of a multi-Φ-bounce solution, $W^{LL}_{3,3}$. The solid line is $W(\Phi)$ and dotted line is $S(\Phi)$. In this case an RG flow connects two UV boundaries on the left UV fixed point after three Φ-bounces. Unlike the previous cases the geometry here has three A-bounces.





(b)and (c) show the behavior of holographic coordinate and scale factor in terms of Φ. Figure (d) is the magnification of the bottom of figure (c). It shows that there are three A-bounces for this RG flow.

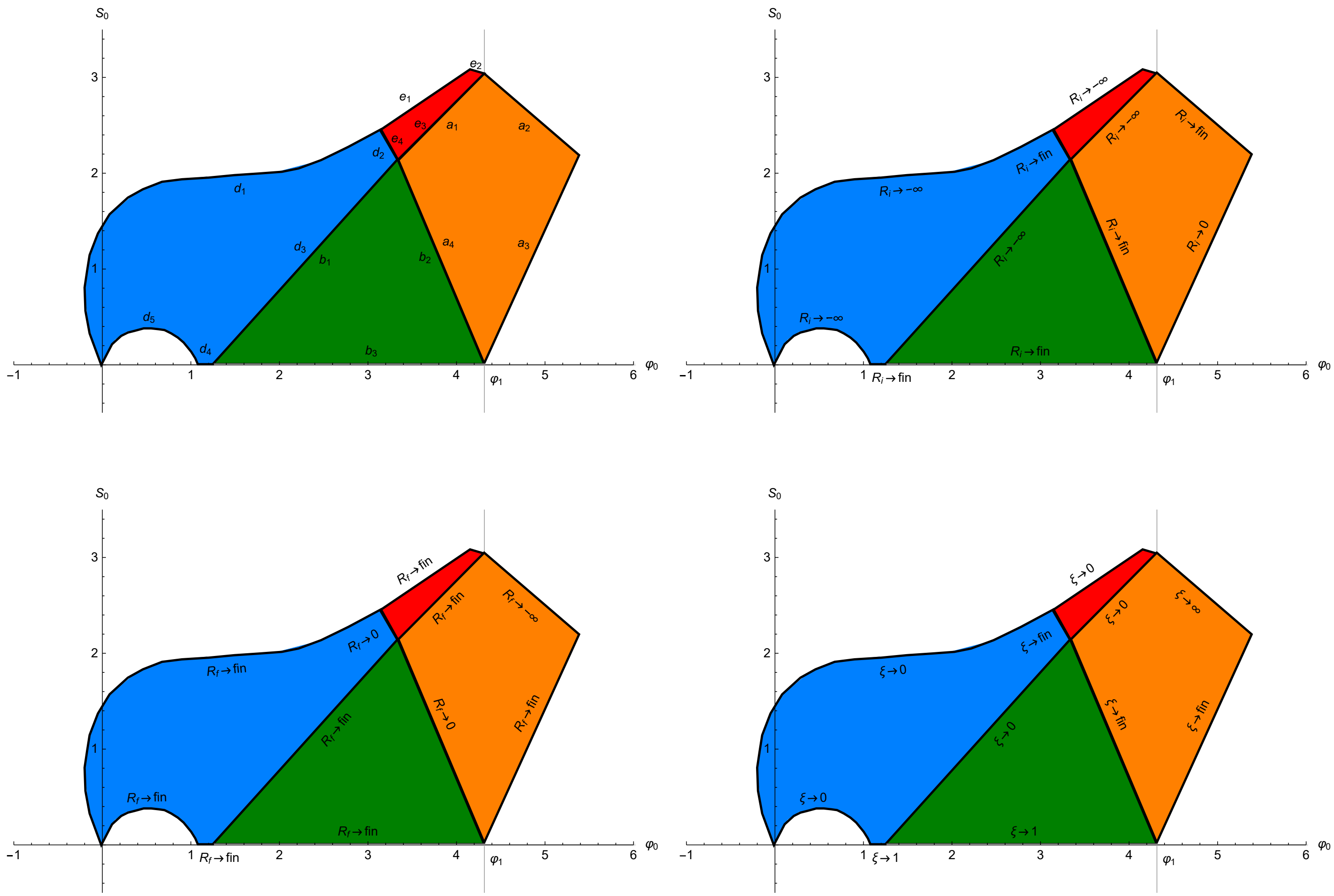


(e): The roots of $\dot{Q}$

$$Q(u) = \frac{1}{2}\dot{\Phi}^2 - V \geq 0\,, \qquad \dot{Q} = \frac{d}{2(d-1)} W S^2\,.$$

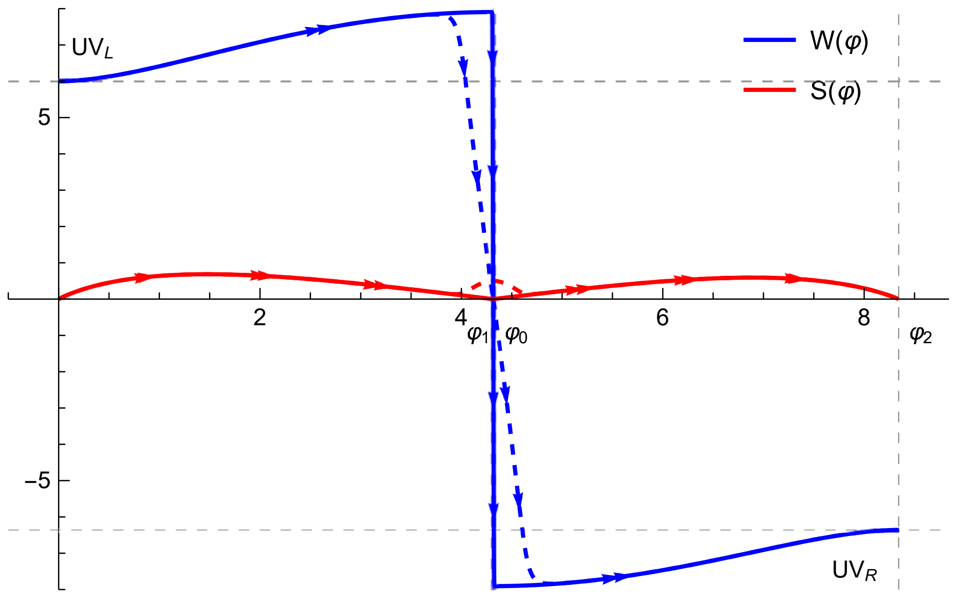
shows the location of Φ-bounces where the color of the graph is changed and location of A-bounces where the blue part of the curve crosses the u axis.

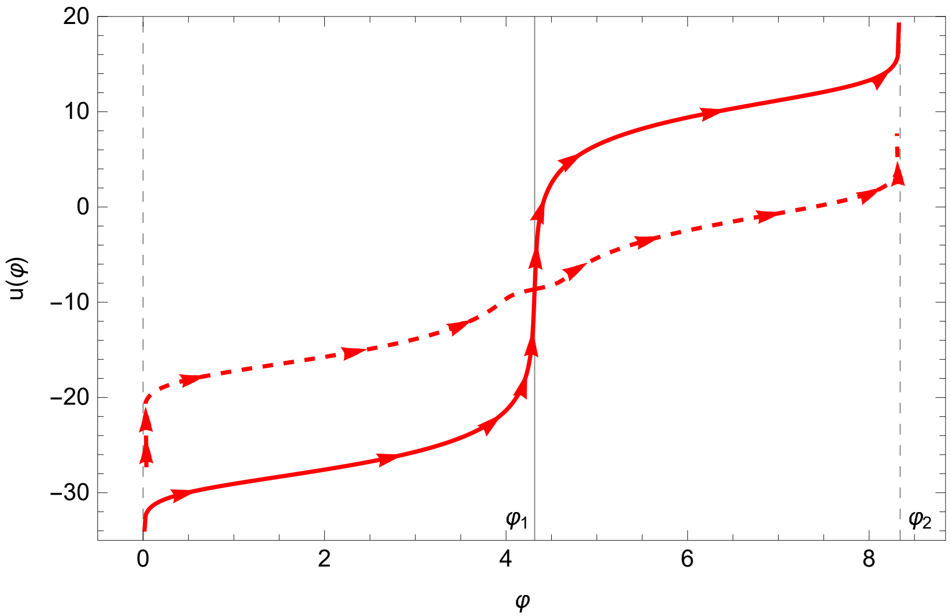
The behavior of relevant couplings

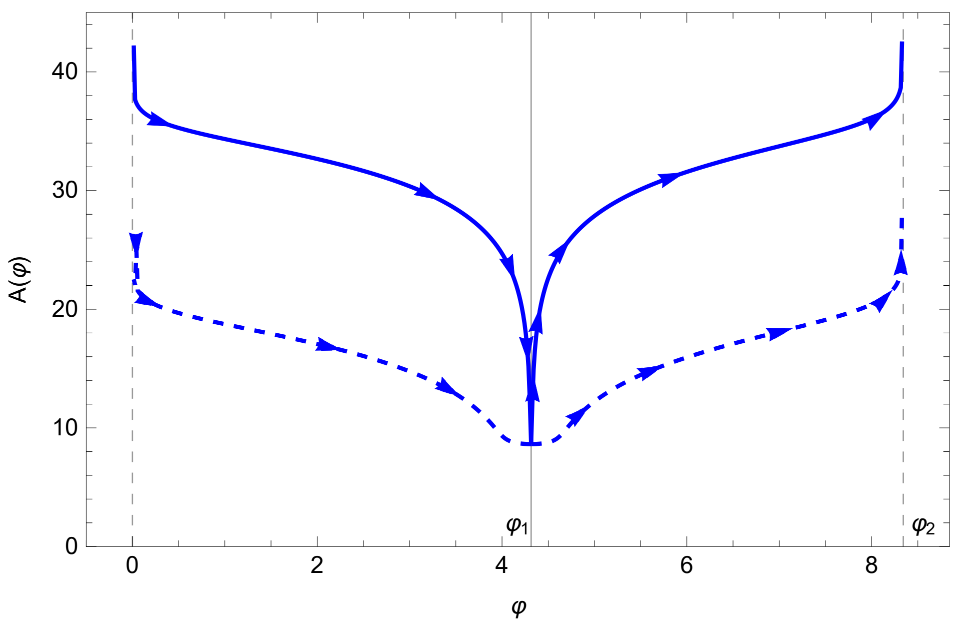


(a) Space of solution with its boundaries. (b) and (c): The behavior of $\mathcal{R}_i$ and $\mathcal{R}_f$ at boundaries. (d): The ratio of two relevant couplings, ξ, at boundaries.

The $a_3 \cup a_4$ solution: triple fragmentation



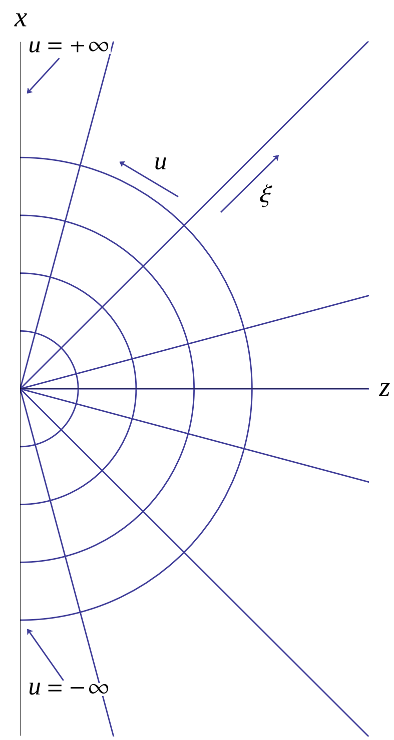




Along the fixed line $\Phi_0 = \Phi_1$ i.e. the minimum of the potential, if we decrease the value of S_0 down to zero, gradually the dashed curves in all figures above move toward the solid curves. In above curves the dashed curves have $S_0 = 0.5$ and the solid ones $S_0 = 0.01$.

Interface correlators

- The picture of overlapping boundaries in AdS-sliced flows is "singular".

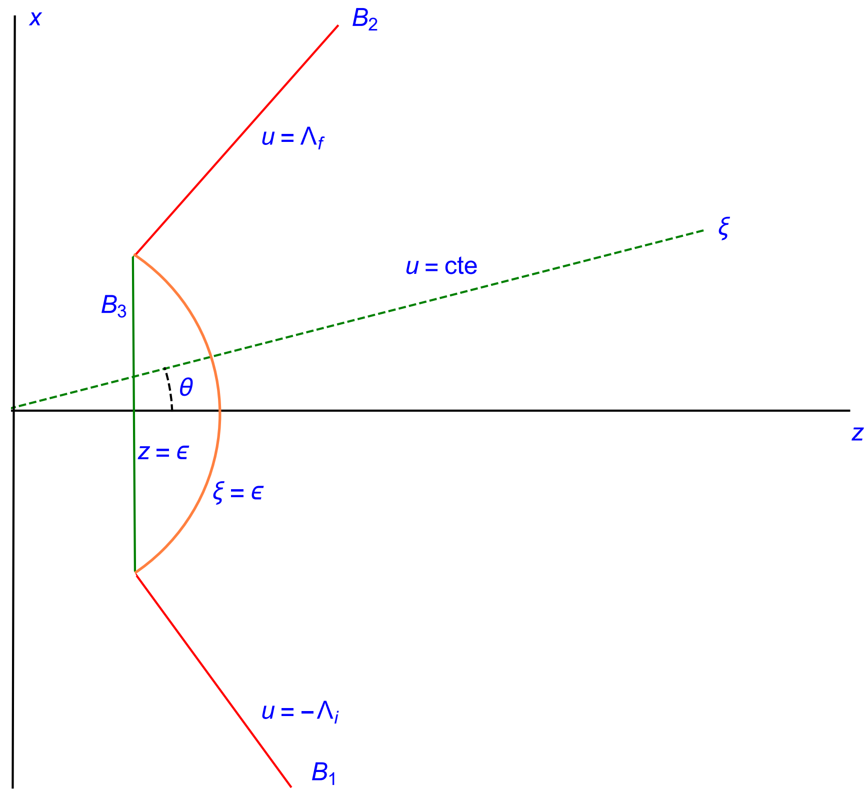


Relation between Poincaré coordinates (x, z) and AdS-slicing coordinates (ξ, u). Constant u curves are half straight lines all ending at the origin ($\xi \to 0^-$); Constant ξ curves are semicircle joining the two halves of the boundary at $u = \pm\infty$.

• The regular picture contains three boundaries:

♠ Two of them ($B_{1,2}$) are at $u = \pm\infty$.

♠ There is a third boundary, B_3, for all values of u that contains the boundaries of AdS slices.



• For a well-defined variational problem apart from the GH term on $B_{1,2,3}$ one needs to add the Hayward term at the two corners, $B_1 \cup B_3$ and $B_2 \cup B_3$.

$$S_H = \frac{1}{8\pi G_N} \int d^{d-1}x \sqrt{-h}\, \text{arccos}(n.\tilde{n})$$

• Correlators of insertions at the $B_{1,2}$ boundaries are done the same way as in standard AdS.

• Calculating correlators on the interface is problematic.

• We could not find a universal form of counterterms on a shifted boundary that removes all divergences from interface correlators.

• This is an open problem.

Details of the confining potential

We consider the following scalar potential

$$V(\Phi) = -\frac{d(d-1)}{\ell^2}\left(b\Phi^2 + \cosh^2(a\Phi)\right) \quad , \quad b = \frac{\Delta(d-\Delta)}{2d(d-1)} - a^2 \,. \qquad (3)$$

As $\Phi \to \pm\infty$, the above potential diverges as

$$V(\Phi) \to -\frac{d(d-1)}{4\ell^2} e^{\pm 2a\Phi} \,, \qquad (4)$$

where we assumed that $a < a_G$, the Gubser's bound.

This potential has a maximum at $\Phi = 0$ (UV fixed point) and near this point, it can be expanded as

$$V(\Phi) = -\frac{d(d-1)}{\ell^2} - \frac{1}{2} m^2 \Phi^2 + \mathcal{O}(\Phi^4) \quad , \quad m^2 = \frac{\Delta(d-\Delta)}{\ell^2} \,. \qquad (5)$$

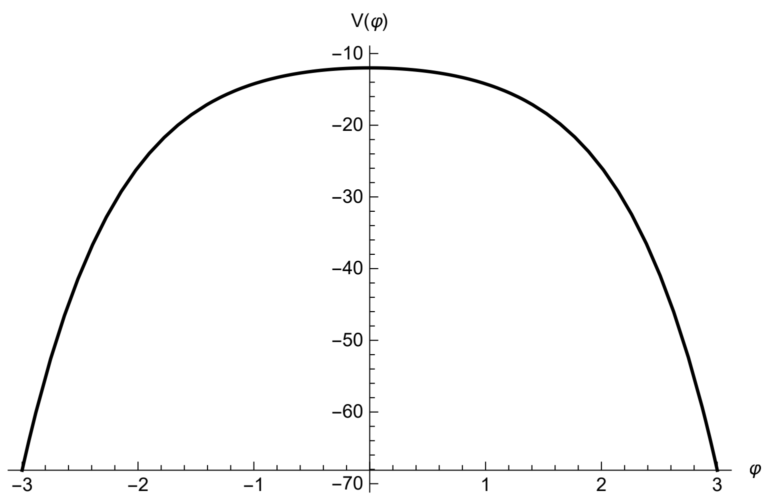
ℓ determines the length scale of asymptotically AdS solutions, Δ determines m^2 and is the scaling dimension of the operator dual to the scalar Φ near the UV fixed point. a determines the asymptotic behavior of the potential (confinement or deconfinement).

For the numerics we fix the constants of the theory as follows

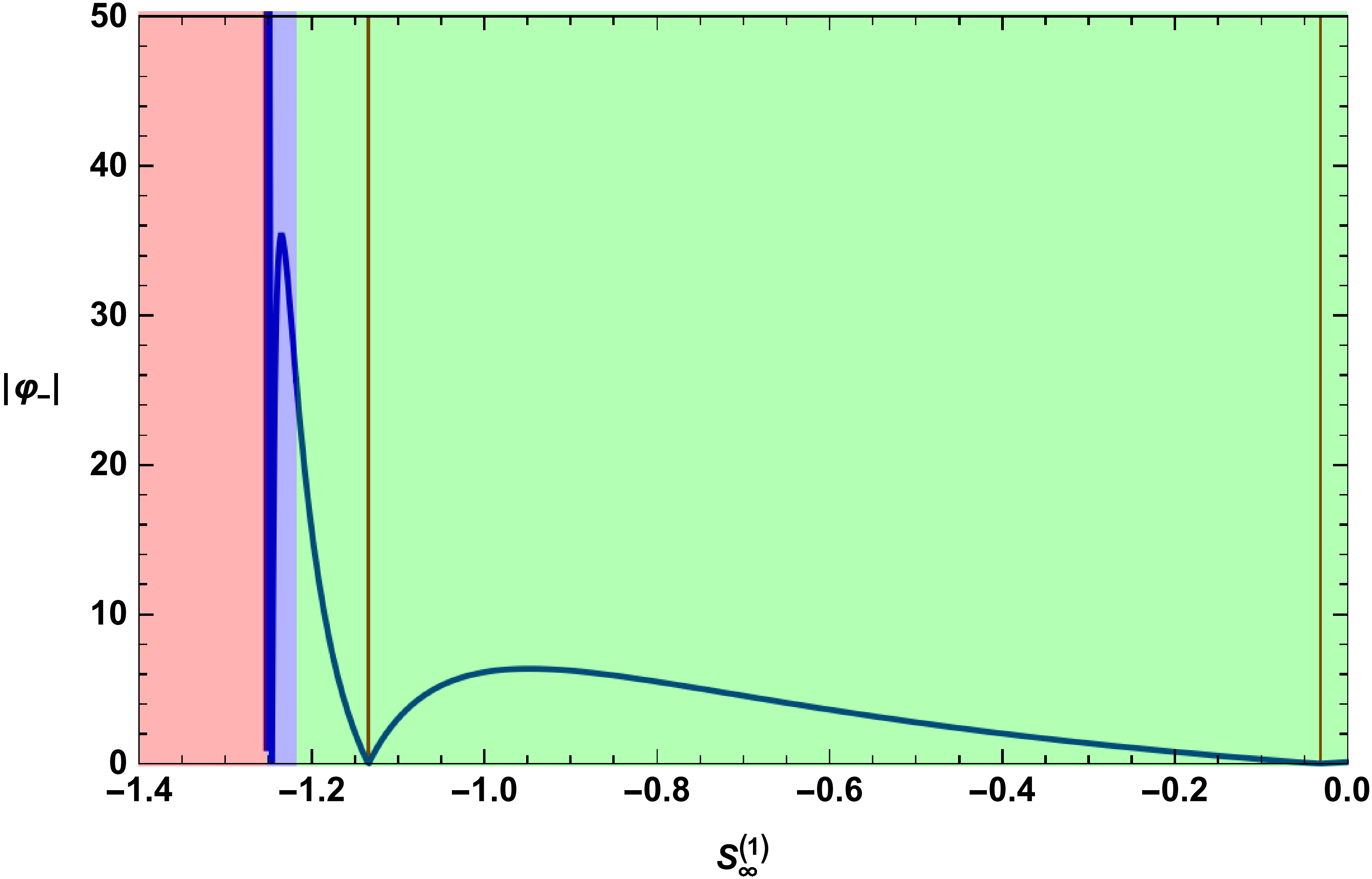
$$d = 4 \quad , \quad \Delta = \frac{3}{2} \quad , \quad \ell = 1 \quad , \quad a = \sqrt{\frac{7}{24}} \quad , \quad b = -\frac{13}{96}. \tag{6}$$

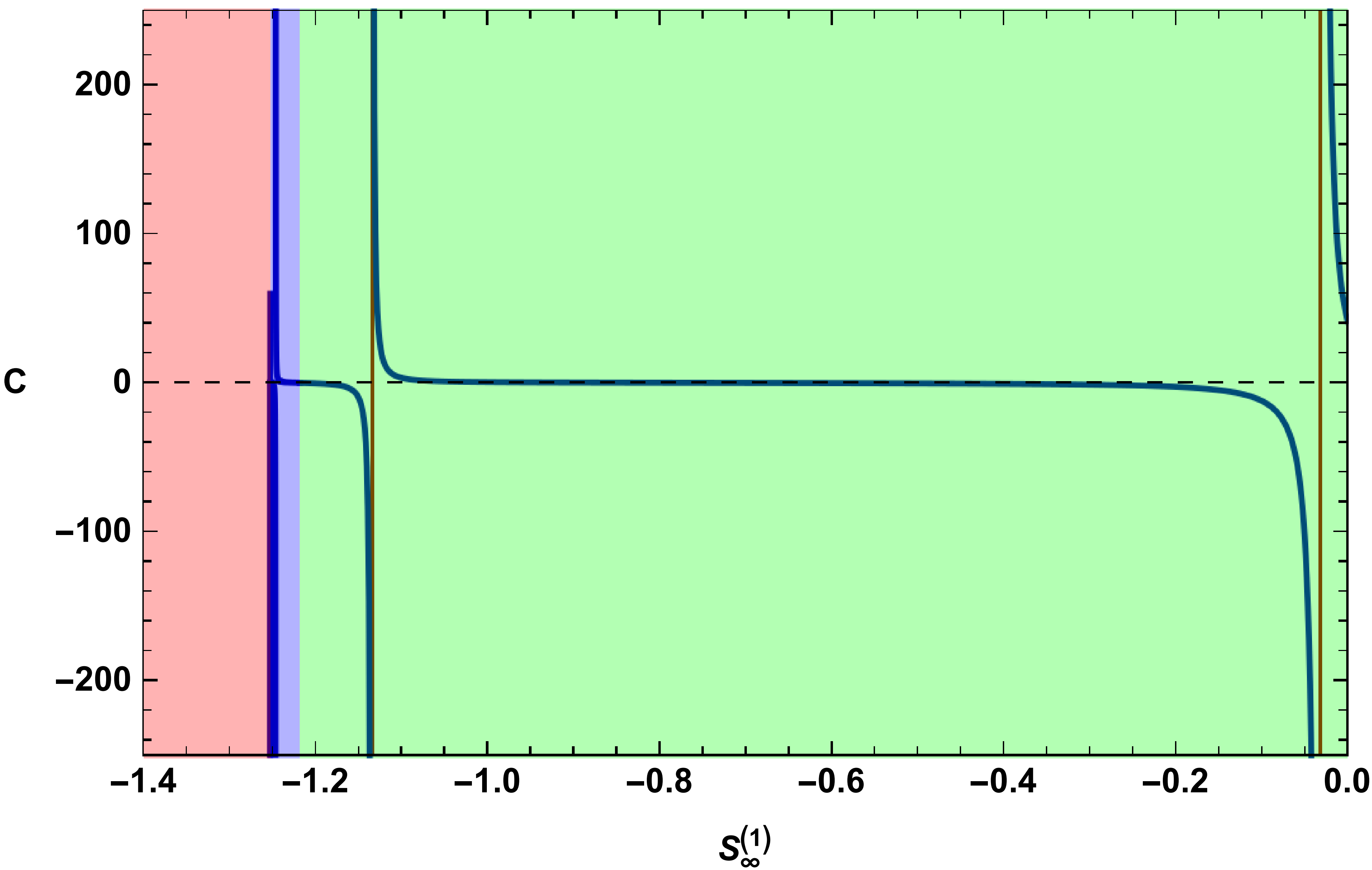
For the specific choice $d = 4$ we have

$$a_C = \frac{1}{\sqrt{6}} \quad , \quad a_G = \frac{2}{\sqrt{6}}, \tag{7}$$



Vevs





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−1.0
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−0.6
−0.4
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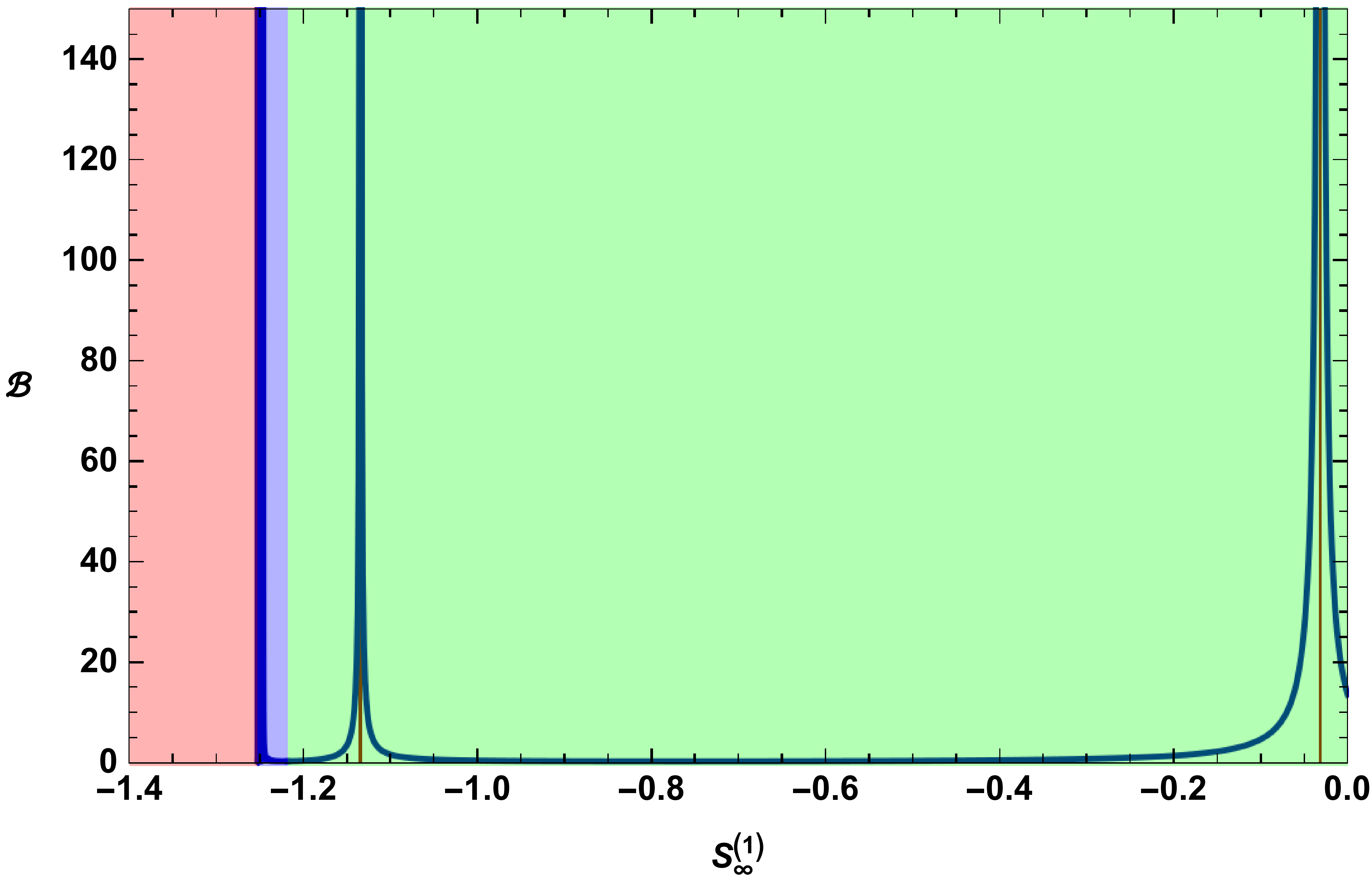
0.0
$S_\infty^{(1)}$



140
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$\mathcal{B}$
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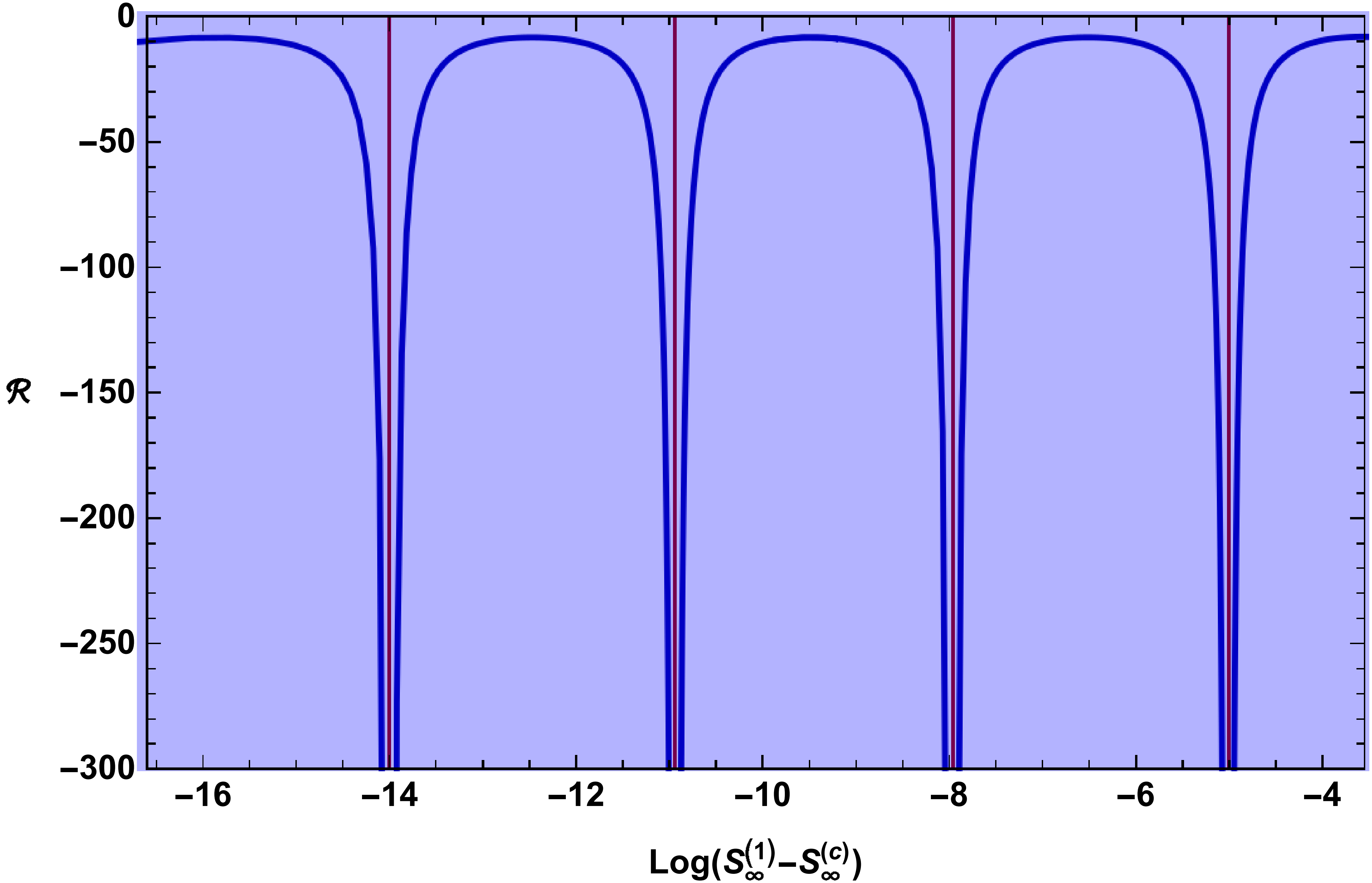
−1.2
−1.0
−0.8
−0.6
−0.4
−0.2
0.0
$S_\infty^{(1)}$

Φ_- the coupling of operator $\mathcal{O}$ at the UV boundary and C parameter of the UV boundary for UV-Reg solutions. All figures are plotted as a function of the free parameter $S_\infty^{(1)}$. In each graph, the green region belongs to the regular solutions without A-bounce and the blue region to solutions with at least one A-bounce. In the red region, we have not solutions with boundary. The vertical dashed line in figure (a) corresponds to the global AdS solution in the uplifted theory and the product solution is the solution right before the blue-red boundary. Figure (d) gives $\mathfrak{B}$ which we need to compute the free energy of the solutions.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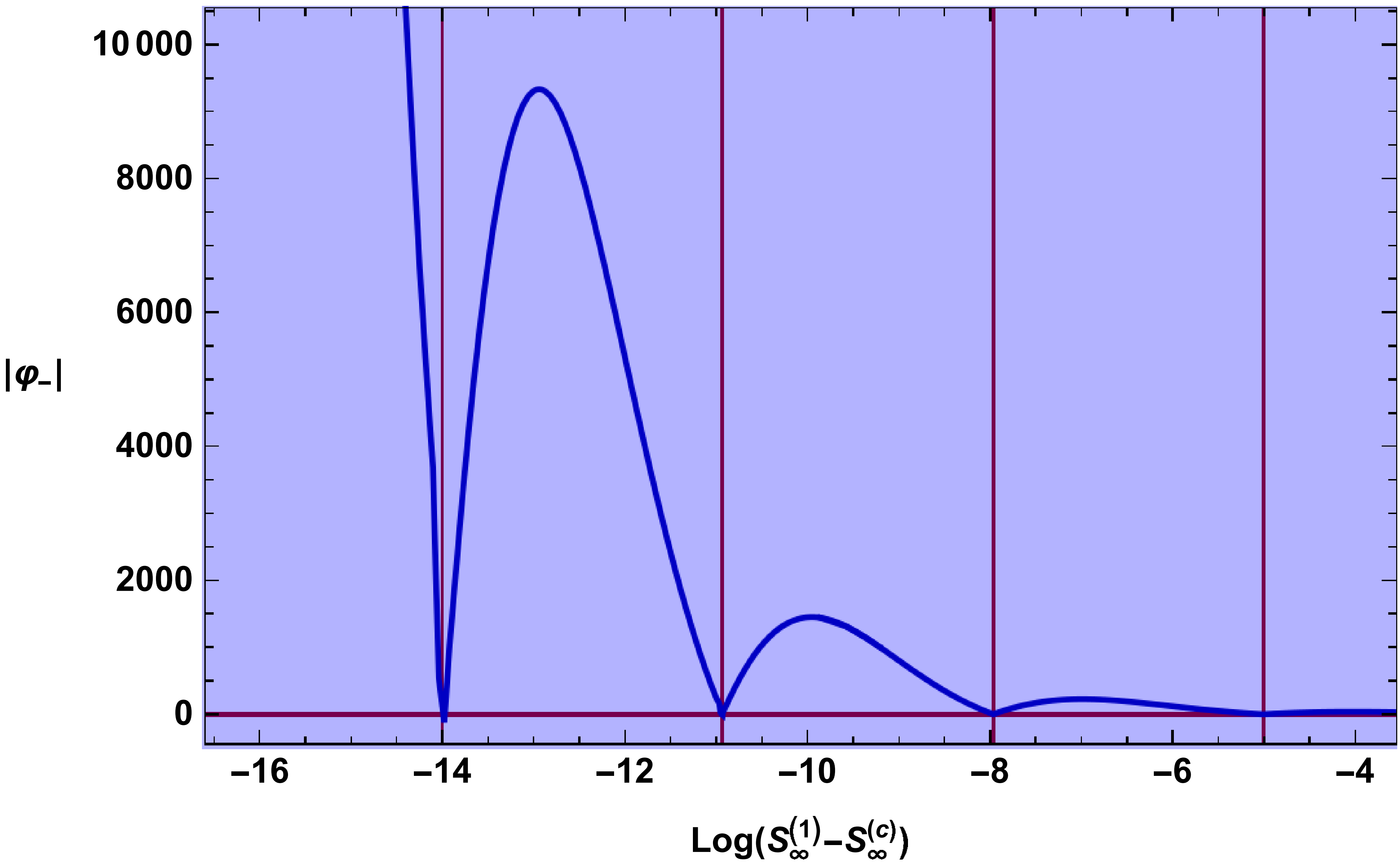
Log($S_\infty^{(1)}$−$S_\infty^{(c)}$)



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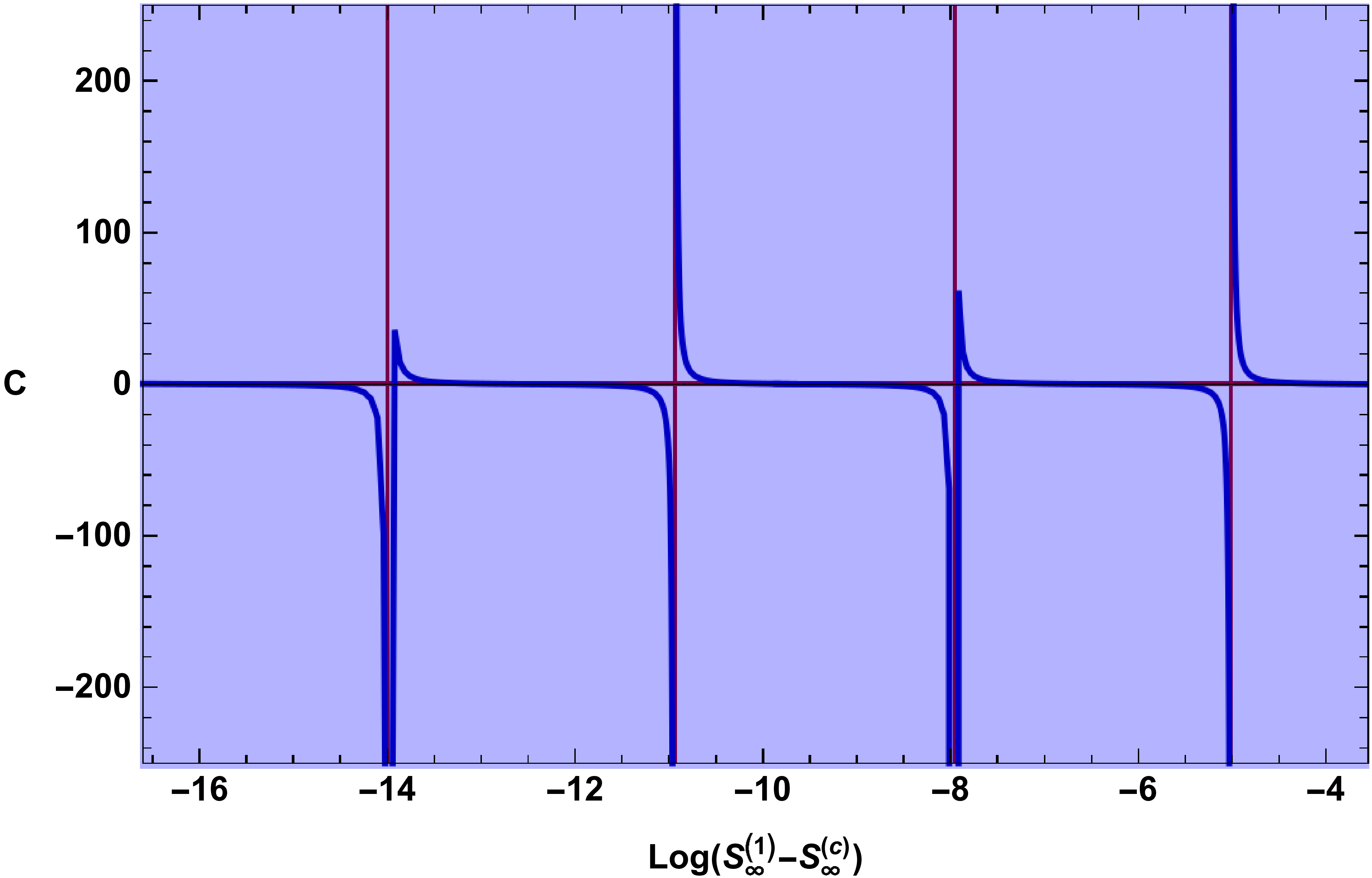
$\mathrm{Log}(S_\infty^{(1)} - S_\infty^{(c)})$



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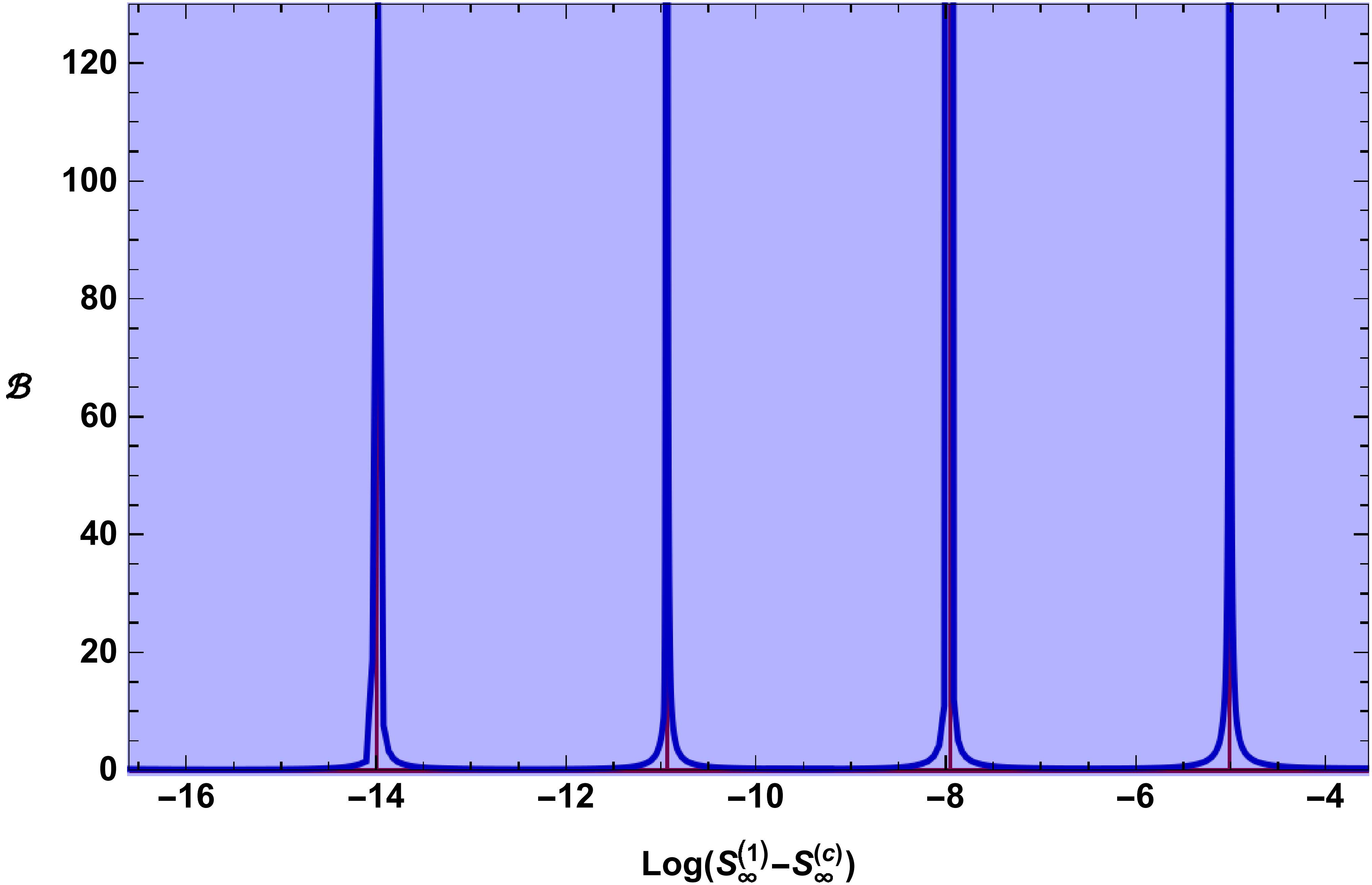
$\mathrm{Log}(S_\infty^{(1)} - S_\infty^{(c)})$



The blue region . The horizontal axis is $\log(S_\infty^{(1)} - S_\infty^{(c)})$, where $S_\infty^{(c)} \approx -1.25$

is the critical value for which we have the UV-Reg solution with infinite numbers of the loops.

Single boundary solutions

• To obtain a single boundary, one can orbifold a symmetric solution.

Aharony+Marolf+Rangamani

• This can be done in the class of solutions we called S. They have $S_0 = 0$ and they are completely symmetric.

• We obtain the half space with $u \in (-\infty, u_0)$.

• We can interpret such solutions by inserting an end-of-the-world brane at u_0.

• But because $\dot{A} = \dot{\Phi} = 0$ at u_0, this brane is both tensionless and chargeless.

• However, a look at correlators indicates that conformal invariance is broken (For AdS-sliced AdS).

• In the two boundary case, we have four possible two-point functions $\langle OO \rangle$: $G_{++}, G_{+-}, G_{-+}, G_{--}$

• The symmetric orbifold gives

$$G = G_{++} + G_{+-} = \frac{1}{2^\Delta}\left[\frac{1}{(\cosh L - 1)^\Delta} + \frac{1}{(\cosh L + 1)^\Delta}\right]$$

$$\cosh L = 1 + \frac{(z-z')^2 + |x-x'|^2}{zz'}$$

• The conformal correlator obtained from a Weyl transformation of flat space is the first piece only.

• This may be due to the fact that most boundary conditions break conformal invariance.

• If instead we insert a brane at $u = u_0$ and impose Dirichlet bc we obtain a similar result with a relative minus sign. (The orbibold corresponds to Neumann)

• Are there bc on the brane so that we obtain a conformal correlator?

• Yes, but they are generically non-local on the brane.

Introduction-I

• QFTs have parameters.

• Some are associated to scalar operators.

♠ Others to the energy-momentum tensor (geometry) or currents (charge densities).

• The latter are always "relevant" (they affect non-trivially the IR physics)

• They are important in cosmology and/or astrophysics and cond-mat physics.

Confining Theories on AdS

• In a single scalar setup, the confining solutions are solutions where the scalar runs off to infinity.

• These are singular solutions (naked singularities)

• But one out of the one-parameter family of solutions is "less" singular.

• This corresponds to a resolvable singularity and can be resolved by KK states.

Gubser: the good, the bad and the naked

• Such solutions correspond to confining ground states in flat space.

• All of their aspects (with flat slices) have been studied extensively in the past.

• In the case of AdS slices new phenomena appear. Unlike non-confining theories , there is an infinite number of solutions with a single AdS boundary.

Confining Theories on AdS-The setup

- We study Einstein Dilaton theory with a potential.

- We parametrize the boundary behavior of the potential (as $\Phi \to +\infty$), as

$$V \simeq -V_\infty e^{2a\Phi} + \cdots ,$$

where V_∞ and a are two positive constants.

- The non-confining range:

$$0 \quad \leq \quad a \quad < \quad a_C \equiv \sqrt{\frac{1}{2(d-1)}}.$$

- The confining range:

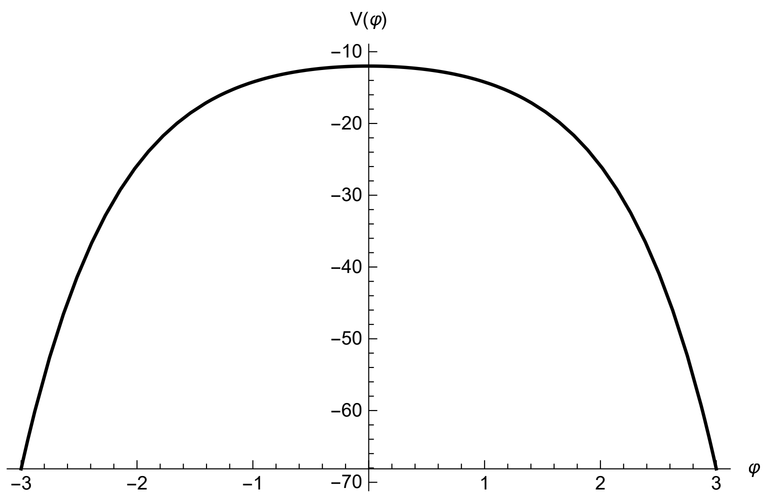
$$a_C \quad < \quad a \quad < \quad a_G \equiv \sqrt{\frac{d}{2(d-1)}}$$

• The Gubser-violating range:

$$a \quad > \quad a_G$$

• We are interested in the confining range.

• We choose a simple potential with the required asymptotics and a single maximum.



• The theory sitting at the maximum is the UV of the confining QFT.

• With flat metric slices, that solution runs from the maximum to $\Phi \to +\infty$ via a "regular" solution and this is the confining ground-state on flat $\mathbb{R}^d$.

• We now consider solutions with AdS$_d$ slices.

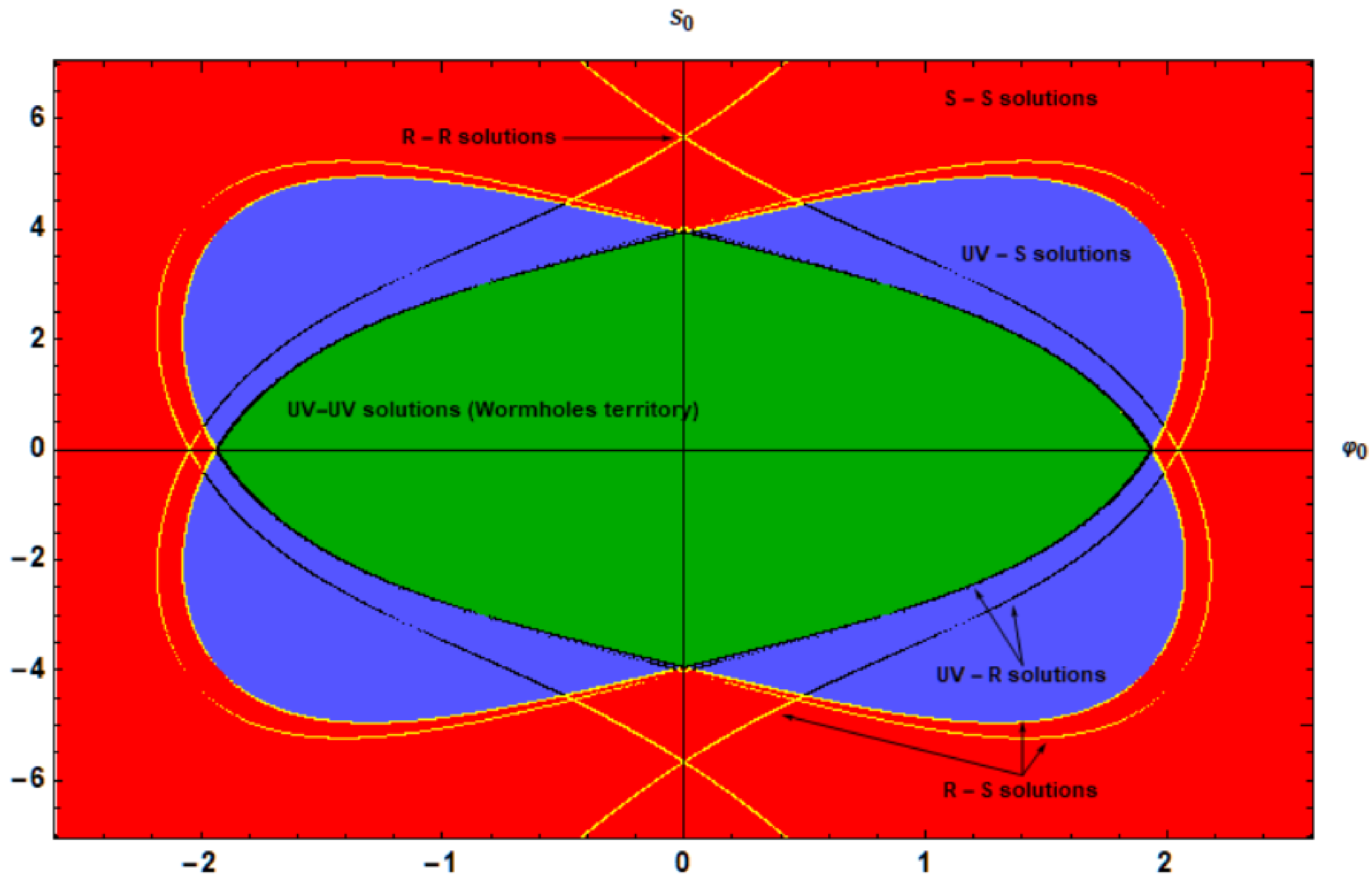
• There are three classes of "regular" solutions

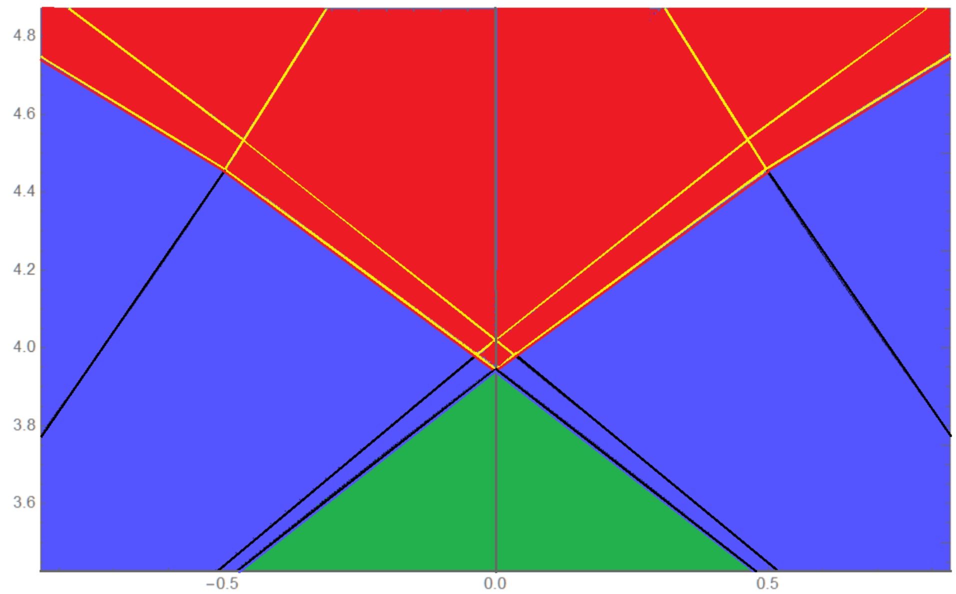
♠ Two-boundary Solutions: They start at $\Phi = 0$ (boundary) and end at $\Phi = 0$ (boundary). These are interface solutions of confining theories.

♠ One-boundary solutions: They start at $\Phi = 0$ (boundary) and end at $\Phi = \pm\infty$ (IR-end point). These are dual to confining theories on AdS$_d$.

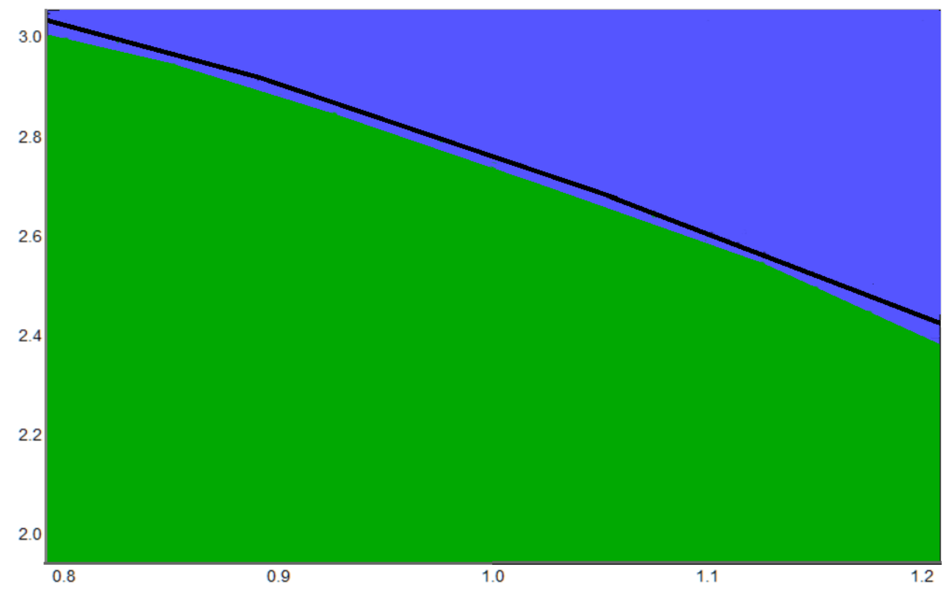
♠ No-boundary solutions: Start at $\Phi = -\infty$ and end at $\Phi = +\infty$ or start at $\Phi = -\infty$ and return back to $\Phi = -\infty$. Interpretation?

Confining Theories on AdS-The space of solutions



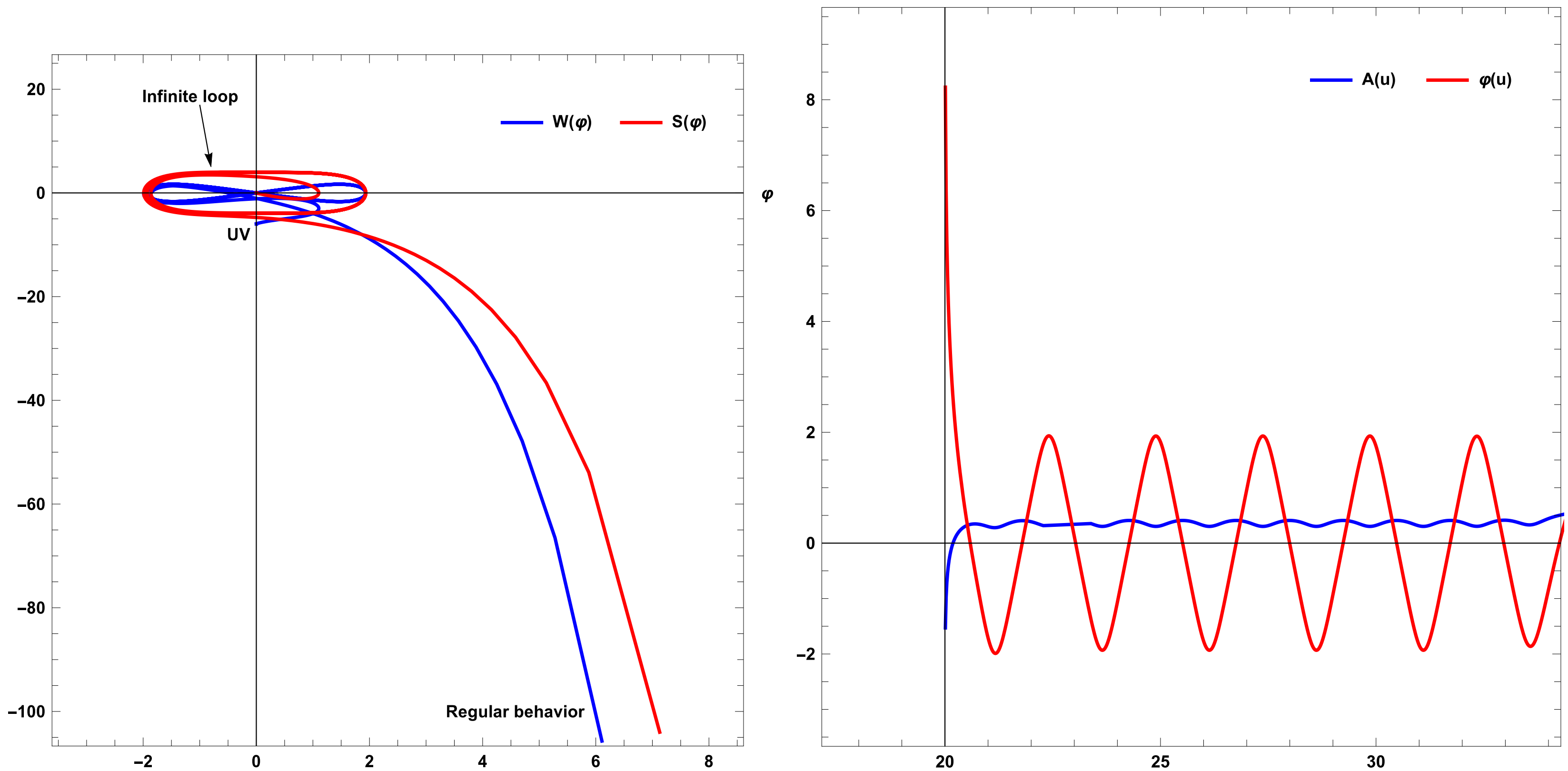


4.8
4.6
4.4
4.2
4.0
3.8
3.6
−0.5
0.0
0.5



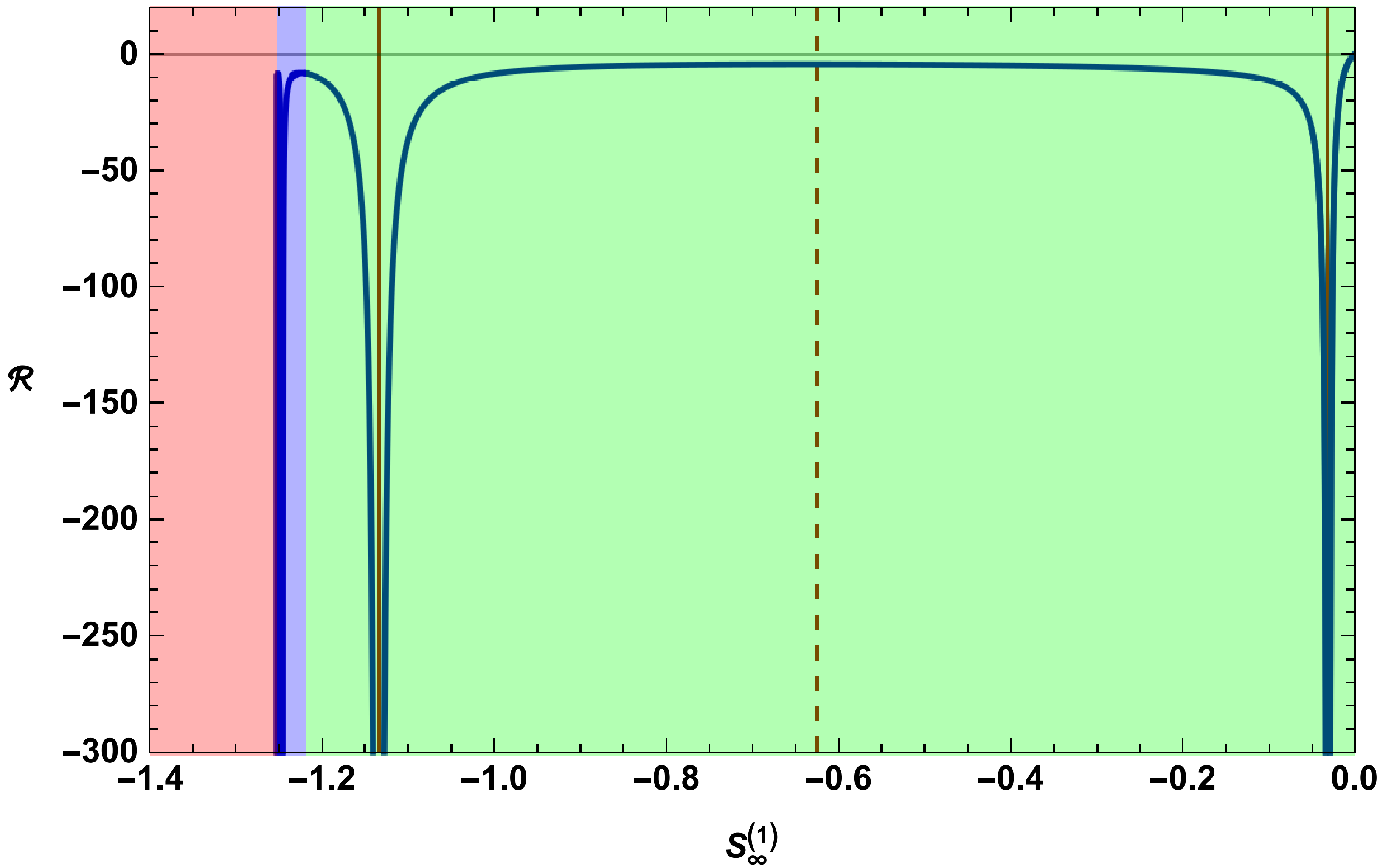
3.0
2.8
2.6
2.4
2.2
2.0
0.8
0.9
1.0
1.1
1.2

Confining Theories on AdS-Critical solutions



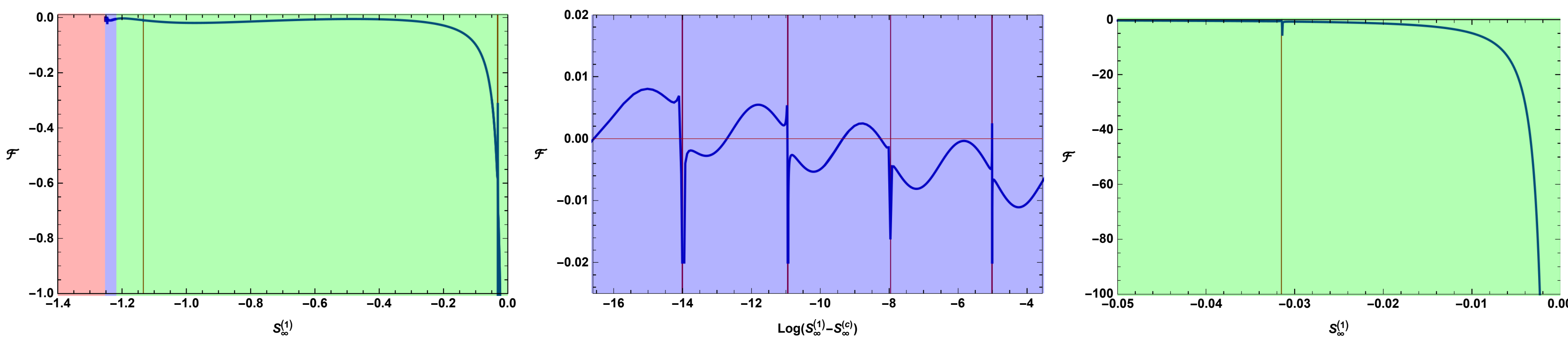
• Similar solutions were found in S-duality orbifold by Arav+Chung+Gauntlett+Roberts+Rosen.

relation to sources



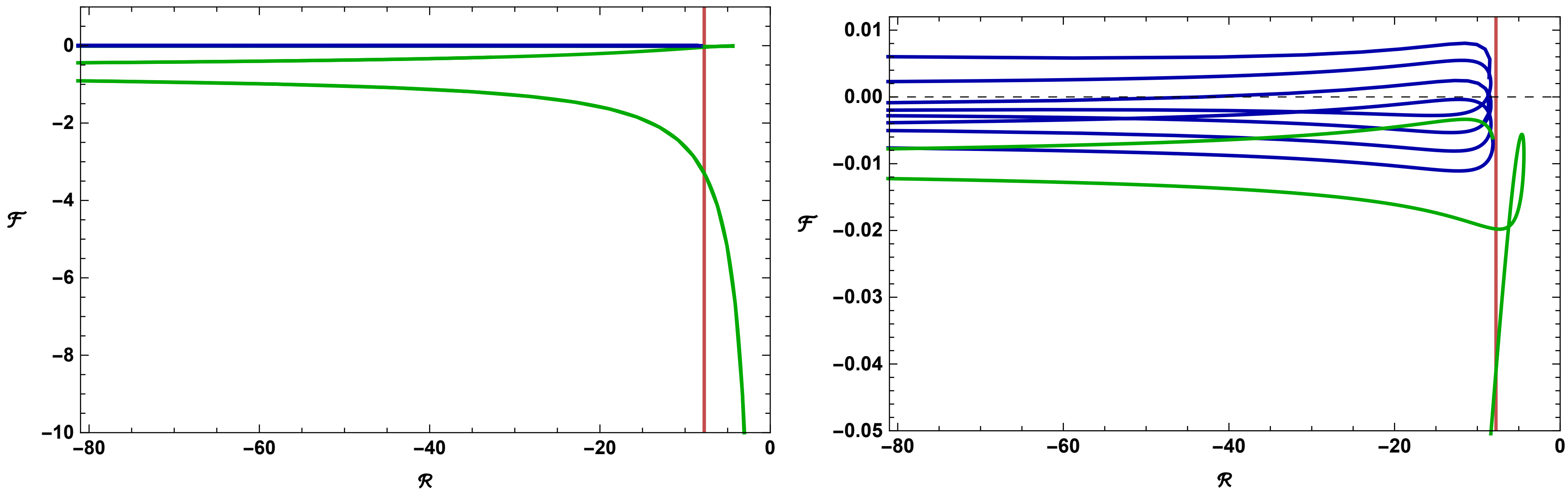
$\mathcal{R}$, the dimensionless curvature, for UV-Reg solutions. All figures are plotted as a function of the free parameter $S_\infty^{(1)}$.

In each graph, the green region belongs to the regular solutions without A-bounce and the blue region to solutions with at least one A-bounce. In the red region, we have solutions without boundary. The vertical dashed line in figure (a) corresponds to the global AdS solution in the uplifted theory and the product solution is the solution right before the blue-red boundary.



(a): The free energy for UV-Reg solutions living on the black curves. (b) The blue region is zoomed in. The horizontal line is now log($S_\infty^{(1)} - S_\infty^{(c)}$), where $S_\infty^{(c)} \approx -1.25$ is the critical value for which we have the UV-Reg solution with infinite numbers of loops. (c): The region near $S_\infty^{(1)} = 0$ is zoomed. In all diagrams, the vertical red lines show the location of Φ-bounces.

The free energy



(a): Free energy in terms of dimensionless curvature. The green/blue curves correspond to the green/blue region in previous plots. Figure (b) is the zoomed region near $\mathcal{F} = 0$. The vertical red line shows for $\mathcal{R} \gtrsim -7.7$ only solutions without A-bounce exist.

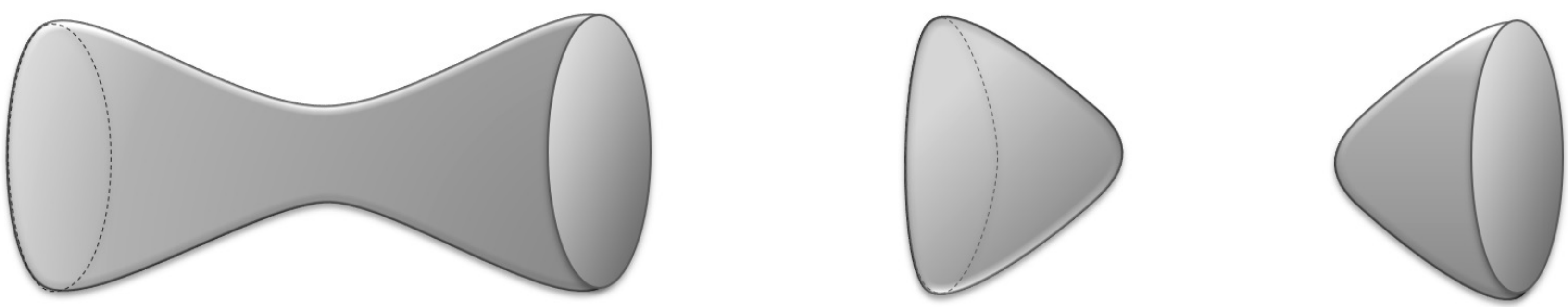
- The solution with no oscillations has the lowest free energy.

- Is there Efimov scaling here?

Two-boundary saddle points

- Similarly, the free energy can be calculated for the two-boundary solutions dual to holographic interfaces.

- In this case, the free energy depends on the data of both theories



- There is always competition from the factorized solutions.

- The factorized solutions have lower-free energy always.

- This implies that cross-correlators are exponentially suppressed in N.

- The physics of this result is not clear to us.

Conclusions

- We have studied (RG) flow solutions with slices that have constant negative curvature manifolds.

- Such solutions have generically two boundaries and can be interpreted as wormholes or interfaces. In confining theories there are also one-boundary solutions.

- We have analysed in detail several types of examples.

- The results suggested that proximity is close to RG Flow connection but its reach is more general.

- We found also many limiting cases where one obtains all possible exotic RG flows.

- Other phenomena found include flow (multi)-fragmentation, walking behavior, and the generation of new boundaries.

- Only in confining examples there are genuine one-boundary geometries.

- We have found an infinite number of saddle points in confining theories on AdS.

- We DID NOT find a phase transition as a function of curvature.

Detailed plan of the presentation

- The Classification of complete flows: $\mathcal{R} > 0$ 28 minutes
- The Classification of complete flows: $\mathcal{R} < 0$ 30 minutes
- A concrete example 31 minutes
- The space of solutions 33 minutes
- The region boundaries and tuned flows 34 minutes
- Flow fragmentation, walking and emergence of boundaries 35 minutes
- Open ends 36 minutes