# ETW brane networks for Calabi-Yau moduli


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Geometry, Strings and the Swampland

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Based on: 2404.xxxxx

2312.16286 with A. Uranga and A. Makridou

2203.11240 with A. Uranga, J. Huertas, M. Delgado and J. Calderon-Infante

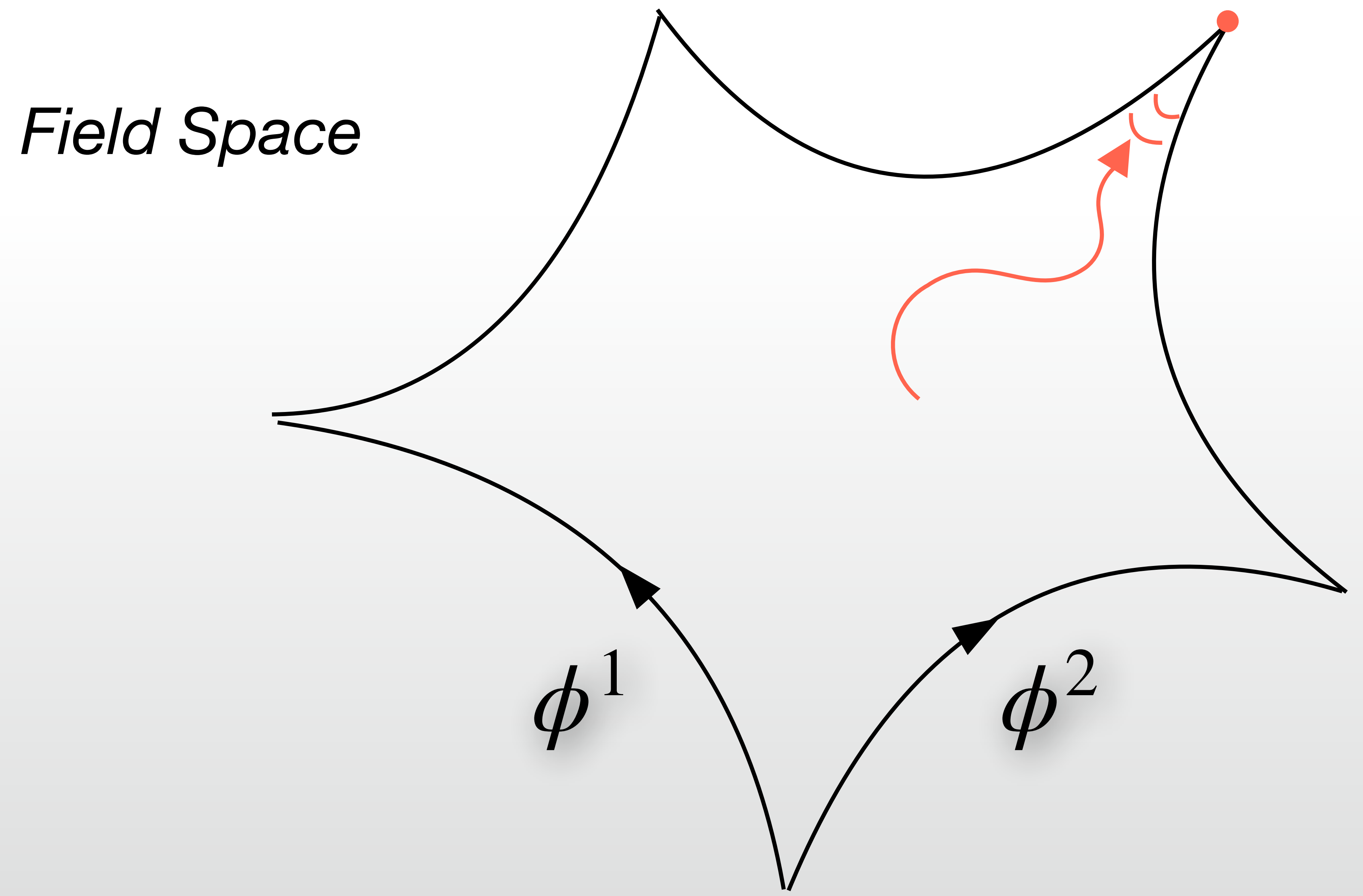
# Dynamical Cobordisms to Nothing

They are realizations of spacetime boundaries through specific solutions of the effective action:

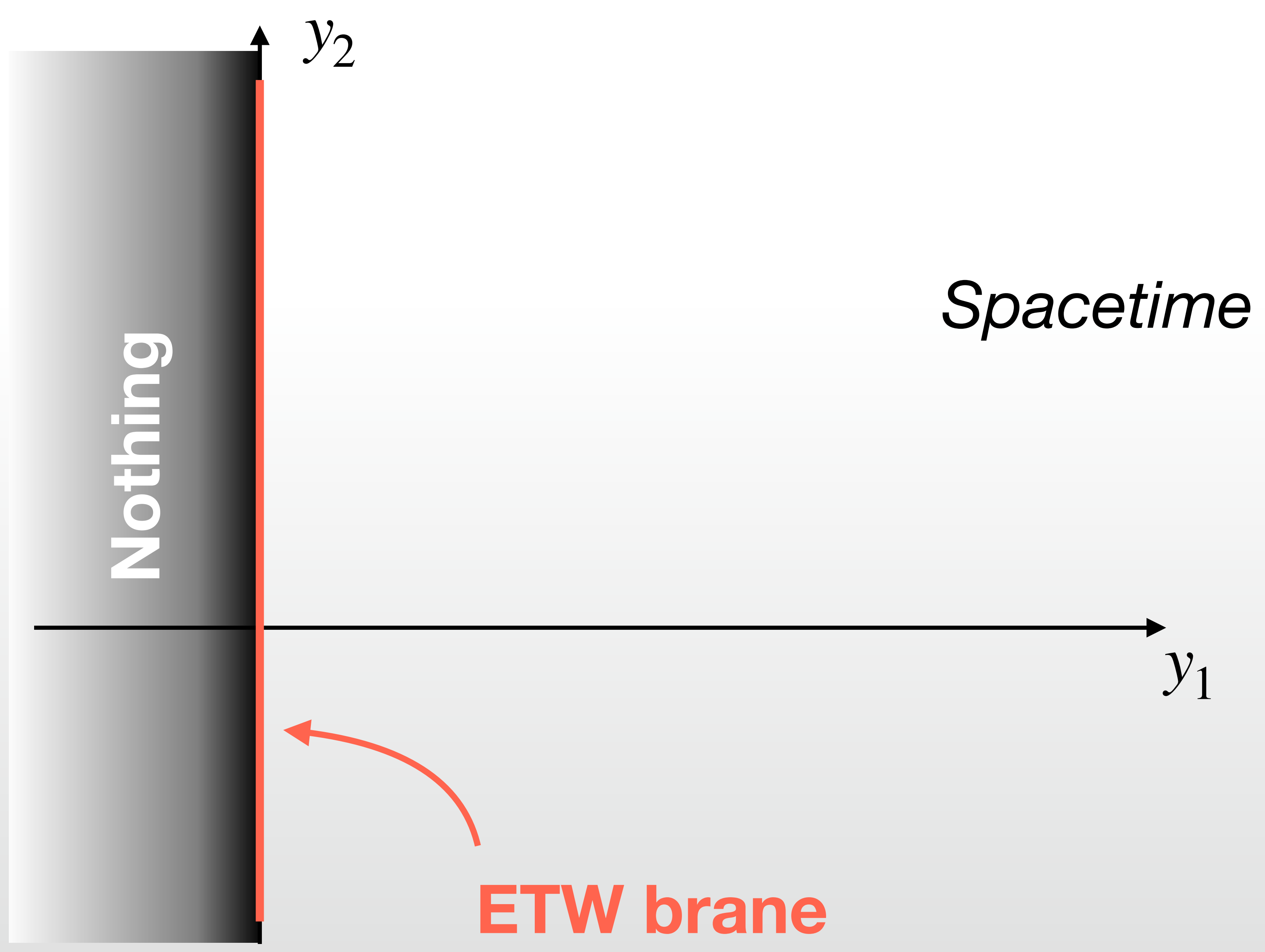
$$S = \int d^d x \sqrt{-g} \left\{ \frac{1}{2} R - \frac{1}{2} G_{ij} \partial \phi^i \partial \phi^j - V(\phi^1, \phi^2, \ldots) \right\}$$

that:

- are spacetime dependent,
- show a Ricci singularity in the metric at finite distance in spacetime,
- explore infinite distance in the field space.




realized as

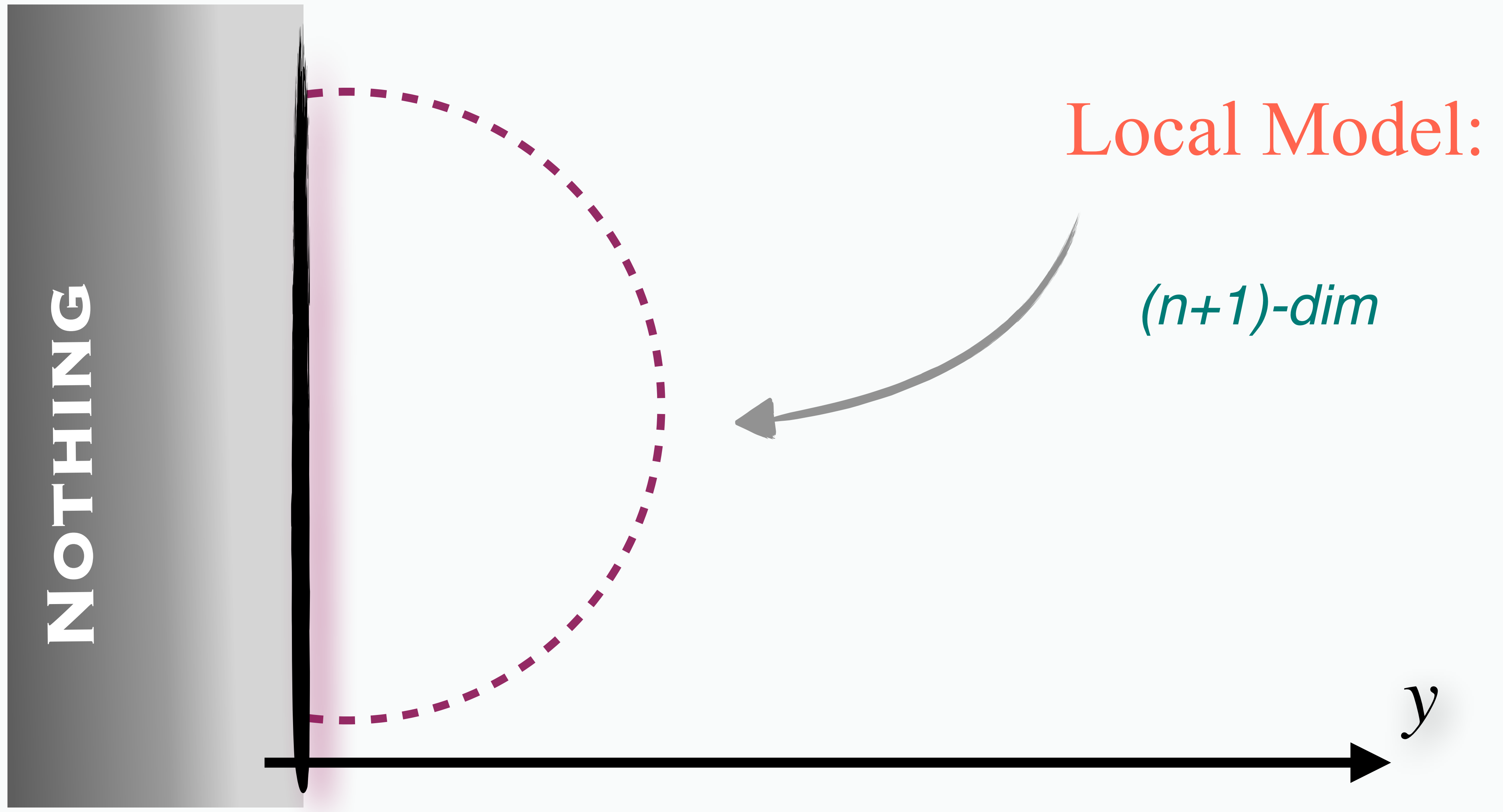


# Codimension 1 ETW

[2203.11240-R.A.,Calderon-Infante, Delgado, Huertas, Uranga]

We describe codimension 1 boundaries in $(n+1)-$*dimensional* spacetime associated to the action:

$$S = \int d^{n+1}x\sqrt{-g}\left\{\frac{1}{2}R - \frac{1}{2}\left(\partial\varphi\right)^2 - V(\varphi)\right\}$$




- $\varphi(y) \simeq -\frac{2}{\delta}\log y$
- $ds^2_{n+1} = e^{-2\sigma(y)}ds^2_n + dy^2$

with $\sigma(y) \simeq \pm\frac{4}{(n-1)\delta^2}\log y$

The class of potentials producing these solutions is: $V(\phi) \simeq -cae^{\delta\phi}$

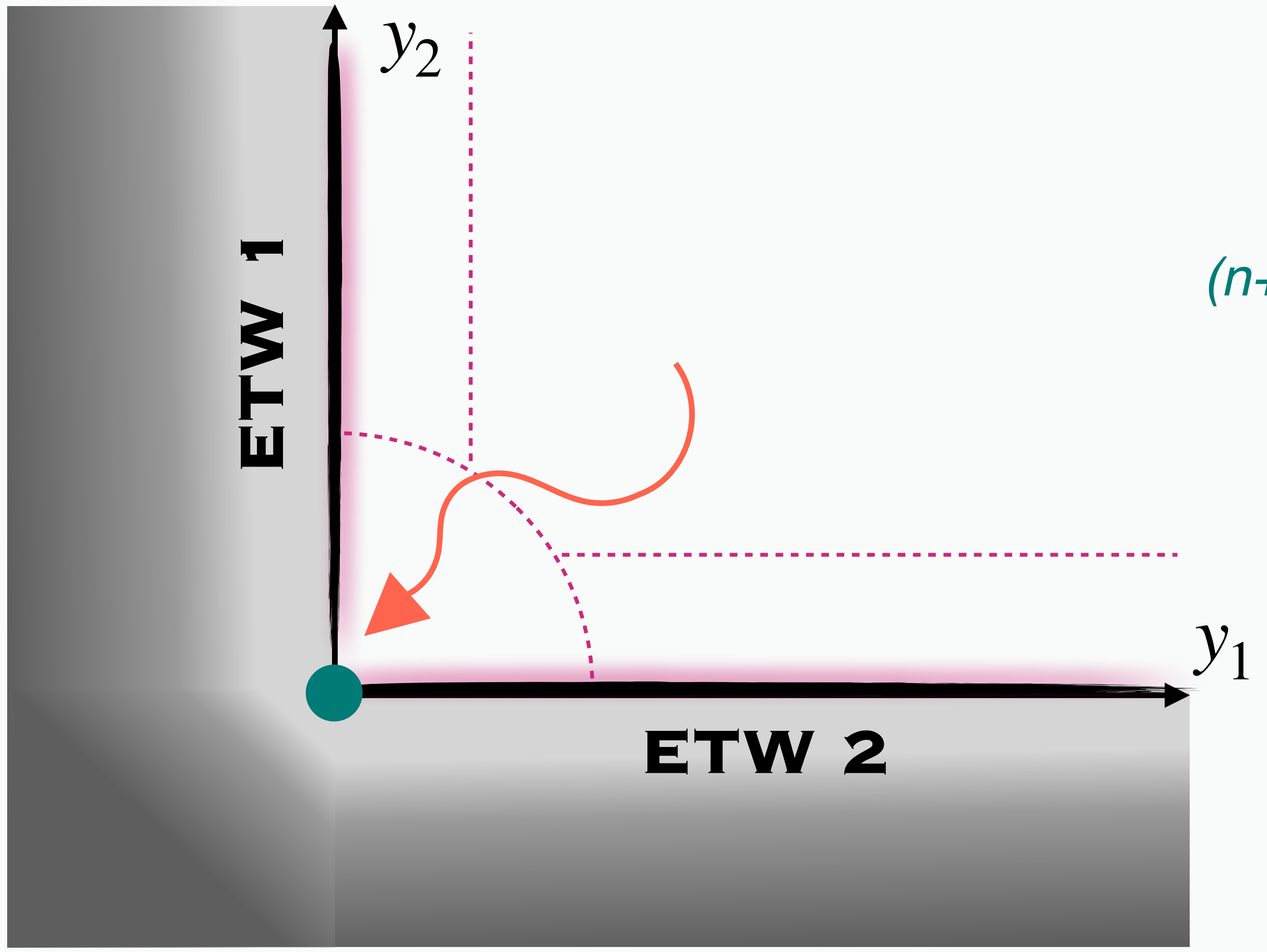
# Intersecting ETWs

[2312.16286 - R.A., Makridou, Uranga]

Andriana's talk

We describe the intersection of multiple codimension 1 boundaries in spacetime associated to the action:

$$S = \int d^{n+2}x\sqrt{-g}\left\{\frac{1}{2}R - \frac{1}{2}\left(\partial\varphi_1\right)^2 - \frac{1}{2}\left(\partial\varphi_1\right)^2 - \frac{\alpha}{2}\partial\varphi_1\partial\varphi_2 - V(\varphi_1,\varphi_2)\right\}$$




(n+2)-dim

- $\varphi_1(y_1) = -\frac{2}{\delta_1}\log y_1$
- $\varphi_2(y_2) = -\frac{2}{\delta_2}\log y_2$
- $ds^2_{n+2} = e^{-2\sigma_1(y_1)-2\sigma_2(y_2)}ds^2_n + e^{-2\sigma_2(y_2)}dy_1^2 + e^{-2\sigma_1(y_1)}dy_2^2$

with $\sigma_1(y_1) = \pm\frac{4}{n\delta_1{}^2}\log y_1$ and $\sigma_2(y_2) = \pm\frac{4}{n\delta_2{}^2}\log y_2$

The class of potentials producing these solutions is: $V(\phi) \simeq -c_1 v_1 e^{\delta_1\varphi_1}e^{a_2\delta_2\varphi_2} - c_2 v_2 e^{a_1\delta_1\varphi_1}e^{\delta_2\varphi_2}$

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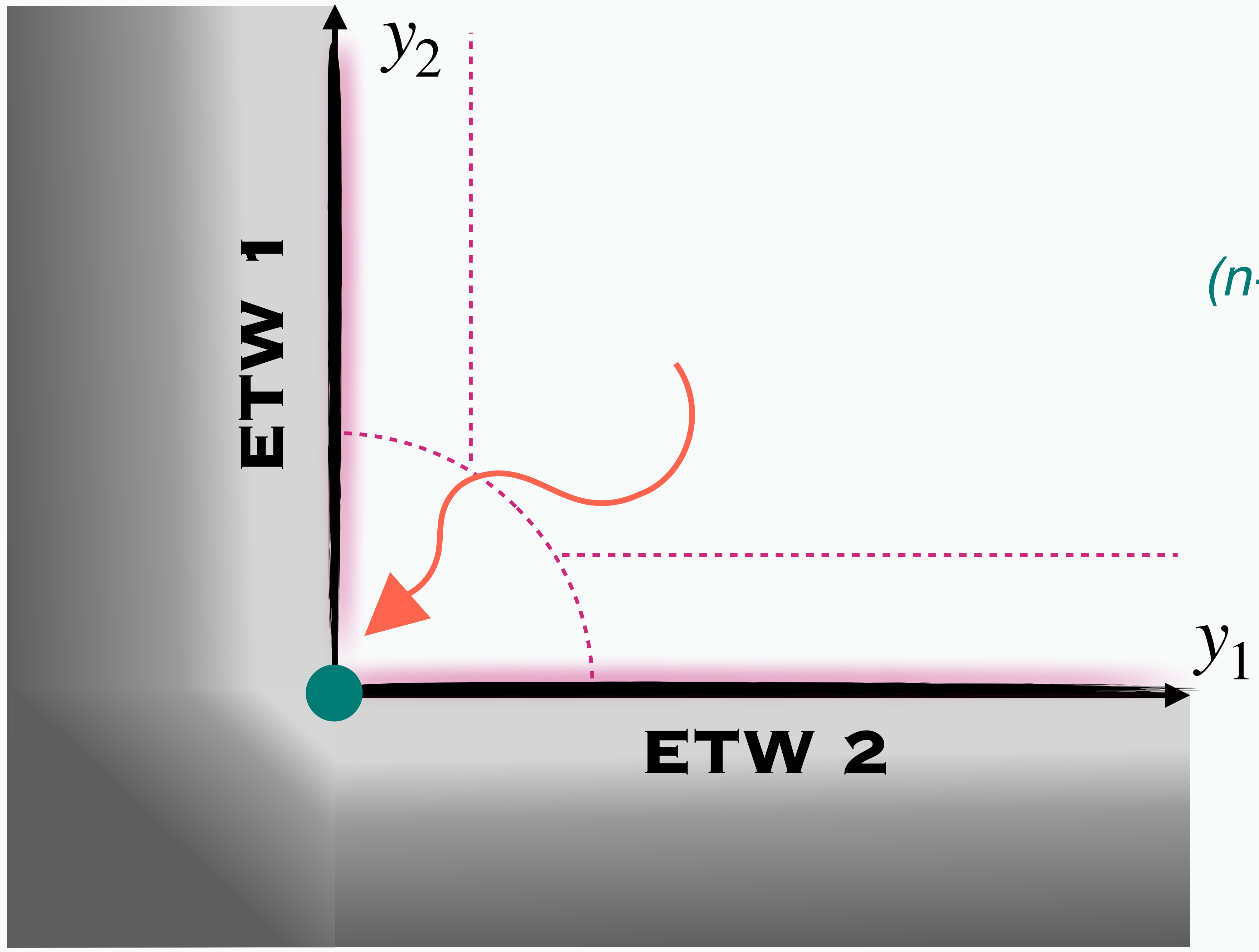


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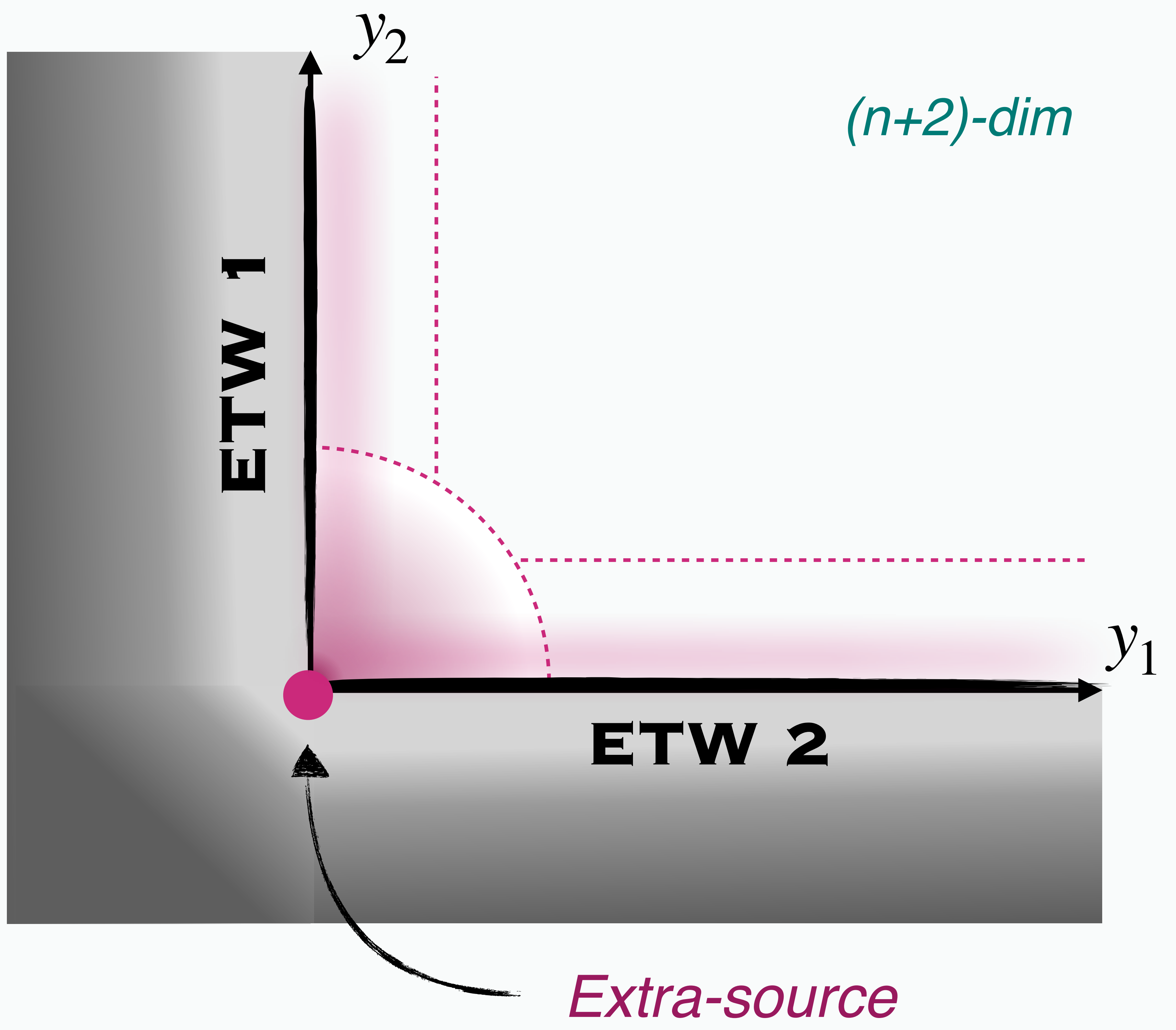
# Beyond the conformal flatness

[2404.xxxxx-R.A.]




**Solutions:**

- $\varphi_1(y_1) = -\sqrt{a_1 n(2 - a_1 - a_1 n)}\,\log y_1 = -\dfrac{2(1 - a_1 n)}{\delta_1}\log y_1$
- $\varphi_2(y_2) = -\sqrt{a_2 n}\,\log y_2 = -\dfrac{2}{\delta_2}\log y_2$
- $ds^2_{n+2} = y_1^{2(1-a_1 n)} y_2^{2a_2}\left[y_1^{4a_1 n - 2} ds_n^2 + y_1^{-2(1-a_1 n)} dy_1^2 - y_2^{-2a_2 n} dy_2^2\right]$

Potential class: $V(\phi) \simeq c \cdot v \cdot e^{\delta_1 \varphi_1} e^{\delta_2 \varphi_2}$

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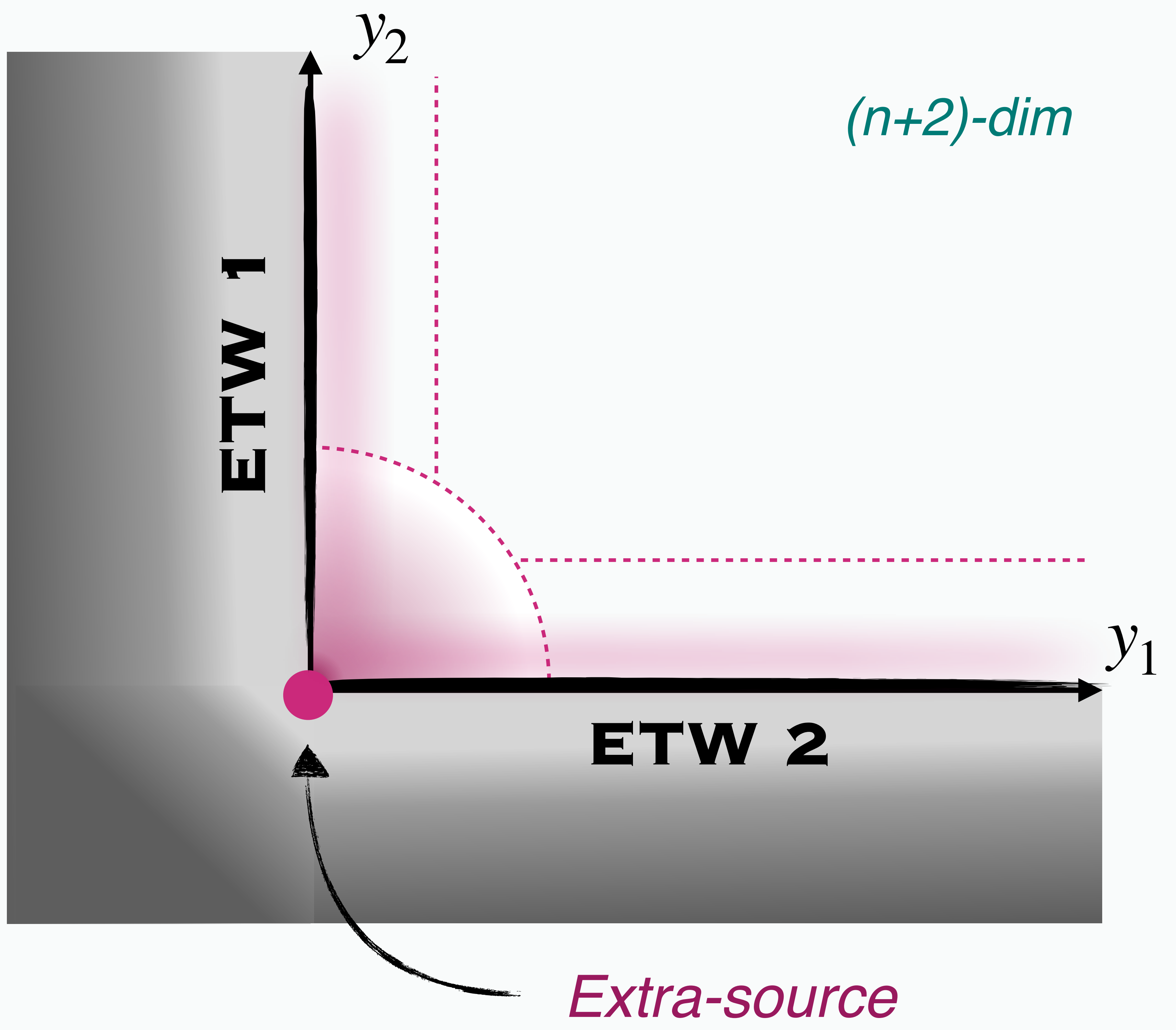


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$$\varphi_1 = \psi_1 - (1 - a_1 n)\sqrt{\frac{a_2}{a_1(2 - a_1 - a_1 n) - a_2(a_1 n - 1)^2}}\psi_2$$

$$\varphi_2 = \left(1 + \frac{a_2(a_1 n - 1)^2}{a_1(-2 + a_1 + a_1 n)}\right)^{-1/2}\psi_2$$

→

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# Complex Structure Moduli Space of $\mathrm{CY}_4$

[Math: Cattani, Kaplan, Schmid ’86; Deligne ’91; Kerr, Pearlstein, Robles ’19 …]

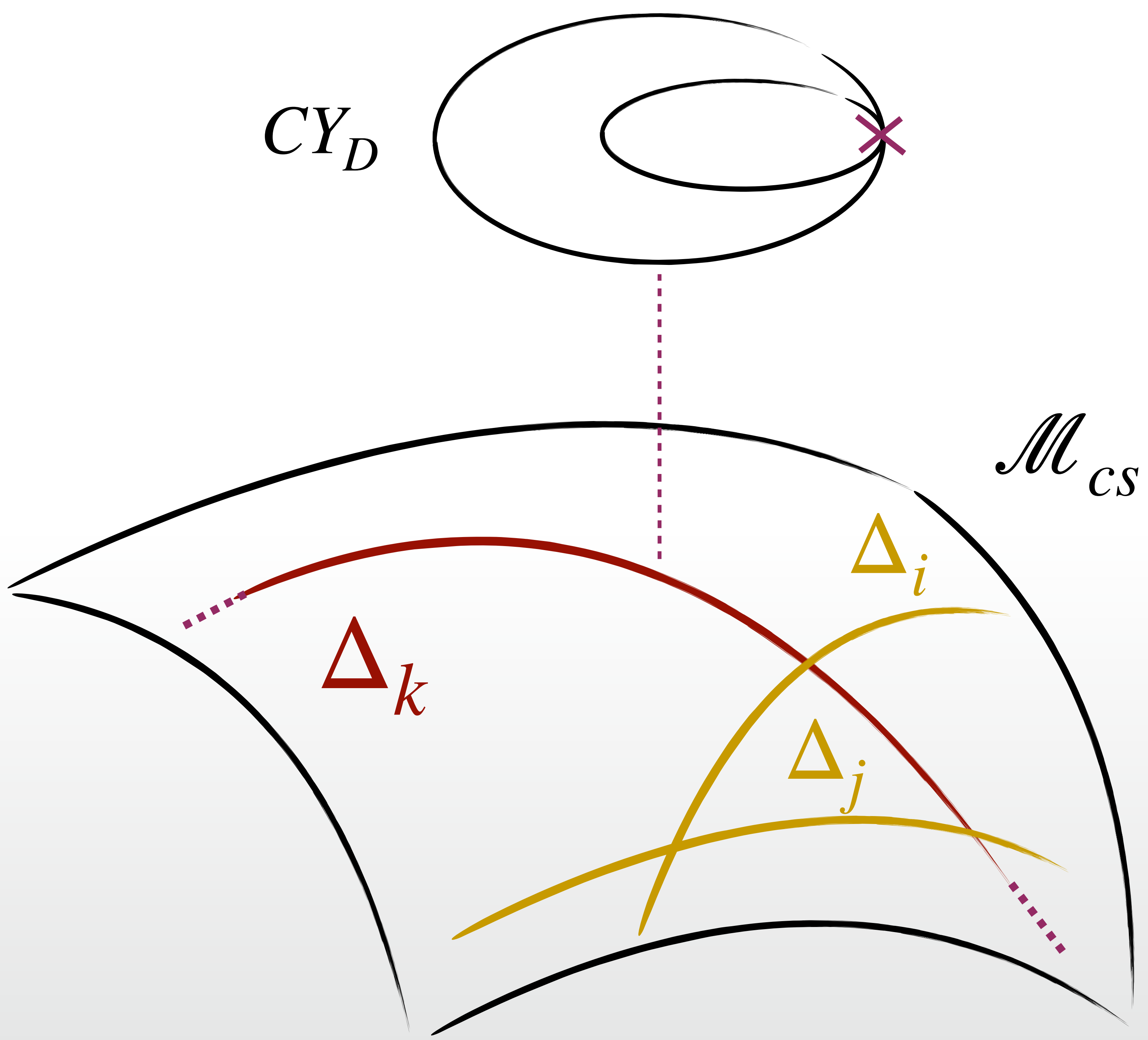
[Phys: Grimm, Palti, Valenzuela ’18; Grimm, Li, Palti ’19; Grimm, Li, Valenzuela ’20… ]

Let $\mathscr{M}_{cs}$ be the complex structure moduli space of $CY_4$:

- It has complex dimension $h^{3,1} = 2$
- It is not smooth nor compact

**Discriminant locus**

$$\Delta = \bigcup_k \Delta_k$$



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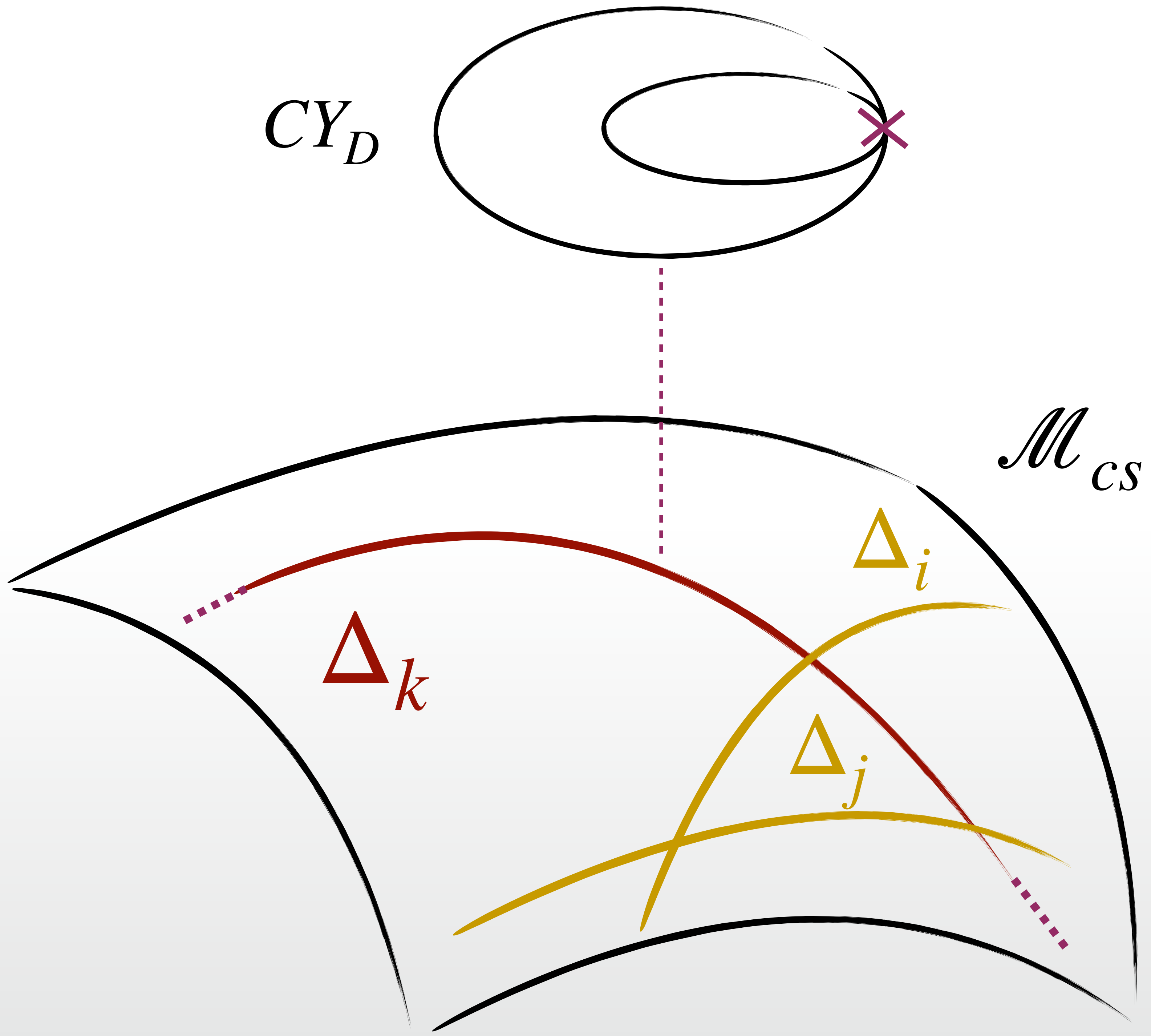
→ Using the mixed Hodge-Deligne theory we get a classification of the singularities and their enhancements

→ Singularities of type II, III, IV, V for $CY_4$ are located at infinite distance



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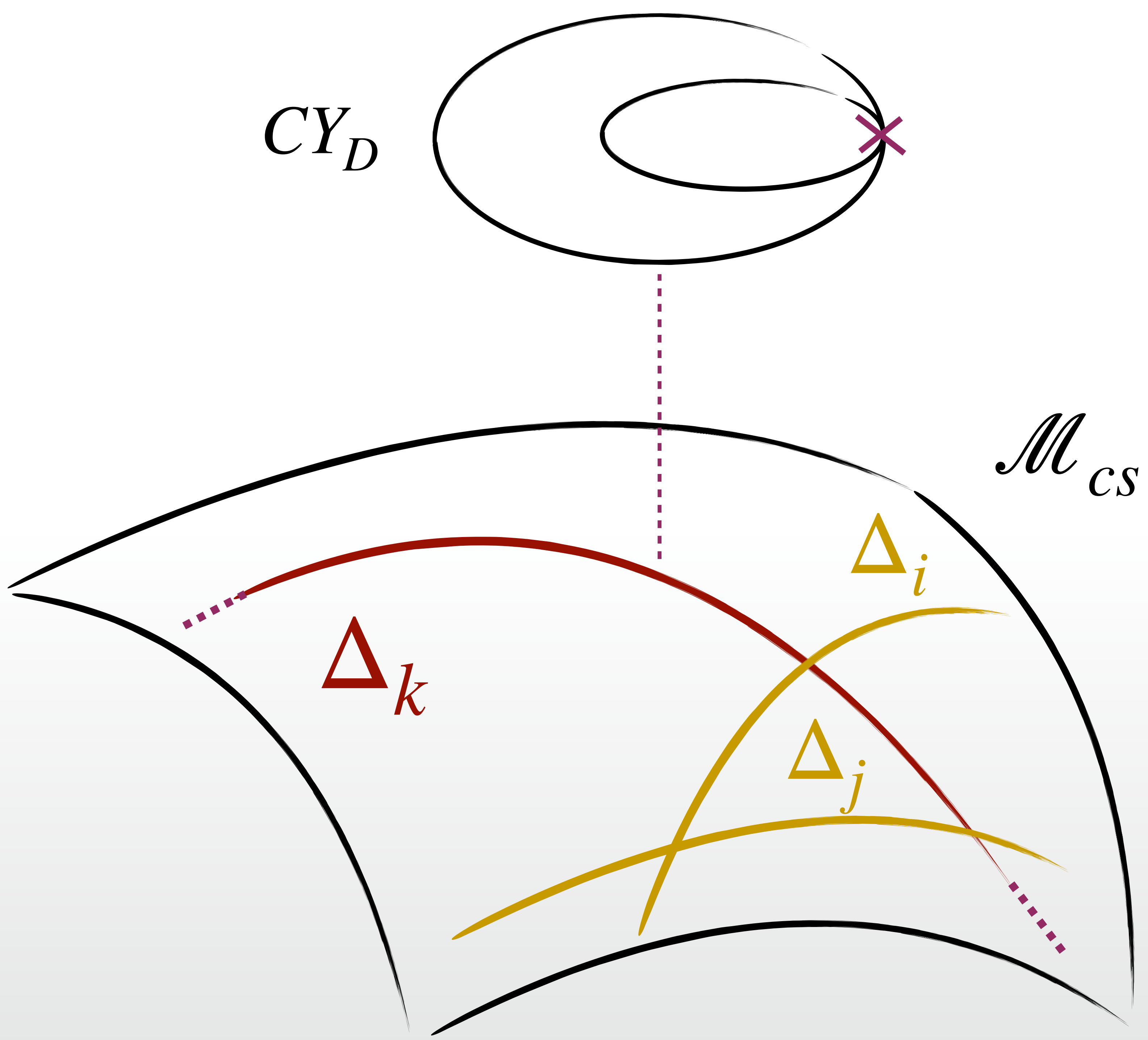
→ Turning on a $G_4 \in H^4_p(Y_4, \mathbb{C})$ we generate the 3d potential:

$$V = \frac{1}{\mathscr{V}_4^3}\left(\int_{Y_4} G_4 \wedge \star \bar{G}_4 - \int_{Y_4} G_4 \wedge G_4\right)$$



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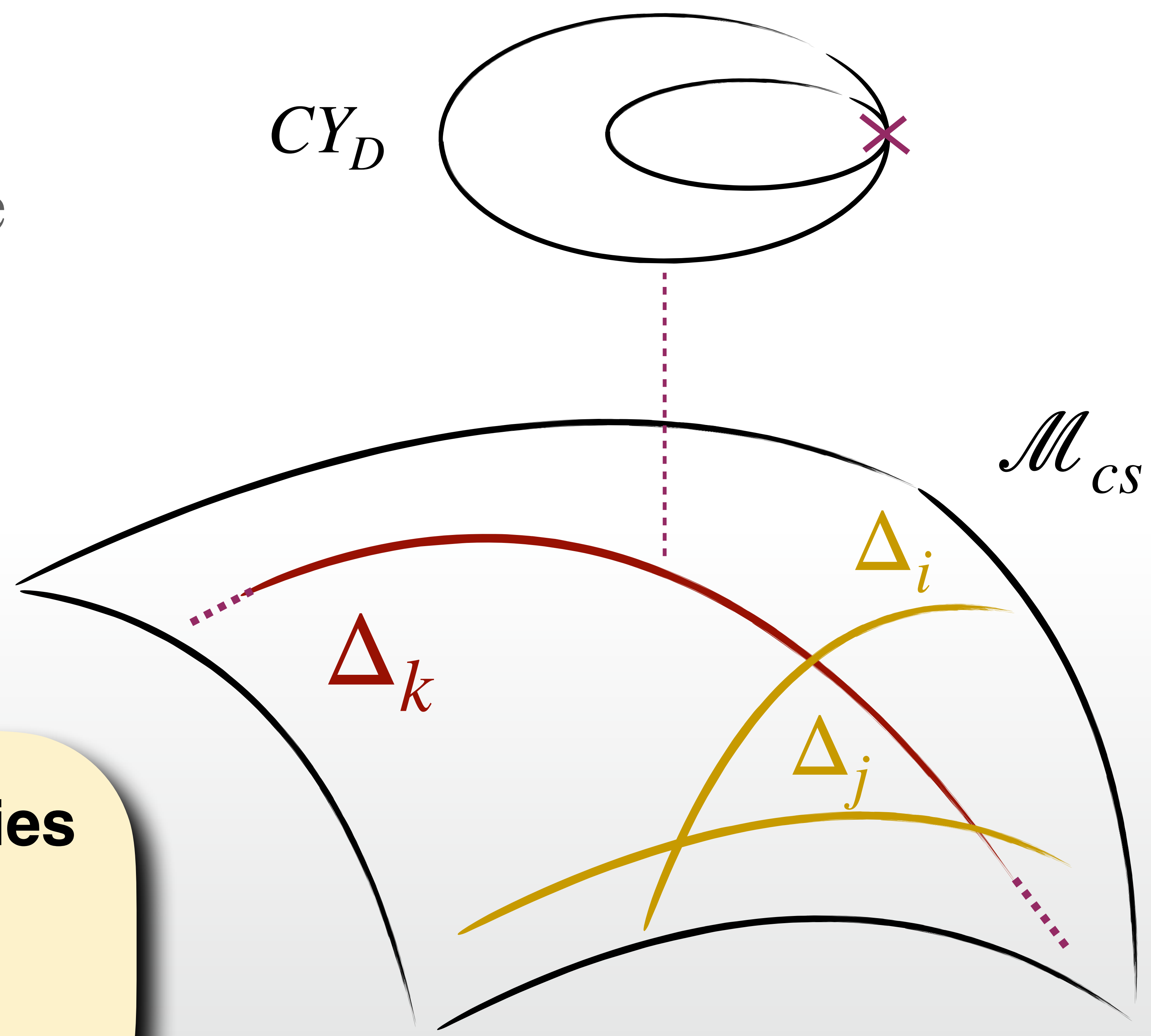


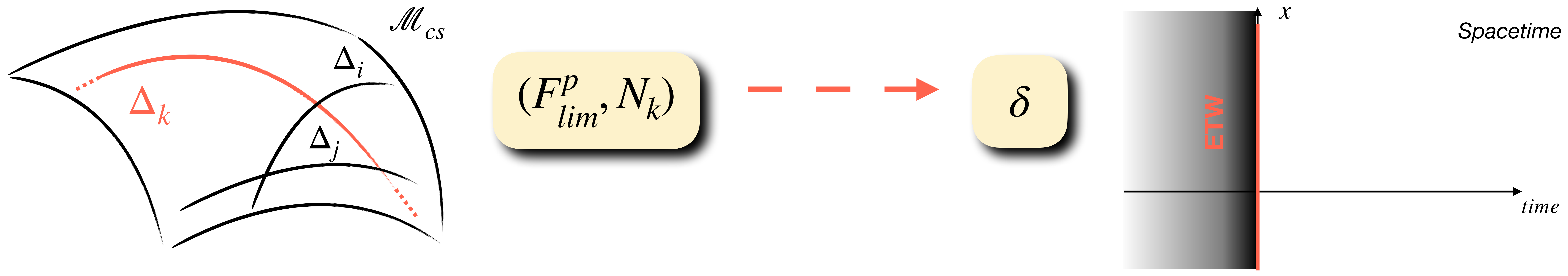

**Effective action for complex structure moduli near the network of singularities**

$$S = \int d^3x\sqrt{-g}\left\{\frac{1}{2}R - G_{kk}\partial\phi^k\phi^k + V(\phi^k)\right\}$$

# ETW networks for $\Delta$

Applying Dynamical Cobordism thecniques in the resulting three-dimensional effective action:

→ We can associate to each singular divisor $\Delta_k \in \Delta$ a specific ETW brane in spacetime



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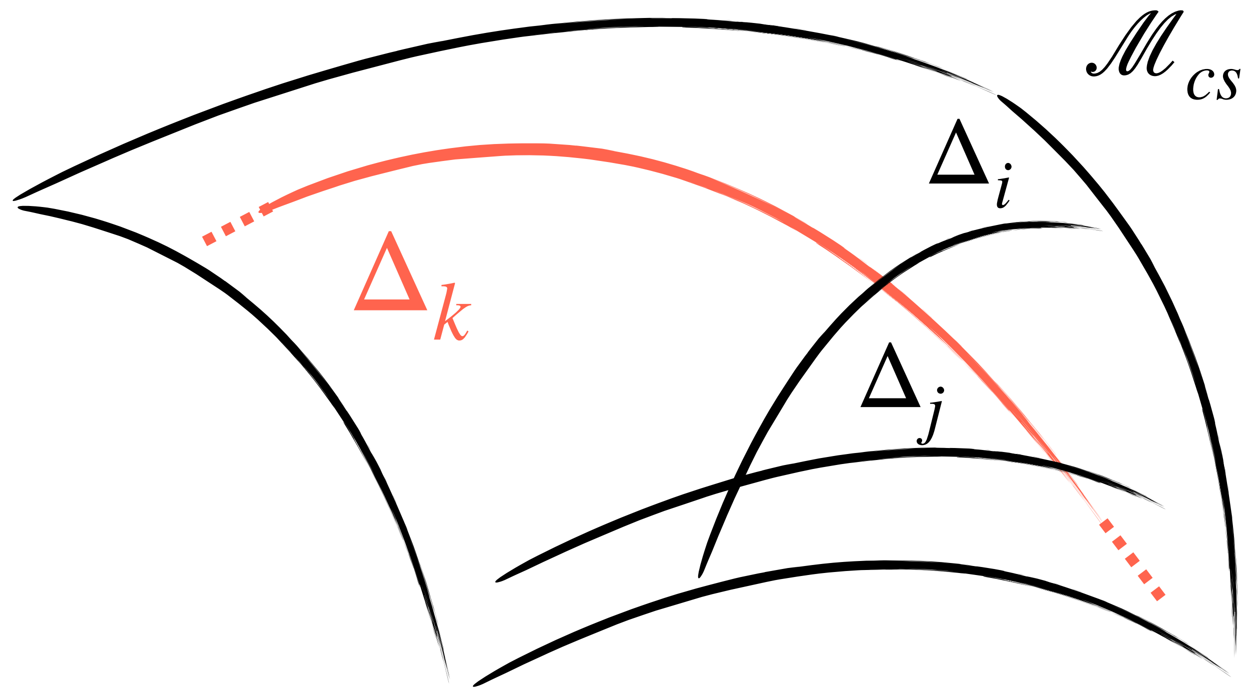
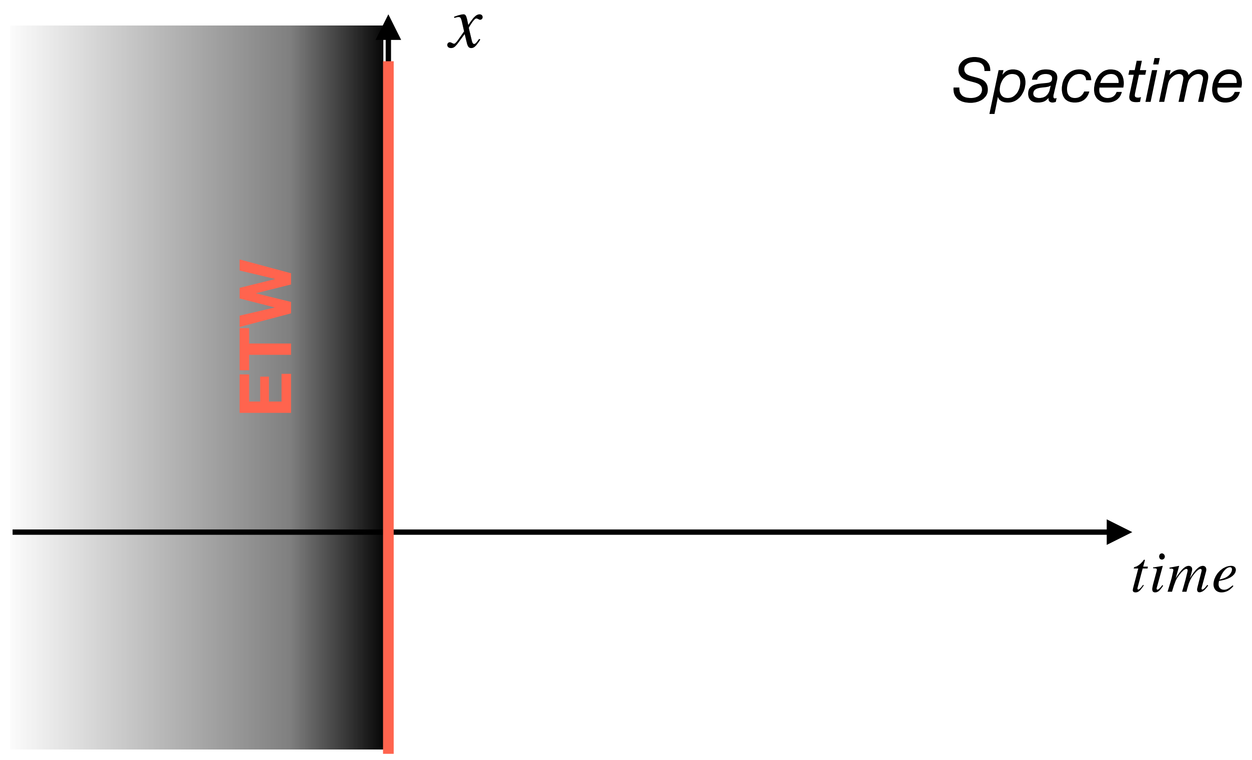
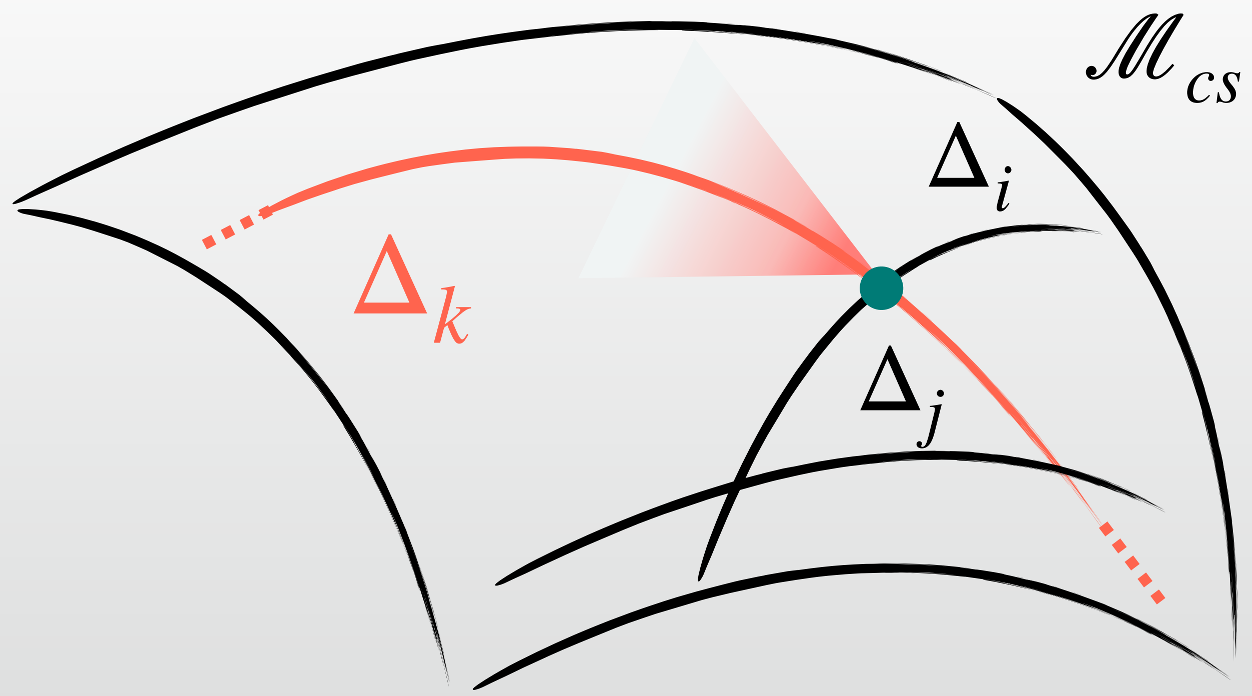



$(F^p_{lim}, N_k)$ → $\delta$




- We can provide a spacetime realization for the Growth Sectors associated to each enhancement as an intersection of two ETW branes: where the first brane represents the first singular divisor and the second brane represents the enhanced singularity

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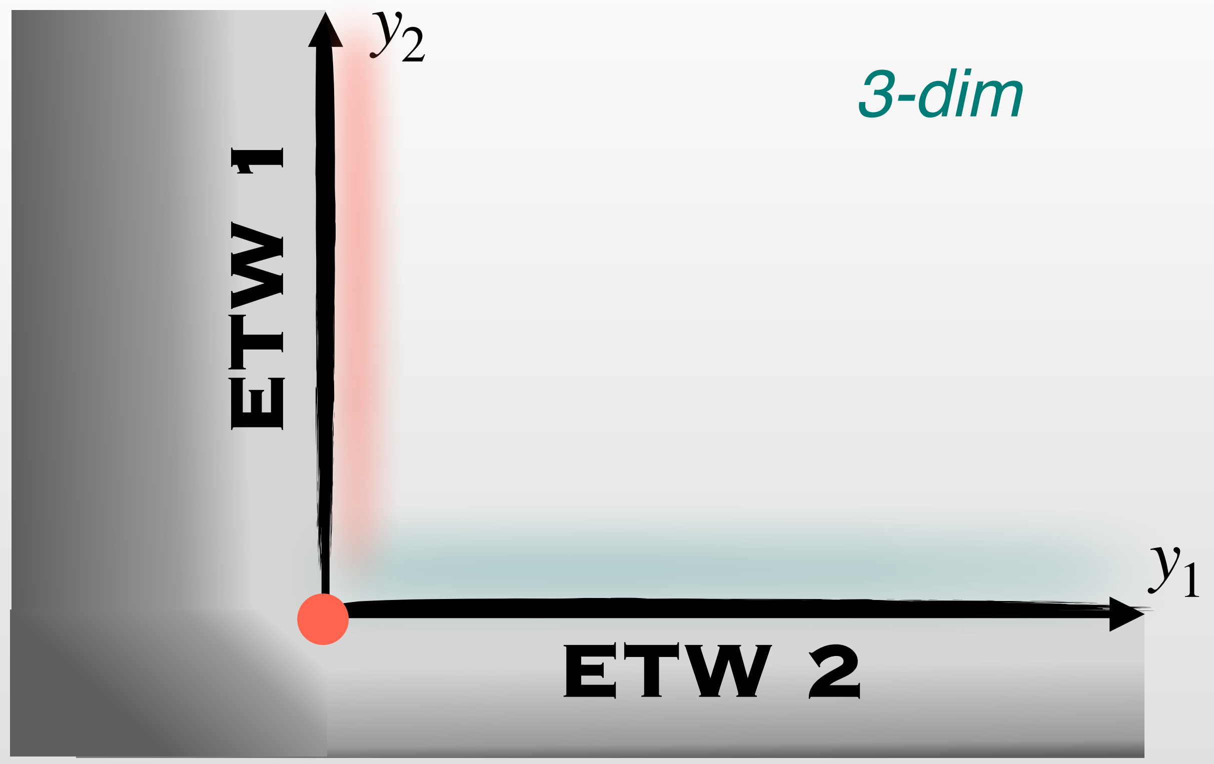

*Thank you for your attention!*