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TWISTED TORI, CASIMIR ENERGY AND ADS CONJECTURE

INTRODUCTION

- Use 4d supergravity and its properties as EFT to learn about the landscape
- The landscape of supergravities seems much larger than ST
  - Example *infinite family of SO(8) N=8 supergravities*
- Simple models are reductions on tori and twisted tori
  - Still many things to understand
  - More theories in the swampland…

TWISTED TORI REDUCTIONS AND GAUGED SUPERGRAVITIES

- Cremmer-Scherk-Schwarz reductions from the 4d perspective [FLAT GROUPS]

$$\mathrm{U}(1) \ltimes T^{27} \qquad \begin{cases} [X_0, X^I] &= Q^I{}_J X^J \\ [X^I, X^J] &= 0 \end{cases}$$

- Gives:
    - Minkowski vacua with *N=0,2,4,6*
    - Gravitino masses $2 \times M_i$
    - Overall sliding scale, but M_i/M_j fixed

TWISTED TORI REDUCTIONS AND GAUGED SUPERGRAVITIES

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- Simple generalisation:

$$[Z_I, X^J] = Q_I{}^J{}_K X^K, \qquad [Z_I, Z_J] = 0, \qquad X^J \ \textit{Nilpotent}$$

- One can classify "flat groups" (either solvable or nilpotent)

TWISTED TORI REDUCTIONS AND GAUGED SUPERGRAVITIES

- Reduce on circle(s) with periodic coordinates $y \sim y + 1$, twisting fields in a $G \subset GL(d, \mathbb{R})$ representation

CREMMER-SCHERK-SCHWARZ
KALOPER-MYERS

$$\Phi(x, y) = \exp(My)[\phi(x)] \qquad M \in \mathfrak{g}$$

- We assume a non-periodic map, with monodromies $\mathscr{M} = e^M \in G(\mathbb{Z})$
  - ***Locally*** the manifold is G/Γ, for some discrete group Γ
- The metric follows from the usual Maurer-Cartan equations for G

$$e^0 = dy \qquad e^a = \exp\left(My\right)^a{}_b\, dz^b \qquad ds^2 = (e^0)^2 + e^a e^b\, \delta_{ab}$$

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$$e^0 = dy \qquad e^a = \exp\left(My\right)^a{}_b\, dz^b \qquad de^a + M^a{}_b e^0 \wedge e^b = 0$$

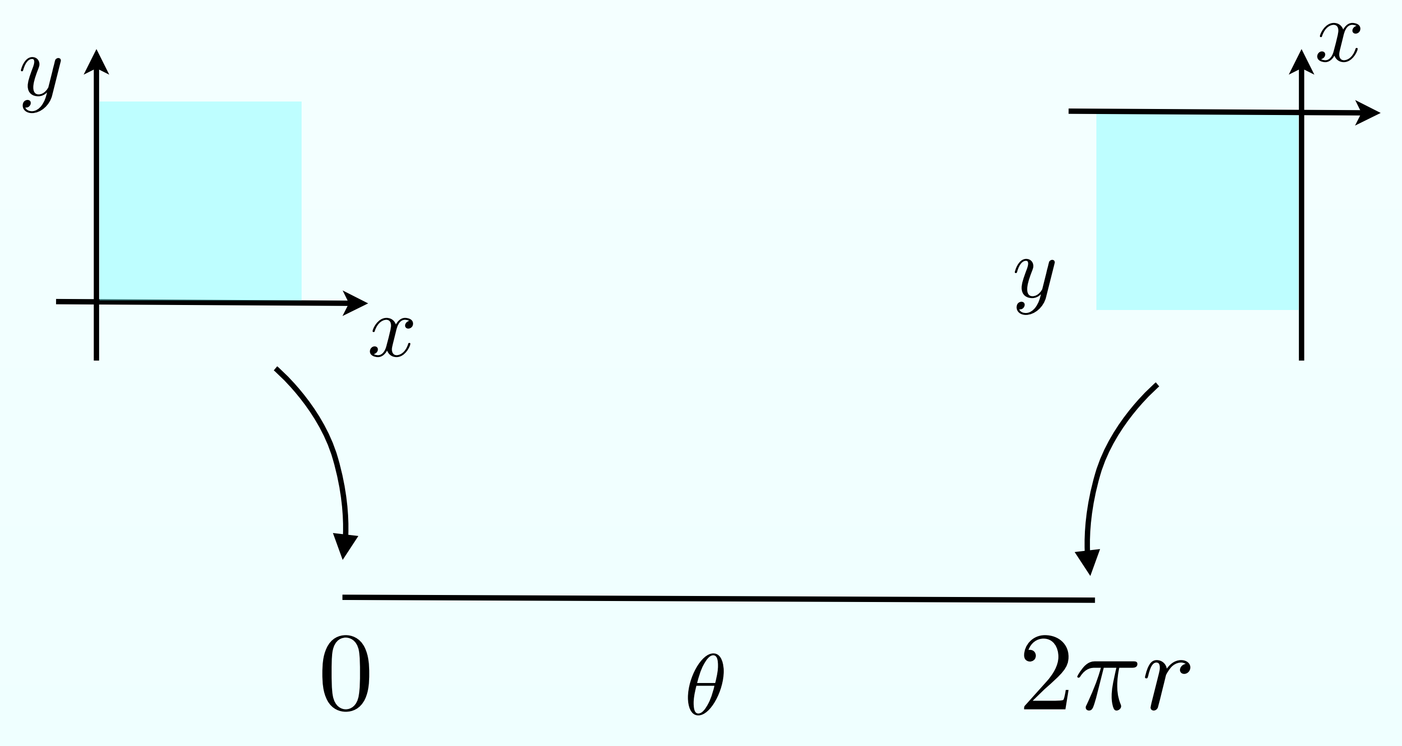
DOUBLED-TWISTED TORI AND GAUGED SUPERGRAVITIES

- One can generalise this to *doubled spacetime* HULL-REID EDWARDS

$$\exp M \in O(d,d,\mathbb{Z})$$

- Double twisted tori, also related to freely acting orbifolds & non-geometric compactifications

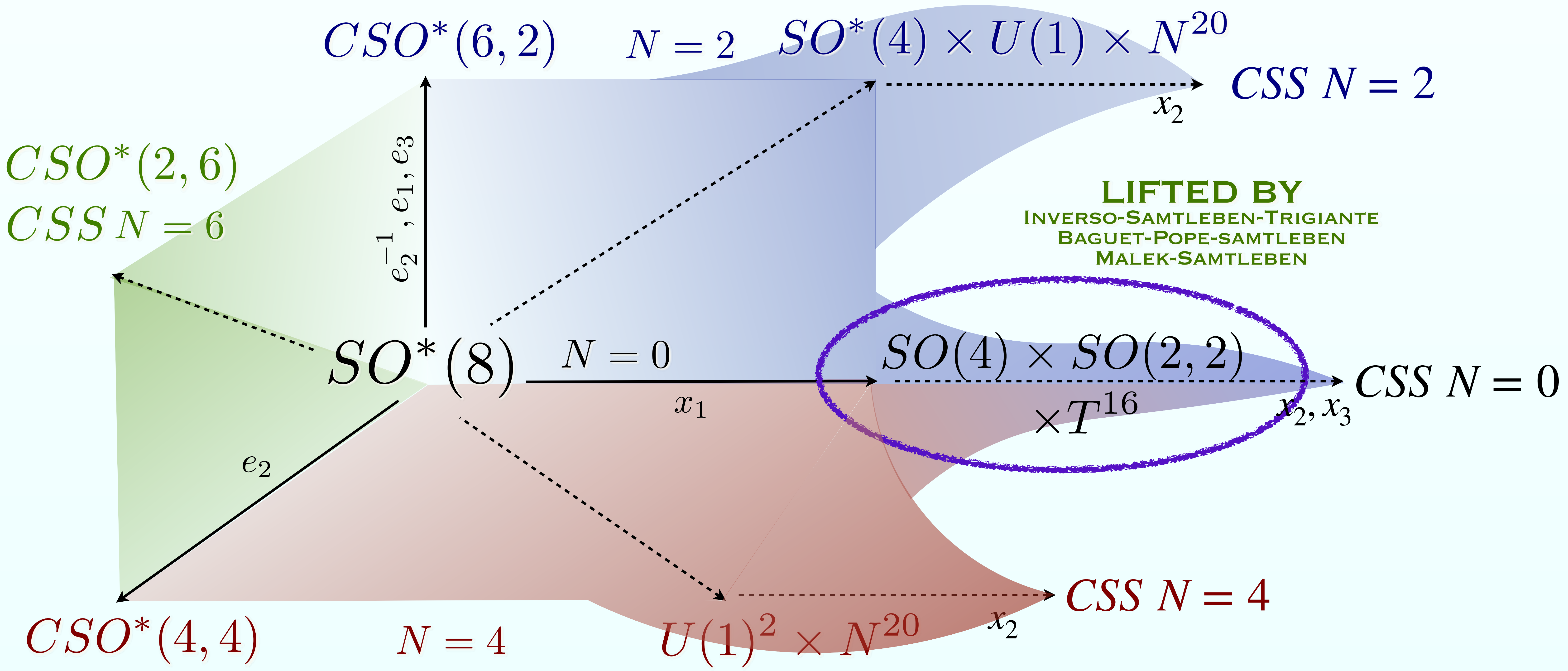
CONDEESCU-KOUNNAS-FLORAKIS-LÜST



MAXIMAL SUPERGRAVITY

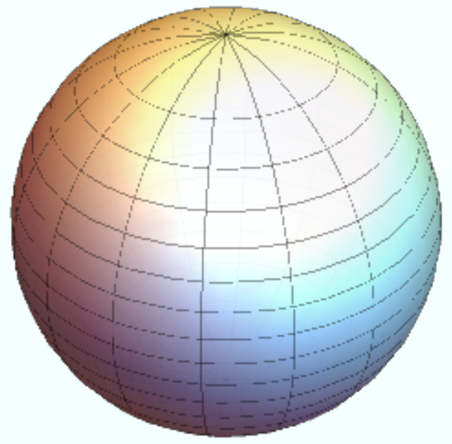
- Gauging N=8 supergravity with G=SO*(8) can lead to Minkowski vacua with an interesting *moduli space* $[SU(1,1)/U(1)]^3$

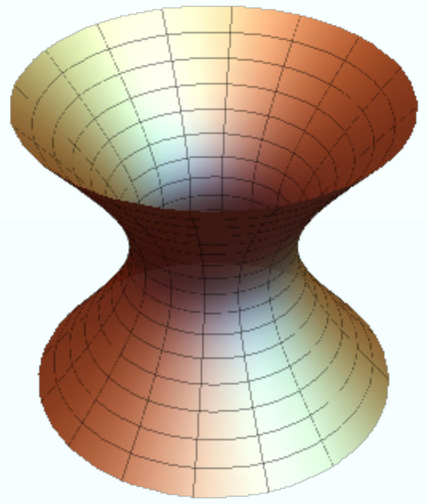
GD-Inverso
Catino-GD-Inverso-Zwirner



MAXIMAL SUPERGRAVITY & FLAT FOLDS

- The uplift uses as internal manifold $S^3 \times H^{2,2}$





- Trick: CSS reduction of DFT ALDAZABAL-BARON-MARQUÉS-NÚÑEZ GEISSBÜHLER

$$g_{\mu\nu} = e^{4\gamma\varphi(x)} g_{\mu\nu}(x) \quad e^{\phi} = \rho^2(y) e^{\varphi(x)}$$

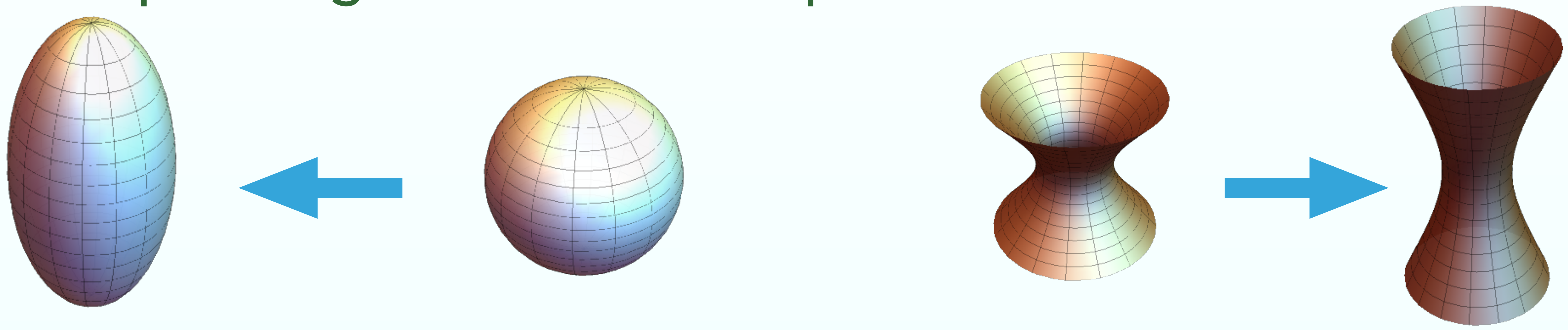
$$\mathcal{H}_{MN} = U_M{}^A(y) M_{AB}(x) U_N{}^B(y)$$

$$\mathcal{A}_\mu^M = U^{-1}{}_A{}^M(y)\, A_\mu^A(x)$$

$$M_{AB}(x) \in \frac{\mathrm{SO}(6,6)}{\mathrm{SO}(6) \times \mathrm{SO}(6)}$$

MAXIMAL SUPERGRAVITY & FLAT FOLDS

- Look at squashing of the internal space



- = vevs for the scalars

$$\langle \mathcal{H} \rangle (y) = U_M{}^A(y) \langle M_{AB} \rangle U_N{}^B(y)$$

MAXIMAL SUPERGRAVITY & FLAT FOLDS

- Follow the deformation to the *boundary of moduli space*

$$\hat{U}_M{}^A(y) = g_M{}^P J_P{}^N U_N{}^B(y') L_B{}^A(\langle\phi_4\rangle)$$

Rescaling of coupling constant

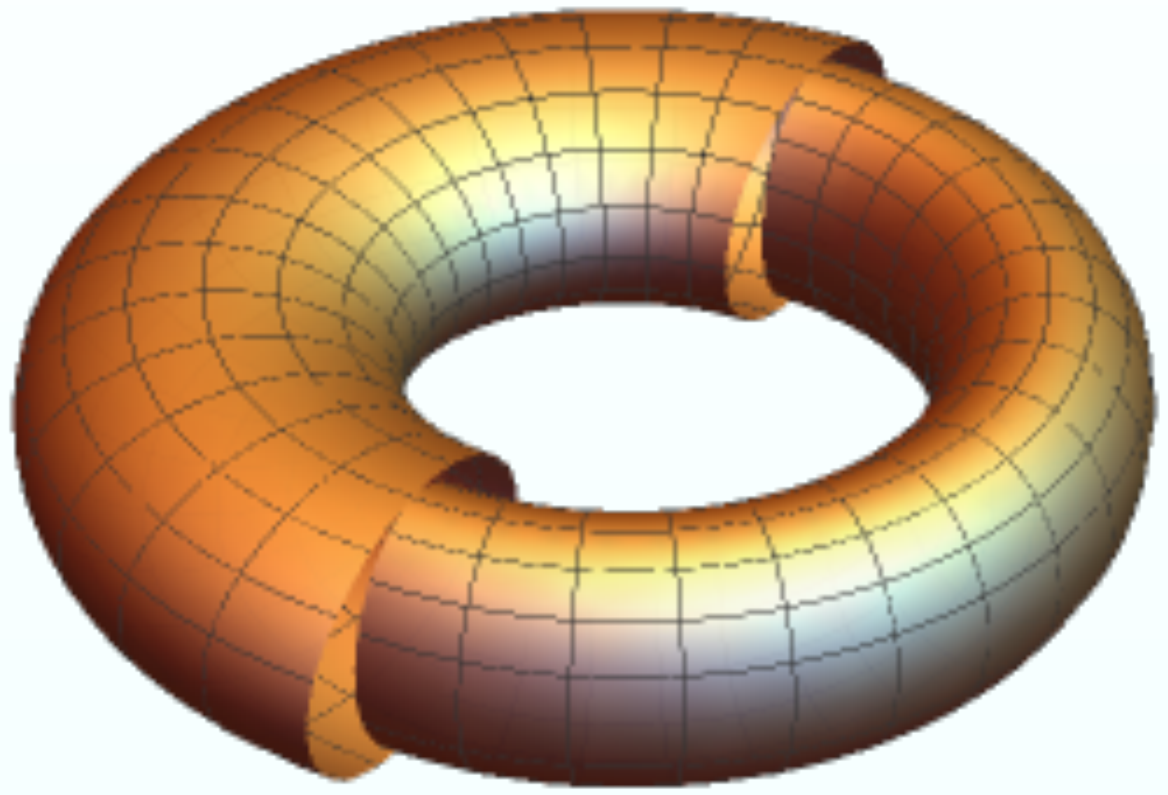
Jacobian of the coordinate change

Vev sent to boundary

- And plug the result back: $\langle\mathcal{H}\rangle(y) = U_M{}^A(y)\langle M_{AB}\rangle U_N{}^B(y)$

MAXIMAL SUPERGRAVITY & FLAT FOLDS

GD-Inverso-Spezzati



- The result is a space with T-duality patching
- Example:

$$ds^2 = dx_1^2 + dx_2^2 + d\theta^2 + dy_1^2 + dy_2^2 + d\psi^2$$

- With patching conditions

$$\theta \sim \theta + \alpha \qquad z_L \sim e^{-i\alpha} z_L \qquad z_R \sim e^{i\alpha} z_R \qquad w \sim e^{i\alpha} w$$

$$\psi \sim \psi + \delta \qquad w_L \sim i\, e^{-i\delta} w_L \qquad w_R \sim -\, i\, e^{i\delta} w_R \qquad z \sim e^{i\delta} z$$

MAXIMAL SUPERGRAVITY & FLAT FOLDS

GD-Inverso-Spezzati

- Q-flux example

$$ds^2 = e^{-\phi/2}\left[dx_1^2 + dx_2^2 + dx_3^2 + \frac{1}{1+y_1^2+y_2^2}\left(dy_1^2 + dy_2^2 + dy_3^2 + (y_1 dy_2 + y_2 dy_1)^2\right)\right]$$

$$e^{\phi} = 1/\sqrt{1+y_1^2+y_2^2}$$

$$B = \frac{2}{1+y_1^2+y_2^2}(y_1 dy_2 - y_2 dy_1)\wedge dy_3$$

- With patching conditions

$$x_3 \sim x_3 + \alpha \qquad z_L \sim e^{-i\alpha} z_L \qquad z_R \sim e^{i\alpha} z_R$$

$$y_1 \sim y_1 + 1 \qquad \beta^{y_2 y_3} = 1$$

$$y_2 \sim y_2 + 1 \qquad \beta^{y_1 y_3} = -1$$

STABILITY

- *Ungauged sugra is finite up to 4 loops (possibly 7)*
- Gauging = new couplings
- One-loop divergencies governed by super traces

$$Str\left(\mathcal{M}^{2k}\right) = \sum_J (-1)^{2J}(2J+1)tr(\mathcal{M}_J)^{2k}$$

- Example: 1-loop potential

$$\begin{aligned} V_{eff} &= \frac{1}{64\pi^2}\mathrm{Str}\mathcal{M}^0\Lambda^4\log\frac{\Lambda^2}{\mu^2} + \frac{1}{32\pi^2}\mathrm{Str}\mathcal{M}^2\Lambda^2 - \frac{1}{64\pi^2}\mathrm{Str}\mathcal{M}^4\log\Lambda^2 \\ &+ \frac{1}{64\pi^2}\mathrm{Str}\left(\mathcal{M}^4\log\mathcal{M}^2\right) \end{aligned}$$

STABILITY

- Using only general identities and the vacuum condition GD-Zwirner

$$Str\,(\mathcal{M}^2) = Str\,(\mathcal{M}^4) = Str\,(\mathcal{M}^6) = 0$$

- **1-loop finiteness**

- 1-loop potential

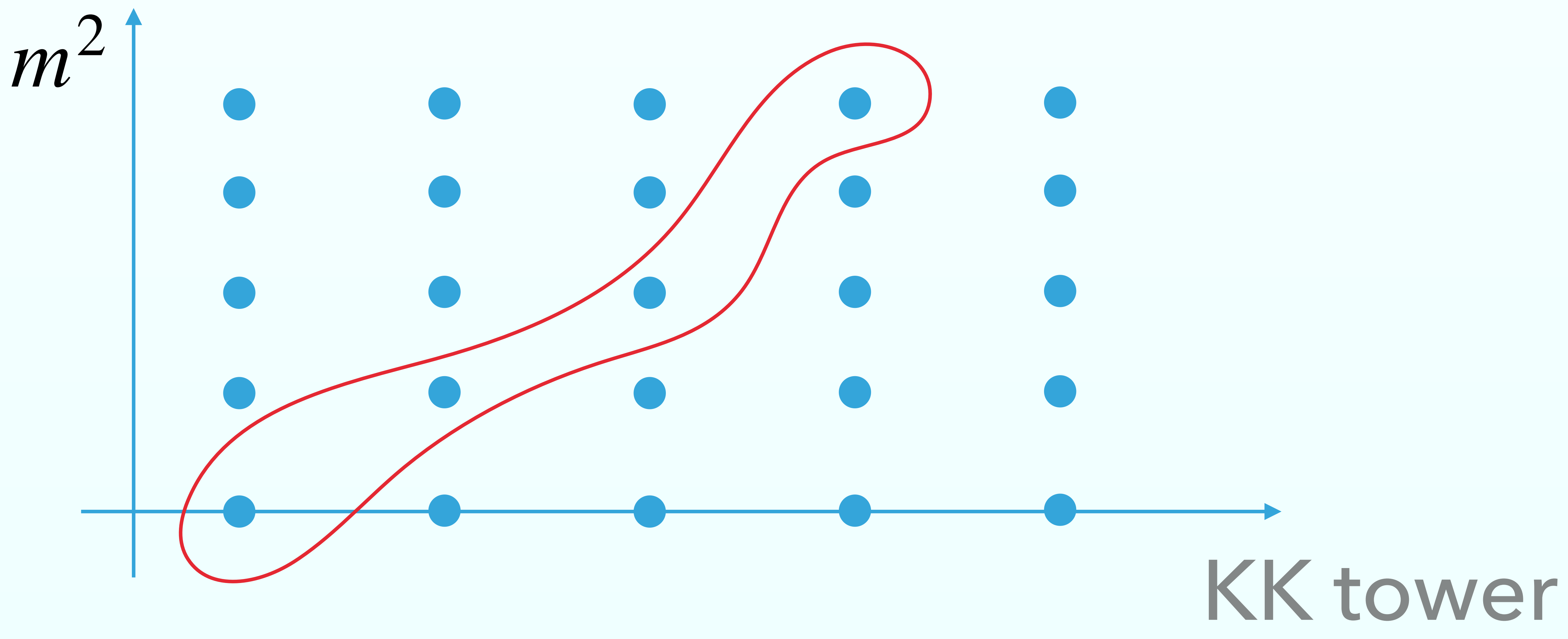
$$V = \frac{1}{64\pi^2} Str\left(\mathcal{M}^4 \log \mathcal{M}^2\right) < 0$$

- Non supersymmetric AdS should decay

WHAT ABOUT TWISTED TORI?

GD-PREZAS
GRAÑA-MINASIAN-TRIENDL

- Wolf: *Any Riemanniann homogeneous flat space is the direct product of the Euclidean plane with the torus*
- Consistent truncations vs EFT



WHAT ABOUT TWISTED TORI?

GD-Prezas
Graña-Minasian-Triendl

$$q = 2\pi\left(k + \frac{1}{n}\right)$$

- For a 3-torus

Consistent truncation

$$y \sim y + m$$

$$x_1 \sim \cos(qy)\, x_1 - \sin(qy)\, x_2$$

$$x_2 \sim \sin(qy)\, x_1 + \cos(qy)\, x_2$$

Homogeneous

Good EFT

$$y \sim y + m$$

$$x_1 \sim \cos(qy)\, x_1 - \sin(qy)\, x_2 + n - \frac{p}{2}$$

$$x_2 \sim \sin(qy)\, x_1 + \cos(qy)\, x_2 + \sqrt{3}\frac{p}{2}$$

Non-Homogeneous

WHAT ABOUT KK STATES? GD-ZWIRNER

- 5d supergravity to 4d SS reduction

$$V_1 = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4} \sum_{n=-\infty}^{+\infty} \sum_i (-1)^{2J_\alpha}(2J_\alpha + 1)\log\left(p^2 + m_{n,\alpha}^2\right) \qquad m_{n,\alpha}^2 = \frac{(n+s_\alpha)^2}{R^2}$$

- Interesting super trace relations for n fixed:
  - $Str\,\mathcal{M}_n^{q<N} = 0,$
  - $Str\,\mathcal{M}_n^{q=N}$ fixed and n-independent

WHAT ABOUT KK STATES? GD-ZWIRNER

- Resumming

$$V_1 = -\frac{3}{128\,\pi^6 R^4} \sum_\alpha (-1)^{2J_\alpha} (2J_\alpha + 1) \left[\mathrm{Li}_5(e^{-2\pi i s_\alpha}) + \mathrm{Li}_5(e^{2\pi i s_\alpha})\right] \quad m^2_{n,\alpha} = \frac{(n + s_\alpha)^2}{R^2}$$

- For small deformation parameters we have corrections to 1-loop eff. Theory
  - Example, N=8

$$V_1 = -\frac{93\,\zeta(5)}{8\,\pi^6 R^4} \simeq -\frac{0.0125}{R^4} \qquad V_{1,red} \simeq -\frac{0.0184}{R^4}$$

WHAT ABOUT KK STATES? GD-ZWIRNER

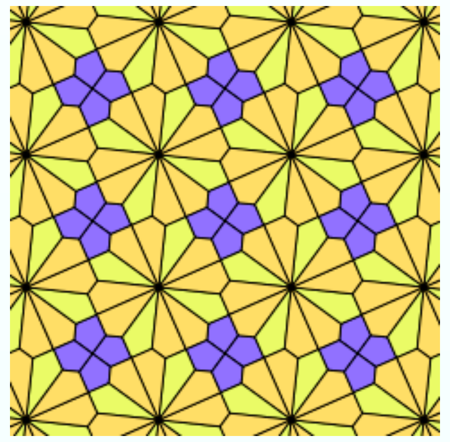
- Higher-dimensional twisted tori classification lacking
- Full classification for 2 and 3-dimensional orbifolds: $\mathbb{R}^2/\Gamma$, $\mathbb{R}^3/\Gamma$
- $\mathbb{R}^2/\Gamma$: 17 wallpaper groups
- $\mathbb{R}^3/\Gamma$: 219 affine space groups as orbifolds CONWAY-FRIEDRICHS-HUSON-THURSTON
- Only 10 freely acting and they are related to wallpaper groups
  - $\Gamma \subset \mathbb{E}_2 = U(1) \ltimes \mathbb{R}^2$

WALLPAPER GROUPS AND FREELY ACTING ORBIFOLDS

- p1 (simple torus)
- pk (k=2,3,4,6) = $\mathbb{Z}^2 \rtimes \mathbb{Z}_k$

p2

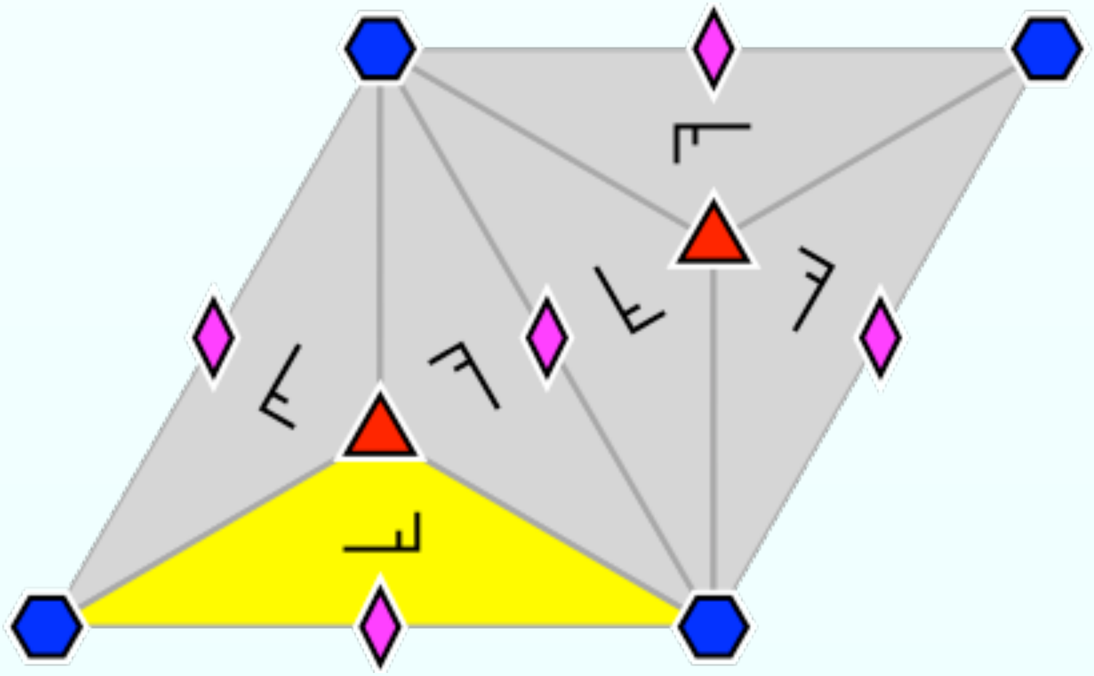
p3



p4

p6

p6 lattice example



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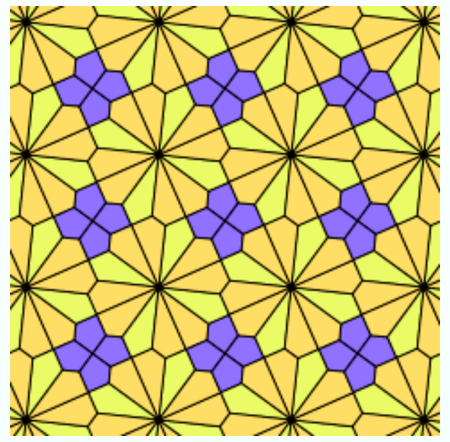
p2

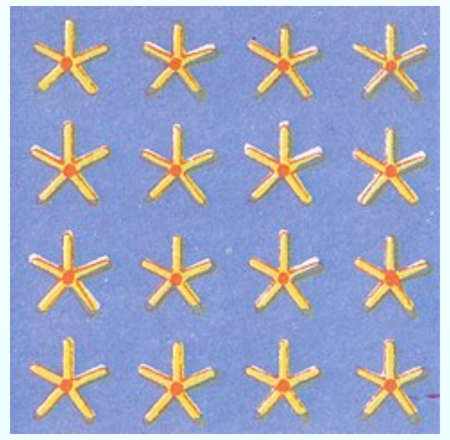
p3



p4

p6

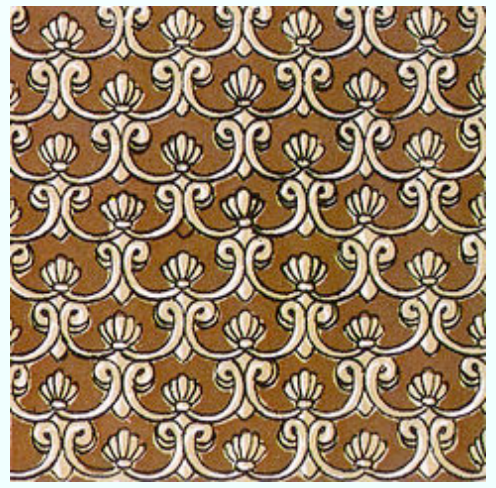
- Non-orientable tori, from wallpapers with parities



pm

pmg

pgg



cm

WALLPAPER GROUPS AND FREELY ACTING ORBIFOLDS

- Presentations (wallpapers without reflections)

$$
\begin{aligned}
\mathbf{p_1} &= \mathbb{Z}\times\mathbb{Z} = \langle a,b \mid ab=ba\rangle\,,\\
\mathbf{p_2} &= \mathbb{Z}^2\rtimes\mathbb{Z}_2 = \left\langle a,b,r \mid ab=ba, r^2=1, rar=a^{-1}, rbr=b^{-1}\right\rangle,\\
\mathbf{p_3} &= \mathbb{Z}^2\rtimes\mathbb{Z}_3 = \left\langle a,b,r \mid ab=ba, r^3=1, r^{-1}ar=a^{-1}b, r^{-1}br=a^{-1}\right\rangle,\\
&= \left\langle a,r \mid r^3=(ar)^3=(ar^2)^3=1\right\rangle,\\
\mathbf{p_4} &= \mathbb{Z}^2\rtimes\mathbb{Z}_4 = \left\langle a,b,r \mid ab=ba, r^4=1, r^{-1}ar=b, r^{-1}br=a^{-1}\right\rangle,\\
&= \left\langle a,r \mid r^4=(ar^3)^4=(ar^2)^2=1\right\rangle,\\
\mathbf{p_6} &= \mathbb{Z}^2\rtimes\mathbb{Z}_6 = \left\langle a,b,r \mid ab=ba, r^6=1, r^{-1}ar=b, r^{-1}br=a^{-1}b\right\rangle,\\
&= \left\langle a,r \mid r^6=(ar^3)^1=(ar^4)^3=1\right\rangle,
\end{aligned}
$$

WALLPAPER GROUPS AND FREELY ACTING ORBIFOLDS

- Presentations (orientable freely acting orbifolds from wallpaper on circle):

$$O_1^3 \qquad \text{(flat torus)} : \Gamma = \mathbb{Z}^3$$

$$O_2^3 : \Gamma = \mathbb{Z}^2 \rtimes \mathbb{Z} = \langle a, b, c, r \,|\, r^2 = c, rar^{-1} = a^{-1}, rbr^{-1} = b^{-1} \rangle$$

$$O_3^3 : \Gamma = \mathbb{Z}^2 \rtimes \mathbb{Z} = \langle a, b, c, r \,|\, r^3 = c, rar^{-1} = b, rbr^{-1} = a^{-1}b^{-1} \rangle$$

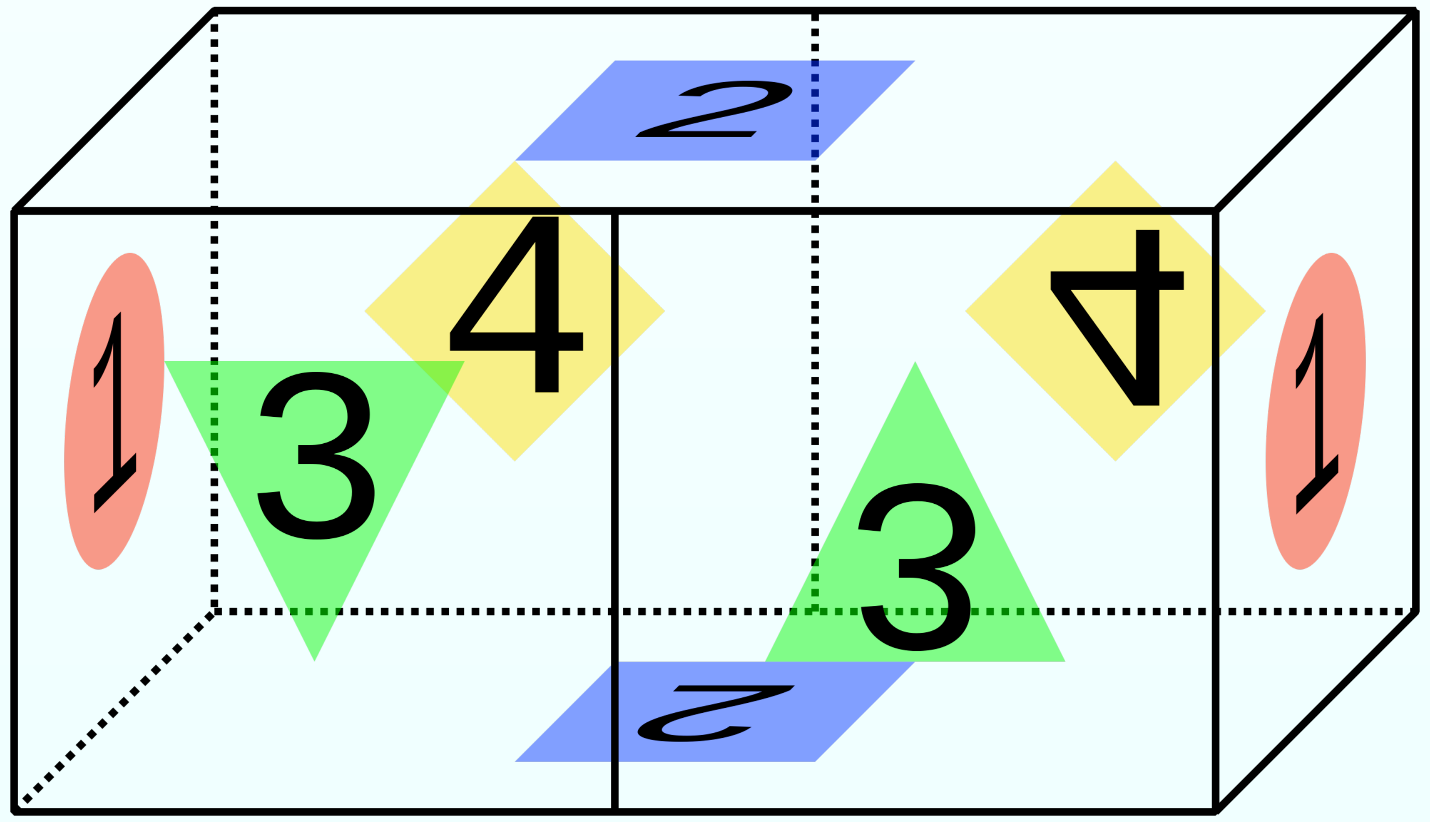
$$O_4^3 : \Gamma = \mathbb{Z}^2 \rtimes \mathbb{Z} = \langle a, b, c, r \,|\, r^4 = c, rar^{-1} = b, rbr^{-1} = a^{-1} \rangle$$

$$O_5^3 : \Gamma = \mathbb{Z}^2 \rtimes \mathbb{Z} = \langle a, b, c, r \,|\, r^6 = c, rar^{-1} = b, rbr^{-1} = a^{-1}b \rangle$$

WALLPAPER GROUPS AND FREELY ACTING ORBIFOLDS

- Presentations (Hantzsche-Wendt 3-manifold):

$$O^3_6 : \Gamma = \mathbb{Z}^2 \rtimes \mathbb{Z} = \langle a, b, c, r, \sigma \quad | \quad r^2 = c, \sigma^2 = a, \sigma r \sigma^{-1} = abr^{-1}, rar^{-1} = a^{-1}, rbr^{-1} = b^{-1}, \sigma a \sigma^{-1} = a^{-1}, \sigma c \sigma^{-1} = c^{-1} \rangle .$$

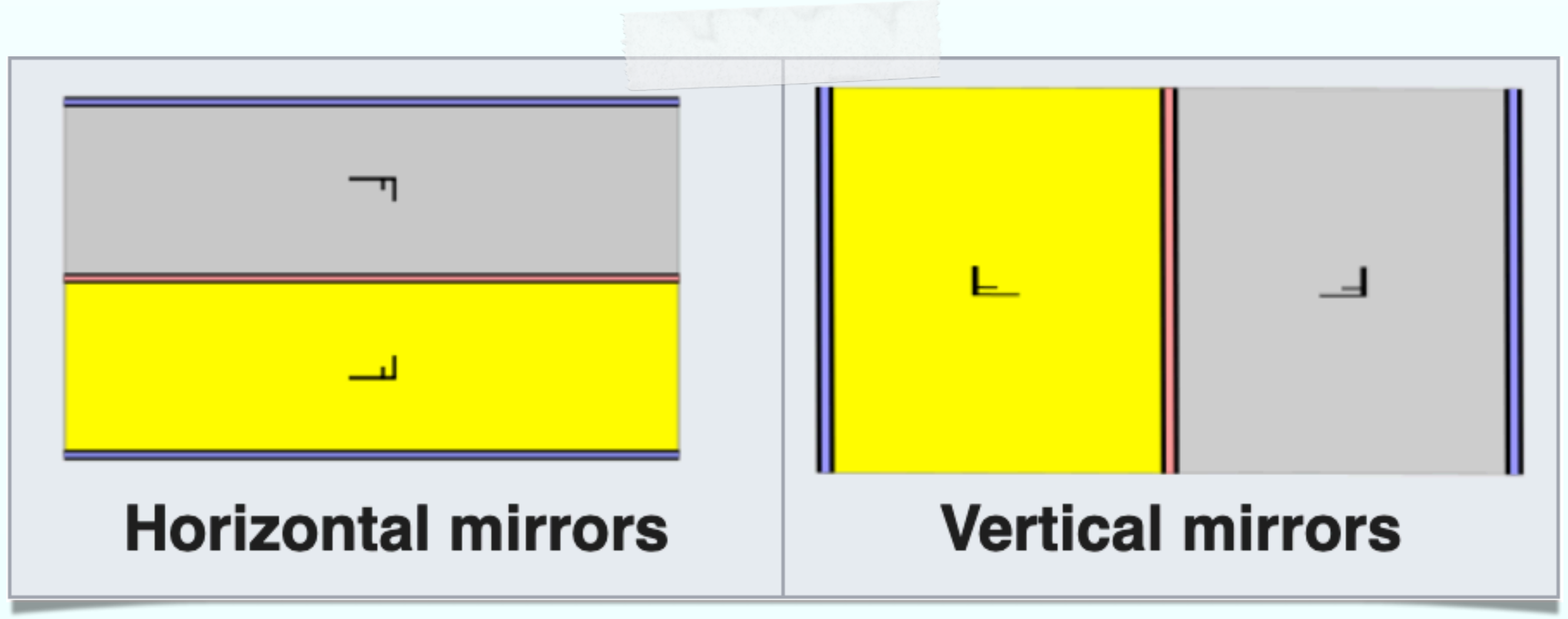


WALLPAPER GROUPS AND FREELY ACTING ORBIFOLDS

- Presentations (non-orientable 3-manifold):

$$N_1^3 : \Gamma = \mathbb{Z}^2 \rtimes \mathbb{Z} = \langle a, b, c, \sigma \,|\, \sigma^2 = c, \sigma a \sigma^{-1} = a, \sigma b \sigma^{-1} = b^{-1} \rangle$$

$$N_2^3 : \Gamma = \mathbb{Z}^2 \rtimes \mathbb{Z} = \langle a, b, c, \sigma \,|\, \sigma^2 = c, \sigma a \sigma^{-1} = a, \sigma b \sigma^{-1} = cab^{-1} \rangle$$



WALLPAPER GROUPS AND FREELY ACTING ORBIFOLDS

- Presentations (non-orientable 3-manifold):
  - Klein-bottle fibrations

$$N^3_3 : \Gamma = \mathbb{Z}^2 \rtimes \mathbb{Z} = \langle a, b, c, r, \sigma \mid r^2 = c, \sigma^2 = a, r\sigma r^{-1} = \sigma^{-1}, \\ rar^{-1} = a^{-1}, rbr^{-1} = b^{-1}, \sigma b \sigma^{-1} = b^{-1}, \sigma c \sigma^{-1} = c \rangle \, .$$

$$N^3_4 : \Gamma = \mathbb{Z}^2 \rtimes \mathbb{Z} = \langle a, b, c, r, \sigma | r^2 = c, \sigma^2 = a, r\sigma r^{-1} = \sigma^{-1} b, \\ rar^{-1} = a^{-1}, rbr^{-1} = b^{-1}, \sigma b \sigma^{-1} = b^{-1}, \sigma c \sigma^{-1} = c \rangle \, .$$

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Homogeneous

Good EFT

$$y \sim y + m$$

$$x_1 \sim \cos(qy)\, x_1 - \sin(qy)\, x_2 + n - \frac{p}{2}$$

$$x_2 \sim \sin(qy)\, x_1 + \cos(qy)\, x_2 + \sqrt{3}\frac{p}{2}$$

Non-Homogeneous

WALLPAPER GROUPS AND FREELY ACTING ORBIFOLDS

- To compute the KK spectrum we need harmonics
- Use unitary reps of $\mathbb{E}_2$ with appropriate boundary conditions?

$$T_R(L)[F(\psi)] = \exp(i\,R\,(y\cos\psi + x\sin\psi))F\left(\psi + \kappa\,z\right)$$

- The matrix elements

$$Y^R_{mn}(x,y,z) = \frac{1}{2\pi}\int d\psi\,\exp(-i\,n\,\psi)T_R(L)[\exp(i\,m\,\psi)]$$

- Are harmonic functions, but difficult construction of invariant ones. For instance on $\mathbb{T}^3$

$(-1)^{(m-n)}e^{i\left[m\,\kappa\,z + (n-m)\arctan\left(\frac{x}{y}\right)\right]}J_{n-m}\left(R\sqrt{x^2+y^2}\right)$ should go into $e^{2\pi i(mx+ny+pz)}$

WALLPAPER GROUPS AND FREELY ACTING ORBIFOLDS

- After some work, we found the proper harmonics and computed the KK spectrum
- M-theory on products of 3d twisted tori or simple generalisations of the freely acting orbifolds described before, with non-homogeneous patches
- Preliminary results keep showing:
    - Supertrace cancellations at fixed level
    - Negative Casimir-energy contribution ⇒ unstable non-susy AdS

SUMMARY

- Minkowski vacua of fully broken sugra theories have a very interesting moduli space (generalised SS)
- *1-loop finite, but in the swampland?*
- Check supergravity reductions on twisted tori
  - EFT vs consistent truncations
- ***Casimir from KK still negative***
- Full string theory analysis?
- Brane/orbifold necessary?