Geometry, Strings and the Swampland Program
Ringberg Castle, 18-22 March 2024

EXPLORING THE LANDSCAPE OF CHL STRINGS

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Based on work in progress with Ioannis Florakis and Diego Perugini

Compactification of heterotic strings is a very old topic,
still we are far for a complete characterisation of its landscape.

Recently, much progress on the exploration of the
moduli space of toroidal compactifications
with and without supersymmetry,
of non-geometrical compactifications,
vacua with reduced rank …

What is a CHL String?

[Chaudhuri, Hockney, Lykken 1995]

It is a 10d or lower dimensional heterotic string vacuum where one gauges an outer isomorphisms ***S***, possibly together with other symmetries

The prototype example is the heterotic string with gauge group

$$\begin{tikzcd} E_8 \arrow[r, phantom, "\times"] \arrow[r, leftrightarrow, bend right, "S"'] & E_8 \end{tikzcd}$$

What is a CHL String?

[Chaudhuri, Hockney, Lykken 1995]

The states (naively) invariant under this automorphism correspond gauge bosons for a single E_8 group

$$\begin{tikzcd} E_8 \arrow[r, phantom, "\times"] \arrow[r, leftrightarrow, bend right, "S"'] & E_8 \end{tikzcd}$$

But, as we know, in string theory one has to add ***twisted states***, which reintroduce the gauge bosons for the other E_8

What is a CHL String?

[Chaudhuri, Hockney, Lykken 1995]

In order to get new vacua, one has to combine ***S*** with other operations which change the twisted sector

$$\begin{tikzcd} E_8 \arrow[r, phantom, "\times"] \arrow[r, bend right, leftrightarrow, "S"'] & E_8 \end{tikzcd}$$

space-time fermion number	shift along compact directions
$(-1)^F$	$\delta \; : \; y \to y + \pi R$

THE E_8 STRING IN D=9?

[Chaudhuri, Polchinski 1995]

$$[E_8 \times E_8 \text{ on } S^1(R)]/S\delta$$

$$\mathcal{Z}\left[{0 \atop 0}\right] + \mathcal{Z}\left[{0 \atop 1}\right] = \tfrac{1}{2}(V_8 - S_8)\ \textstyle\sum_{m,n} \left[\bar{\chi}_8(\bar{q})\bar{\chi}_8(\bar{q})\Lambda_{m,n} + \bar{\chi}_8(\bar{q}^2)(-1)^m \Lambda_{m,n}\right]$$

untwisted sector

$$\simeq (8_v - 8_s + O(q))(\bar{q}^{-1} + 8_v + 248 + O(\bar{q}))$$

$$\mathcal{Z}\left[{1 \atop 0}\right] + \mathcal{Z}\left[{1 \atop 1}\right] = \tfrac{1}{2}(V_8 - S_8)\ \textstyle\sum_{m,n} \left[\bar{\chi}_8(\sqrt{\bar{q}})\Lambda_{m,n+\frac{1}{2}} + \hat{\bar{\chi}}_8(-\sqrt{\bar{q}})(-1)^m \Lambda_{m,n+\frac{1}{2}}\right]$$

twisted sector

The twisted sector is now massive and one is left with a single E_8 factor

How Many Strings Admit Outer Automorphisms?

Supersymmetric:	$\boxed{E_8 \times E_8}$	$SO(32)$
Non-supersymmetric:	$\boxed{SO(16) \times SO(16)}$	$SO(32)$
	$\boxed{E_7 \times SU(2) \times E_7 \times SU(2)}$	$SO(8) \times SO(24)$
	$SO(16) \times E_8$	$SU(16) \times U(1)$

Orbifold actions: $g_1 = (-1)^F\, S$ $\quad g_2 = S\,\delta$ $\quad g_3 = (-1)^F\, S\,\delta$

	$E_8 \times E_8$	$SO(16) \times SO(16)$	$E_7 \times SU(2) \times E_7 \times SU(2)$
$g_1 = (-1)^F S$	E_8 [Kawai, Lewellen, Tye 1986]	$SO(16) \times SO(16)$	$E_7 \times SU(2) \times E_7 \times SU(2)$
$g_2 = S\,\delta$	E_8 [Chaudhuri, Polchinski 1995]	$SO(16)$	$E_7 \times SU(2)$
$g_3 = (-1)^F S\,\delta$	E_8 [Nakajima 2023]	$SO(16)'$	$E_7 \times SU(2)$

Supersymmetric

See also Parra De Freitas, 02/2024

Some Details on the 9D Vacua

$$\mathcal{Z}_{O(16)}\left[{0 \atop 0}\right] = \frac{1}{2}\Big[V_8(\bar{O}_{16}\bar{O}_{16} + \bar{S}_{16}\bar{S}_{16}) - C_8(\bar{V}_{16}\bar{V}_{16} + \bar{C}_{16}\bar{C}_{16})$$
$$+ O_8(\bar{V}_{16}\bar{C}_{16} + \bar{C}_{16}\bar{V}_{16}) - S_8(\bar{O}_{16}\bar{S}_{16} + \bar{S}_{16}\bar{O}_{16})\Big]\,\Lambda_{m,n}$$

$$\mathcal{Z}^{2,3}_{O(16)}\left[{0 \atop 1}\right] = \frac{1}{2}\Big[V_8(\bar{O}_{16}(\bar{q}^2) + \bar{S}_{16}(\bar{q}^2)) \mp C_8(\bar{V}_{16}(\bar{q}^2) + \bar{C}_{16}(\bar{q}^2))\Big](-1)^m\Lambda_{m,n}$$

$$\mathcal{Z}_{E_7}\left[{0 \atop 0}\right] = \frac{1}{2}\Big[V_8(\bar{\chi}_0\bar{\xi}_0\,\bar{\chi}_0\bar{\xi}_0 + \bar{\chi}_v\bar{\xi}_v\,\bar{\chi}_v\bar{\xi}_v) - S_8(\bar{\chi}_0\bar{\xi}_0\,\bar{\chi}_v\bar{\xi}_v + \bar{\chi}_v\bar{\xi}_v\,\bar{\chi}_0\bar{\xi}_0)$$
$$+ O_8(\bar{\chi}_0\bar{\xi}_v\,\bar{\chi}_0\bar{\xi}_v + \bar{\chi}_v\bar{\xi}_0\,\bar{\chi}_v\bar{\xi}_0) - C_8(\bar{\chi}_0\bar{\xi}_v\,\bar{\chi}_v\bar{\xi}_0 + \bar{\chi}_v\bar{\xi}_0\,\bar{\chi}_0\bar{\xi}_v)\Big]\Lambda_{m,n}$$

$$\mathcal{Z}^{2,3}_{E_7}\left[{0 \atop 1}\right] = \frac{1}{2}\Big[V_8(\bar{\chi}_0\bar{\xi}_0(\bar{q}^2) + \bar{\chi}_v\bar{\xi}_v(\bar{q}^2)) + O_8(\bar{\chi}_0\bar{\xi}_v(\bar{q}^2) + \bar{\chi}_v\bar{\xi}_0(\bar{q}^2))\Big](-1)^m\Lambda_{m,n}$$

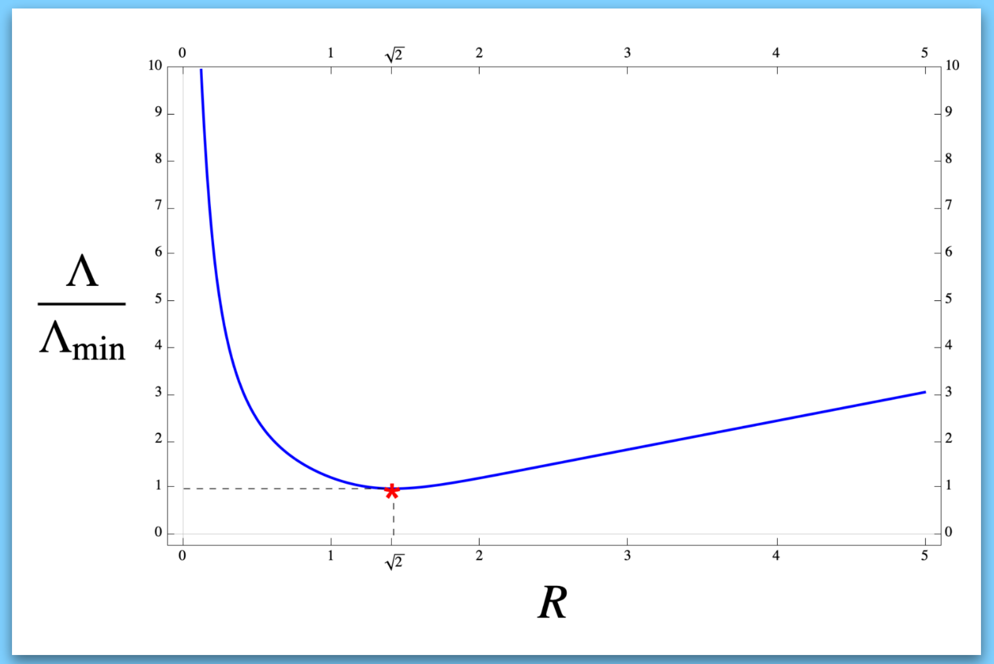
Some Details on the 9D Vacua

$$\mathcal{Z}^2_{O(16)} \simeq V_8(8_v + 120 + \ldots) - S_8(128 + \ldots) - C_8(136 + \ldots)$$

at $R = \sqrt{2}$ $\quad U(1) \to SU(2) \quad O_8 \sim \mathbf{128} \quad 2\,C_8$

$$\begin{array}{ccc} O(16)\times O(16) & O(16) & O(16)\times O(16) \\ R=0 & \longrightarrow & R\to\infty \end{array}$$

1-loop vacuum energy

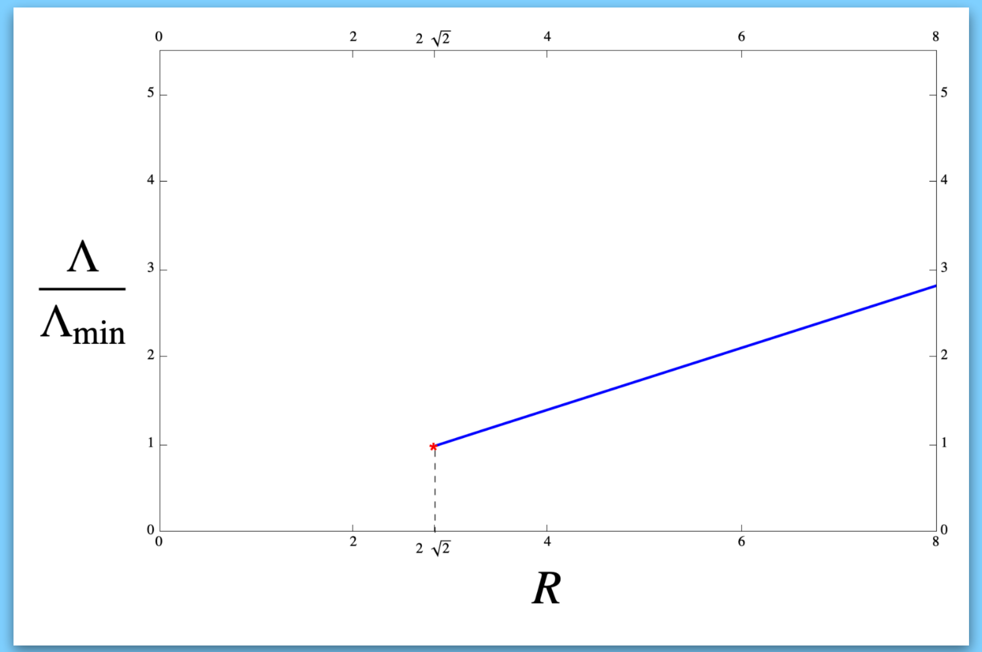


$$\mathcal{Z}^3_{O(16)} \simeq V_8(8_v + 120 + \ldots) - S_8(128 + \ldots) - C_8(120 + \ldots)$$

tachyons for $\quad R < 2\sqrt{2}$

$$\begin{array}{ccc} E_8 & O(16) & O(16)\times O(16) \\ R=0 & \longrightarrow & R\to\infty \end{array}$$

1-loop vacuum energy



TURNING ON WILSON LINES IN D=8

Upon compactification to D=8 on a spectator circle, all vacua (*seem*) to be connected via Wilson lines

What about the genera classification of Höhn & Möller for these non-supersymmetric vacua with reduced rank?

CHL Vacua with Eight Supercharges

Involve compactification on K3 $\sim T^4/\mathbb{Z}_N$ orbifolds

The orbifold has a non-trivial action on the gauge bundle

To build a CHL vacuum we need the presence of
an outer automorphism in D=6

ELSEVIER

Nuclear Physics B 519 (1998) 239–281

NUCLEAR PHYSICS B

Non-perturbative heterotic $D = 6, 4$, $N = 1$ orbifold vacua

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Table 1
Perturbative $\mathbb{Z}_2$ and $\mathbb{Z}_3$, $E_8 \times E_8$, orbifold models. The asterisk indicates twisted states involving left-handed oscillators. The last column shows which smooth $K3$ compactification yields a similar massless spectrum *after Higgsing*

Shift V / Group	Untwisted matter	Twisted matter	(k_1, k_2)
$\frac{1}{2}(1,1,0,\ldots,0) \times (0,\ldots,0)$ $E_7 \times SU(2) \times E_8$	(56,2)+4(1,1)	8(56,1)+32(1,2)*	(24,0)
$\frac{1}{2}(1,0,\ldots,0) \times (1,1,0\ldots,0)$ $SO(16) \times E_7 \times SU(2)$	(1,56,2)+4(1,1,1) + (128,1,1)	8(16,1,2)	(16,8)
$\frac{1}{3}(1,1,0,\ldots,0) \times (0,\ldots,0)$ $E_7 \times U(1) \times E_8$	(56,1)+3(1,1)	9(56,1)+18(1,1)* + 45 (1,1)*	(24,0)
$\frac{1}{3}(2,0,\ldots,0) \times \frac{1}{3}(2,0\ldots,0)$ $SO(14) \times SO(14) \times U(1)^2$	(14,1)+(64,1) + (1,14)+(1,64) + 2(1,1)	9(14,1)+9(1,14) + 18(1,1)*	(12,12)
$\frac{1}{3}(1,1,1,1,2,0,0,0) \times (0,\ldots,0)$ $SU(9) \times E_8$	(84,1)+2(1,1)	9(36,1)+18(9,1)*	(24,0)
$\frac{1}{3}(1,1,2,0,\ldots,0) \times \frac{1}{3}(1,1,0\ldots,0)$ $E_6 \times SU(3) \times E_7 \times U(1)$	(27,3,1) + (1,1,56) + 3(1,1,1)	9(27,1,1)+9(1,3,1) + 18(1,3,1)*	(18,6)
$\frac{1}{3}(1,1,1,1,2,0\ldots,0) \times \frac{1}{3}(1,1,2,0,0,0)$ $SU(9) \times E_6 \times SU(3)$	(1,27,3) + (84,1,1) + 2(1,1,1)	9(9,1,3)	(15,9)

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$\frac{1}{3}(1,1,0,\ldots,0) \times (0,\ldots,0)$ $E_7 \times U(1) \times E_8$	(56,1)+3(1,1)	9(56,1)+18(1,1)* + 45 (1,1)*	(24,0)
$\frac{1}{3}(2,0,\ldots,0) \times \frac{1}{3}(2,0\ldots,0)$ $SO(14) \times SO(14) \times U(1)^2$	(14,1)+(64,1) + (1,14)+(1,64) + 2(1,1)	9(14,1)+9(1,14) + 18(1,1)*	(12,12)
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$\frac{1}{3}(1,1,1,1,2,0\ldots,0) \times \frac{1}{3}(1,1,2,0,0,0)$ $SU(9) \times E_6 \times SU(3)$	(1,27,3) + (84,1,1) + 2(1,1,1)	9(9,1,3)	(15,9)

Smooth K3 compactifications with instanton number k_i=(12,12)

CHL Vacua with Eight Supercharges

The orbifold action is encoded in the vectors

$$v = (0, 0, \tfrac{1}{3}, -\tfrac{1}{3}) \qquad \text{spacetime}$$

$$V = (\tfrac{2}{3}, \mathbf{0}_7; \tfrac{2}{3}, \mathbf{0}_7) \qquad \text{gauge group}$$

The partition function in D=6 (rank 16)

$$Z = \frac{1}{\tau_2 \eta^2 \bar\eta^2} \frac{1}{3} \sum_{h,g\in\mathbb{Z}_3} \frac{1}{2} \sum_{a,b} e^{2\pi i h g/3} (-1)^{a+b+ab}$$

$$\times \frac{\vartheta\left[{a \atop b}\right]^2 \vartheta\left[{a+2h/3 \atop b+2g/3}\right] \vartheta\left[{a-2h/3 \atop b-2g/3}\right]}{\eta^4} \frac{\Gamma_{4,4}\left[{h \atop g}\right]}{\eta^4\bar\eta^4} \frac{\Gamma_{2,2}}{\eta^2\bar\eta^2}$$

$$\times \frac{1}{2}\sum_{k,\ell} e^{2\pi i h\ell/3} \frac{\bar\vartheta\left[{k \atop \ell}\right]^7 \bar\vartheta\left[{k+4h/3 \atop \ell+4g/3}\right]}{\bar\eta^8} \frac{1}{2}\sum_{\rho,\sigma} e^{2\pi i h\sigma/3} \frac{\bar\vartheta\left[{\rho \atop \sigma}\right]^7 \bar\vartheta\left[{\rho+4h/3 \atop \sigma+4g/3}\right]}{\bar\eta^8}$$

invariant under the exchange of the two group factors

The partition function in D=6 (rank 16)

$$
\begin{aligned}
Z = \frac{1}{\tau_2 \eta^2 \bar\eta^2} \frac{1}{3} \sum_{h,g\in\mathbb{Z}_3} \frac{1}{2} \sum_{a,b} e^{2\pi i h g/3} (-1)^{a+b+ab} \\
\times \frac{\vartheta\begin{bmatrix}a\\b\end{bmatrix}^2 \vartheta\begin{bmatrix}a+2h/3\\b+2g/3\end{bmatrix} \vartheta\begin{bmatrix}a-2h/3\\b-2g/3\end{bmatrix}}{\eta^4} \frac{\Gamma_{4,4}\begin{bmatrix}h\\g\end{bmatrix}}{\eta^4\bar\eta^4} \frac{\Gamma_{2,2}}{\eta^2\bar\eta^2} \\
\times \frac{1}{2}\sum_{k,\ell} e^{2\pi i h\ell/3} \frac{\bar\vartheta\begin{bmatrix}k\\\ell\end{bmatrix}^7 \bar\vartheta\begin{bmatrix}k+4h/3\\\ell+4g/3\end{bmatrix}}{\bar\eta^8} \frac{1}{2}\sum_{\rho,\sigma} e^{2\pi i h\sigma/3} \frac{\bar\vartheta\begin{bmatrix}\rho\\\sigma\end{bmatrix}^7 \bar\vartheta\begin{bmatrix}\rho+4h/3\\\sigma+4g/3\end{bmatrix}}{\bar\eta^8}
\end{aligned}
$$

$\chi\begin{bmatrix}h\\g\end{bmatrix}$

$\zeta\begin{bmatrix}h\\g\end{bmatrix}$

The supersymmetric CHL construction with eight supercharges

modding out by *S* δ

$$Z_{\mathbb{Z}_3/\mathrm{CHL}} = \frac{1}{2} \sum_{H,G} Z \begin{bmatrix} H \\ G \end{bmatrix} \frac{\Gamma^{\mathrm{shift}}_{2,2} \begin{bmatrix} H \\ G \end{bmatrix}}{\eta^2 \bar{\eta}^2}$$

$$Z \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\tau_2 \eta^2 \bar{\eta}^2} \frac{1}{3} \sum_{h,g \in \mathbb{Z}_3} \chi \begin{bmatrix} h \\ g \end{bmatrix} \frac{\Gamma_{4,4} \begin{bmatrix} h \\ g \end{bmatrix}}{\eta^4 \bar{\eta}^4} \zeta \begin{bmatrix} h \\ 2g \end{bmatrix} (\bar{q}^2) \qquad \Gamma^{\mathrm{shift}}_{2,2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \sum_{m,n} (-1)^m \, q^{\frac{1}{4} p_{\mathrm{L}}^2} \, \bar{q}^{\frac{1}{4} p_{\mathrm{R}}^2}$$

The twisted sector is obtained by imposing modular invariance.

It yields massive states only because of the shift δ.

The supersymmetric CHL construction with eight supercharges

modding out by ***S*** δ

$$Z_{\mathbb{Z}_3/\mathrm{CHL}} = \frac{1}{2} \sum_{H,G} Z \begin{bmatrix} H \\ G \end{bmatrix} \frac{\Gamma_{2,2}^{\mathrm{shift}} \begin{bmatrix} H \\ G \end{bmatrix}}{\eta^2 \bar{\eta}^2}$$

The light spectrum comprises the gravitational multiplet, one tensor multiplet, neutral hypers and

$G = SO(14) \times U(1)$ with charged hypers in $10 \cdot \mathbf{14} + \mathbf{64}$

NON-SUPERSYMMETRIC VACUA

Similar CHL construction with $(-1)^F$ ***S* (δ)**

Start from 10D non-supersymmetric strings
with outer automorphism compactified on K3
with symmetric embedding and mod-out by
$(-1)^F$ ***S δ*** or ***S δ***

Conclusions and Outlook

We have started our journey through the landscape of CHL vacua with and without supersymmetry on tori and K3 orbifold

Are these (non-supersymmetric) vacua in the same moduli space or not?

Duality with orientifold vacua?

THANK YOU