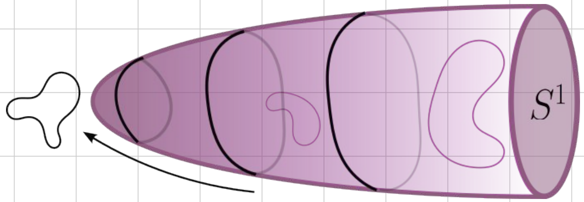
THE BUBBLE OF NOTHING
UNDER T-DUALITY



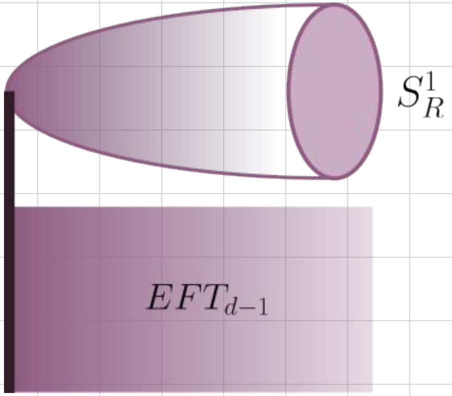
Based on:
[2312.09291] Matilda Delgado

NOTHING IS ALWAYS AROUND THE CORNER

[McNamara, Vafa '19]

The cobordism conjecture implies that spacetime can always end in a theory of QG

There always exists a physical process that allows you to **shrink the compact space to a point**



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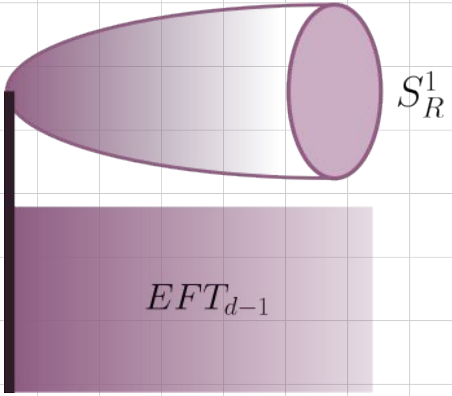
What is the simplest compactification?

The circle is a **consistent background** for all known string theories,

There must always exist a process that shrinks it to a point.



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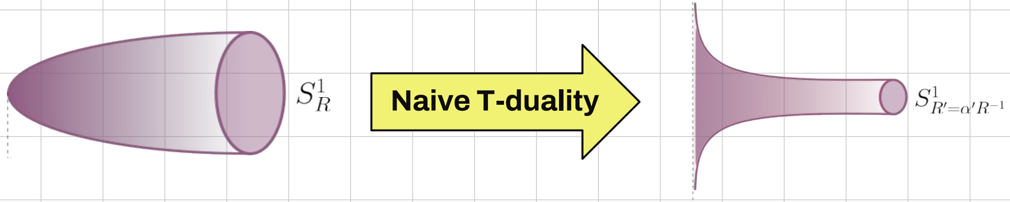

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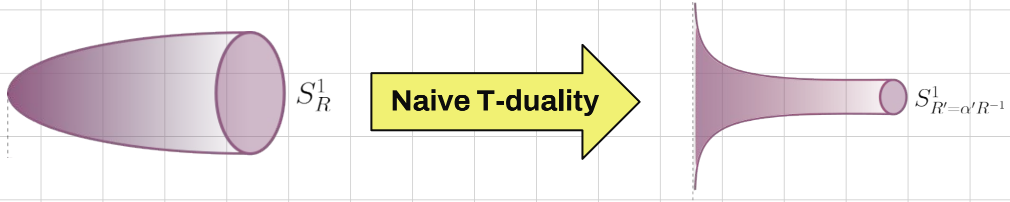

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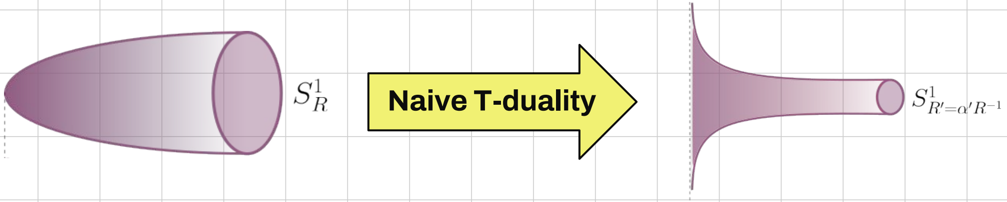

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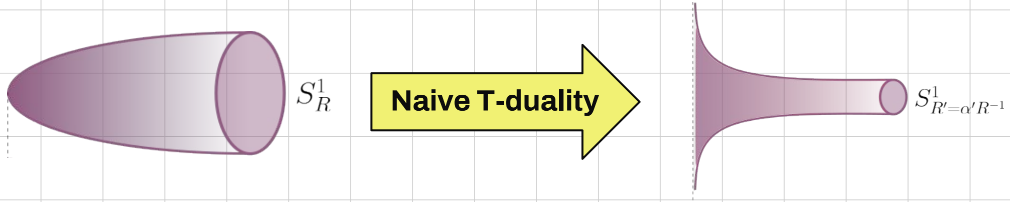

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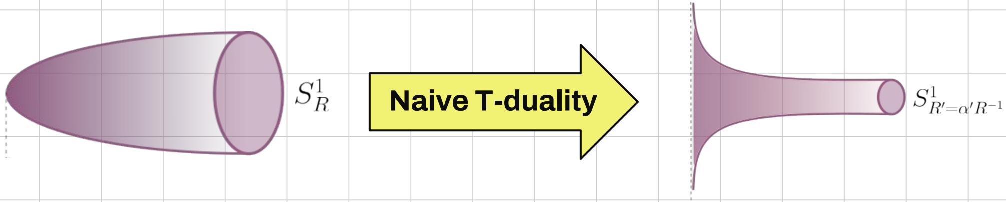

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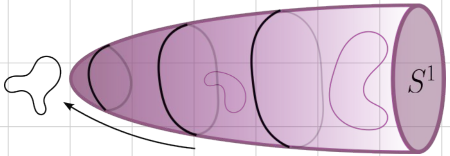

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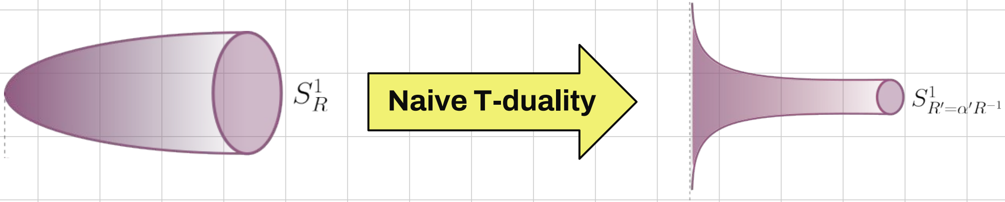

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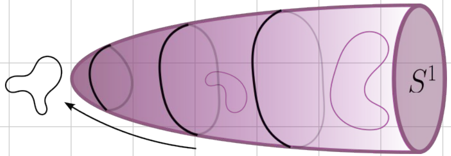

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[Gregory, Harvey, Moore, `97]

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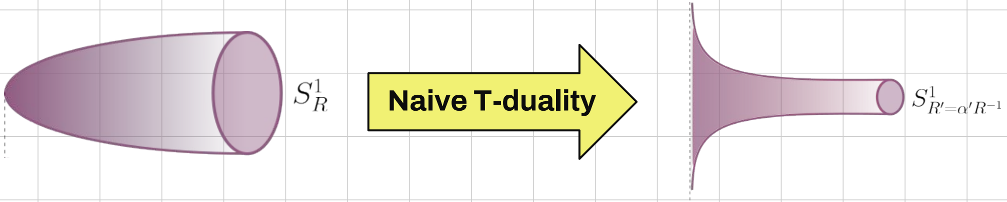

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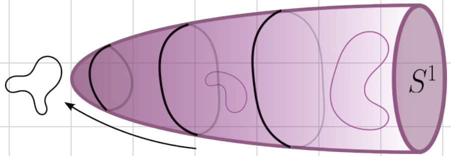

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Let's look at an example..

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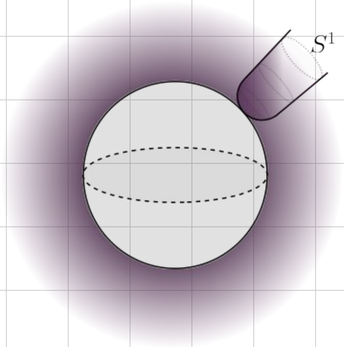
What about the simplest example, Witten's Bubble of Nothing ? [Witten '82]



Witten's bubble of nothing is a smooth cobordism to nothing

It is topologically allowed *in the absence of fermions* (so embedit in bosonic string theory)

$$ds^2 = f(r)^{-1}dr^2 + r^2[-dt^2 + \cosh(t)^2 d\Omega_{23}^2] + R_0^2 f(r) d\theta^2 \qquad f(r) = 1 - \left(\frac{r_0}{r}\right)^{23}$$

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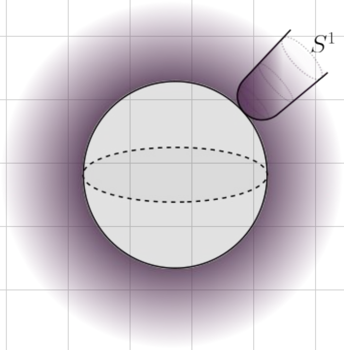

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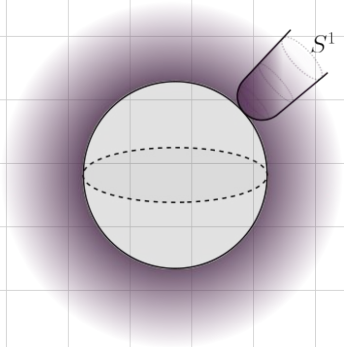

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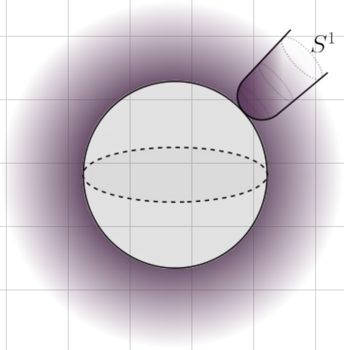

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TODAY: what is this mysterious dual object in (bosonic) String Theory?

UNDER NAIVE T-DUALITY

T-duality trades a string theory on a circle of radius R for one on a radius $\alpha' R^{-1}$

The Buscher rules dictate how the zero-modes of the metric and dilaton change

$$(\tilde{G}_s)_{\theta\theta} = \frac{1}{(G_s)_{\theta\theta}} \qquad (\tilde{G}_s)_{\mu\nu} = (G_s)_{\mu\nu} \qquad \tilde{\phi} = \phi - \frac{1}{2}\log|(G_s)_{\theta\theta}|$$

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Applying these rules to the BoN, we get:
$$d\tilde{s}^2 = \left(e^{\frac{\phi_0}{6}} R_0^2 f(r)\right)^{\frac{1}{12}} \left[f(r)^{-1}dr^2 + (g_{bon})_{ij}dx^i dx^j\right] + e^{-\frac{23}{72}\phi_0}\left(R_0^2 f(r)\right)^{-\frac{11}{12}} d\tilde{\theta}^2$$
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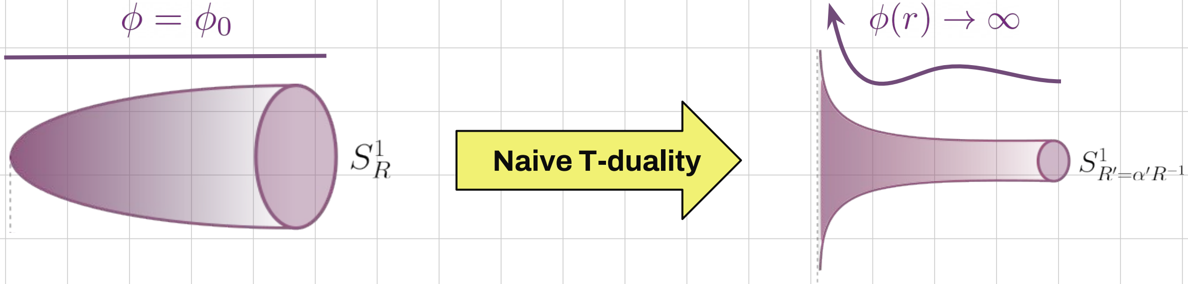

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$\Rightarrow$ Fitting the defect with a delta source: **the solution has finite tension**

$\Rightarrow$ The metric seems to preserve a momentum U(1): **The dependence on the dual angle is smeared**

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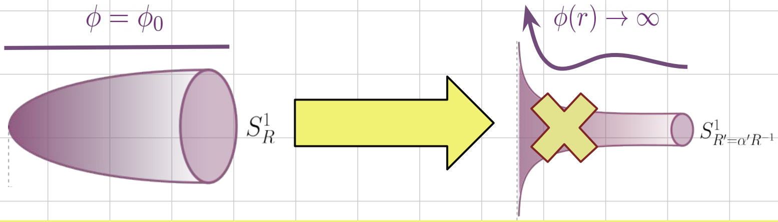

So the dual is some sort of 24-dimensional object with finite tension …
What else can we say?

BEYOND GRAVITY

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Other ideas?

THANKS!

EXTRA SLIDES

BoN & CIGAR CFT

- **The cigar CFT describes the worldsheet of the cigar geometry** [Witten '91]

$$ds^2_{cigar} = dr^2 + \tanh(r)^2 d\theta^2$$

It is based on the **Sl(2,R)/U(1) gauged WZW (coset) model**

This also describes a circle that pinches-off

But it is **not the same solution as the BoN**:

Main difference: the cigar is not Ricci-flat→ you need to introduce a dilaton background for conformal invariance

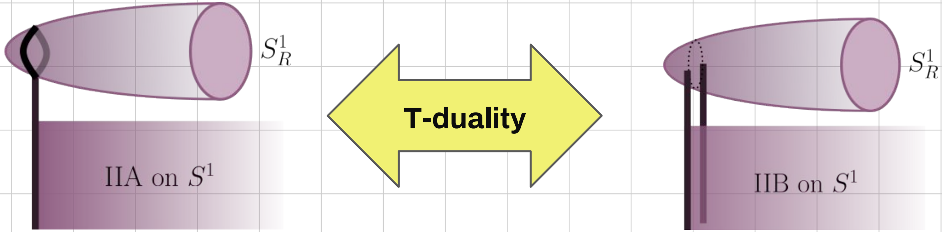
$$\phi(r) = 2 \ln \cosh r + \text{constant}$$

PLUS: the BoN has a time-dependent sphere transverse to this plane..

UNDER T-DUALITY: EXAMPLES

- **The Bordism defects of type IIA and IIB on a circle** [McNamara, Vafa '19]

An **O8** wrapping the circle (+D8's)



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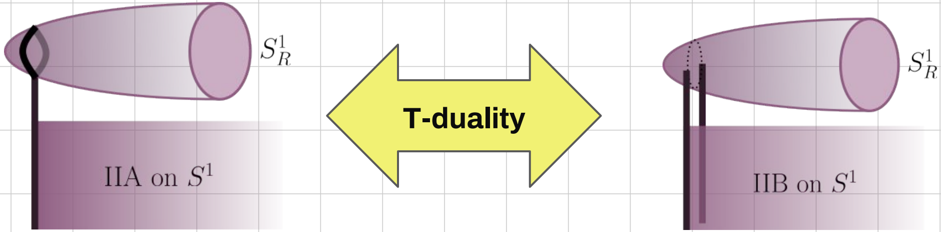

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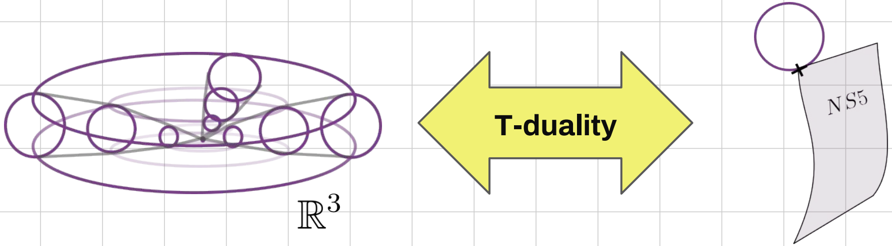
- **The T-duality between the Taub-NUT and the NS5-brane** [Gregory, Harvey, Moore, `97] [Tong '02]

The **Taub-NUT** describes a circle that pinches off smoothly
"pure geometry"



An **NS5-brane** localized on the dual circle

- Broken momentum U(1) ✅
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CLUES IN GRAVITY

After dimensional reduction, the dual object has **finite tension**

Perform the dimensional reduction along the dual circle.

Introduce a codimension 1 source and solve the equations of motion:

$$S_{25} \supset -T \int d^{25}x \sqrt{-g_{24}} e^{a\tilde{\phi}} e^{b\omega}\ \delta(r - r_0)$$

You get: $T \sim O(1) M_{p,25}^{24}$

The dual solution describes an object that is **smeared** along the circle

We know the momentum U(1) of the dual has to be broken

And yet the metric obtained by naive T-duality exhibits this symmetry:

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