Symmetries from Branes
&
Branes from Symmetries

Jonathan J. Heckman

University of Pennsylvania

Alternative Title:

Alternative Title:

What Happens To Topological Symmetry Operators In Quantum Gravity?

Based On

A series of papers with (in various permutations)

Baume, Cvetič, Debray, Dierigl, Hübner,

McNamara, Montero, Murdia, Sharon, Torres,

Turner, Valenzuela, Vafa, Yu, Zhang

Symmetries

Symmetries

- Are interesting...

Symmetries

- Are interesting...
- Are useful (selection rules, RG flow constraints...)

Symmetries

- Are interesting...
- Are useful (selection rules, RG flow constraints...)
- Generalized Symms:
  Recent (and vast) generalization
  New Physical & Mathematical Structures

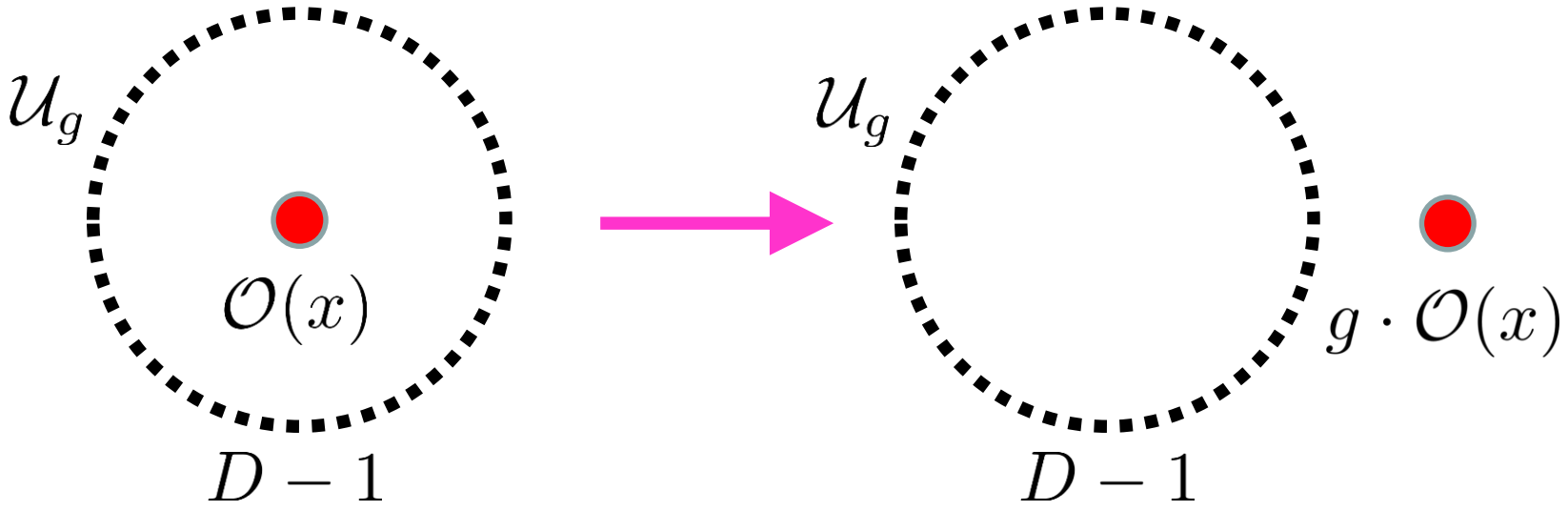
Gen$^{\text{ed}}$ Symms in QFT

Gaiotto Kapustin Seiberg Willett '14

Main Idea: Global Symms are *Topological*

Consider D-dim^{l} QFT & Charged Local Op:

"Zero-Form Symmetry"



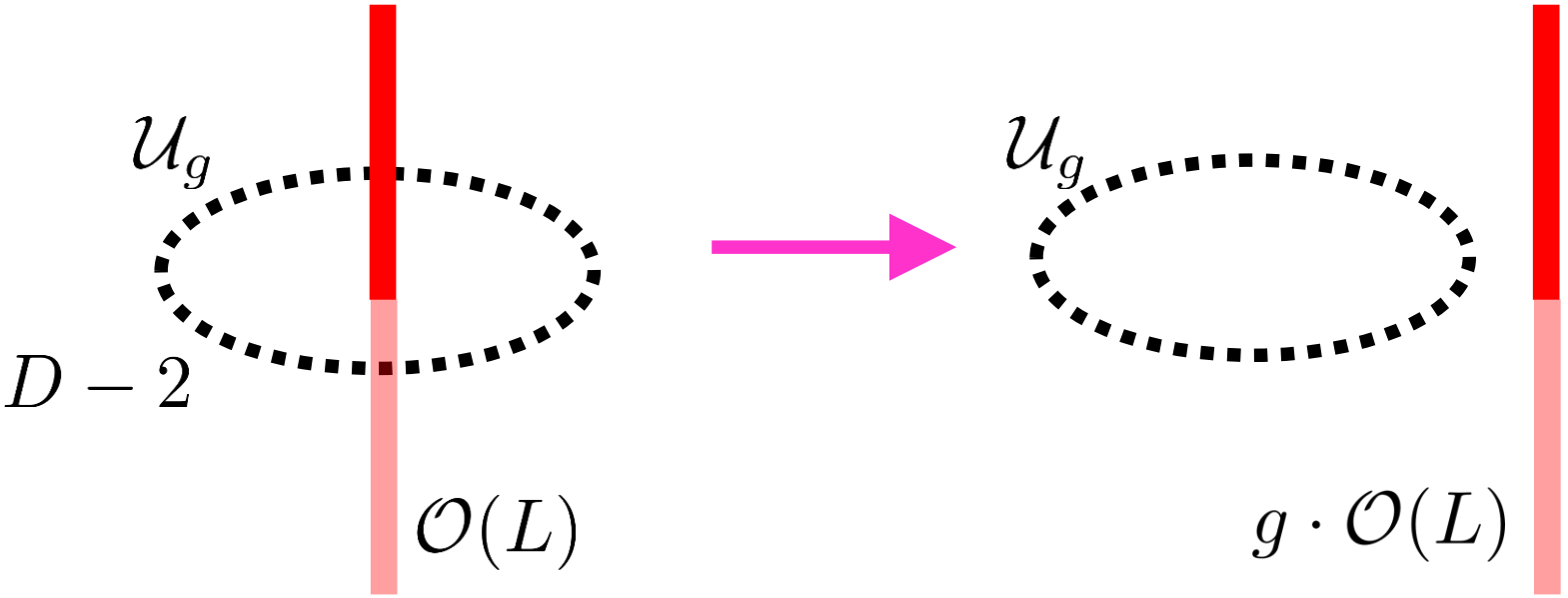
Gen$^{\text{ed}}$ Symms in QFT

Gaiotto Kapustin Seiberg Willett '14

Main Idea: Global Symms are *Topological*

Consider D-dim^{l} QFT & Charged Line Op:

"One-Form Symmetry"



Many Generalizations...

- Higher p-Form Symmetries
- Higher Group Symmetries
- Non-Invertible / Fusion / Categorical Symms

¿2014 $\rightarrow$ 2024?

Gen. Symms. most extensively examined in $D \leq 4$ Lagrangian QFTs

How About:

- "Non-Lagrangian" QFTs?
- Interacting $D > 4$ CFTs?
- Consequences of $G_N \neq 0$?

¿2014 $\rightarrow$ 2024?

Gen. Symms. most extensively examined in $D \leq 4$ Lagrangian QFTs

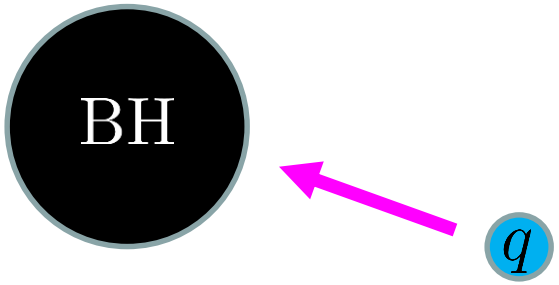
How About:

- "Non-Lagrangian" QFTs?
- Interacting $D > 4$ CFTs?
- Consequences of $G_N \neq 0$?

} Use String Theory!!!

Symmetries and $G_N \neq 0$

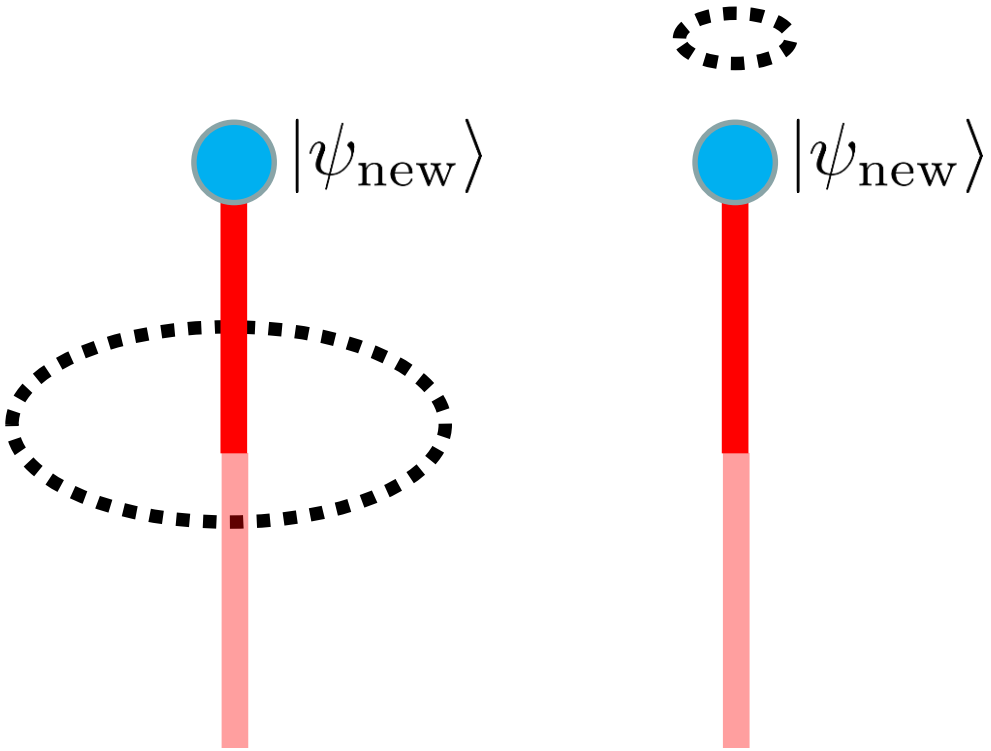
Lore: "Gauged or Broken"



Question: How does this work with
topological symmetry operators???

Breaking: Bottom Up View

Breaking a p-form symmetry: Introduce new states



Breaking: Bottom Up View

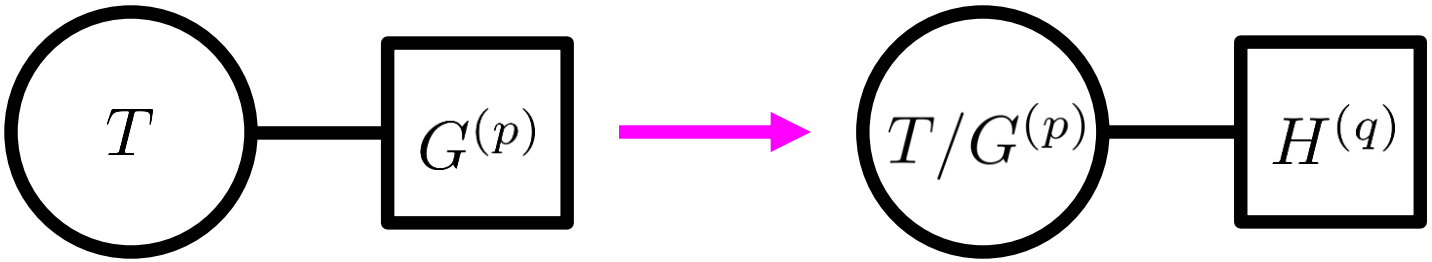
Breaking a symmetry: Introduce new states

Precisely which states needed for "completeness"?

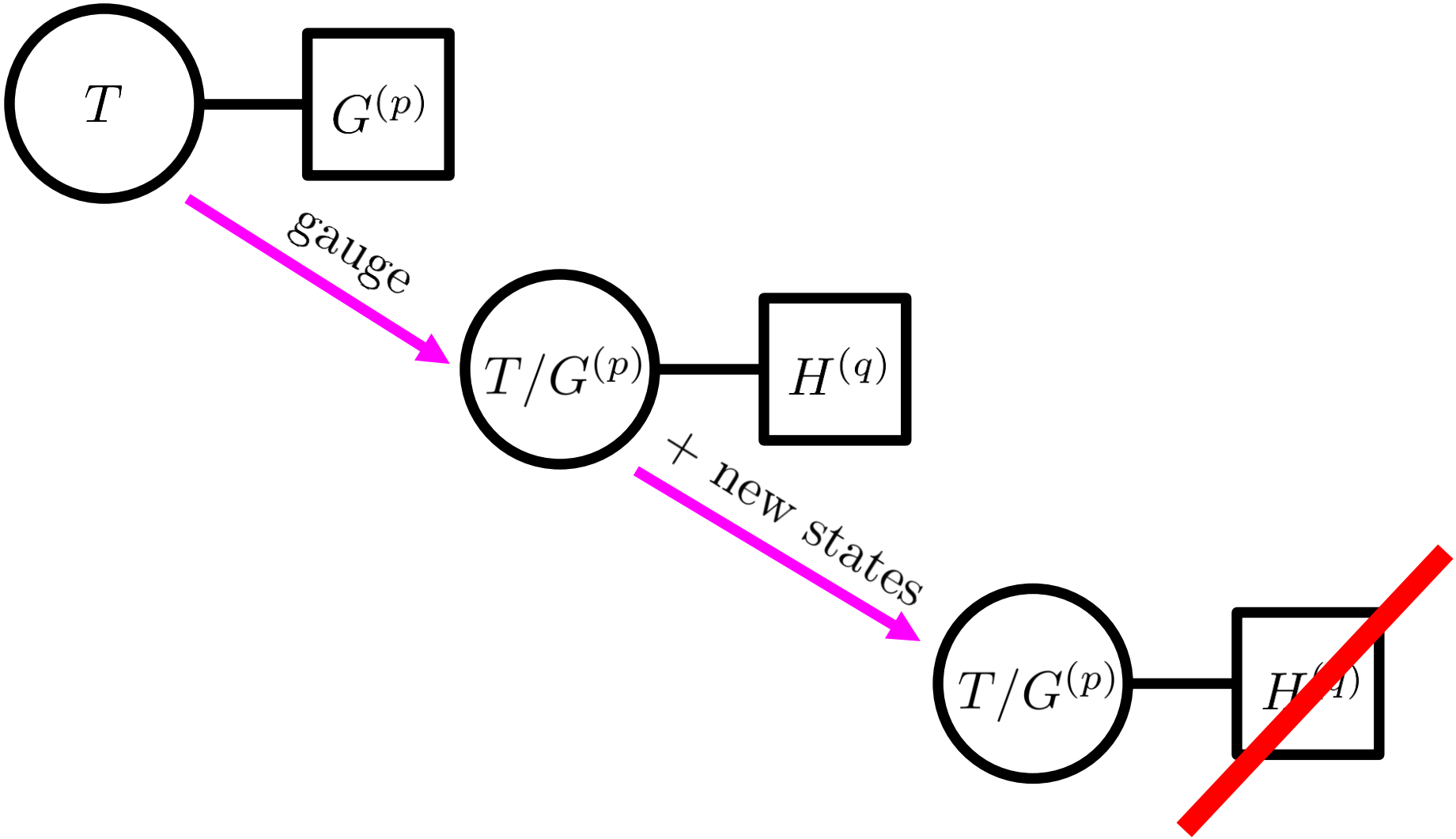
Gauging: Bottom Up View

Gauging a symmetry:
(Assuming non-anomalous, makes sense in general)

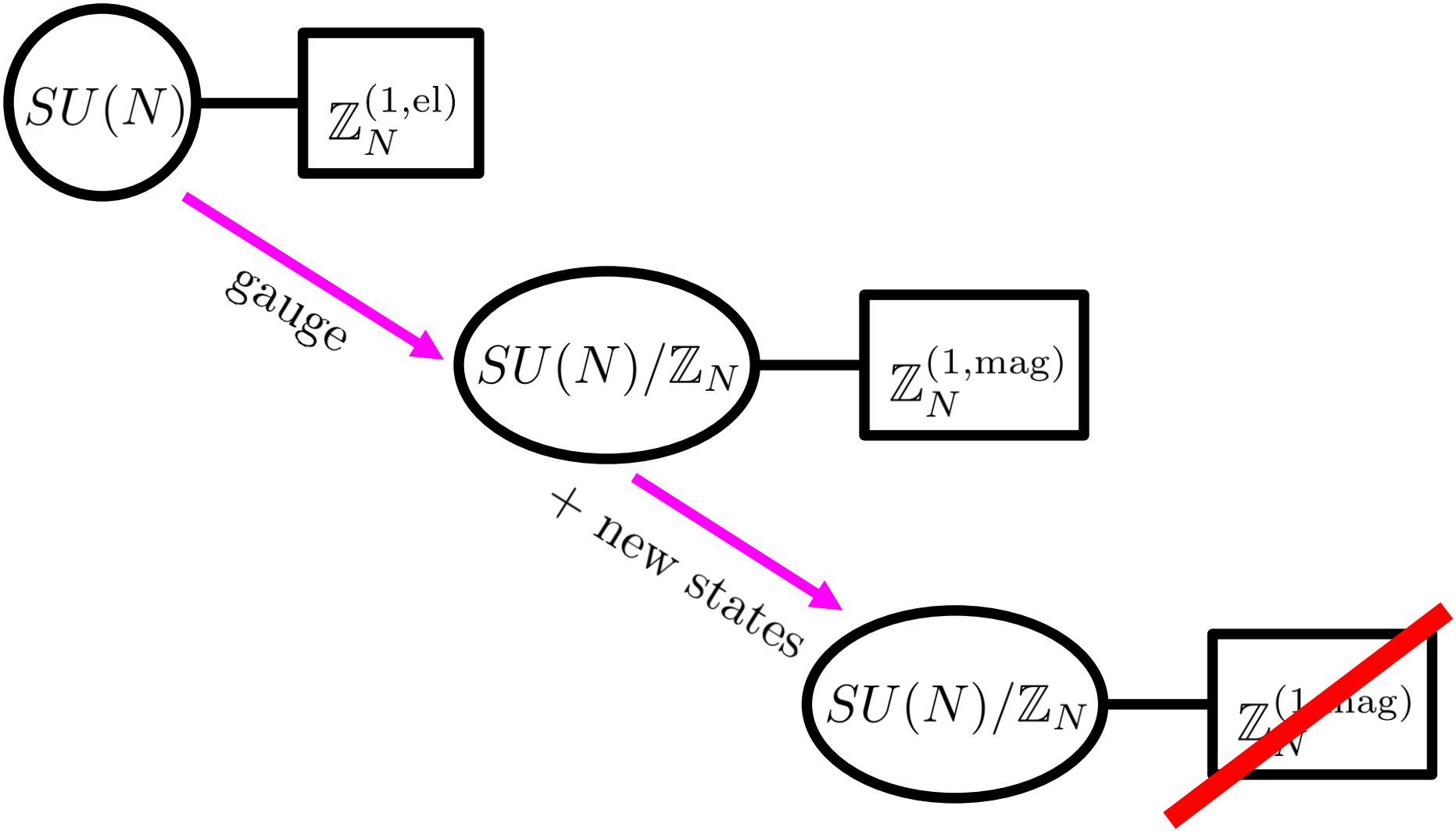
Example issue: gauge a finite p-form symm,
get a magnetic q-form symm
$(p + q = D - 2)$



Gauging + Breaking



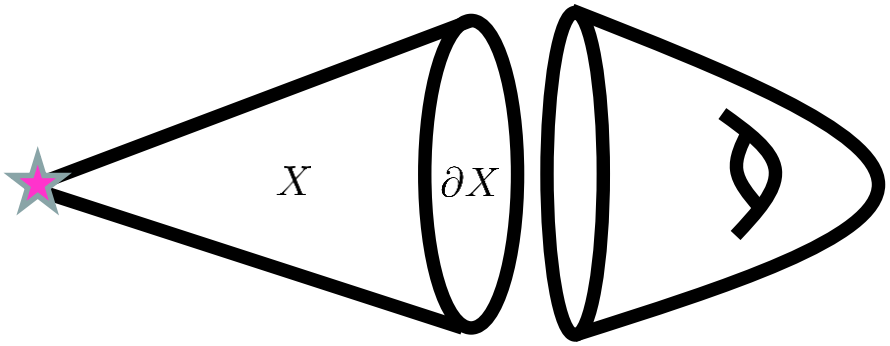
Example


$SU(N)$
$\mathbb{Z}_N^{(1,\mathrm{el})}$
gauge
$SU(N)/\mathbb{Z}_N$
$\mathbb{Z}_N^{(1,\mathrm{mag})}$
+ new states
$SU(N)/\mathbb{Z}_N$
$\mathbb{Z}_N^{(1,\mathrm{mag})}$

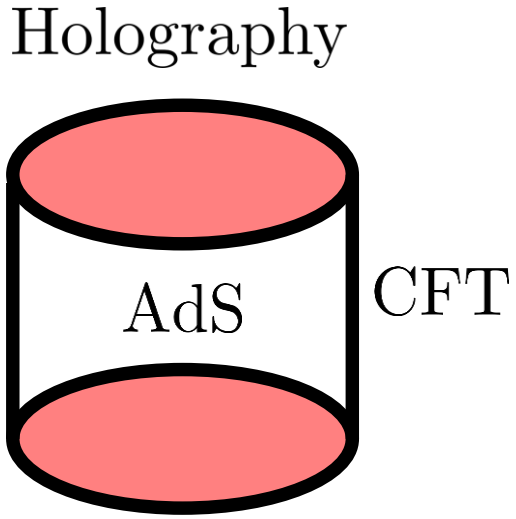
UV Motivated Approaches

UV Motivated Approaches

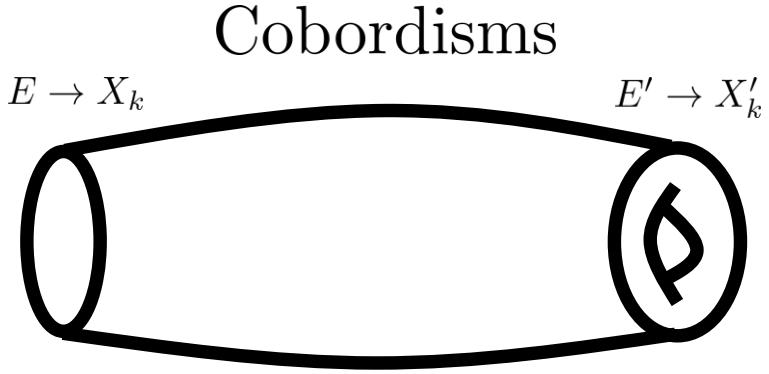
String Compactification



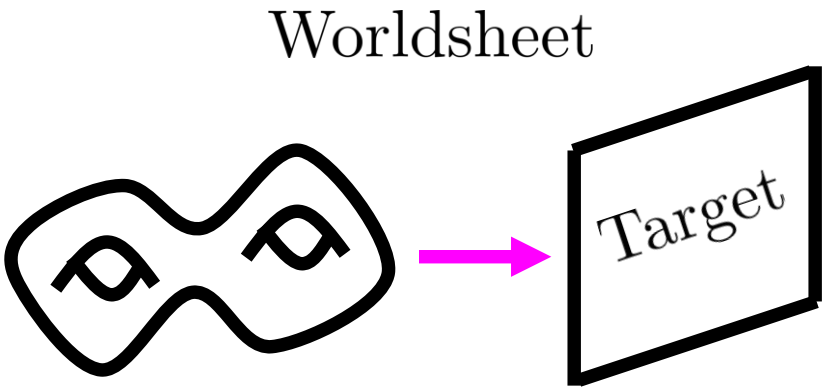
UV Motivated Approaches



UV Motivated Approaches

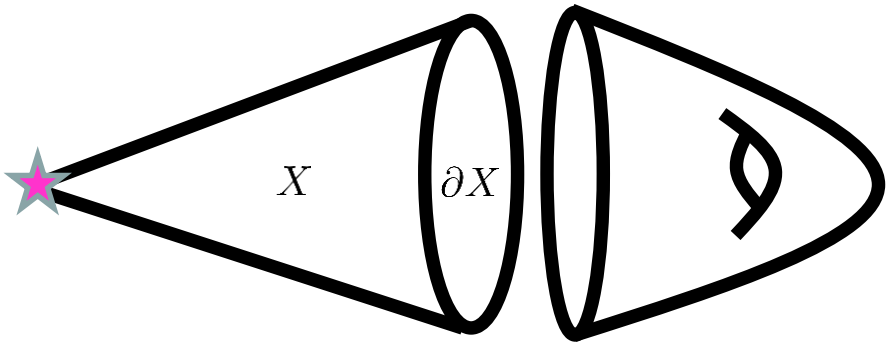


UV Motivated Approaches

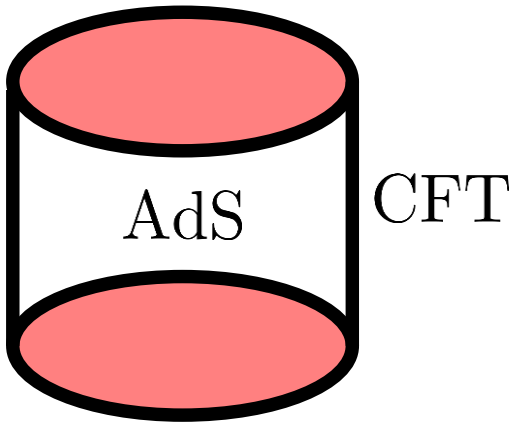


UV Motivated Approaches

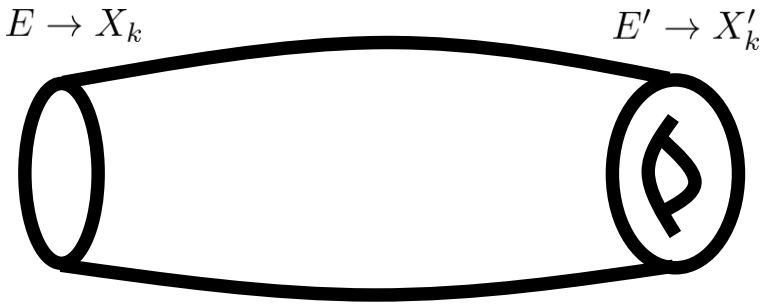
String Compactification



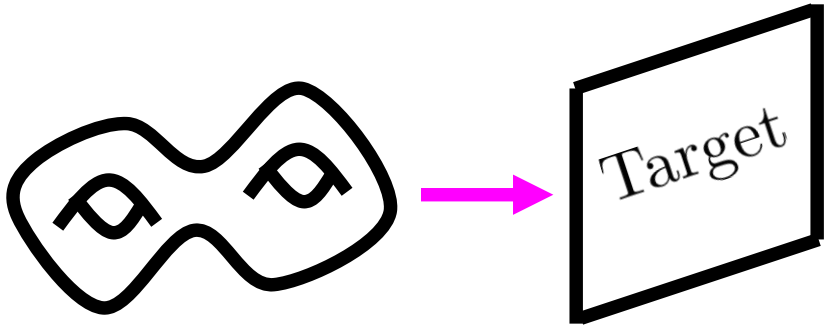
Holography



Cobordisms

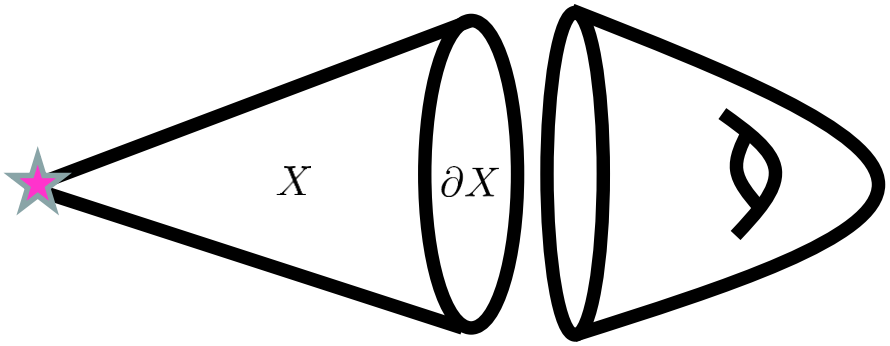


Worldsheet

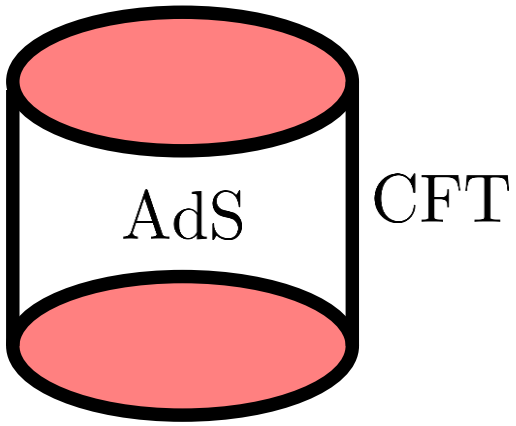


Focus For Today:

String Compactification



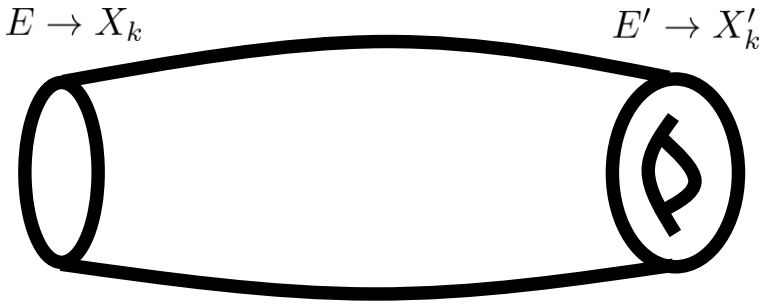
Holography



For Another Time...

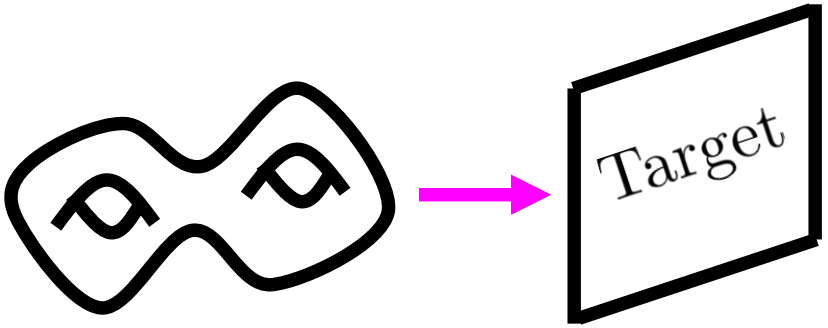
Markus' Talk

Cobordisms

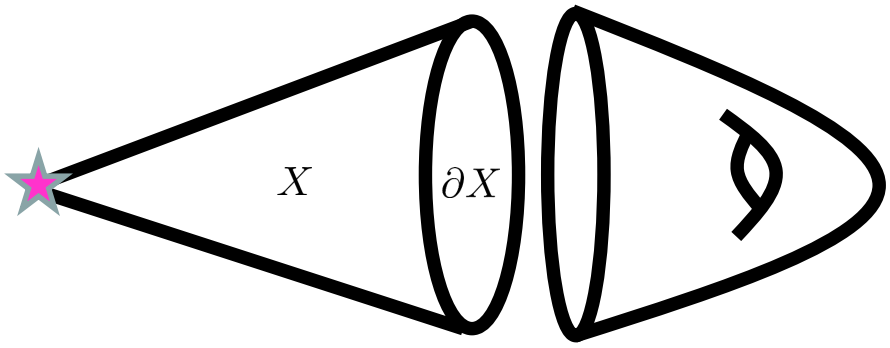


Jake's Talk

Worldsheet



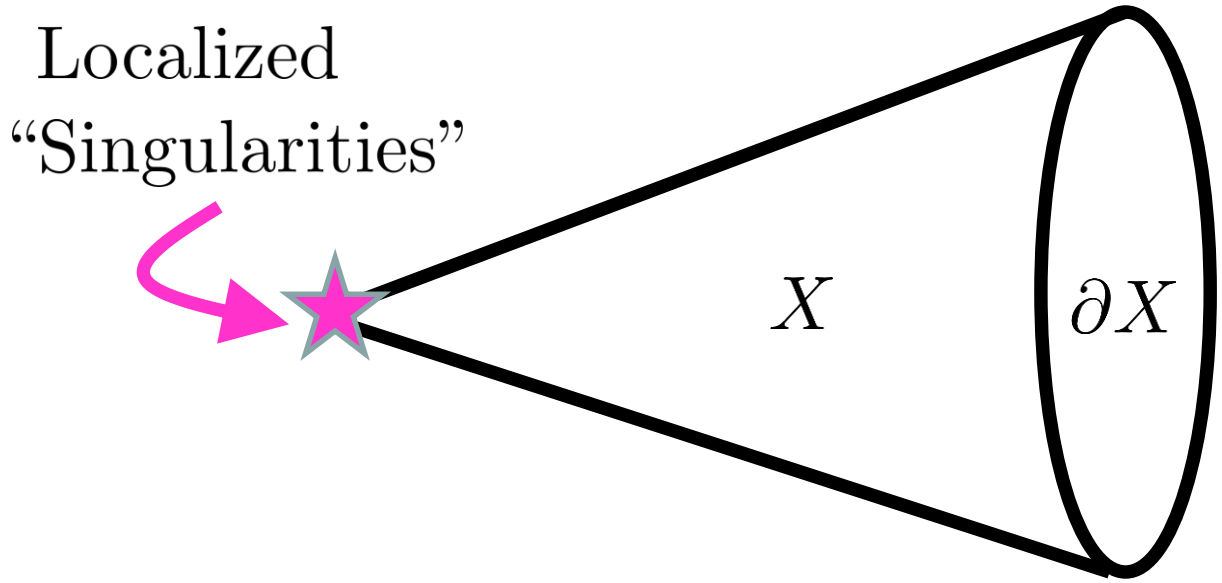
Approach 1:
String Compactification



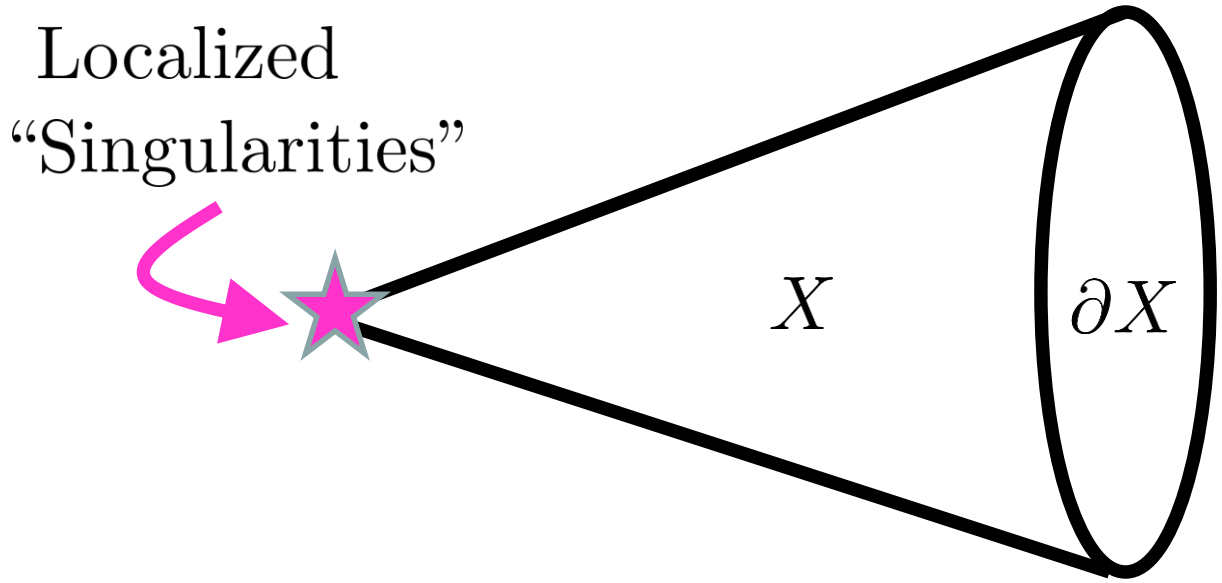
Stringy Generalized Symms

- Engineer a QFT (via Geometry)
- Defects (via Branes / Geometry / ...)
- Symmetry Ops (via Branes / Geometry / ...)

Step 1: Engineering QFTs



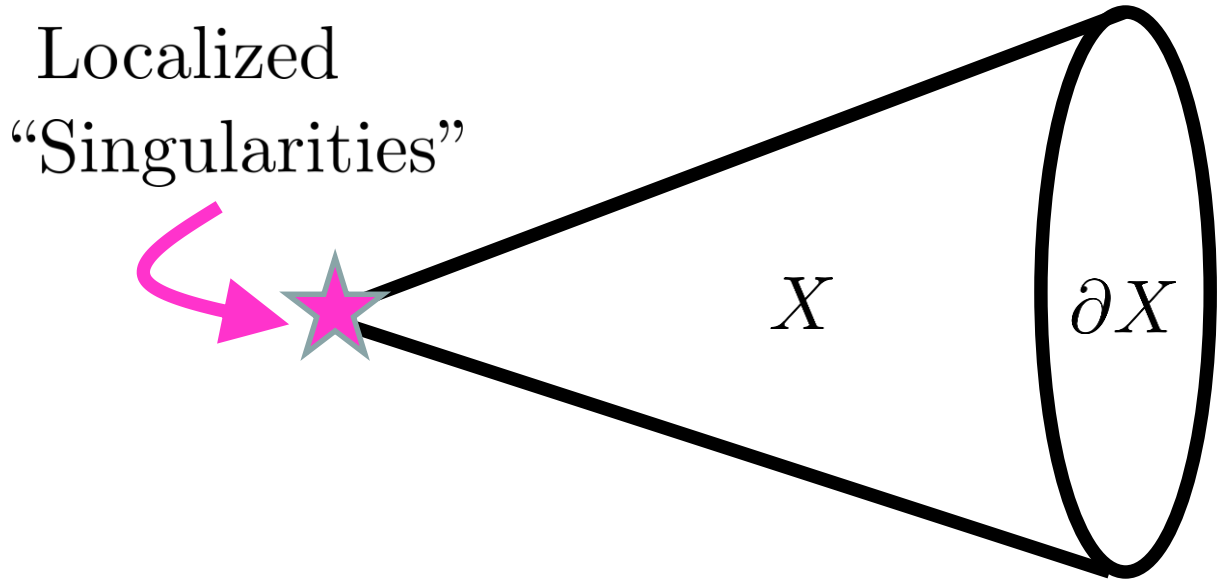
Step 1: Engineering QFTs



Example: 6D SYM via IIA on $\mathbb{R}^{5,1} \times X$,

$$X = \mathbb{C}^2/\Gamma_{SU(2)},\ \partial X = S^3/\Gamma_{SU(2)}$$

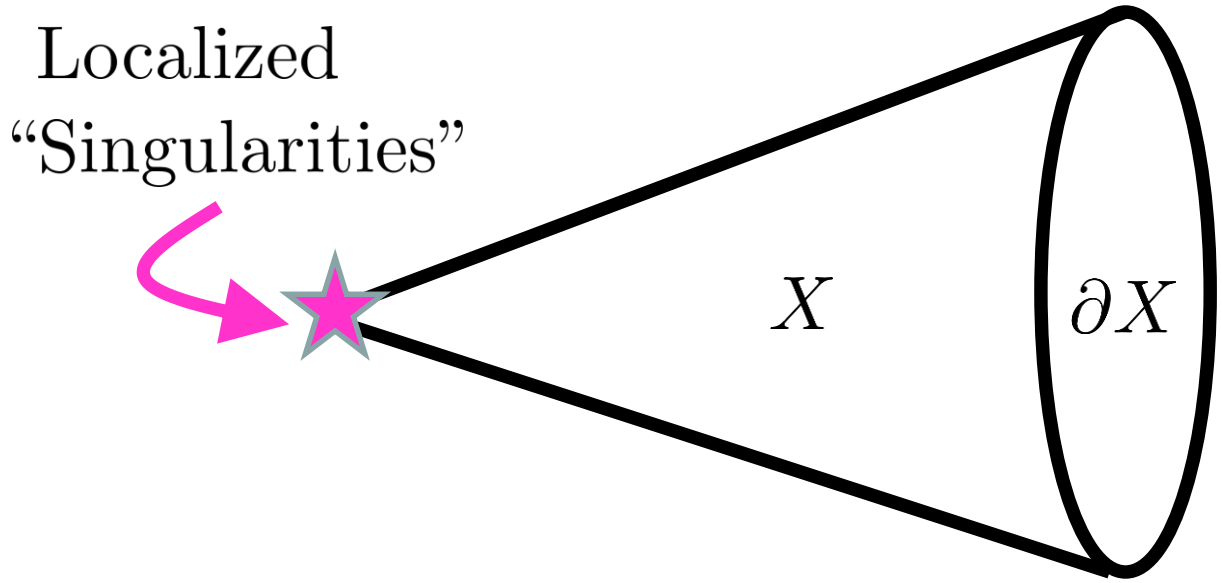
Step 1: Engineering QFTs



Example: 6D SCFTs via IIB on $\mathbb{R}^{5,1} \times X$,

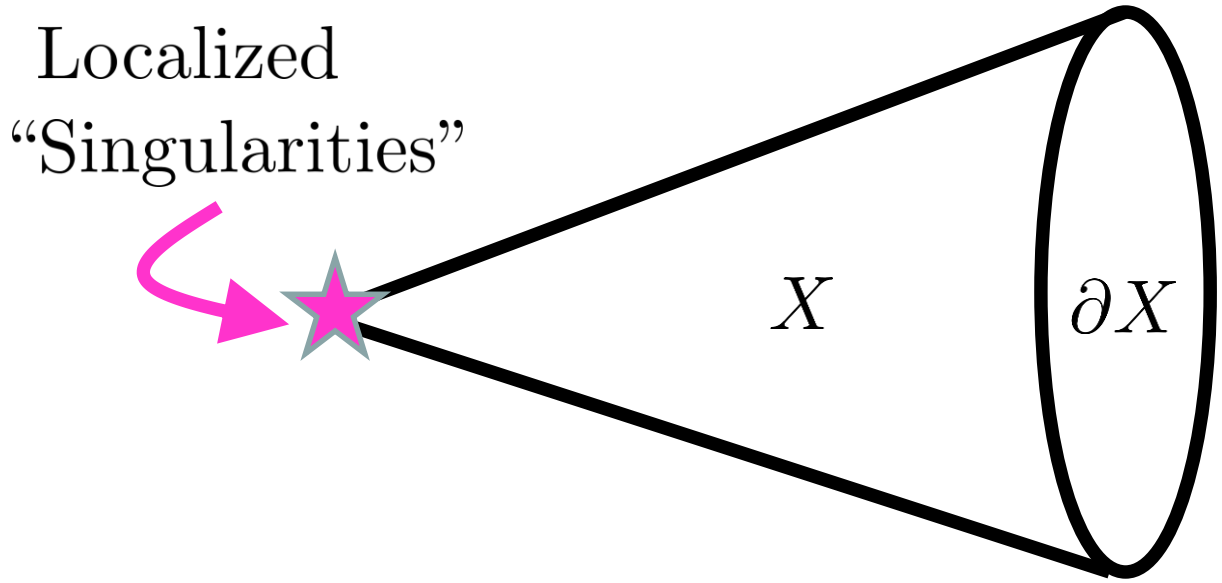
$$X = \mathbb{C}^2/\Gamma_{SU(2)},\ \partial X = S^3/\Gamma_{SU(2)}$$

Step 1: Engineering QFTs



Example: 5D SCFTs via M-th on $\mathbb{R}^{4,1} \times X$,
$X = \mathbb{C}^3/\Gamma_{SU(3)}$, $\partial X = S^5/\Gamma_{SU(3)}$

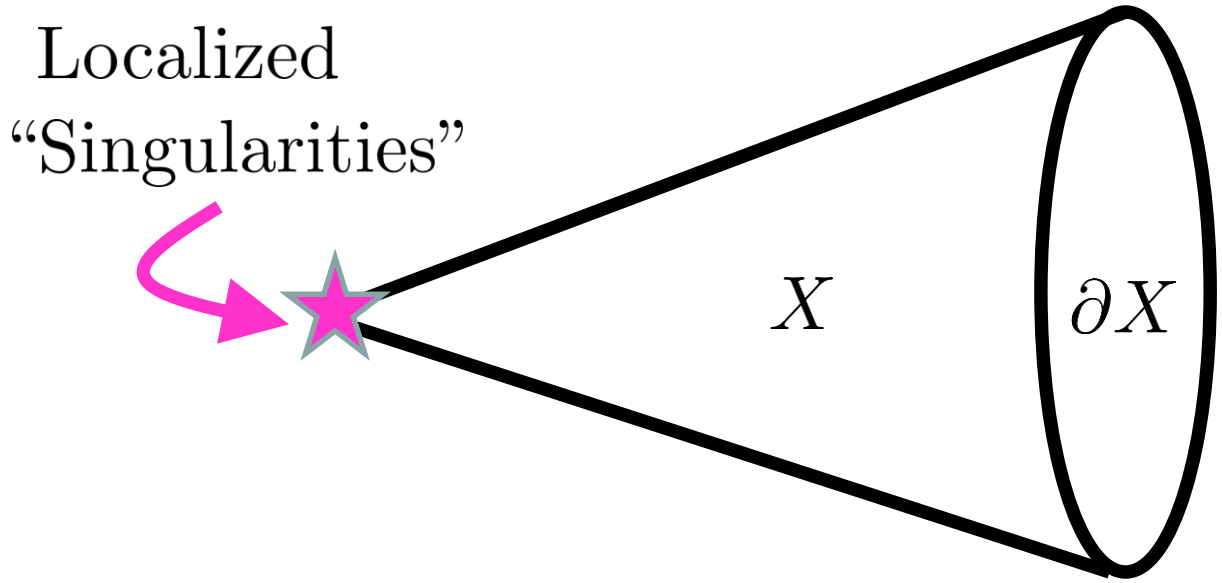
Step 1: Engineering QFTs



Example: 4D SCFTs via IIB on $\mathbb{R}^{3,1} \times X$ w/ D3's,
$X = K \to dP_8$ "Standard Model Quiver"

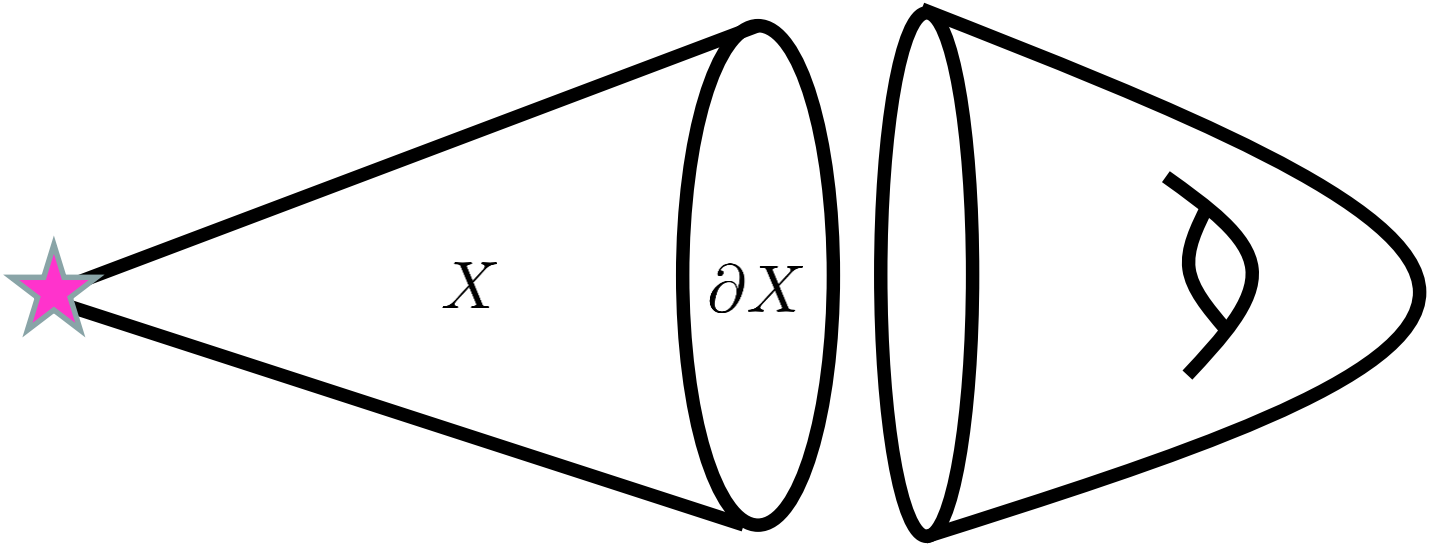
Verlinde Wijnholt '05

Step 1: Engineering QFTs



Example: Many many many more

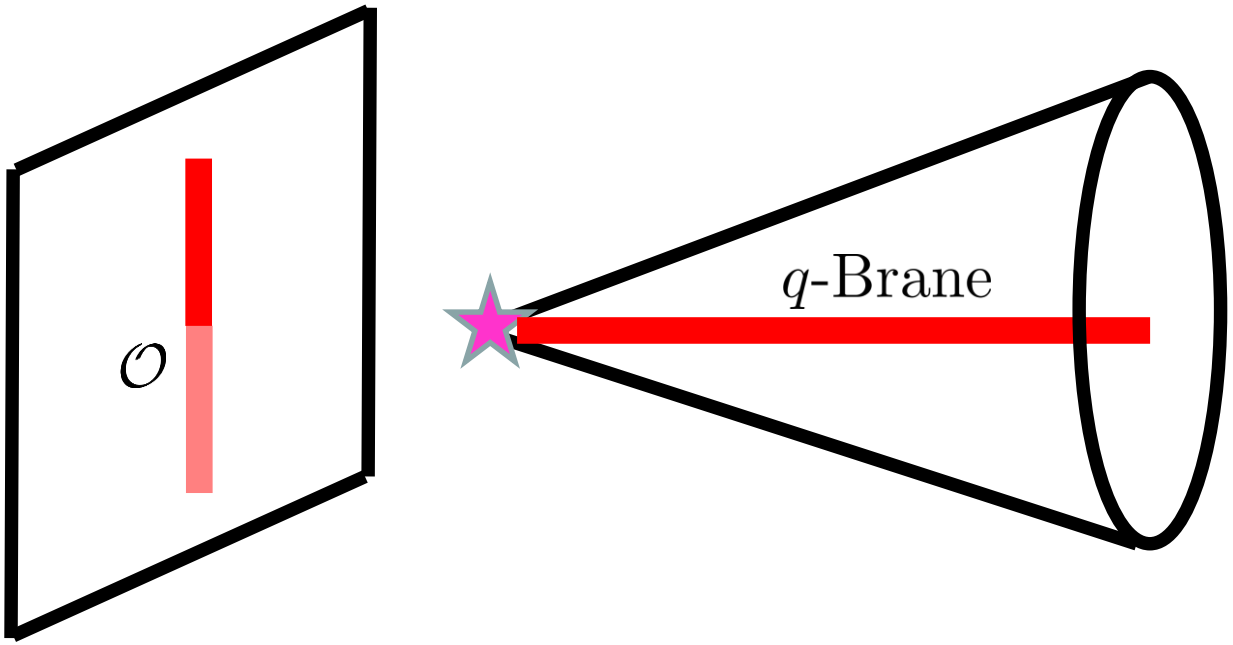
Engineering QFTs + Gravity



Stringy Generalized Symms

- Engineer a QFT (via Geometry)
- Defects (via Branes / Geometry / ...)
- Symmetry Ops (via Branes / Geometry / ...)

Step 2: Engineering Defects

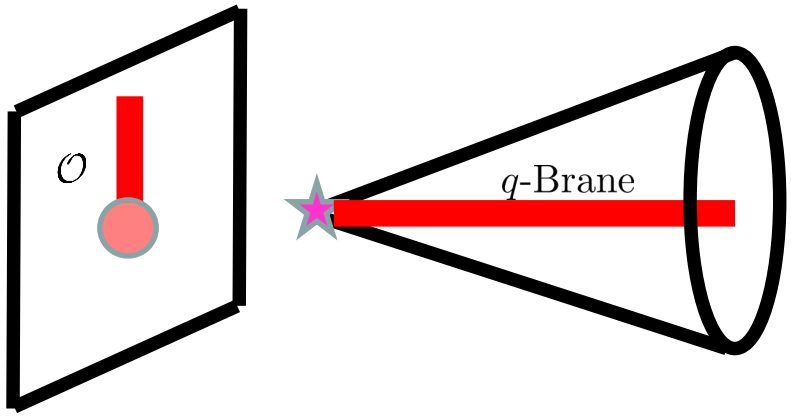


Wrap Brane on Cycle $\in H_{k+1}(X, \partial X)$

$\Rightarrow (q+1) - (k+1)$ dimensional defect in spacetime

Del Zotto JJH Park Rudelius '15 Albertini Del Zotto Garcia Etxebarria Hosseini '20
Morrison Schafer-Nameki Willett '20

Screening



Wrap Brane on Cycle $\in H_{k+1}(X, \partial X)$

(Partially Screened) By q-Brane on $H^{\text{cpct}}_{k+1}(X)$

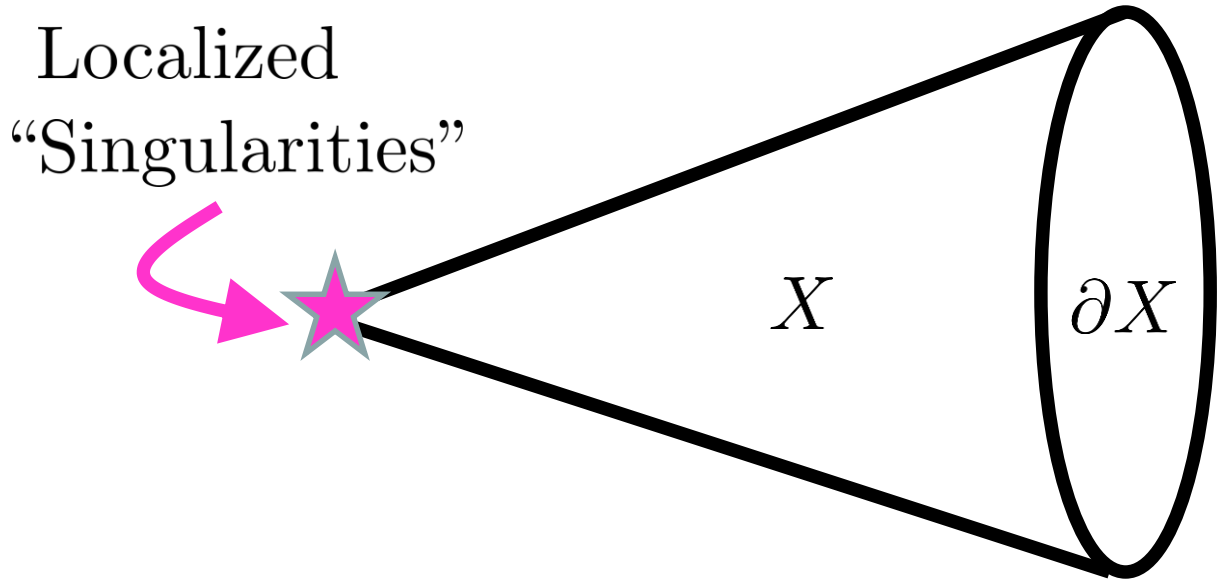
QFT Characterization: 

$((q+1)-(k+1))$-Form Symm: $\frac{H_{k+1}(X,\partial X)}{H^{\text{cpct}}_{k+1}(X)}$

Stringy Generalized Symms

- Engineer a QFT (via Geometry)
- Defects (via Branes / Geometry / ...)
- Symmetry Ops (via Branes / Geometry / ...)

Step 3: Symmetry Operators



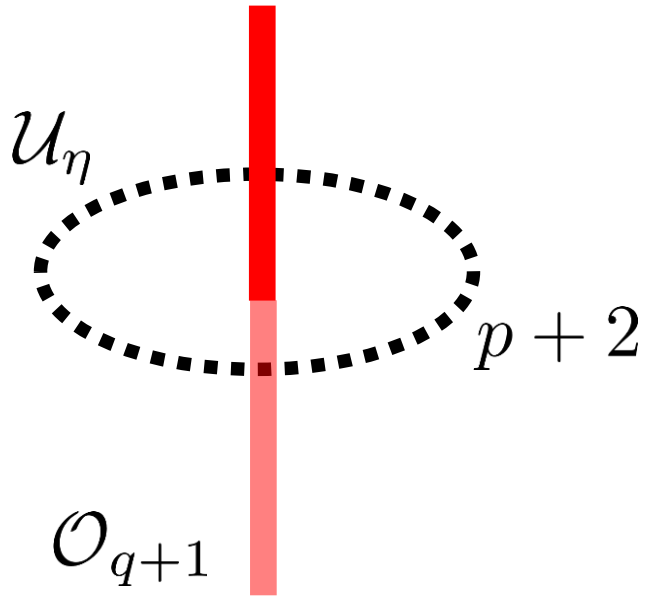
Step 3: Symmetry Operators

Apruzzi Bah Bonetti Schafer-Nameki '22; Garcia-Etxebarria '22
JJH Hubner Torres Zhang '22; Cvetic JJH Hubner Torres '23

In SUGRA, q-brane links with $\mathcal{U}_\eta = e^{i\eta \int_{p+2} \widetilde{F}_{p+2}}$

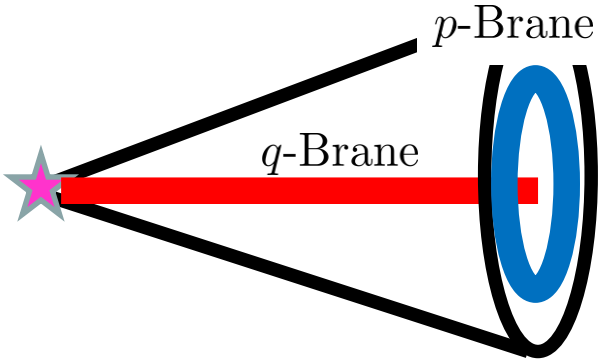
$(p+2) + (q+2) = \mathcal{D}_{\text{SUGRA}}$

Locally, $*dC_{q+1} = \widetilde{F}_{p+2}$

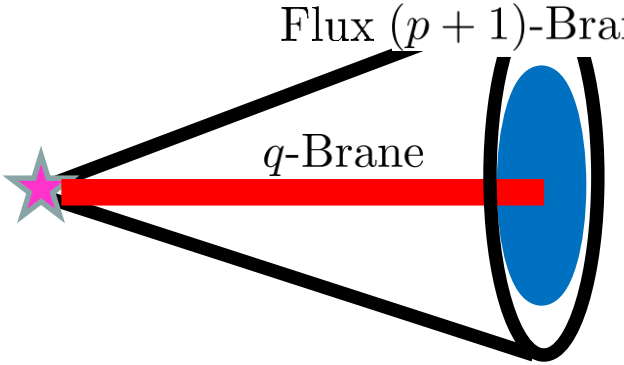


"Branes at Infinity"

- Discrete Symms



- Continuous Symms



Apruzzi Bah Bonetti Schafer-Nameki '22; Garcia-Etxebarria '22
JJH Hubner Torres Zhang '22; Cvetic JJH Hubner Torres '23

Topological Operators

Brane Worldvolume: $S_{\text{brane}} = S_{\text{dyn}} + S_{\text{top}}$

Brane on $\Sigma_{\text{QFT}} \times \gamma_{\text{internal}}$ (reduction)

$\int [dA] e^{S_{\text{top}}(\Sigma \times \gamma)} \to \mathcal{U}(\Sigma_{\text{QFT}})$

Examples

Some Examples...

- 6D SCFTs and 2-form symms
  JJH Hubner Torres Zhang '22

- 4D Duality Defects via D3's on $\mathbb{C}^3/\Gamma$
  JJH Hubner Torres Yu Zhang '22

- Charge Conjugation of 6D $\mathcal{N} = (2,0)$ SCFTs
  Dierigl JJH Montero Torres '23

- Verlinde's Monopole & $U(1)^{(1)}_{\text{mag}}$
  Cvetic JJH Hubner Torres '23

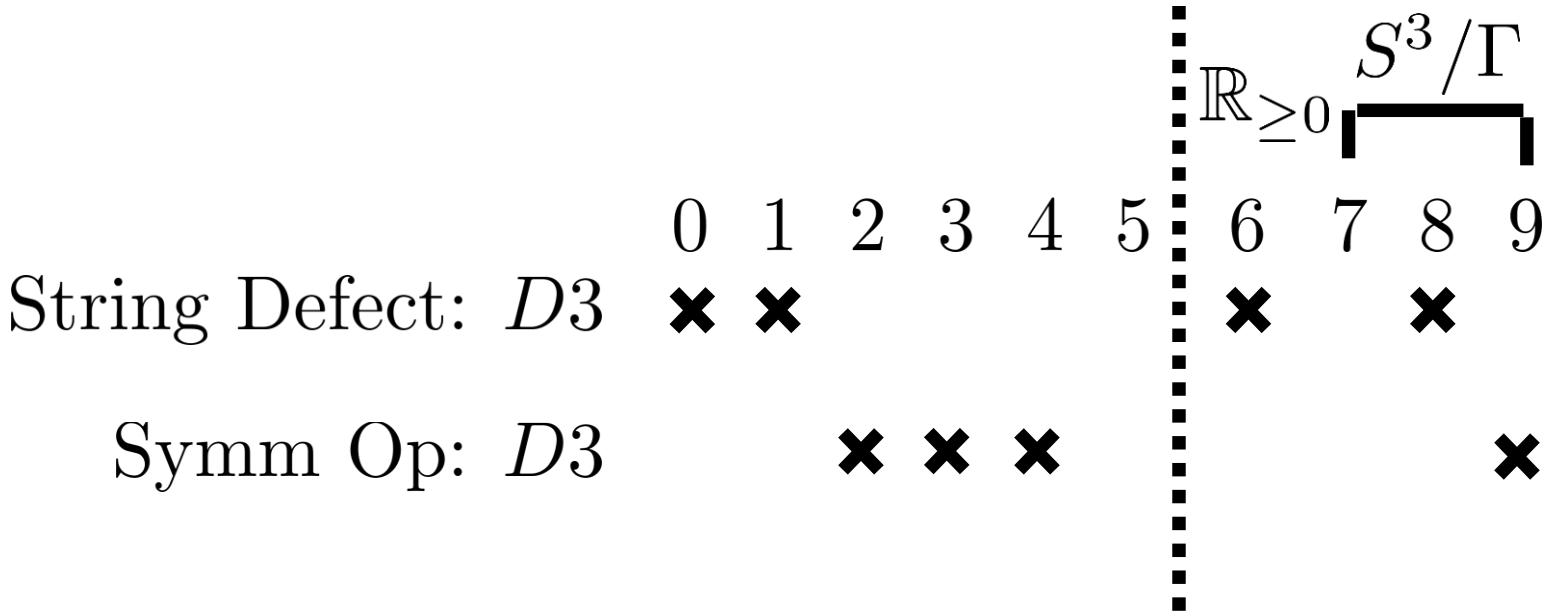
Some Examples...

- 6D SCFTs and 2-form symms
  Wrapped D3-Branes on 1-cyc $\in \partial X = S^3/\Gamma$
- 4D Duality Defects via D3's on $\mathbb{C}^3/\Gamma$
  Wrapped 7-Branes$_{\tau_{\text{IIB}}\,\text{const}}$ on $\partial X = S^5/\Gamma$
- Charge Conjugation of 6D $\mathcal{N} = (2,0)$ SCFTs
  Wrapped R7's on $\partial X = S^3/\Gamma$
- Verlinde's Monopole & $U(1)^{(1)}_{\text{mag}}$
  Wrapped Flux 4-Branes

Example: 6D SCFTs

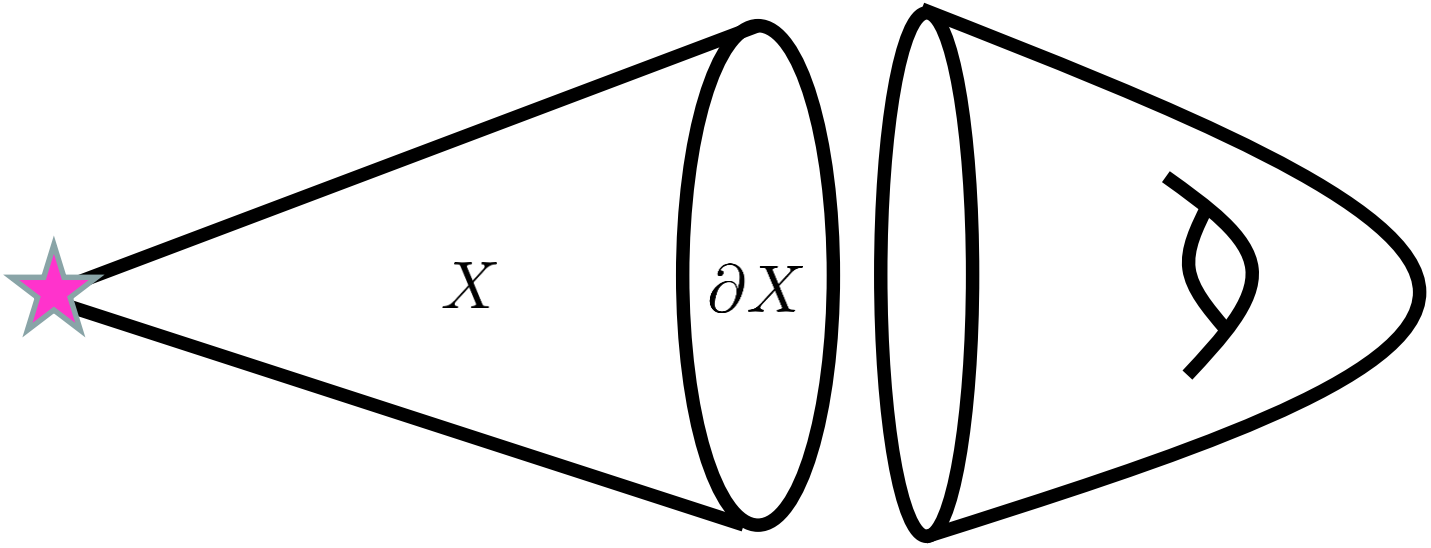
F-theory on $T^2 \to \mathbb{C}^2/\Gamma_{U(2)}$, $\partial\mathbb{C}^2/\Gamma_{U(2)} = S^3/\Gamma_{U(2)}$

JJH Morrison Vafa '13



JJH Hubner Torres Zhang '22

Engineering QFTs + Gravity



Defects & $G_N \neq 0$

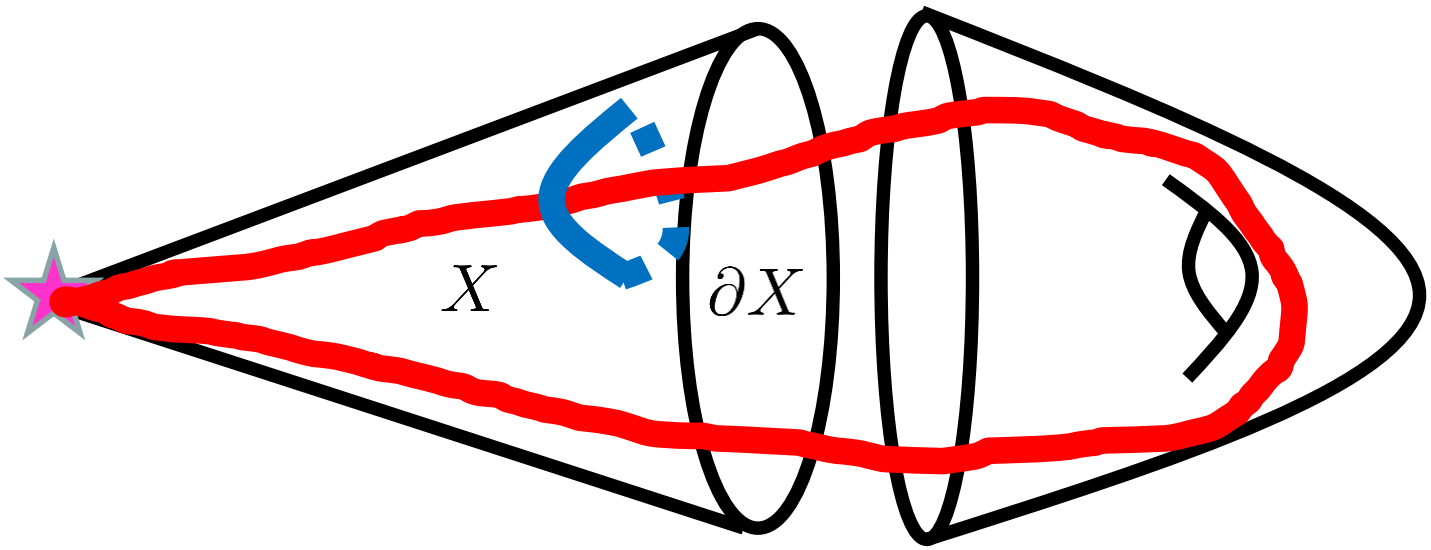
1) The defect persists (but now it's dynamical)
   Cvetic JJH Hubner Torres '23

2) The defect is metastable (unwinds in bulk)
   Verlinde '06, Cvetic JJH Hubner Torres '23

3) The defect is shared (multiple throats)
   Baume JJH Hubner Torres Turner Yu '23

Defects & $G_N \neq 0$

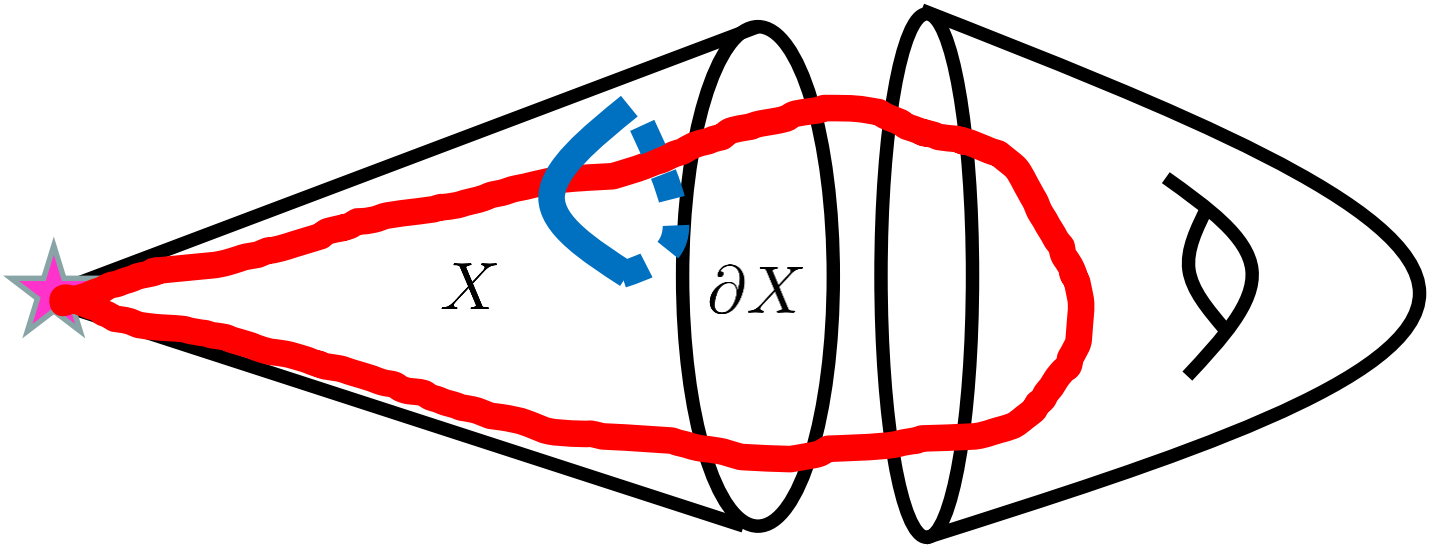
1) The defect persists (but now it's dynamical)



Defects & $G_N \neq 0$

2) The defect is metastable (unwinds in bulk)

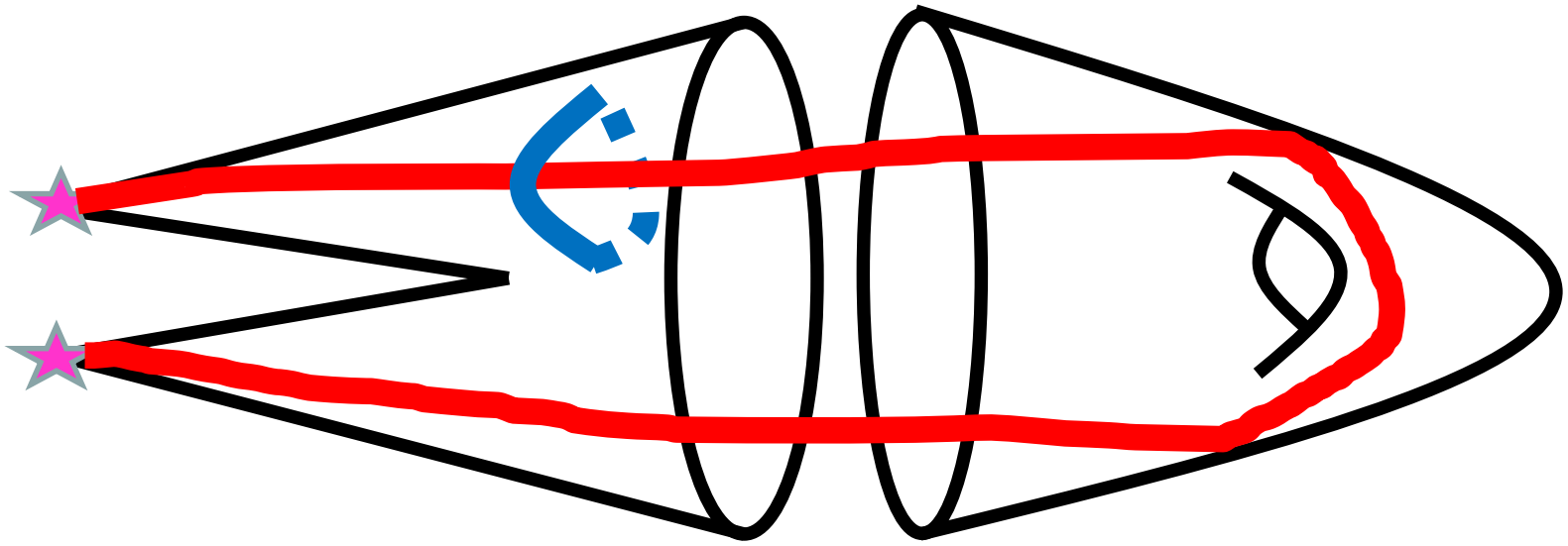
Verlinde '06, Cvetic JJH Hubner Torres '23



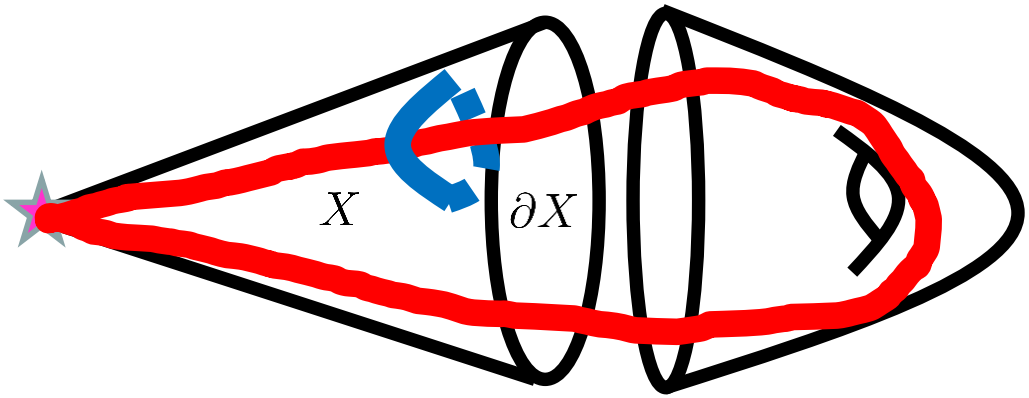
Defects & $G_N \neq 0$

3) The defect is shared (multiple throats)

Baume JJH Hubner Torres Turner Yu '23



¿Fate Of Symmetry Operators?



QFT ⟶ QFT + Gravity

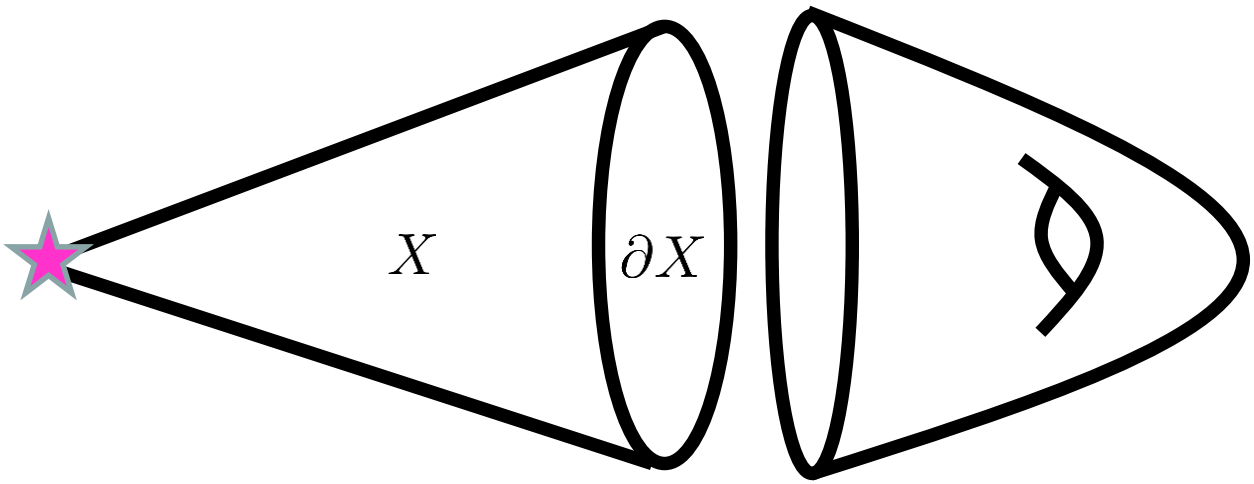
Top. Symm Op. ⟶ Dynamical Brane

Gluing into the Bulk

Gluing Boundary Conditions

Use Mayer-Vietoris:

$$\ldots \to H_{n+1}(Y) \to H_n(U \cap V) \to H_n(U) \oplus H_n(V) \to H_n(Y) \to \ldots$$

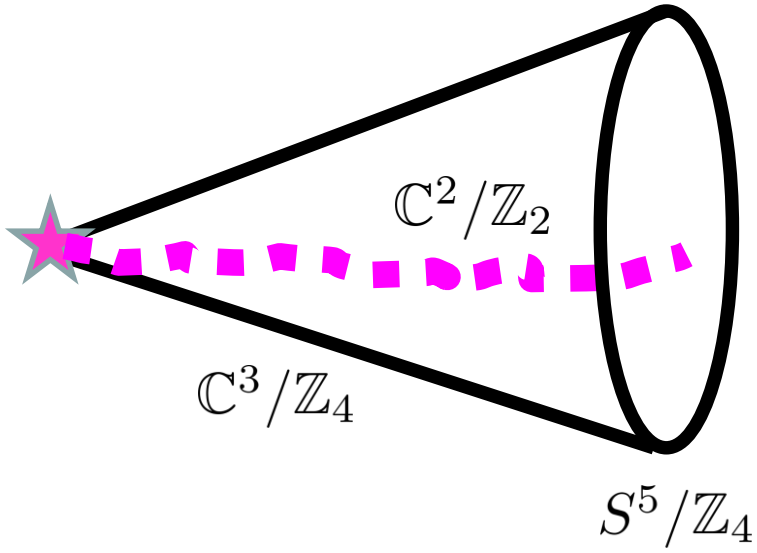


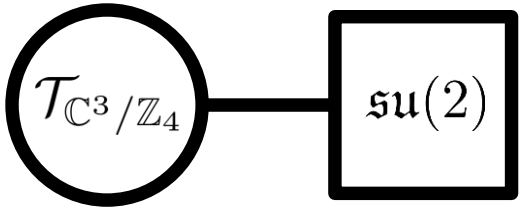
Example:
5D SCFTs + Gravity

Cvetic JJH Hubner Torres '23

M-th on $\mathbb{C}^3/\mathbb{Z}_4$ (Local)

Local Model:
5D SCFT
$\mathcal{T}_{\mathbb{C}^3/\mathbb{Z}_4}$

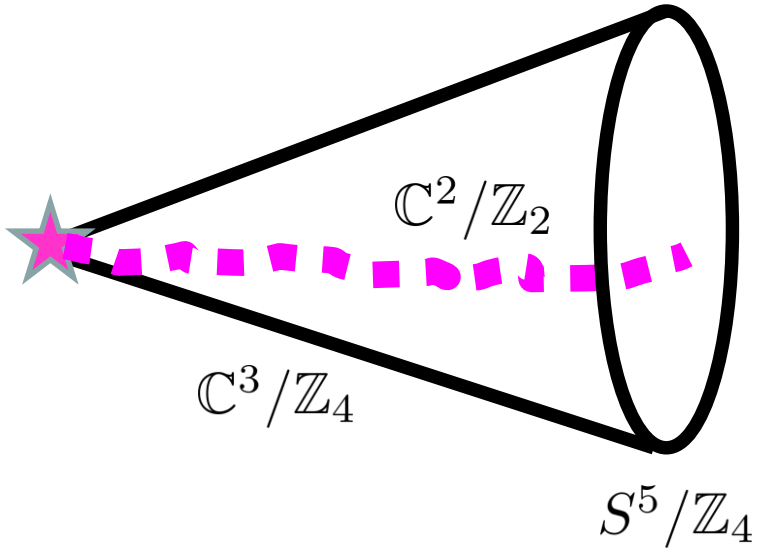




Apruzzi Bhardwaj Schafer Nameki Oh '21; Del Zotto JJH Meynet Moscrop Zhang '22;
Cvetic JJH Hubner Torres '22; Del Zotto Garcia Etxebarria Schafer Nameki '22

M-th on $\mathbb{C}^3/\mathbb{Z}_4$ (Local)

Local Model:
5D SCFT
$\mathcal{T}_{\mathbb{C}^3/\mathbb{Z}_4}$



$SO(3)_{\text{flav}}$ 0-form symm
$\mathbb{Z}_2$ 1-form symmetry (acts on M2 line defects)
2-group (entwined 0- and 1-form symms)

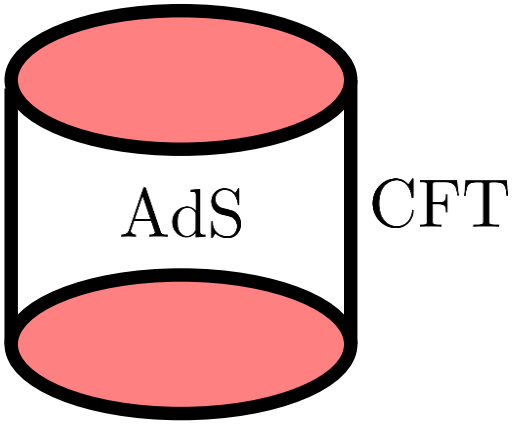
M-th on $T^6/\mathbb{Z}_4$ (Global)

16 $\mathcal{T}_{\mathbb{C}^3/\mathbb{Z}_4}$'s 10 $\mathfrak{su}(2)$'s and 5 $\mathfrak{u}(1)$'s

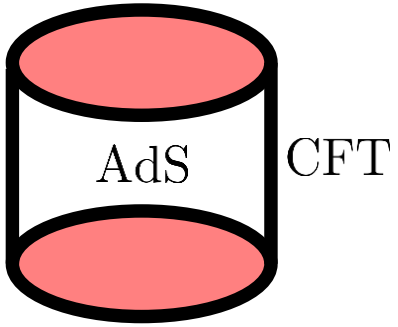
Full Gauge Group:

$$\frac{U(1)^5 \times \frac{[\mathbb{Z}_2^3 \times (\mathbb{Z}_4 \times SU(2))/\mathbb{Z}_2]^4 \times SU(2)^6}{\mathbb{Z}_4 \times \mathbb{Z}_2^2}}{\mathbb{Z}_4 \times \mathbb{Z}_2}$$

Approach 2: Holography



Dictionary Entries

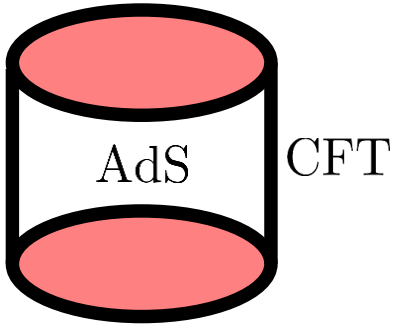


AdS_{D+1} CFT_D

Gauge Symm. ⟷ Global Symm.

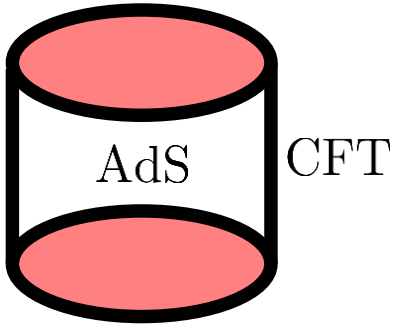
• •
• •
• •

Dictionary Entries



AdS_{D+1}		CFT_D
		Hamilton Kabat Lifschytz Lowe '05 '06
Radial Evol^n	$\longleftrightarrow$	Smearing
Gauge Symm.	$\longleftrightarrow$	Global Symm.
$\bullet$		$\bullet$
$\bullet$		$\bullet$
$\bullet$		$\bullet$

Dictionary Entries



AdS_{D+1}		CFT_D
Radial Evol^n	$\longleftrightarrow$	Smearing
Gauge Symm.	$\longleftrightarrow$	Global Symm.
$\widetilde{\mathcal{U}}_{\text{Brane}}$	$\longleftarrow$	$\mathcal{U}_{\text{Symmetry Operator}}$

Top Down Approach: Apruzzi Bah Bonetti Schafer-Nameki '22;
Garcia Etxebarria '22;
JJH Hubner Torres Zhang '22 + ...

Smearing

AdS_{D+1} $\qquad\qquad$ CFT_D

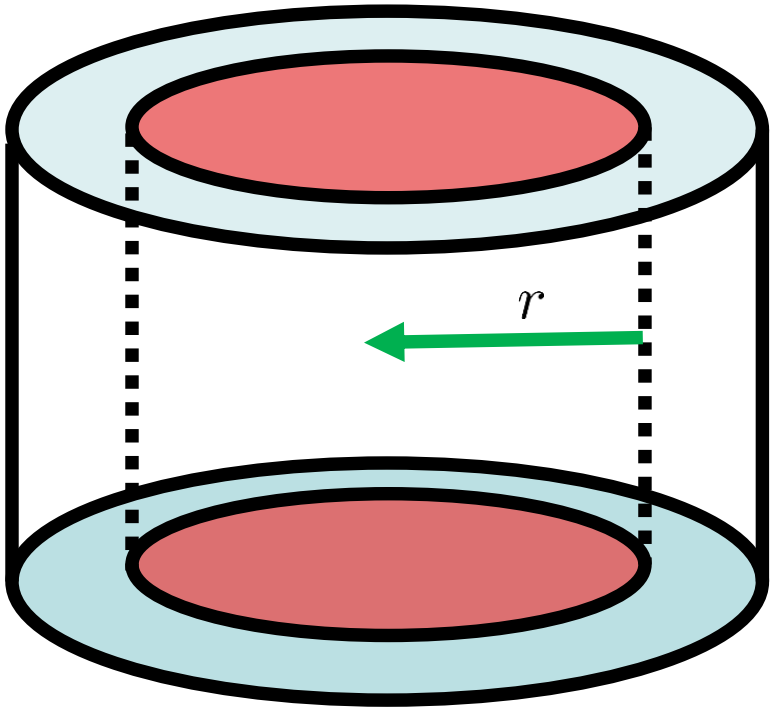
$$\widetilde{\mathcal{O}}(y;r) = \int_x \mathcal{K}(x,y;r)\mathcal{O}(x) \xleftarrow{\mathcal{K}*\mathcal{O}} \mathcal{O}(x)$$

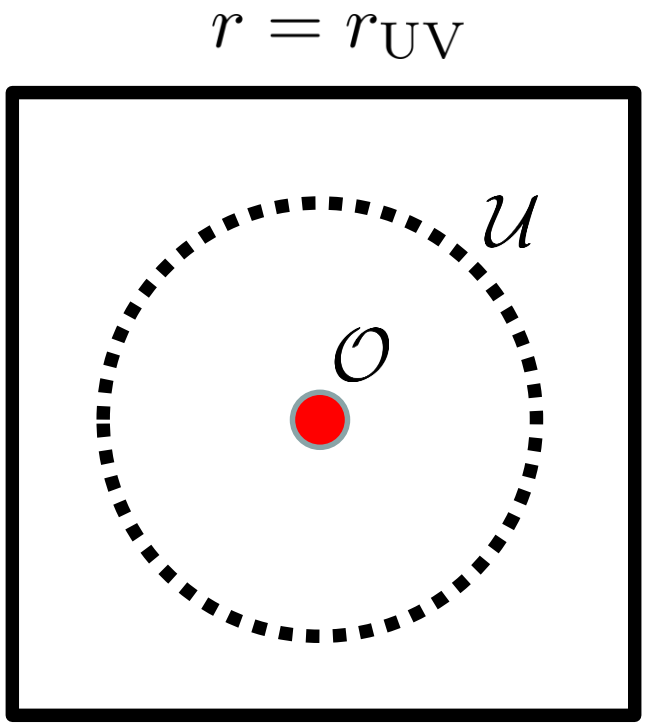
$$??? \xleftarrow{\mathcal{K}*\mathcal{U}} \mathcal{U}$$

Smearing $\mathcal{U} \oplus \mathcal{O}$

Look at how $\mathcal{U}$ acts on charged $\mathcal{O}$:

CFT + $r_{\mathrm{UV-cutoff}}$

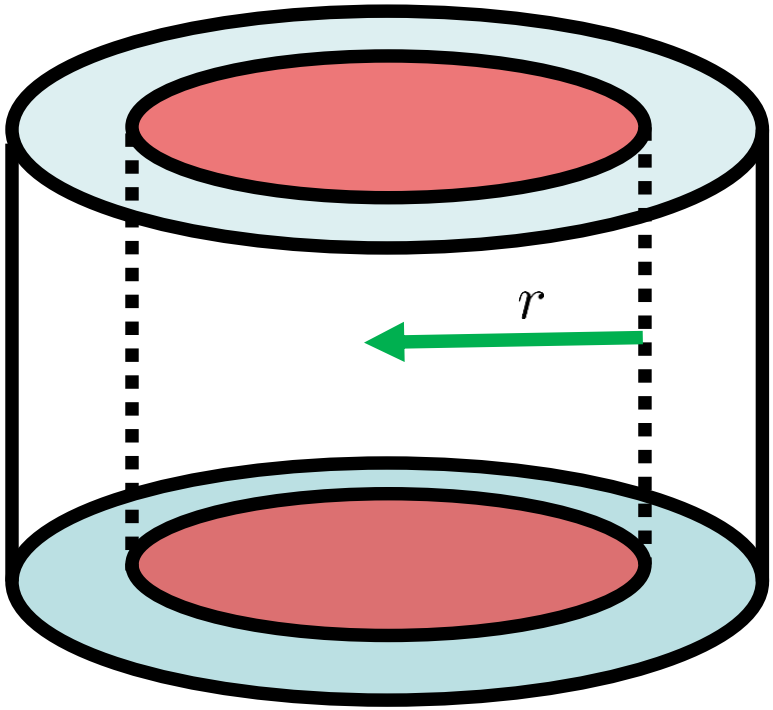


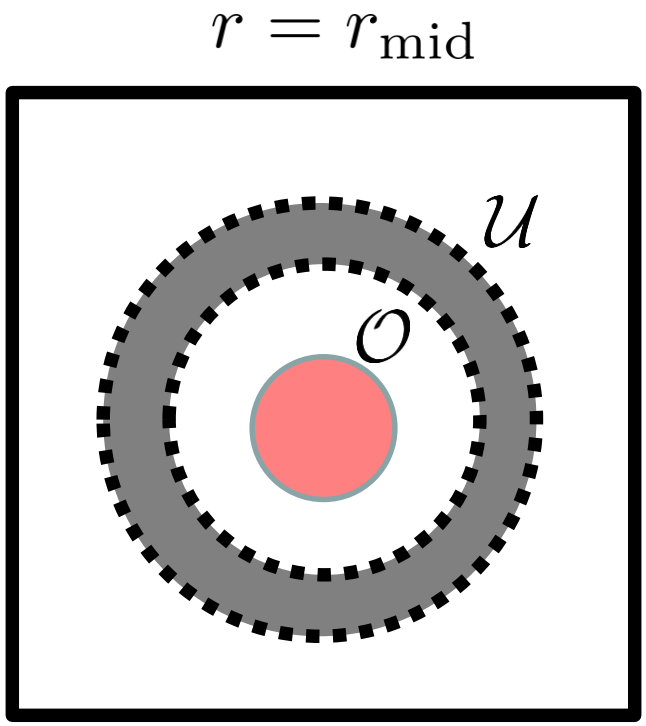


Smearing $\mathcal{U} \oplus \mathcal{O}$

Look at how $\mathcal{U}$ acts on charged $\mathcal{O}$:

CFT + $r_{\mathrm{UV-cutoff}}$

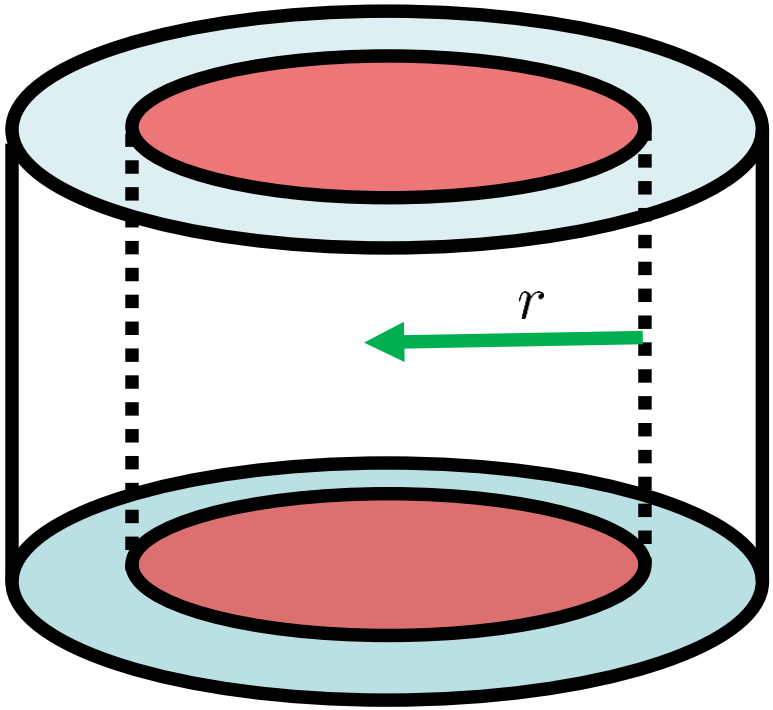


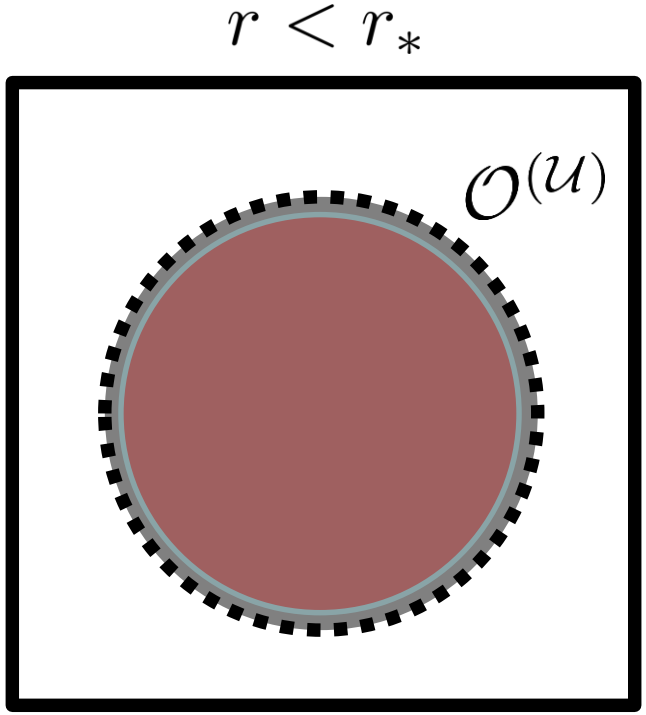


Smearing $\mathcal{U} \oplus \mathcal{O}$

Look at how $\mathcal{U}$ acts on charged $\mathcal{O}$:

CFT + $r_{\mathrm{UV-cutoff}}$

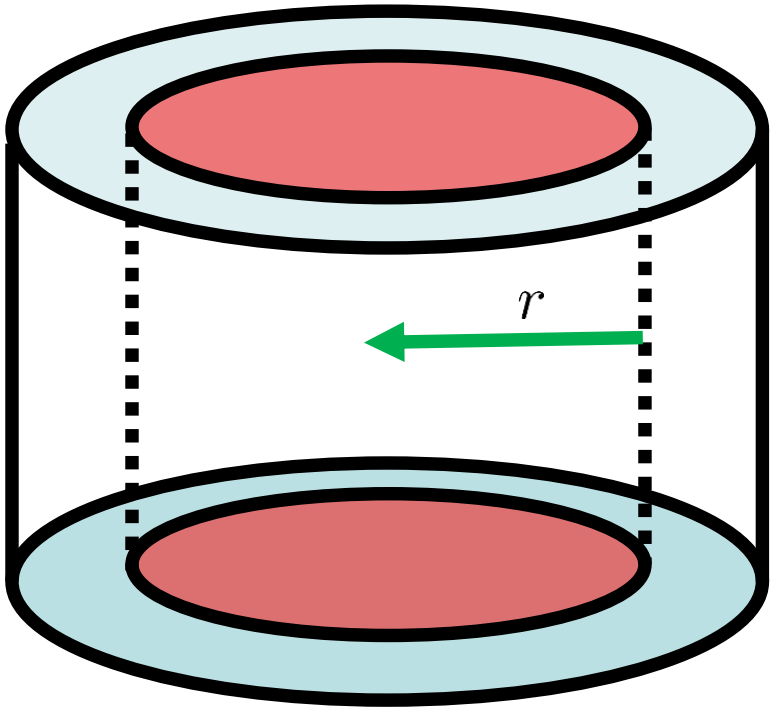


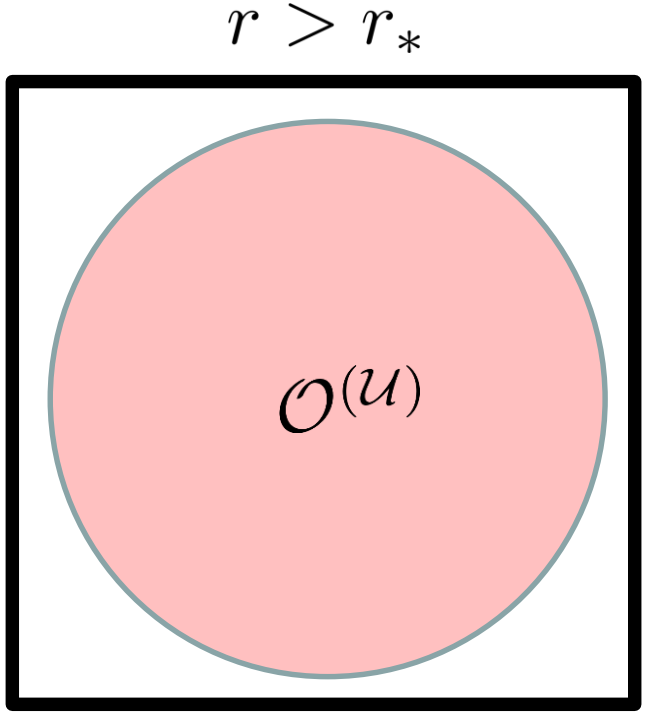


Smearing $\mathcal{U} \oplus \mathcal{O}$

Look at how $\mathcal{U}$ acts on charged $\mathcal{O}$:

CFT + $r_{\mathrm{UV-cutoff}}$

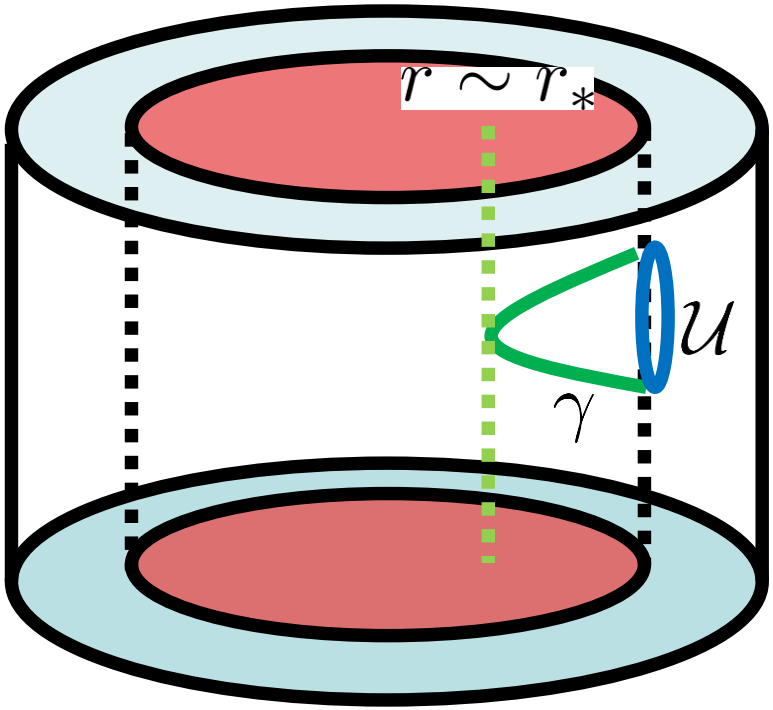




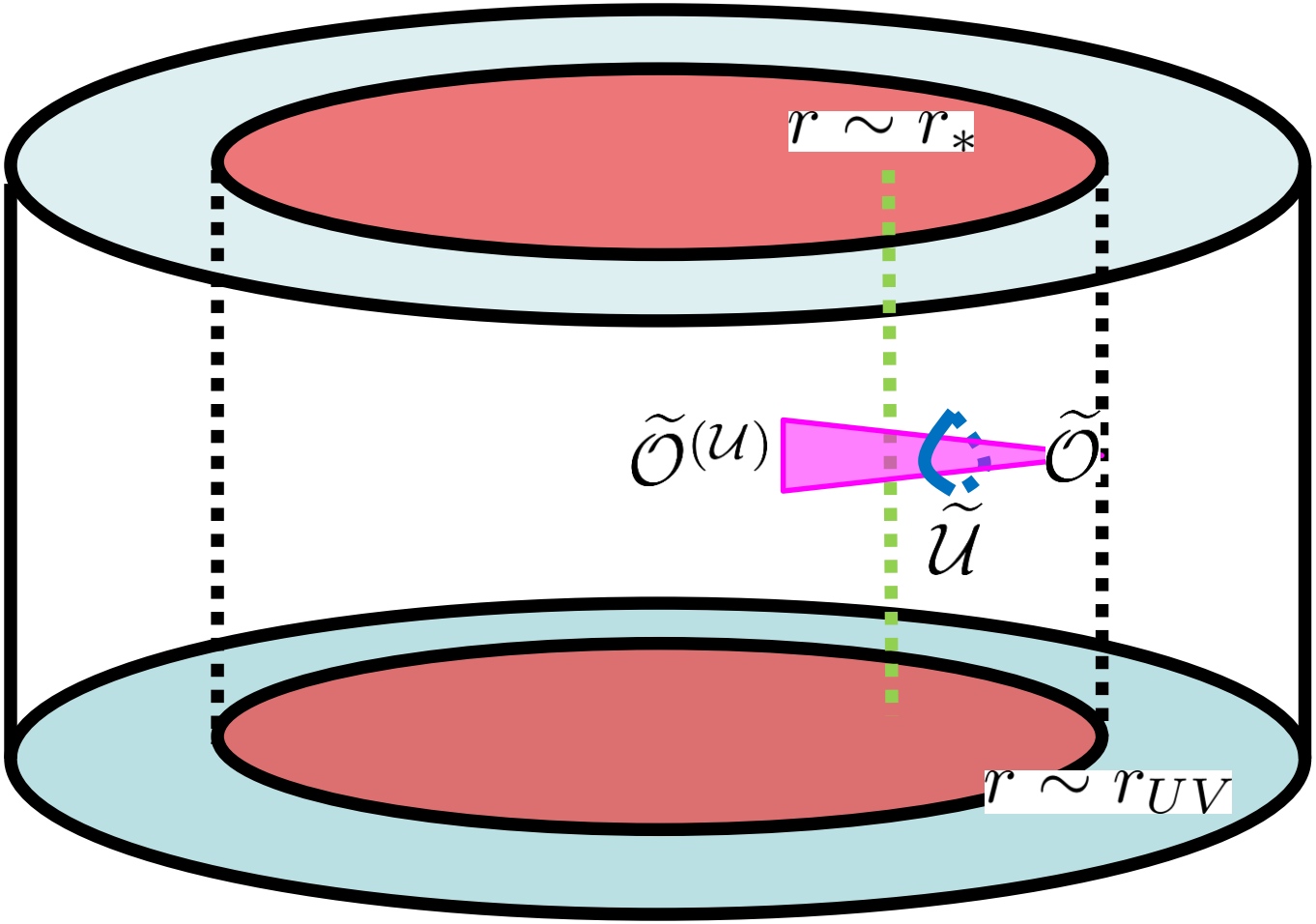
What is r_*?

Set by extremal surface anchored to boundary!

Ryu Takayanagi '06; Hubeny Rangamani Takayanagi '07

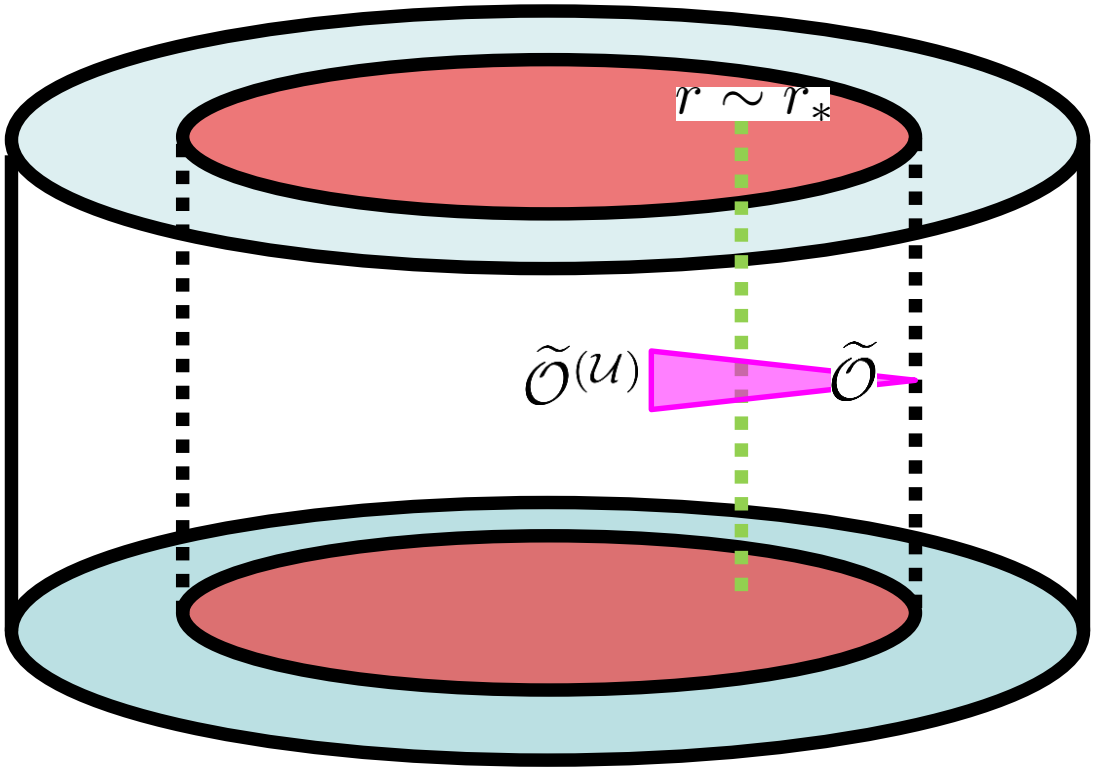


View From The Bulk



So Does $\widetilde{\mathcal{U}}$ Have A Tension?

Yes! Otherwise jump in $T^{\rm bulk}_{\mu\nu}$ from $\widetilde{\mathcal{O}}^{(\mathcal{U})} \leftarrow \widetilde{\mathcal{O}}$!



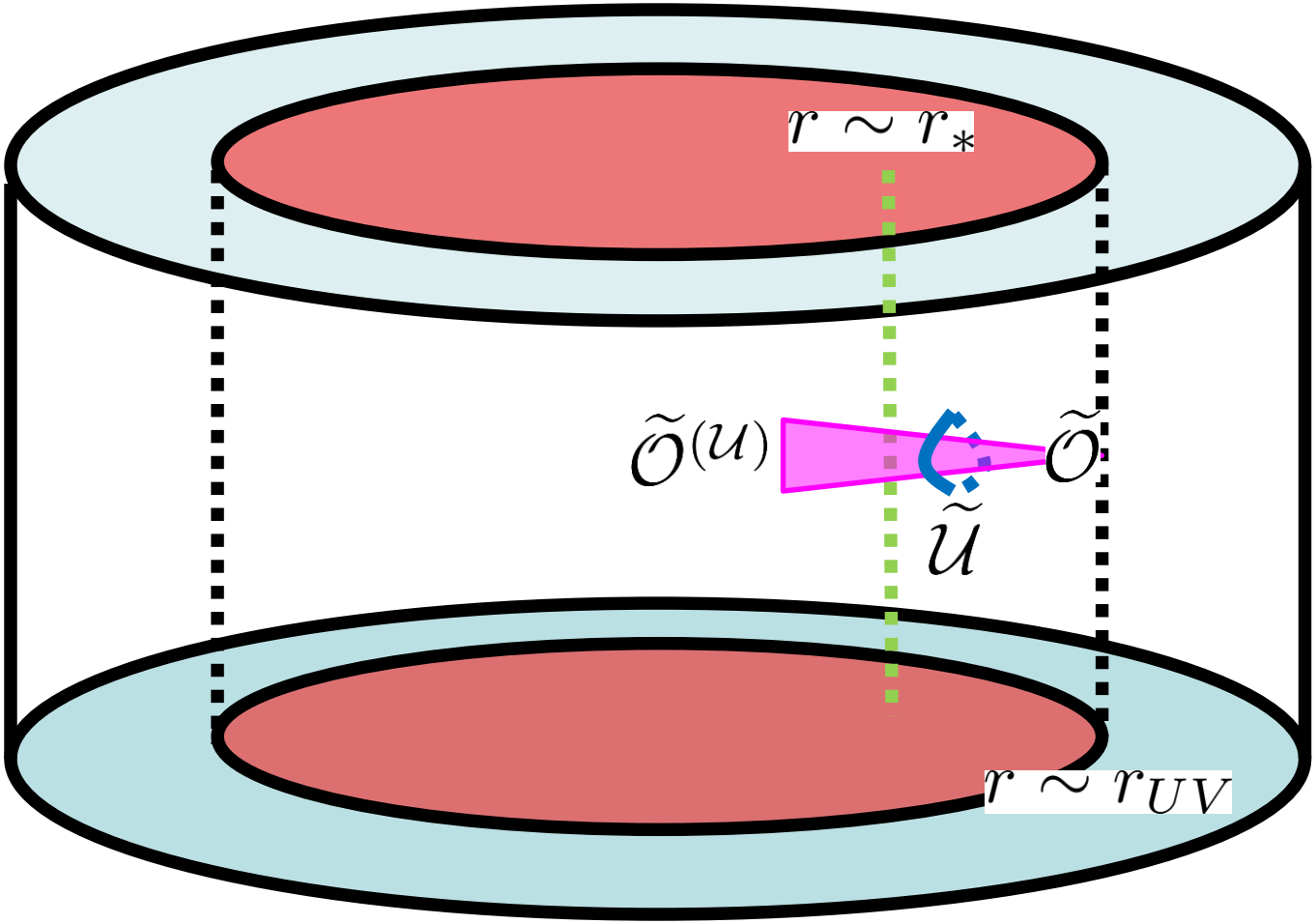
So Does $\widetilde{\mathcal{U}}$ Have A Tension?

Yes! Otherwise jump in $T^{\rm bulk}_{\mu\nu}$ from $\widetilde{\mathcal{O}}^{(\mathcal{U})} \leftarrow \widetilde{\mathcal{O}}$!

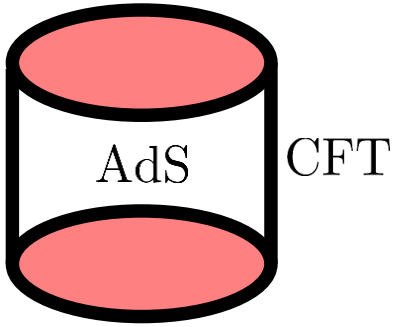
$\Rightarrow \widetilde{\mathcal{U}}$ Non-Topological: "Dynamical Brane"

$S = S_{\rm dyn} + S_{\rm top}$ (recall D-branes)

View From The Bulk

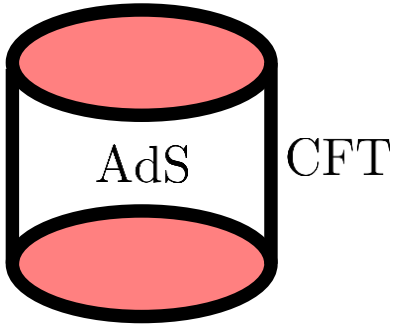


Dictionary Entries



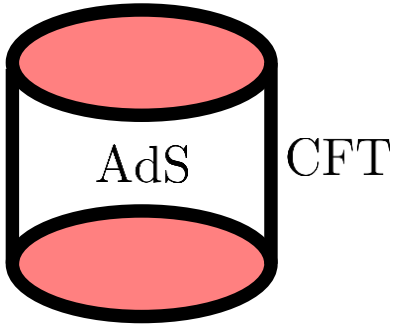
AdS_{D+1}		CFT_D
Radial Evol^n	$\longleftrightarrow$	Smearing
Gauge Symm.	$\longleftrightarrow$	Global Symm.
$\widetilde{\mathcal{U}}_{\mathrm{Brane}}$	$\longleftrightarrow$	$\mathcal{U}_{\mathrm{Symmetry\ Operator}}$

Dictionary Entries



AdS_{D+1}		CFT_D
Radial Evol^n	$\longleftrightarrow$	Smearing
Gauge Symm.	$\longleftrightarrow$	Global Symm.
$\widetilde{\mathcal{U}}_{\mathrm{Brane}}$	$\longleftrightarrow$	$\mathcal{U}_{\mathrm{Symmetry\ Operator}}$

Dictionary Entries



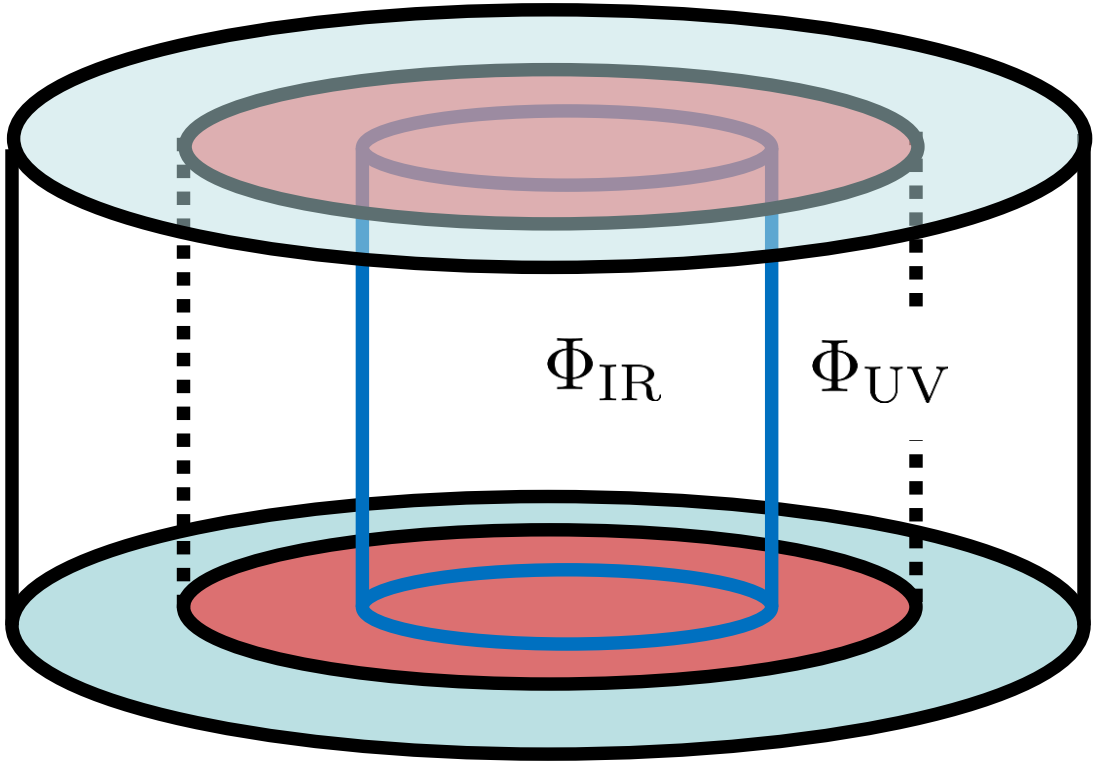
Smearing works for $n \geq 1$

AdS_{D+1}	CFT_D
$\mathrm{Codim}^n\ n+1$	$\mathrm{Codim}^n\ n$
$\widetilde{\mathcal{U}}_{\mathrm{Brane}}$	$\longleftrightarrow$ $\mathcal{U}_{\mathrm{Symmetry\ Operator}}$

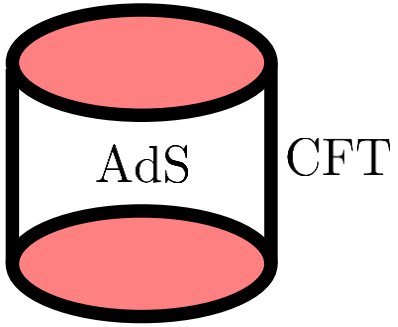
$¿n = 0?$

In CFT_D (-1)-form symm $\Rightarrow$ parameters

$\Phi_{\mathrm{blk}} \to \phi_{\partial} \sim \lambda_{\mathrm{CFT}}$ Gubser Klebanov Polyakov '98



Dictionary Entries



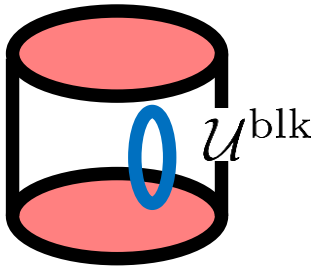
Works for $n \geq 0$

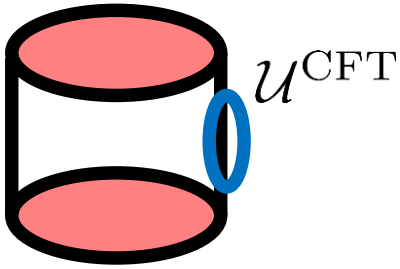
AdS_{D+1} CFT_D

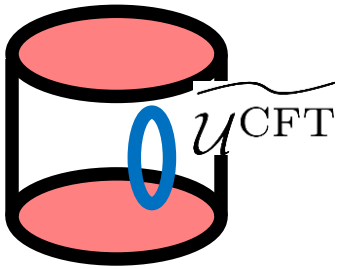
$\mathrm{Codim}^n\ n+1$	$\mathrm{Codim}^n\ n$
$\widetilde{\mathcal{U}}_{\mathrm{Brane}}$	$\longleftrightarrow$ $\mathcal{U}_{\mathrm{Symmetry\ Operator}}$

JJH Hubner Murdia '24; complementary to Harlow Ooguri '18

No Global Symmetries!

Suppose $\mathcal{U}^{\text{blk}}$ topological 

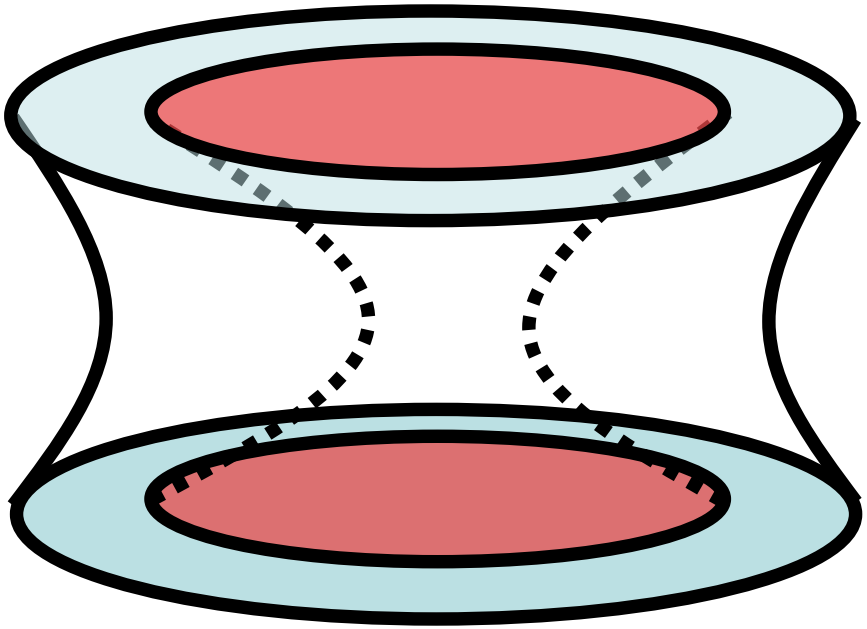
$\Rightarrow \mathcal{U}^{\text{CFT}}$ symmetry operator in CFT 

$\Rightarrow \widetilde{\mathcal{U}^{\text{CFT}}}$ Non-Topological in Bulk! 

More Generally...

Bulk Reconstruction ⇒ No Global Symms!

JJH Hubner Murdia '24



Bulk Reconstruction for general spacetimes:
Miyaji Numasawa Shib Takayangi Watanabe '15; Miyaji Takayangi '15;
Nomura Salzetta Sanches Weinberg '16; Nomura Rath Salzetta '17

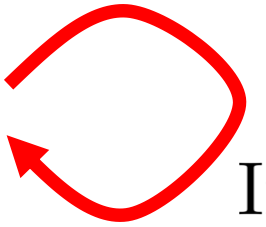
Symmetries $\Rightarrow$ Branes

Sometimes "known objects" (e.g., D-branes)

Apruzzi Bah Bonetti Schafer Nameki '22; Garcia Etxebarria '22; JJH Hubner Torres Zhang '22

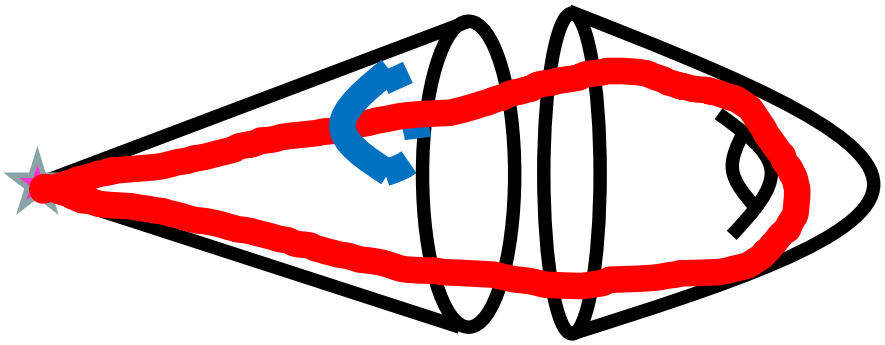
Sometimes "new objects"
(e.g., R7-Branes / Charge Conjugation)

Dierigl JJH Montero Torres '22; Debray Dierigl JJH Montero '23; Dierigl JJH Montero Torres '23

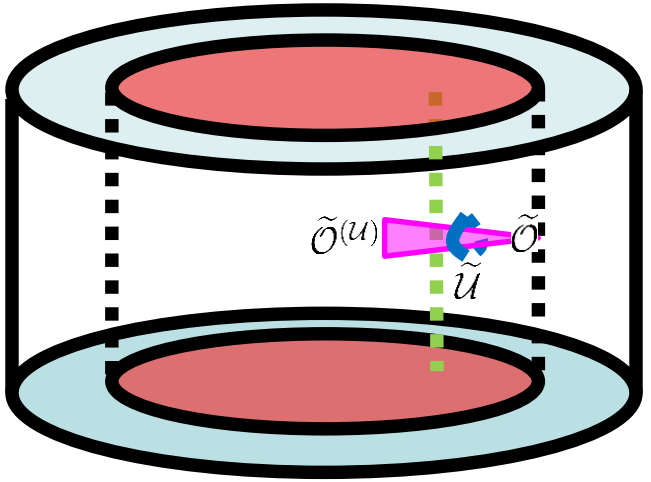
$AdS_7 \times S^4$  Internal Reflection

Summary / Future

Summary / Future



"Symmetries From Branes"



"Branes From Symmetries"

- More General Spacetimes?
- Non-SUSY String Backgrounds?