Geometry, Strings and the Swampland Program
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Higher Symmetries in (Non-)Compact String Models

Mirjam Cvetič

Penn
UNIVERSITY of PENNSYLVANIA

CAMTP
University of Maribor

Univerza v Ljubljani
Fakulteta za matematiko in fiziko

Motivation

M/string theory on singular special holonomy spaces X:

- Non-compact spaces X →
  Geometric engineering of supersymmetric quantum field theories (SQFTs) by studying String Theory on X

- Build dictionary:{operators, symmetries} ⬌ {geometry,topology}
- Focus: higher-form global symmetries ⬌ topology
  (associated with ``flavor'' branes)

- Compact spaces X →
  Quantum field theory (QFT) w/ gravity →
  Higher-form symmetries gauged or broken
  Physical consistency conditions → swampland program

Higher-form symmetries in (S)QFT - active field of research

[Gaiotto, Kapustin, Seiberg, Willet, 2014],…

Higher-form symmetries & geometric engineering

[Del Zotto, Heckman, Park, Rudelius, 2015],…
[Morrison, Schäfer-Nameki, Willett, 2020],
[Albertini, Del Zotto, Garcia Etxebarria, Hosseini, 2020],…
[M.C., Dierigl, Lin, Zhang, 2020],..
[Apruzzi,Bhardwaj, Oh, Schäfer-Nameki, 2021],…
[M.C., Dierigl, Lin, Zhang, 2021],…
[M.C., Heckman, Hübner, Torres, 2203.10102],
[Del Zotto, Garcia Etxebarria, Schäfer-Nameki, 2022],…
[Hübner, Morrison, Schäfer-Nameki,Wang, 2022],…
[Heckman, Hübner, Torres, Zhang, 2023],…
[M.C.,Heckman, Hübner, Torres, Zhang, 2023],..

Higher-form symmetries & and compact geometry

[M.C., Dierigl, Lin, Zhang, 2020, 2021,2022],…
[M.C., Heckman, Hübner, Torres, 2307.1023], & work in progress
[Gould, Lin, Sabag, 2312.02131],…

Goals

- Identify geometric origin of higher-form symmetries (0-form, 1-form & 2-group) for M-/string theory on non-compact & compact special holonomy [Calabi-Yau (CY)] spaces

  Punchline:

  - Higher-form symmetries via cutting & gluing of singular boundary of non-compact X^{loc}
  - Study the fate of higher-form symmetries (gauged or broken) via gluing of X^{loc} to compact X

- Examples: M-theory on (non-)compact
  - Calabi-Yau n-folds (n=2,3)
  - Focus: Toroidal Orbifolds
  - For elliptically fibered CY n-folds, dual to F-theory, confront results against those, obtained in F-theory via resolutions & arithmetic properties of elliptic curves

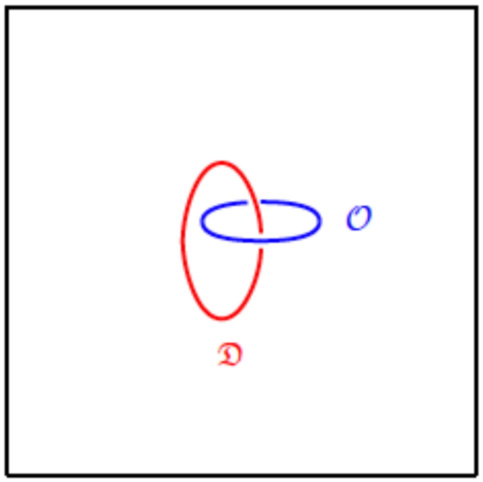
Outline

- Defect & Symmetry operators via geom. engineering
- Defect ops. and higher-form symmetries → Topology of flavor symmetry group
- Compact examples → fate of higher-form symmetries
- Outlook & Concluding remarks

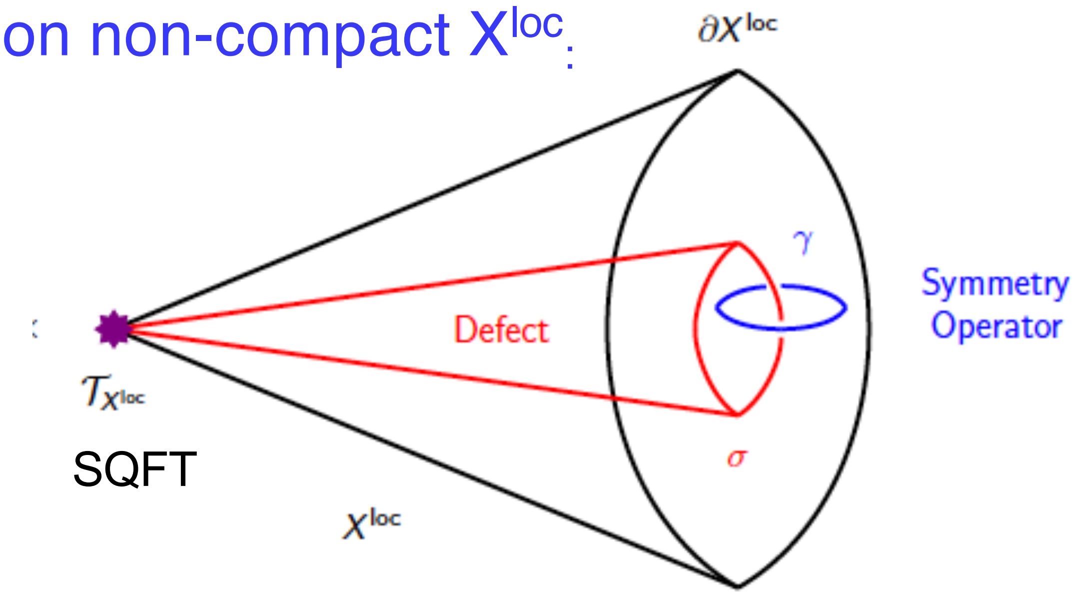
Defect $\mathfrak{D}$ & Symmetry $\mathcal{O}$ operators in SQFT

In String Theory: defect & symmetry operators constructed from branes wrapping (non-)compact cycles on non-compact X^{loc}:

Schematically



Space-time



[Morrison, Schäfer-Nameki, Willett, 2020], [Albertini, Del Zotto, Garcia Etxebarria, Hosseini, 2020]

Defect operators from branes wrapped on relative homology quotients:

$$\mathbb{D}^{(k)} \cong \frac{H_k(X^{\text{loc}}, \partial X^{\text{loc}})}{H_k(X^{\text{loc}})} \cong H_{k-1}(\partial X^{\text{loc}})|_{\text{triv.}} \qquad (\text{ cone } \sigma)$$

Symmetry operators from (flux-)branes wrapped on asymptotic cycles:

$$\mathbb{O}^{(\ell)} \cong H_\ell(\partial X^{\text{loc}}) \qquad (\text{cycle on the boundary } \gamma)$$

1. Two cases for symmetry operators from branes:

- γ is torsional → wrap membrane - focus
- γ is free → wrap ``fluxbrane” [M.C., Heckman, Hübner, Torres, 2023]

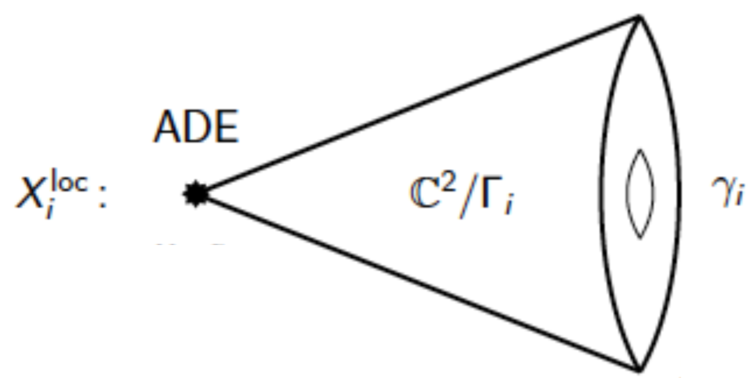
2. Examples in M-theory: Defect operators are extended p-dim ops. associated with M2 and M5 branes wrapping relative cycles in X:

$$\mathcal{D}_p^{\mathrm{M2}} = \frac{H_{3-p}(X, \partial X)}{H_{3-p}(X)} \cong H_{3-p-1}(\partial X)|_{\mathrm{triv}}$$

$$\mathcal{D}_p^{\mathrm{M5}} = \frac{H_{6-p}(X, \partial X)}{H_{6-p}(X)} \cong H_{6-p-1}(\partial X)|_{\mathrm{triv}}$$

Focus on (Wilson) lines:
p=1 electric operators of M2

3. Typical Examples: Singular conical K3 surfaces $\mathbb{C}^2/\Gamma_i$ & $\mathbb{C}^3/\Gamma_i$

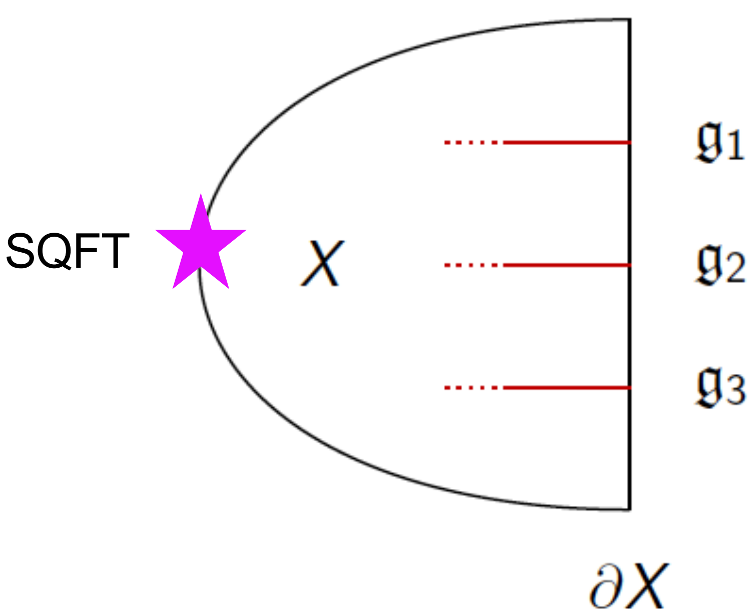


Defect Operators: M2-/M5-brane wrapped on Cone(γ_i)

II. Geometrizing Topology of Flavor Group

Non-compact ADE loci ≡ flavor branes → flavor symmetries

Naïve flavor symmetry $\widetilde{G}_F$ (simply connected w/ Lie Algebra $\mathfrak{g}_i$)



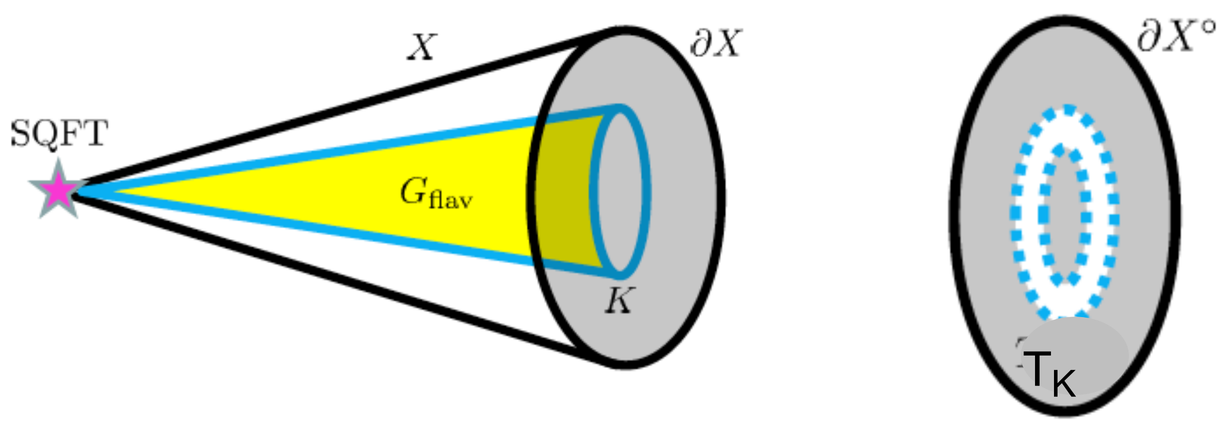
[From now on, only ADE's in ∂X]

$$\widetilde{G}_F = \widetilde{G}_1 \times \widetilde{G}_2 \times \widetilde{G}_3 \times \dots$$

(Flavor Wilson) lines → fix topology of favor symmetry G_F from singular boundary topology

III. Boundary geometry of flavor branes:

- Singular non-compact space X w/ [Example: $X \cong \mathbb{C}^3/\Gamma_i$]
  $K = \cup_i K_i$ - ADE loci (of flavor branes) in the boundary **∂**X
- Define a smooth boundary **∂**X° = **∂**X \ *K*
  & a tubular region T_K (excise K)
- Locally $T_K \cap \partial X^\circ \cong \cup_i K_i \times S^3/\Gamma_i$

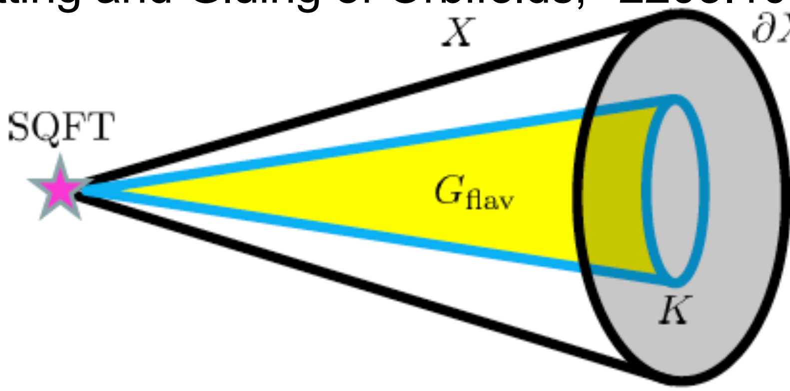


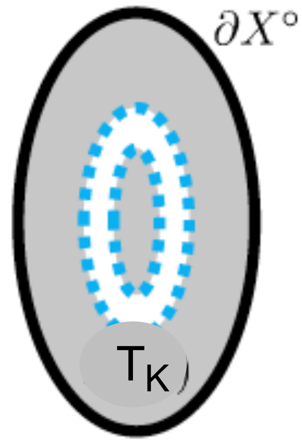
- Naïve flavor center symmetry:

$$Z_{\widetilde{G}_F} = \mathrm{Tor}\, H_1(T_K \cap \partial X^\circ) \cong Z_{\widetilde{G}_1} \oplus Z_{\widetilde{G}_2} \oplus Z_{\widetilde{G}_3} \oplus \ldots$$

Boundary geometry of true flavor center symmetry Z_{G_F}

M. C., J. J. Heckman, M. Hübner and E. Torres: "0-Form, 1-Form and 2-Group Symmetries via Cutting and Gluing of Orbifolds," 2203.10102





[Mayer, 1929], [Vietoris, 1930]

- Key: Mayer-Vietoris sequence in homology for singular boundary

$$\partial X = \partial X^\circ \cup T_K$$

true flavor center naïve flavor center

$$Z_{\text{0-form}}: \quad 0 \to \ker(\iota_1) \to H_1(\partial X^\circ \cap T_K) \xrightarrow{\iota_1} \frac{H_1(\partial X^\circ \cap T_K)}{\ker(\iota_1)} \to 0\,,$$

$$\text{1-form:} \quad 0 \to \frac{H_1(\partial X^\circ \cap T_K)}{\ker(\iota_1)} \to \underbrace{H_1(\partial X^\circ) \oplus H_1(T_K)}_{\text{naïve 1-form}} \to \underbrace{H_1(\partial X)}_{\text{true 1-form}} \to 0$$

$$Z_{G_F} = \operatorname{Ker}\left(\iota_1 : Z_{\widetilde{G}_F} \cong H_1(\partial X^\circ \cap T_K) \to H_1(\partial X^\circ) \oplus H_1(T_K)\right)$$

When the bottom sequence does not split → 0-form and 1-form intertwined: 2-group!

[Benini, Cordova, Hsin, 2019], [Lee, Ohmori, Tachikawa, 2021], ...

Examples: noncompact orbifolds, elliptically fibered CY's, G_2, etc.

IV. Compact Models

- Compact singular space X → theory that includes quantum gravity & global symmetries gauged or broken
- What is M-theory gauge group?
- For elliptically fibered geometries via M/F-theory duality:

- Non-Abelian group algebras – ADE Kodaira classification

group topology → Mordell-Weil torsion

[Aspinwall, Morrison, 1998], [Mayrhofer, Morrison,Till, Weigand, 2014], [M.C., Lin, 2017]

- Abelian groups → Mordell-Weil ``free'' part

[Morrison, Park 2012], [M.C., Klevers, Piragua, 2013], [Borchmann, Mayrhofer, Palti, Weigand, 2013]…

- Total group topology → Shoida map of Mordell-Weil

[M.C., Lin, 2017]

$$\frac{U(1)^r \times G_{\text{non-ab}}}{\prod_{i=1}^{r} \mathbb{Z}_{m_i} \times \prod_{j=1}^{t} \mathbb{Z}_{k_j}}$$

How to relate these results, encoded in resolution & arithmetic structure of elliptic curve, due Mordell-Weil, to higher-form symmetries and singular geometry?

Some progress already on the fate of higher-form symmetries for compact models:

- Global symmetries, including higher-form ones should be gauged or broken [No Global Symmetry Hypothesis]

…[Harlow, Ooguri 2018]

- In 8D N =1 Supergravity:
  quantified conditions under which no anomalies due to gauged 1-form symmetry
  [magnetic version – preferred polarization!]

[M.C., Diriegl, Ling, Zhang, 2020]

- True for all 8D N=1 string compactifications (beyond F-theory) via (refined) string junction construction

[M.C., Diriegl, Ling, Zhang, 2022]
& study of ``frozen-singularities’’
[M.C., Diriegl, Ling, Torres, Zhang, 2024]

c.f, [Font, Fraiman, Graña, Nuñez, de Freitas, 2021]… from heterotic side

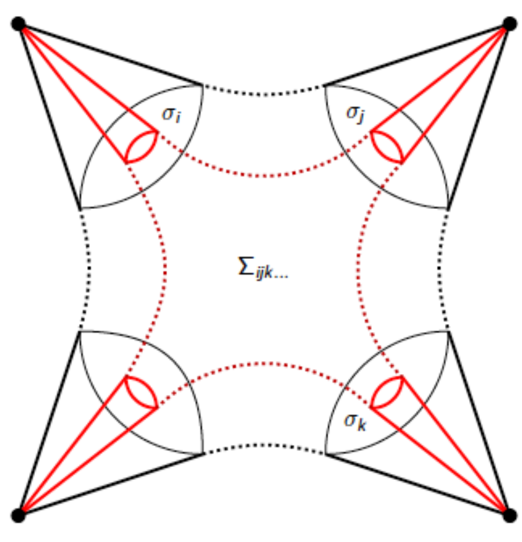
Fate of higher symmetries via cutting and gluing

M. C., Heckman, Hübner, Torres,
Generalized Symmetries, Gravity, and the Swampland, 2307.1023

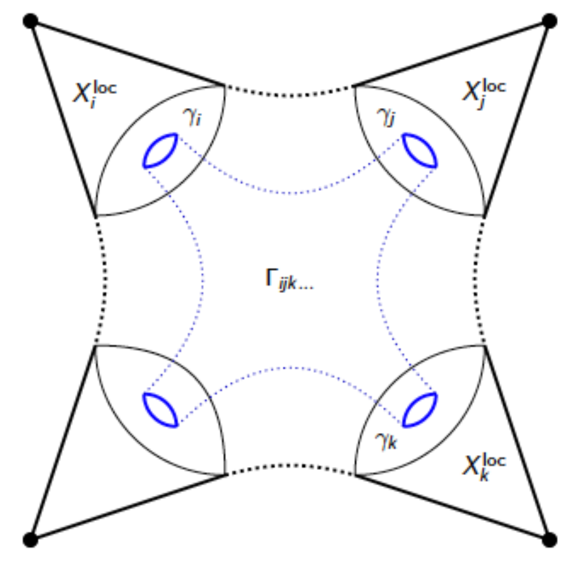
- Glue local $U_i X_i^{loc}$ of local models X_i^{loc} into X
  [Physics: Couple $\{SQFT_i\}$ to a resulting QFT & include gravity]

- Gluing Schematically:

Σ_{ijk} – two-chain
glues defects



Defect Operators
(some compactified [massive matter])



Symmetry Operators
(some identified → symmetry trivialized)

- Some relative cycles in X_i^{loc} survive & compactify
  [Physics: some defects in $SQFT_i$ become dynamical - gauged]
  → determine the gauge group in compact models

Fate of higher symmetries in compact geometries (continued):

- Quantify: Mayer-Vietoris long exact sequence

$$\dots \xrightarrow{\jmath_n} H_n(X) \xrightarrow{\partial_n} H_{n-1}(\partial X^{\mathrm{loc}}) \xrightarrow{\imath_{n-1}} H_{n-1}(X^\circ) \oplus H_{n-1}(X^{\mathrm{loc}}) \to \dots$$

w/ respect to the covering: $X = X^{\mathrm{loc}} \cup X^\circ\,, \qquad X^\circ = X \setminus X^{\mathrm{loc}}$

w/ total local model: $X^{\mathrm{loc}} = \coprod_i X_i^{\mathrm{loc}}$

- Example of K3 Surfaces → Mayer-Vietoris exact (sub-)sequence

$$\begin{array}{ccccccccccc} & & \text{Extra } U(1)\text{'s} & & \text{Massive Matter} & & \text{Local Model Defect/Symmery Ops} & & \text{Emergent/Broken Symmetries} & & \\ & & \vdots & & \vdots & & \vdots & & \vdots & & \\ 0 & \to & H_2(X^\circ) & \to & H_2(X) & \to & H_1(\partial X^{\mathrm{loc}}) & \to & H_1(X^\circ) & \to & 0 \\ & & & & & \vdots & & & \cong (\mathrm{Tor}\, H_2(X))^\vee & & \\ & & & & & \text{Gauging/Trivializing Symmetries} & & & & & \end{array}$$

Decomposition of compact two-cycles into a sum of relative cycles associated with each local model ∂_2: $H_2(X) \to \oplus_i H_1(\partial X_i^{\mathrm{loc}})$

Examples: Toroidal Orbifolds

$X = T^4/\mathbb{Z}_n$ with $n = 2, 3, 4, 6$. Evaluate

$$0 \to H_2(X^\circ) \xrightarrow{\jmath_2} H_2(X) \xrightarrow{\partial_2} H_1(\partial X^{\mathrm{loc}}) \xrightarrow{\imath_1} H_1(X^\circ) \to 0$$

$$T^4/\mathbb{Z}_2 : \quad 0 \xrightarrow{\imath_2} \mathbb{Z}^6 \xrightarrow{\jmath_2} \mathbb{Z}^6 \oplus \mathbb{Z}_2^5 \xrightarrow{\partial_2} \mathbb{Z}_2^{16} \xrightarrow{\imath_1} \mathbb{Z}_2^5 \xrightarrow{\jmath_1} 0$$

$$T^4/\mathbb{Z}_3 : \quad 0 \xrightarrow{\imath_2} \mathbb{Z}^4 \xrightarrow{\jmath_2} \mathbb{Z}^4 \oplus \mathbb{Z}_3^3 \xrightarrow{\partial_2} \mathbb{Z}_3^9 \xrightarrow{\imath_1} \mathbb{Z}_3^3 \xrightarrow{\jmath_1} 0$$

$$T^4/\mathbb{Z}_4 : \quad 0 \xrightarrow{\imath_2} \mathbb{Z}^4 \xrightarrow{\jmath_2} \mathbb{Z}^4 \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_2^2 \xrightarrow{\partial_2} \mathbb{Z}_4^4 \oplus \mathbb{Z}_2^6 \xrightarrow{\imath_1} \mathbb{Z}_4 \oplus \mathbb{Z}_2^2 \xrightarrow{\jmath_1} 0$$

$$T^4/\mathbb{Z}_6 : \quad 0 \xrightarrow{\imath_2} \mathbb{Z}^4 \xrightarrow{\jmath_2} \mathbb{Z}^4 \oplus \mathbb{Z}_6 \xrightarrow{\partial_2} \mathbb{Z}_6 \oplus \mathbb{Z}_3^4 \oplus \mathbb{Z}_2^5 \xrightarrow{\imath_1} \mathbb{Z}_6 \xrightarrow{\jmath_1} 0$$

Computations built on: [Spanier, 1956], [Nikulin, 1975], [Shioda, Inose, 1977], [Nahm, Wendland, 2001], [Wendland, 2002]

Also examples of non-abelian quotients of tori

$$0 \to H_2(X^\circ) \xrightarrow{\jmath_2} H_2(X) \xrightarrow{\partial_2} H_1(\partial X^{\mathrm{loc}}) \xrightarrow{\imath_1} H_1(X^\circ) \to 0$$

determines the gauge group G: $G = G_{ADE} \times U(1)^{b_2} / Im\ \partial 2$

$$T^4/\mathbb{Z}_2 : \qquad G = \frac{\left(SU(2)^{16}/\mathbb{Z}_2^5\right) \times U(1)^6}{\mathbb{Z}_2^6}$$

$$T^4/\mathbb{Z}_3 : \qquad G = \frac{\left(SU(3)^{9}/\mathbb{Z}_3^3\right) \times U(1)^4}{\mathbb{Z}_3^3}$$

$$T^4/\mathbb{Z}_4 : \qquad G = \frac{\left(SU(4)^{4}/\mathbb{Z}_4 \times \mathbb{Z}_2^2\right) \times SU(2)^6 \times U(1)^4}{\mathbb{Z}_4^2 \times \mathbb{Z}_2^2}$$

$$T^4/\mathbb{Z}_6 : \qquad G = \frac{\left([SU(6) \times SU(3)^4 \times SU(2)^5]/\mathbb{Z}_3 \times \mathbb{Z}_2\right) \times U(1)^4}{\mathbb{Z}_6^3 \times \mathbb{Z}_2}$$

b_2-second Betti number

Calabi-Yau Threefold Example: $T^6/\mathbb{Z}_3$

Local Models: $27 \times \mathbb{C}^3/\mathbb{Z}_3$

Cutting and gluing gives two exact sequences:

$$0 \to H_4(X^\circ) \xrightarrow{\jmath_4} H_4(X) \xrightarrow{\partial_4} H_3(\partial X^{\mathrm{loc}}) \xrightarrow{\imath_3} \mathit{Tor}\, H_3(X^\circ) \to 0$$

$$0 \to \mathbb{Z}^9 \xrightarrow{\jmath_4} \mathbb{Z}^9 \oplus \mathbb{Z}_3^4 \xrightarrow{\partial_4} \mathbb{Z}_3^{27} \xrightarrow{\imath_3} \mathbb{Z}_3^{17} \to 0$$

$$0 \to H_2(X^\circ) \xrightarrow{\jmath_2} H_2(X) \xrightarrow{\partial_2} H_1(\partial X^{\mathrm{loc}}) \xrightarrow{\imath_1} \mathrm{Tor}\, H_1(X^\circ) \to 0$$

$$0 \to \mathbb{Z}^9 \xrightarrow{\jmath_2} \mathbb{Z}^9 \oplus \mathbb{Z}_3^{17} \xrightarrow{\partial_2} \mathbb{Z}_3^{27} \xrightarrow{\imath_1} \mathbb{Z}_3^4 \to 0\,.$$

No global symmetries?

Further Analysis: global symmetry either broken or gauged!

Starting Point: electric frame $\rightarrow \mathbb{Z}_3^{27}$ 1-form symmetry group

Defect operators and Symmetry operators (in local geometry)

$$0 \rightarrow H_4(X^\circ) \xrightarrow{\jmath_4} H_4(X) \xrightarrow{\partial_4} H_3(\partial X^{\mathrm{loc}}) \xrightarrow{\imath_3} \mathit{Tor}\, H_3(X^\circ) \rightarrow 0$$

$$0 \rightarrow \mathbb{Z}^9 \xrightarrow{\jmath_4} \mathbb{Z}^9 \oplus \mathbb{Z}_3^4 \xrightarrow{\partial_4} \mathbb{Z}_3^{27} \xrightarrow{\imath_3} \mathbb{Z}_3^{17} \rightarrow 0$$

$$0 \rightarrow H_2(X^\circ) \xrightarrow{\jmath_2} H_2(X) \xrightarrow{\partial_2} H_1(\partial X^{\mathrm{loc}}) \xrightarrow{\imath_1} \mathrm{Tor}\, H_1(X^\circ) \rightarrow 0$$

$$0 \rightarrow \mathbb{Z}^9 \xrightarrow{\jmath_2} \mathbb{Z}^9 \oplus \mathbb{Z}_3^{17} \xrightarrow{\partial_2} \mathbb{Z}_3^{27} \xrightarrow{\imath_1} \mathbb{Z}_3^4 \rightarrow 0\,.$$

Further Analysis: global symmetry either broken or gauged!

Consider purely electric frame: $\mathbb{Z}_3^{27}$ 1-form symmetry group

Defect operators and Symmetry operators

Add matter and break symmetries

Gauge/trivialize symmetries

$$0 \to H_4(X^\circ) \xrightarrow{\jmath_4} H_4(X) \xrightarrow{\partial_4} H_3(\partial X^{\mathrm{loc}}) \xrightarrow{\imath_3} \mathrm{Tor}\, H_3(X^\circ) \to 0$$

$$0 \to \mathbb{Z}^9 \xrightarrow{\jmath_4} \mathbb{Z}^9 \oplus \mathbb{Z}_3^4 \xrightarrow{\partial_4} \mathbb{Z}_3^{27} \xrightarrow{\imath_3} \mathbb{Z}_3^{17} \to 0$$

$$0 \to H_2(X^\circ) \xrightarrow{\jmath_2} H_2(X) \xrightarrow{\partial_2} H_1(\partial X^{\mathrm{loc}}) \xrightarrow{\imath_1} \mathrm{Tor}\, H_1(X^\circ) \to 0$$

$$0 \to \mathbb{Z}^9 \xrightarrow{\jmath_2} \mathbb{Z}^9 \oplus \mathbb{Z}_3^{17} \xrightarrow{\partial_2} \mathbb{Z}_3^{27} \xrightarrow{\imath_1} \mathbb{Z}_3^4 \to 0\,.$$

Resulting gauge group: $G = \mathbb{Z}_3^{17} \times U(1)^9$

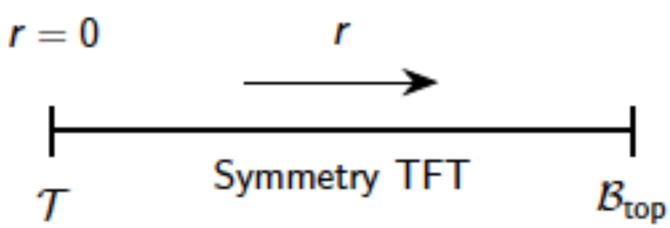
- Study of elliptically fibered CY two- and three-folds: complementary results to those obtained in F-theory (arithmetic structure of elliptic curves) c.f., also J. Heckman's talk

V. Outlook & Work in Progress

Novel developments: Symmetry TFTs

Given a QFT we can often isolate its symmetry structures into a topological field theory (TFT) → Introduction of the so-called symmetry TFT [Freed, Teleman, Moore, 2022]

Present the QFT as:

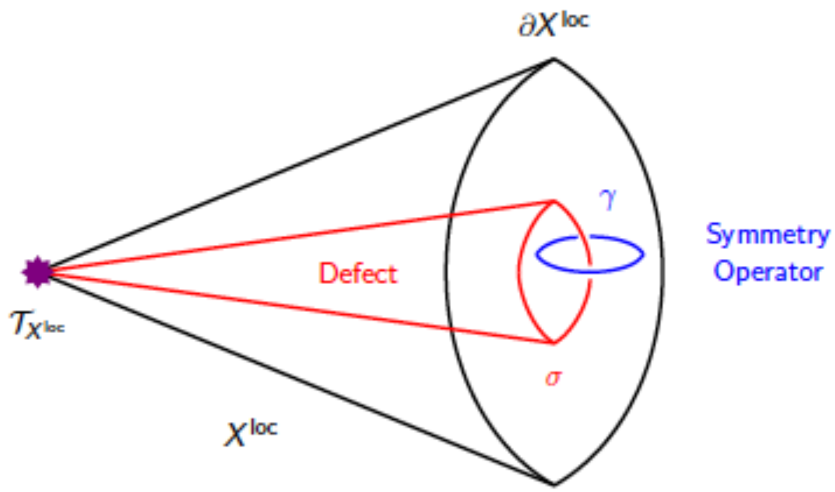


indicating physical boundary conditions (b.c.) at $\mathcal{T}$ and topological b.c. at $\mathcal{B}_{top}$ for the symmetry TFT.

Many symmetry structures depend only on $\mathcal{B}_{top}$ and the TFT.

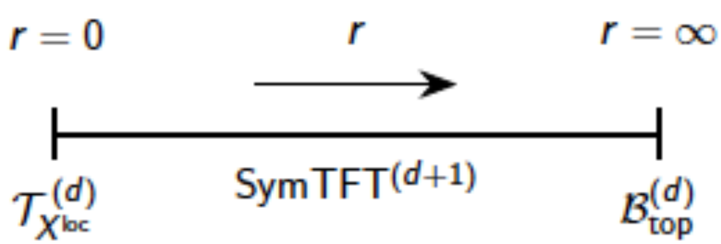
Symmetry TFTs from String Theory (Geometric Engineering)

Consider the QFT $\mathcal{T}_{X^{loc}}$ with string theory construction on conical X^{loc}



Its symmetry TFT follows via compactification over radial slices:

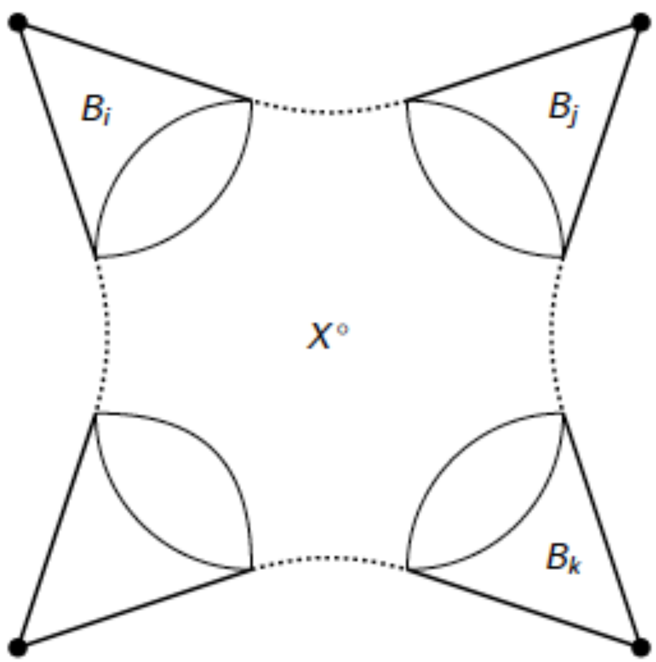
[Apruzzi, Bonetti, Garcia Etxebarria, Hosseini, Schäfer-Nameki, 2021]

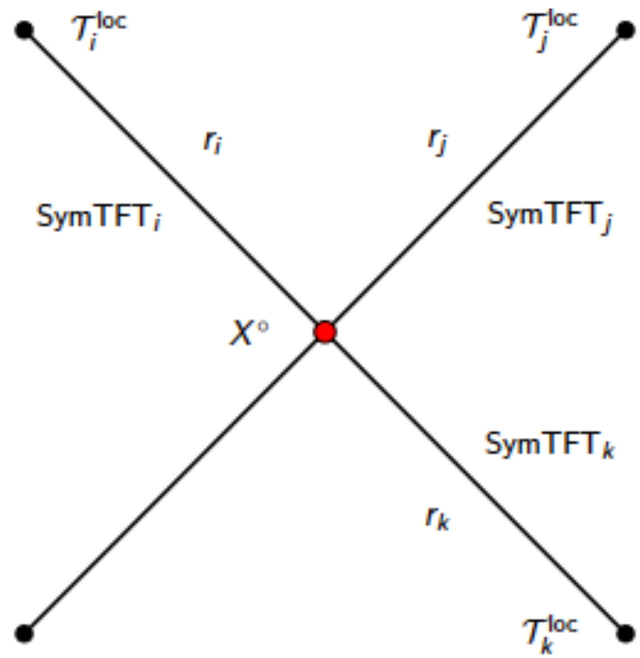


Note: Branes support worldvolume TFT w/ Non-invertible Fusion Rules

Compactification: SymTFTs + Junctions = SymTrees

Gluing these local constructions one can assign a junction of symmetry TFTs to the field theory sector of M-theory on compact K3 surface.





Physical boundary conditions for $\mathcal{T}_i^{\mathrm{loc}}$ at $r_i = 0$.
At the central node; partially topological and partially physical b.c.

[M.C., Heckman, Hübner,Torres, 2023],
[Baume, Heckman,Hübner, Torres, Turner, Yu, 2023],
[M.C., Donagi, Heckman, Hübner, Torres, work in progress]

Summary

Question addressed in this talk:
Given a QFT $\mathcal{T}_X$ engineered by a geometry X in string theory with categorical symmetries S[$\mathcal{T}_X$] $\equiv \mathcal{S}_X$,
What is the mapping of geometry/topology of X in $\mathcal{S}_X$?

Strong Activity:
[Del Zotto, Heckman, Park, Rudelius, 2015],[Morrison, Schäfer-Nameki, Willet, 2020], [Albertini, Del Zotto, Garcıa Etxebarria, Hosseini, 2020], [Apruzzi, Bonetti, Garcia Etxebarria, Hosseini, Schafer-Nameki, 2021], [Garcıa Etxebarria, 2022], [Apruzzi, Bah, Bonetti, Schäfer-Nameki, 2022], [Heckman, Hübner, Torres, Zhang, 2022], [M.C., Heckman, Hübner, Torres, 2022], [Acharya, Del Zotto, Heckman, Hübner, Torres, 2023], [M.C., Heckman, Hübner, Torres, 2023],[Apruzzi, Bonetti, Gould, Schäfer-Nameki, 2023], [Bah, Leung, Waddleton, 2023], [Gould, Lin, Sabag, 2023], [Bonetti, Del Zotto, Minasian, 2024], [Del Zotto, Meynet,Moscrop, 2024],...

[M.C., Donagi, Heckman, Hübner, Torres, work in progress],...

Future Directions:

- Purely geometric string backgrounds relate to non-geometric backgrounds via dualities → geometric analysis is the starting point to understanding all possible string backgrounds

- Swampland Program: No-Global-Symmetries Conjecture, understand how string theory avoids global symmetries & improve on analysis presented here.

Thank you!