Comments on asymmetric heterotic orbifolds

Anamaría Font V.

Facultad de Ciencias, Universidad Central de Venezuela

in collaboration with:

G. Aldazabal, E. Andrés, K. Narain, I. Zadeh. Work in progress

Aims and motivation

- go beyond geometric compactifications

  ∗ orbifold action can include elements of duality group

  ∗ moduli stabilization (action exists at special values of moduli)

  ∗ new ground to look for universal properties and test swampland conjectures

- provide explicit world-sheet description, using methods developed for heterotic non-supersymmetric $\mathbb{T}^3/\mathbb{Z}_2$ asymmetric orbifolds   Acharya, Aldazabal, AF, Narain, Zadeh '22

  e.g. for heterotic on $\mathbb{T}^4/\mathbb{Z}_M$, with 8 and 16 supercharges

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∗ Some recent work on asymmetric heterotic orbifolds:

Groot-Nibbelink, Vaudrevange '17
Harvey, Moore '18
Groot-Nibbelink '21
Kaan Baykara, Harvey '21
Faraggi, Groot-Nibbelink, Percival '23
Kaan Baykara, Hamada, Tarazi, Vafa '23

Outline

- Aims and Motivation ✓
- Basics of heterotic asymmetric orbifolds
- Heterotic on $\mathbb{T}^4/\mathbb{Z}_2$ with 8 supercharges
- Heterotic on $\mathbb{T}^4/\mathbb{Z}_M$ with 16 supercharges and rank reduction
- Summary and Outlook

Basics of heterotic asymmetric orbifolds

Heterotic in 10 dim

Gross, Harvey, Martinec, Rohm '85

- R-movers (superstring) $+$ L-movers (bosonic)

  $\psi_R^M,\, X_R^M,\;\; M=0,\ldots,9$ $\qquad$ $X_L^M,\; Y_L^{\mathrm{I}},\;\; \mathrm{I}=1,\ldots,16$

- modular invariance $\Rightarrow$ Y_L^{I} must live on a 16-dim torus with even selfdual lattice

- only 2 such lattices

  $\Gamma_8 \oplus \Gamma_8\,,$ for the $\mathrm{E}_8 \times \mathrm{E}_8$ heterotic

  $\Gamma_{16}\,,$ for the $\mathrm{Spin}(32)/\mathbb{Z}_2$ heterotic

Γ_8 : root lattice of E_8

$$\Gamma_{8q} = \left\{ (m_1,\ldots,m_{8q}), (m_1+\tfrac{1}{2},\ldots,m_{8q}+\tfrac{1}{2}) | m_k \in \mathbb{Z}, \sum_{k=1}^{8q} m_k = \text{even} \right\}$$

Compactification on $\mathbb{T}^d$ and the lattice $\Gamma(16+d,d)$ Narain '86

- R-movers (superstring) $+$ L-movers (bosonic)

$\psi_R^\mu,\ X_R^\mu,\ \ \mu = 0,\ldots,9\text{-}d$ X_L^μ

$\psi_R^j,\ X_R^j,\ \ j = 1,\ldots,d$ $X_L^j,\ Y_L^{\mathrm{I}},\ \ \mathrm{I} = 1,\ldots,16$

- modular invariance $\Rightarrow$ $(Y_L^{\mathrm{I}}, X_L^j; X_R^j)$ must have momenta in an even selfdual lattice $\Gamma(16+d,d)$, signature $(16+d,d)$

- infinite such lattices, parametrized by $d(16+d)$ continuous moduli

background values of metric g_{ij}, Kalb-Ramond field b_{ij}, Wilson lines A_i^{I}

Narain, Sarmadi, Witten '86

▷ $\Gamma(16+d,d)$ lattice vectors = momenta of $(Y_L^{\mathrm{I}}, X_L^j; X_R^j) = (P_L; P_R)$

▷ toroidal partition function

see e.g. Blumenhagen, Lüst, Theisen

$$\mathcal{Z} = \frac{1}{\left(\sqrt{\tau_2}\eta\bar{\eta}\right)^{8-d}} \times \frac{1}{2\,\bar{\eta}^4}\left[\bar{\vartheta}_3^4 - \bar{\vartheta}_4^4 - \bar{\vartheta}_2^4 + \bar{\vartheta}_1^4\right] \times \frac{1}{\eta^{16+d}\bar{\eta}^d} \sum_{(P_L,P_R)\in\Gamma(16+d,d)} q^{\frac{1}{2}P_L^2}\,\bar{q}^{\frac{1}{2}P_R^2}$$

▷ $(10-d)$-dimensional theory with 16 supercharges

massless gravity multiplet (with d graviphotons)

massless gauge multiplets of $\mathrm{U}(1)^{16+d}$ at generic moduli

supersymmetry breaking, rank reduction ⟶ asymmetric orbifolds

Asymmetric orbifolds 1

Narain, Sarmadi, Vafa '87

- in closed strings orbifold action can be different on L and R
- need to specify orbifold action on $\Gamma(16+d,d) \equiv \Gamma$

go to pt in moduli space where Γ admits authomorphism Θ not mixing L and R

Invariant lattice: $\mathrm{I} = \{P \in \Gamma \,||\, \Theta P = P\}$

Normal lattice: $\mathrm{N} = \{P \in \Gamma \,||\, P \cdot X = 0,\ \forall X \in \mathrm{I}\}$ = orthogonal complement of I in Γ
a.k.a. coinvariant lattice

$$\mathrm{N}^*/\mathrm{N} = \mathrm{I}^*/\mathrm{I}$$

$$\forall P \in \Gamma, \quad P = (P_\mathrm{N}, P_\mathrm{I}),\ P_\mathrm{N} \in \mathrm{N}^*,\ P_\mathrm{I} \in \mathrm{I}^*$$

$$\Gamma = (\mathrm{N}, \mathrm{I}) + \coprod_{w \in \mathrm{N}^*/\mathrm{N}} (w, \varsigma(w))$$

$$\varsigma : \mathrm{N}^*/\mathrm{N} \to \mathrm{I}^*/\mathrm{I}$$

glue vectors $(w, \varsigma(w))$ have even norm and integer scalar product with each other

Example

$$\Gamma = \mathrm{E}_8, \qquad \Theta(P_1, \dots, P_6, P_7, P_8) = (P_1, \dots, P_6, P_8, P_7)$$

$$\mathrm{I} = \mathrm{E}_7, \qquad \mathrm{N} = \mathrm{A}_1$$

$$\mathrm{I}^*/\mathrm{I} = \mathbb{Z}_2, \qquad \mathrm{N}^*/\mathrm{N} = \mathbb{Z}_2$$

$$\mathrm{E}_8 = (\mathrm{A}_1, \mathrm{E}_7) + \coprod_{n \in \mathbb{Z}_2} n(w_{\mathbf{2}}, w_{\mathbf{56}})$$

$$\mathbf{248} = (\mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{133}) + (\mathbf{2}, \mathbf{56})$$

Recall orbifold partition function:

- in Abelian $\mathbb{Z}_K$ orbifolds with generator g

$$\mathcal{Z}(\tau,\bar\tau) = \sum_{\ell=0}^{K-1} \left[\frac{1}{K} \sum_{m=0}^{K-1} \mathcal{Z}(g^{\ell}, g^{m}) \right] ; \qquad \mathcal{Z}(g^{\ell}, g^{m}) = \mathrm{Tr}_{\mathcal{H}_\ell}\Big(g^{m}\, q^{L_0}\, \bar q^{\bar L_0} \Big)$$

$\mathcal{H}_\ell$: g^{ℓ}-twisted Hilbert space

$$X(t, \sigma + 2\pi) = g^{\ell} X(t,\sigma)$$

- sum over ℓ is a sum over twisted sectors
- sum over m enforces the orbifold projection: $\displaystyle \frac{1}{K}\sum_{m=0}^{K-1} g^{m}$ inserted in trace
- double sum $\Leftrightarrow$ modular invariance

Asymmetric orbifolds 2

Narain, Sarmadi, Vafa '87

- orbifold generator g

on Γ, $\quad g|P_{\rm N}, P_{\rm I}\rangle = e^{2i\pi P_{\rm I}\cdot v}|\Theta P_{\rm N}, P_{\rm I}\rangle$, $\quad v$: constant shift along $\rm I$ directions

g on $(Y_L, X_L; X_R)$ along $\rm N$ given by $\Theta = (\Theta_L, \Theta_R)$

g on ψ_R given by Θ_R to preserve world-sheet supersymmetry

defines action in untwisted sector $\longrightarrow$ $\mathcal{Z}(\mathbb{1}, g) = \mathrm{Tr}_{\mathcal{H}_0}\Big(g\, q^{L_0}\, \bar{q}^{\bar{L}_0}\Big) \supset \sum_{P\in \rm I} q^{\frac{1}{2}P_L^2}\, \bar{q}^{\frac{1}{2}P_R^2}\, e^{2i\pi P\cdot v}$

- modular transformations $\to$ action on twisted sectors, e.g.

$$\mathcal{Z}(\mathbb{1}, g) \xrightarrow{\tau\to -\frac{1}{\tau}} \mathcal{Z}(g, \mathbb{1}) \supset \sum_{P\in {\rm I}^*} q^{\frac{1}{2}(P+v)_L^2}\, \bar{q}^{\frac{1}{2}(P+v)_R^2}$$

- modular invariance puts conditions on g

$$\mathcal{Z}(g, \mathbb{1}) \xrightarrow{\tau\to\tau+K} \mathcal{Z}(g, g^K) \equiv \mathcal{Z}(g, \mathbb{1}) \quad \text{for } g^K = \mathbb{1}$$

Heterotic on $\mathbb{T}^4/\mathbb{Z}_2$ with 8 supercharges

Setup

- $\mathbb{Z}_2$: reflection of s from 20 L-movers and 4 R-movers (superstring sector)
  $r = (20 - s)$ L-movers are invariant

- on (bosonized) world-sheet fermions, with r an $\mathrm{SO}(8)$ weight

  $$\mathbb{Z}_2 : \; |r\rangle \to e^{-2\pi i r\cdot v_f}|r\rangle, \qquad v_f = (0, 0, \tfrac{1}{2}, -\tfrac{1}{2})$$

  breaks half supersymmetries

- on $\Gamma(20, 4)$, $\mathbb{Z}_2 : \; \Theta(P_{\mathrm{N}}, P_{\mathrm{I}}) = (-P_{\mathrm{N}}, P_{\mathrm{I}})$, $\quad \det{}'(1 - \Theta) = 2^s 2^4$

  invariant lattice I, signature $(r, 0)$,

  normal lattice $\mathrm{N} = \mathrm{I}^{\perp}$, signature $(s, 4)$

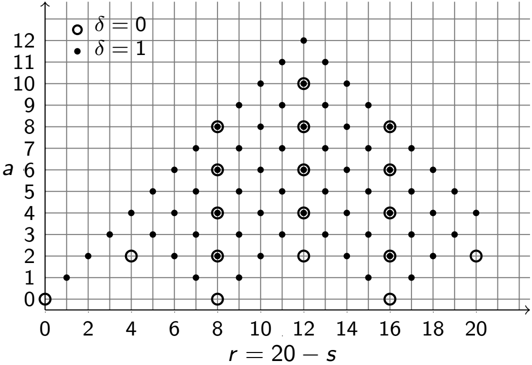
  $$\mathrm{I}^*/\mathrm{I} = \mathrm{N}^*/\mathrm{N} = \mathbb{Z}_2^a$$

  there exist 84 such involutions

  classified using results of Nikulin '80

Classification of half-supersymmetric involutions of $\Gamma(20,4)$

▷ characterized by (r,a,δ), $r = \mathrm{rk}(\mathrm{I})$, $\mathrm{I}^*/\mathrm{I} = \mathbb{Z}_2^a$, $\delta = \begin{cases} 0 & \text{if } P_\mathrm{I}^2 \in \mathbb{Z} \quad \forall P_\mathrm{I} \in \mathrm{I}^* \\ 1 & \text{otherwise} \end{cases}$



▷ e.g. $(r,a,\delta) = (12,10,0) \Rightarrow \mathrm{I} = \mathrm{E}_8(2) \oplus \mathrm{D}_4,\ \mathrm{N} \sim \mathrm{E}_8(2) \oplus \mathrm{D}_4(-1)$

$(r,a,\delta) = (7,1,1) \Rightarrow \mathrm{I} = \mathrm{E}_7,\ \mathrm{N} \sim \mathrm{E}_8 \oplus \mathrm{A}_1 \oplus 4\mathrm{U}$

$\mathrm{L}(n)$: lattice with Gram matrix rescaled by n, U: even, selfdual, signature (1,1)

- N unique up to $\mathrm{SO}(s,4)$

- I has no moduli   no Coulomb branch in 6d with (0,1) supersymmetry

- symmetric orbifolds: $(r,a,\delta)=(16,0,0)$, $\mathrm{N}=4\mathrm{U}$, $\mathrm{I}=2\mathrm{E}_8$ or Γ_{16}

- degeneracy factor in twisted sector: $\sqrt{\dfrac{\det'(1-\Theta)}{|\mathrm{I}^*/\mathrm{I}|}}=4\times 2^{\frac{s-a}{2}}\in\mathbb{Z}$

- $\delta=0,\quad e^{2i\pi P^2}=1,\ \forall\, P\in\mathrm{I}^*$

- $\delta=1,\quad \exists\, w\in\mathrm{I}^*,\ w^2+\dfrac{s}{2}\in 2\mathbb{Z}\ ||\ e^{2i\pi P^2}=e^{2i\pi P\cdot w},\ \forall\, P\in\mathrm{I}^*$

Modular invariance

▷ $\mathcal{Z}(g, g^2) \stackrel{?}{=} \mathcal{Z}(g, \mathbb{1})$

$$e^{2i\pi(v^2+\frac{s}{8})} \sum_{P\in \mathrm{I}^*} q^{\frac{1}{2}(P+v)_L^2}\, \bar{q}^{\frac{1}{2}(P+v)_R^2}\, e^{2i\pi P\cdot 2v} e^{2i\pi P^2} \stackrel{?}{=} \sum_{P\in \mathrm{I}^*} q^{\frac{1}{2}(P+v)_L^2}\, \bar{q}^{\frac{1}{2}(P+v)_R^2}$$

▷ $\delta = 0,\ e^{2i\pi P^2} = 1,\ \forall\, P \in \mathrm{I}^*, \quad 2v \in \mathrm{I}, \quad v^2 + \frac{s}{8} \in \mathbb{Z} \ \Rightarrow\ \mathcal{Z}(g, g^2) = \mathcal{Z}(g, \mathbb{1})$

$$g|P_{\mathrm{N}}, P_{\mathrm{I}}\rangle = e^{2i\pi P_{\mathrm{I}}\cdot v}|-P_{\mathrm{N}}, P_{\mathrm{I}}\rangle$$

▷ $\delta = 1, e^{2i\pi P^2} = e^{2i\pi P\cdot w},\ \forall\, P \in \mathrm{I}^*, \quad 2v + w \in \mathrm{I}, \quad v^2 + \frac{s}{8} \in \mathbb{Z} \ \Rightarrow\ \mathcal{Z}(g, g^2) = \mathcal{Z}(g, \mathbb{1})$

Q : but how can it be $g^2|P_{\mathrm{N}}, P_{\mathrm{I}}\rangle = |P_{\mathrm{N}}, P_{\mathrm{I}}\rangle$?

A : $g|P_{\mathrm{N}}, P_{\mathrm{I}}\rangle = f(P_{\mathrm{N}}) e^{2i\pi P_{\mathrm{I}}\cdot v}|-P_{\mathrm{N}}, P_{\mathrm{I}}\rangle$ Acharya, Aldazabal, AF, Narain, Zadeh '22

$$f(0) = 1, \quad f(P_{\mathrm{N}}) f(-P_{\mathrm{N}}) = e^{2i\pi P_{\mathrm{N}}^2} = e^{2i\pi P_{\mathrm{I}}^2} = e^{2i\pi P_{\mathrm{I}}\cdot w}$$

$$g^2|P_{\mathrm{N}}, P_{\mathrm{I}}\rangle = e^{2i\pi P_{\mathrm{I}}\cdot(2v+w)}|P_{\mathrm{N}}, P_{\mathrm{I}}\rangle = |P_{\mathrm{N}}, P_{\mathrm{I}}\rangle$$

▷ $2v + w \in \mathrm{I}, \quad v^2 + \frac{s}{8} \in \mathbb{Z} + \frac{1}{2} \;\Rightarrow\; \mathcal{Z}(g, g^4) = \mathcal{Z}(g, \mathbb{1})$

gives $\mathcal{Z} = \mathcal{Z}(\mathbb{1}, \mathbb{1})$, i.e. $\mathbb{T}^4$ compactification with 16 supercharges

▷ $2v + w \notin \mathrm{I}, \quad 4v \in I, \quad 2v^2 + \frac{s}{4} \in \mathbb{Z} \;\Rightarrow\; \mathcal{Z}(g, g^4) = \mathcal{Z}(g, \mathbb{1})$

twisted sectors can be rearranged into a $\mathbb{Z}_2$ with I', N'

▷ end result: full modular invariant partition function $\longrightarrow$ spectrum of states

* in cases with 8 supercharges, massless matter in 1 tensor multiplet,
  n_V vector multiplets of some G, n_H hypermultiplets transforming under G
* can check cancellation of tr R^4 anomalies, $n_H - n_V = 244$
  and cancellation of tr F^4 anomalies

Examples

▷ $(r,a,\delta) = (20,2,0)$, $\mathrm{I} = 2\mathrm{E}_8 + \mathrm{D}_4$, $\mathrm{N} = \mathrm{D}_4(-1)$, $v = (1,0^7) \times (1,0^7) \times (0^4)$

$2v \in \mathrm{I}$

$\mathrm{G} = \mathrm{SO}(16) \times \mathrm{SO}(16) \times \mathrm{SO}(8)$

hypers: $(\mathbf{128},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{128},\mathbf{1}) + (\mathbf{16},\mathbf{16},\mathbf{1})$

no neutral hypermultiplets

Kaan Baykara, Hamada, Tarazi, Vafa '23

▷ $(r,a,\delta) = (20,4,1)$, $\mathrm{I} = 2\mathrm{E}_8 + 4\mathrm{A}_1$, $\mathrm{N} = 4\mathrm{A}_1(-1)$, $v = (0^8) \times (\frac{1}{8}^7, \frac{5}{8}) \times (0^4)$

$w = (0^8) \times (0^8) \times (\frac{1}{\sqrt{2}}^4)$, $2v + w \notin \mathrm{I}$

$\mathrm{G} = \mathrm{E}_8 \times \mathrm{U}(1) \times \mathrm{SU}(8) \times \mathrm{SU}(2)^4$

hypers: no neutral

$(\mathbf{28},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1}) + (\overline{\mathbf{28}},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1}) + (\mathbf{56},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1}) + (\mathbf{28},\underline{\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1}}) + (\mathbf{8},\underline{\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{1}}) + (\mathbf{1},\underline{\mathbf{2},\mathbf{2},\mathbf{2},\mathbf{1}}) + (\mathbf{8},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$

▷ $(r,a,\delta) = (6,2,1)$, $\mathrm{I} = \mathrm{D}_6$, $\mathrm{N} = 2\mathrm{A}_1 + \Gamma(12,4)$, $v = (\frac{1}{2}, 0^5), w = (1,0^5)$

$2v + w \in \mathrm{I}$, $v^2 + \frac{s}{8} \in \mathbb{Z}$

$\mathrm{G} = \mathrm{E}_7 \times \mathrm{SU}(2)$

hypers: $12(\mathbf{1},\mathbf{1}) + (\mathbf{56},\mathbf{2}) + 128(\mathbf{1},\mathbf{2})$

Heterotic on $\mathbb{T}^4/\mathbb{Z}_K$ with 16 supercharges and rank reduction

Setup

▷ $\mathbb{Z}_K$: acts on s (even) of 20 L-movers, eigenvalues $e^{\pm 2\pi i t_m}$, $m = 1, \ldots, \frac{s}{2}$

leaves invariant $(20 - s)$ L-movers and all 4 right movers

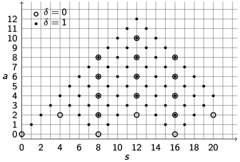
no action on world-sheet fermions, all 16 supersymmetries unbroken

▷ on $\Gamma(20, 4)$, $\mathbb{Z}_K$ automorphism Θ, $\det{}'(1 - \Theta) = \prod_m 4 \sin^2 \pi t_m, \quad t_m = \frac{\ell}{K}$

invariant lattice I, signature $(20 - s, 4)$

normal lattice $\mathrm{N} = \mathrm{I}^{\perp}$, signature $(s, 0)$

▷ full classification for $\mathbb{Z}_2$ (exchange I with N in $\mathbb{T}^4/\mathbb{Z}_2$ with 8 supercharges)



Rank reduction

▷ typical example: CHL $\Gamma(20,4) = \mathrm{E}_8 + \mathrm{E}_8 + 4\mathrm{U}$ (with Θ exchanging the two E_8 factors) Chaudhuri, Polchinski '95

$\mathrm{I} = \mathrm{E}_8(2) + 4\mathrm{U}, \quad \mathrm{N} = \mathrm{E}_8(2)$

N does not have roots, i.e. vectors of squared norm 2

Θ kills massless states from oscillators along N directions

invariant combinations $|P_\mathrm{N}\rangle + \Theta|P_\mathrm{N}\rangle + \cdots + \Theta^{K-1}|P_\mathrm{N}\rangle$ allowed, but $\nexists\, P_\mathrm{N} \,||\, P_\mathrm{N}^2 = 2$

rank reduction requires absence of roots in N

▷ other examples: 7d asymmetric $\mathbb{T}^3/\mathbb{Z}_K$, $K = 2,3,4,5,6$ de Boer et al '01

∗ explicitly constructed as $(\mathbb{T}^2 \times S^1)/\mathbb{Z}_K$ Fraiman, Parra De Freitas '21

$\mathbb{Z}_K$ automorphism of $\Gamma(18,2)$ + translation by $\frac{2\pi\mathcal{R}}{K}$ in S^1

all have N without roots e.g. $\mathbb{Z}_3$, $\mathrm{I} = 2\mathrm{A}_2 + 2\mathrm{U}(3) + \mathrm{U}$, $\mathrm{N} = \mathrm{K}_{12}$ (Coxeter-Todd lattice)

∗ equivalently $\mathbb{T}^3/\mathbb{Z}_K$ with $\mathbb{Z}_K$ automorphism of $\Gamma(19,3)$

▷ $\mathbb{Z}_K$ automorphisms of $\Gamma(19,3)$ with I of signature $(19-s,3)$,

N of signature $(s,0)$, and N without roots, have been classified Nikulin '80

a.k.a. symplectic automorphisms of $\mathrm{K3}$ surfaces

▷ also $K=7,8$ allowed, but no rank reduction in 7d asymmetric $\mathbb{T}^3/\mathbb{Z}_7$ and $\mathbb{T}^3/\mathbb{Z}_8$

extra massless vectors in twisted sectors $\Leftarrow$ $v\in \mathrm{I}^*$, $\forall v$ satisfying modular invariance

still can construct $(\mathbb{T}^3\times S^1)/\mathbb{Z}_7$ and $(\mathbb{T}^3\times S^1)/\mathbb{Z}_8$ with rank reduction in 6d

including a translation by $\frac{2\pi\mathcal{R}}{K}$ in S^1

▷ for rank reduction in 6d asymmetric $\mathbb{T}^4/\mathbb{Z}_K$ want automorphisms of $\Gamma(20,4)$

with with I of signature $(20-s,4)$, N of signature $(s,0)$, and N without roots

automorphisms of this type are symmetries of $\mathrm{K3}$ sigma models

Gaberdiel, Hohenegger, Volpato '11

Rank reduction in $\mathbb{T}^4/\mathbb{Z}_K$ via Leech lattice Λ

▷ N without roots $\rightsquigarrow$ look at automorphisms of Λ — Gaberdiel, Hohenegger, Volpato '11

Λ: even self-dual $(24, 0)$ without roots, automorphism group = Conway group Co_0

▷ sublattices of Λ fixed by elements of Co_0 classified up to conjugacy — Höhn, Mason '16

$\exists$ 290 distinct invariant lattices $\tilde{\mathrm{I}}$ $(r, 0)$, with normal N $(s, 0)$, $r + s = 24$

- if $r \geq 4$ look for I of signature $(r-4, 4)$ || $\mathrm{N}^*/\mathrm{N} = \mathrm{I}^*/\mathrm{I}$ — Kaan Baykara, Harvey '21

  and there exist glue vectors to construct $\Gamma(20, 4)$

- $\Theta : \mathbb{Z}_K$ automorphism of N that preserves correlated classes of N^*/N

▷ $\mathbb{T}^4/\mathbb{Z}_K$, $K = 2(\mathrm{CHL}), 3, 4, 5, 6, 7, 8$, with rank reduction $\longleftrightarrow$ HM# 2,4,9,20,18,52,55

▷ other components of the moduli space of 6d theories with 16 supercharges ?

Fraiman, Parra De Freitas '22

$\mathbb{T}^4/\mathbb{Z}_2$ with rank 8

▷ HM5, $s = 12$, $\mathrm{N} = \mathrm{D}_{12}^{+}(2)$, $\mathrm{N}^*/\mathrm{N} = \mathbb{Z}_2^{12}$, $\tilde{\mathrm{I}} = \mathrm{D}_{12}^{+}(2)$ $\qquad \mathrm{D}_{12}^{+} = \mathrm{D}_{12} + (\mathsf{Sp})$

$\mathrm{I} = 8\mathrm{A}_1 + 4\mathrm{A}_1(-1)$ or $\mathrm{I} = \mathrm{E}_8(2) + 4\mathrm{A}_1(-1)$ or $\mathrm{I} \sim 7\mathrm{A}_1 + \mathrm{U}(2) + 3\mathrm{A}_1(-1)$

$\Theta = -\mathbb{1}$ acting on N

▷ $e^{2i\pi P^2} \neq 1,\ \forall P \in \mathrm{I}^*$, but $\exists\ w \in \mathrm{I}^*,\ w^2 + \frac{s}{2} \in 2\mathbb{Z} \ ||\ e^{2i\pi P^2} = e^{2i\pi P\cdot w},\ \forall P \in \mathrm{I}^*$

▷ $\exists v \ ||\ 2v + w \in I,\ 2v^2 + \frac{s}{4} \in 2\mathbb{Z}$

▷ can construct modular invariant partition function with $\mathcal{Z}(g, g^2) = \mathcal{Z}(g, \mathbb{1})$

$$g|P_{\mathrm{N}}, P_{\mathrm{I}}\rangle = f(P_{\mathrm{N}})e^{2i\pi P_{\mathrm{I}}\cdot v}|-P_{\mathrm{N}}, P_{\mathrm{I}}\rangle, \qquad f(0) = 1,\ f(P_{\mathrm{N}})f(-P_{\mathrm{N}}) = e^{2i\pi P_{\mathrm{I}}\cdot w}$$

$$g^2|P_{\mathrm{N}}, P_{\mathrm{I}}\rangle = e^{2i\pi P_{\mathrm{I}}\cdot(2v+w)}|P_{\mathrm{N}}, P_{\mathrm{I}}\rangle = |P_{\mathrm{N}}, P_{\mathrm{I}}\rangle$$

▷ partition function

$$\mathcal{Z} = \frac{1}{\left(\sqrt{\tau_2}\eta\bar{\eta}\right)^4} \times \frac{1}{2\,\bar{\eta}^4}\left[\bar{\vartheta}_3^4 - \bar{\vartheta}_4^4 - \bar{\vartheta}_2^4 + \bar{\vartheta}_1^4\right] \times \frac{1}{2}\sum_{\ell=0}^{1}\sum_{m=0}^{1}\mathcal{Z}_\Gamma(g^\ell, g^m)$$

untwisted sector

$$\mathcal{Z}_\Gamma(\mathbb{1},\mathbb{1}) = \frac{1}{\eta^{20}\bar{\eta}^4}\sum_{P\in\Gamma} q^{\frac{1}{2}P_L^2}\bar{q}^{\frac{1}{2}P_R^2}$$

$$\mathcal{Z}_\Gamma(\mathbb{1},g) = \left(\frac{2\eta}{\vartheta_2}\right)^{s/2}\frac{1}{\eta^8\bar{\eta}^4}\sum_{P\in\mathrm{I}} q^{\frac{1}{2}P_L^2}\bar{q}^{\frac{1}{2}P_R^2}\,e^{2\pi i P\cdot v} \qquad s = 12$$

twisted sector

$$\mathcal{Z}_\Gamma(g,\mathbb{1}) = \left(\frac{\eta}{\vartheta_4}\right)^{s/2}\frac{1}{\eta^8\bar{\eta}^4}\sum_{P\in\mathrm{I}^*} q^{\frac{1}{2}(P+v)_L^2}\bar{q}^{\frac{1}{2}(P+v)_R^2}$$

$$\mathcal{Z}_\Gamma(g,g) = e^{i\pi(v^2+\frac{s}{8})}\left(\frac{\eta}{\vartheta_3}\right)^{s/2}\frac{1}{\eta^8\bar{\eta}^4}\sum_{P\in\mathrm{I}^*} q^{\frac{1}{2}(P+v)_L^2}\bar{q}^{\frac{1}{2}(P+v)_R^2}\,e^{2\pi i P\cdot v}e^{i\pi P^2}$$

▷ partition function encodes spectrum

* can choose v satisfying modular invariance || $(P+v)_R \neq 0, \forall P \in \mathrm{I}^*$

  $\Rightarrow$ no massless states in twisted sector

* massless states only in untwisted sector

  from R-movers: $\mathrm{NS}\ \mathbf{8}_v$, $\mathrm{R}\ \mathbf{8}_s$, $P_R = 0$

  from L-movers: $\frac{1}{2}P_L^2 + N_L - 1 = 0$

  vector multiplets from oscillator number $N_L = 1$ along 8 left I directions

  at generic points of I, group $\mathrm{U}(1)^8$

  enhancement at special points, e.g. $\mathrm{SO}(9)\times\mathrm{SO}(9)$, $\mathrm{SU}(9)$

  Fraiman, Parra De Freitas '22

Heterotic island in 6d ?

▷ HM149, $s = 20$, $\mathrm{N}^*/\mathrm{N} = \mathbb{Z}_2^2 \times \mathbb{Z}_{10}^2$, $\quad \mathrm{N} =$

$$\begin{pmatrix}
4 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 2 & 1 & 2 & 2 & 0 & -2 & 1 & -2 & -2 & -1 & -2 & -1 \\
1 & 4 & -2 & -1 & 1 & -2 & 1 & -2 & 2 & -1 & 2 & 0 & 1 & -1 & 2 & -1 & -2 & -2 & 0 & -2 \\
-1 & -2 & 4 & -1 & -1 & 2 & 1 & 0 & -2 & -1 & -1 & 0 & 1 & 0 & -1 & 2 & 1 & 0 & -1 & 2 \\
1 & -1 & -1 & 4 & 0 & 0 & 0 & 2 & 1 & 2 & -1 & 1 & -2 & 0 & -1 & 0 & -1 & 0 & -1 & -1 \\
-1 & 1 & -1 & 0 & 4 & -2 & 0 & 1 & 1 & 1 & -1 & -2 & 0 & 2 & 0 & 0 & 1 & 1 & 2 & -1 \\
-1 & -2 & 2 & 0 & -2 & 4 & 0 & 1 & -1 & 0 & -1 & 1 & -1 & 0 & 0 & 2 & 1 & 1 & 0 & 2 \\
-1 & 1 & 1 & 0 & 0 & 0 & 4 & 0 & -1 & -2 & -1 & -1 & 0 & 0 & -1 & 2 & 0 & -2 & 0 & 0 \\
-1 & -2 & 0 & 2 & 1 & 1 & 0 & 4 & 0 & 2 & -2 & -1 & -2 & 1 & -2 & 1 & 1 & 2 & 1 & 0 \\
2 & 2 & -2 & 1 & 1 & -1 & -1 & 0 & 4 & 1 & 2 & 1 & 0 & -1 & 2 & -1 & -2 & 0 & 0 & -2 \\
1 & -1 & -1 & 2 & 1 & 0 & -2 & 2 & 1 & 4 & -1 & 1 & -2 & 1 & 0 & -1 & 0 & 2 & 0 & -1 \\
2 & 2 & -1 & -1 & -1 & -1 & -1 & -2 & 2 & -1 & 4 & 1 & 2 & -2 & 2 & -2 & -2 & -1 & -1 & -1 \\
2 & 0 & 0 & 1 & -2 & 1 & -1 & -1 & 1 & 1 & 1 & 4 & 0 & -1 & 1 & -1 & -2 & -1 & -2 & 0 \\
0 & 1 & 1 & -2 & 0 & -1 & 0 & -2 & 0 & -2 & 2 & 0 & 4 & -1 & 1 & 0 & -1 & -1 & 0 & 0 \\
-2 & -1 & 0 & 0 & 2 & 0 & 0 & 1 & -1 & 1 & -2 & -1 & -1 & 4 & 0 & 0 & 2 & 2 & 2 & 0 \\
1 & 2 & -1 & -1 & 0 & 0 & -1 & -2 & 2 & 0 & 2 & 1 & 1 & 0 & 4 & -1 & -1 & 0 & 0 & -1 \\
-2 & -1 & 2 & 0 & 0 & 2 & 2 & 1 & -1 & -1 & -2 & -1 & 0 & 0 & -1 & 4 & 1 & 0 & 1 & 1 \\
-2 & -2 & 1 & -1 & 1 & 1 & 0 & 1 & -2 & 0 & -2 & -2 & -1 & 2 & -1 & 1 & 4 & 2 & 2 & 2 \\
-1 & -2 & 0 & 0 & 1 & 1 & -2 & 2 & 0 & 2 & -1 & -1 & -1 & 2 & 0 & 0 & 2 & 4 & 2 & 0 \\
-2 & 0 & -1 & -1 & 2 & 0 & 0 & 1 & 0 & 0 & -1 & -2 & 0 & 2 & 0 & 1 & 2 & 2 & 4 & 0 \\
-1 & -2 & 2 & -1 & -1 & 2 & 0 & 0 & -2 & -1 & -1 & 0 & 0 & 0 & -1 & 1 & 2 & 0 & 0 & 4
\end{pmatrix}$$

$\mathrm{I} = 2\mathrm{A}_1(-1) + 2\mathrm{A}_1(-5)$

Θ acting on N: $\mathbb{Z}_{10}$ with eigenvalues $e^{\pm 2\pi i t_i}$, $t = \frac{1}{10}(1^2, 2^2, 3^2, 4^2, 5^2)$

▷ $e^{10 i\pi P^2} \neq 1,\ \forall\, P \in \mathrm{I}^*$, but $\exists\, w \in \mathrm{I}^* \,||\, e^{10 i \pi P^2} = e^{2 i \pi P\cdot w},\ \forall\, P \in \mathrm{I}^*$

▷ $\exists\, v \,||\, 10v + w \in \mathrm{I},\ 10(v^2 + t^2) \in 2\mathbb{Z},\ v, 2v, 5v \notin \mathrm{I}^*$

▷ $\mathcal{Z}(g, g^{10}) = \mathcal{Z}(g, \mathbb{1})$, but not clear how to define g such that

$$g^{10}|P_\mathrm{N}, P_\mathrm{I}\rangle = e^{2 i\pi P_\mathrm{I}\cdot(10v+w)}|P_\mathrm{N}, P_\mathrm{I}\rangle = |P_\mathrm{N}, P_\mathrm{I}\rangle$$

$\mathbb{T}^4/\mathbb{Z}_{10}$ with rank 4

▷ HM100, $s = 20$, $\mathrm{N}^*/\mathrm{N} = \mathbb{Z}_2^3 \times \mathbb{Z}_{10}$, $\quad \mathrm{N} = \begin{pmatrix} 4 & -1 & 2 & -2 & 2 & -2 & 2 & 2 & -2 & -2 & -2 & -2 & 1 & -2 & 2 & 1 & 2 & 1 & 1 & 1 \\ -1 & 4 & -1 & -1 & 1 & 2 & -2 & -2 & 2 & 0 & 1 & -1 & -2 & 1 & -1 & 1 & 0 & -2 & 1 & -2 \\ 2 & -1 & 4 & -1 & 0 & 0 & 0 & 2 & -2 & -2 & -2 & 0 & 2 & -2 & 2 & -1 & 2 & 0 & 1 & 1 \\ -2 & -1 & -1 & 4 & -2 & 1 & 0 & 0 & 1 & 1 & 2 & 2 & 1 & 2 & -2 & -1 & -1 & 1 & -2 & -1 \\ 2 & 1 & 0 & -2 & 4 & 0 & 1 & 1 & 0 & -1 & 0 & -1 & -1 & -1 & 0 & 2 & 0 & -1 & 2 & -1 \\ -2 & 2 & 0 & 1 & 0 & 4 & -2 & -1 & 2 & 0 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -2 \\ 2 & -2 & 0 & 0 & 1 & -2 & 4 & 1 & -2 & 0 & -1 & -1 & 0 & -1 & 1 & 0 & 1 & 2 & -1 & 0 \\ 2 & -2 & 2 & 0 & 1 & -1 & 1 & 4 & -1 & -2 & -1 & 0 & 2 & -2 & 1 & 0 & 0 & 0 & 0 & 1 \\ -2 & 2 & -2 & 1 & 0 & 2 & -2 & -1 & 4 & 0 & 2 & 1 & -1 & 1 & -2 & 1 & -2 & -1 & 0 & -2 \\ -2 & 0 & -2 & 1 & -1 & 0 & 0 & -2 & 0 & 4 & 2 & 0 & -1 & 1 & -1 & 0 & -1 & 1 & 0 & 0 \\ -2 & 1 & -2 & 2 & 0 & 1 & -1 & -1 & 2 & 2 & 4 & 1 & 0 & 2 & -2 & 1 & -2 & 0 & 0 & -2 \\ -2 & -1 & 0 & 2 & -1 & 1 & -1 & 0 & 1 & 0 & 1 & 4 & 1 & 1 & -2 & -1 & -1 & -1 & -1 & 0 \\ 1 & -2 & 2 & 1 & -1 & -1 & 0 & 2 & -1 & -1 & 0 & 1 & 4 & 0 & 0 & -1 & 1 & 1 & -1 & 1 \\ -2 & 1 & -2 & 2 & -1 & 1 & -1 & -2 & 1 & 1 & 2 & 1 & 0 & 4 & -2 & 0 & -1 & 0 & -1 & -1 \\ 2 & -1 & 2 & -2 & 0 & -1 & 1 & 1 & -2 & -1 & -2 & -2 & 0 & -2 & 4 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 2 & -1 & 0 & 0 & 1 & 0 & 1 & -1 & -1 & 0 & 0 & 4 & -1 & 0 & 1 & -1 \\ 2 & 0 & 2 & -1 & 0 & -1 & 1 & 0 & -2 & -1 & -2 & -1 & 1 & -1 & 1 & -1 & 4 & 0 & 0 & 1 \\ 1 & -2 & 0 & 1 & -1 & -1 & 2 & 0 & -1 & 1 & 0 & -1 & 1 & 0 & 1 & 0 & 0 & 4 & -1 & 0 \\ 1 & 1 & 1 & -2 & 2 & 1 & -1 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 1 & 1 & 0 & -1 & 4 & 0 \\ 1 & -2 & 1 & -1 & -1 & -2 & 0 & 1 & -2 & 0 & -2 & 0 & 1 & -1 & 1 & -1 & 1 & 0 & 0 & 4 \end{pmatrix}$

$\mathrm{I} = \mathrm{A}_4(-2)$

Θ acting on N: $\mathbb{Z}_{10}$ with eigenvalues $e^{\pm 2\pi i t_i}$, $t = \frac{1}{10}(1^3, 2, 3^3, 4, 5^2)$

▷ $e^{10 i\pi P^2} = 1,\ \forall\, P \in \mathrm{I}^*, \quad e^{5 i\pi P^2} = 1,\ \forall\, P \in \mathrm{I}_2^*, \quad e^{2 i\pi P^2} = 1,\ \forall\, P \in \mathrm{I}_5^*$

▷ can define order 10 action

$$g|P_{\mathrm{N}}, P_{\mathrm{I}}\rangle = e^{2i\pi P_{\mathrm{I}}\cdot v}|\Theta P_{\mathrm{N}}, P_{\mathrm{I}}\rangle, \qquad 10v \in \mathrm{I}$$

and construct the full partition function

▷ modular invariance $\Rightarrow$ $10(v^2 + t^2) \in 2\mathbb{Z}$

all solutions have $5v \in \mathrm{I}^*$ $\Rightarrow$ massless states in g^5 sector

▷ partition function $\longrightarrow$ spectrum

- no massless vectors in untwisted sector (no invariant L directions, $P_L^2 \geq 4$)
- 20 massless vectors in twisted sectors when $v = 0$
- can choose $v \neq 0$ such that there are no massless vectors in twisted sectors, except 4 in g^5 sector

▷ can give masses to twisted states adding a spectator circle, i.e. going to 5d

- $(\mathbb{T}^4 \times S^1)/\mathbb{Z}_{10}, \qquad \Gamma(21,5) = \Gamma(20,4) + \mathrm{U}$
- $\mathbb{Z}_{10}$: Θ on $\Gamma(20,4)$ + translation by $\frac{2\pi\mathcal{R}}{10}$ in S^1
- translation (equiv. to order 10 shift in U) $\Rightarrow$ rational windings $\Rightarrow$ massive states
- only 1 vector multiplet from untwisted sector, 5d theory with rank 1

at $\frac{\mathcal{R}}{\sqrt{\alpha'}} = \sqrt{10}$, twisted massless states enhance $\mathrm{U}(1)$ to $\mathrm{SU}(2)$

Summary and Outlook

- Studied heterotic asymmetric $\mathbb{T}^4/\mathbb{Z}_2$ with 8 superchrages

* classified the relevant automorphisms of the $\Gamma(20,4)$ lattice
* constructed the partition function, with novel ways to define the $\mathbb{Z}_2$ action
* worked out several examples, rediscovered some without neutral hypermultiplets

◇ extensions: $\mathbb{T}^5/\mathbb{Z}_2, \mathbb{T}^6/\mathbb{Z}_2, \mathbb{Z}_2 \to \mathbb{Z}_p$
also type II Gkountoumis, Hull, Stemerdink, Vandoren '23, Gkountoumis, Hull, Vandoren '24

◇ to do: examine dualities

- Studied heterotic asymmetric $\mathbb{T}^4/\mathbb{Z}_K$ with 16 superchrages and rank reduction

* noticed that rank reduction requires N lattice of $\mathbb{Z}_K$ in $\Gamma(20,4)$ with no roots
* constructed examples via Leech lattice
* unable to confirm heterotic island in 6d, but a 5d $\mathbb{T}^5/\mathbb{Z}_{10}$ with rank 1

◇ to do: go to 4d, $(\mathbb{T}^4 \times \tilde{S}^1 \times S^1)/\mathbb{Z}_K$ or $\mathbb{T}^6/\mathbb{Z}_K$

Persson, Volpato '15, Bossard, Cosnier-Horeau, Pioline '17, Harvey, Moore '18

Fin