News on de Sitter and quintessence from string theory

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2309.03938 (with D. Tsimpis, T. Wrase)

2403.07065 (with F. Ruehle)

2403….. (with S. Parameswaran, D. Tsimpis, T. Wrase, I. Zavala)

Geometry, Strings
and the Swampland Program

21/03/24
Ringberg Castle, Tegernsee

Introduction

Motivation: describe **Dark Energy** (today)

Our universe is currently expanding + expansion is **accelerating**

⟶ Energy responsible for this acceleration? ⟶ **Dark energy**

Nature is unknown / not understood

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Cosmological model: 4d theory of scalar fields φ^i min. coupled to gravity
with a scalar potential $V > 0$
(+ matter…)

$$\int \mathrm{d}^4x \sqrt{|g_4|} \left(\frac{M_p^2}{2} \mathcal{R}_4 - \frac{1}{2} g_{ij} \partial_\mu \varphi^i \partial^\mu \varphi^j - V \right)$$

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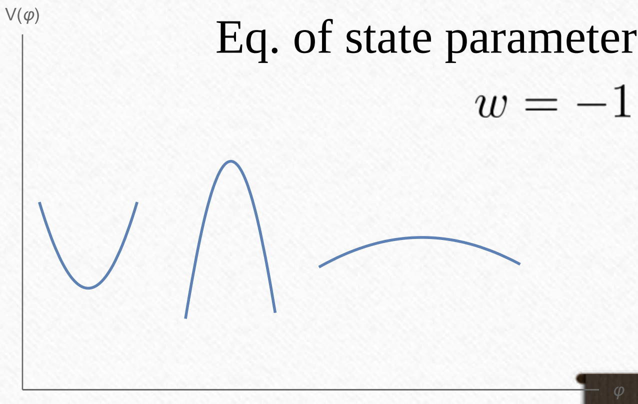
De Sitter solution:
critical point:

$V' \equiv \partial_\varphi V = 0$

$\dot{\varphi} = 0$

⟶ $\Lambda = \frac{V}{M_p^2}$ = constant
cosmological constant



Eq. of state parameter:

$$w = -1$$

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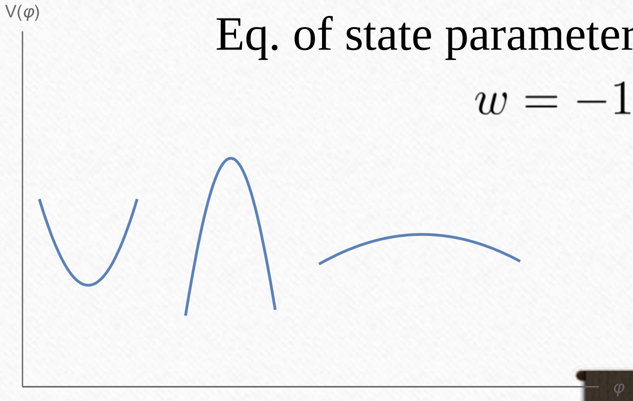
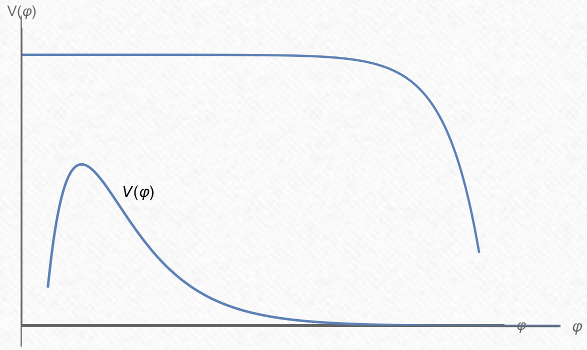


Quintessence:
rolling field:

$V' \neq 0, \ \dot\varphi \neq 0$

Varying e.o.s.

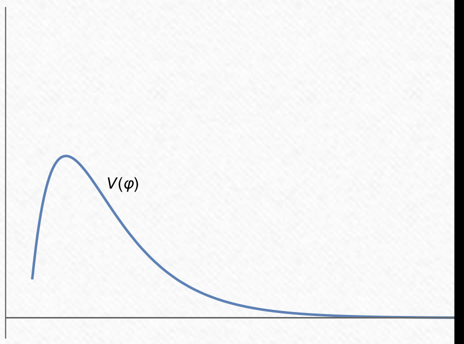
$-1 \leq w \leq 1$

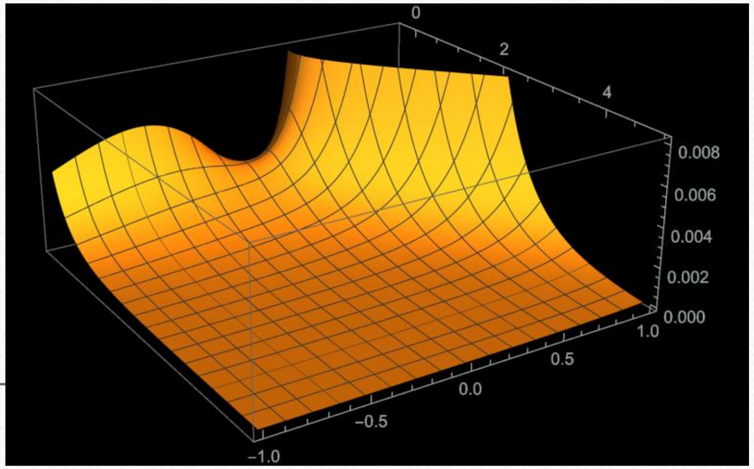


De Sitter: **minimum**/vacuum: gives Λ ✓

But difficult to achieve from string theory…

Here: tachyonic/**maximum** along 1 direction





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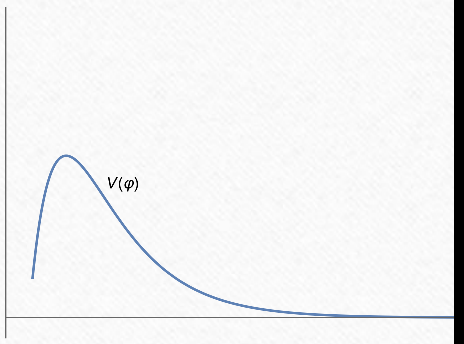

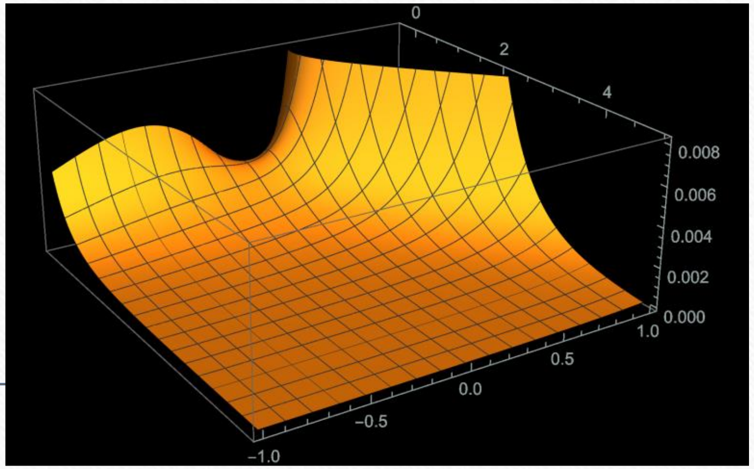

Friction holds field at maximum for $H > H_0$, then field starts rolling

Ok even with $\eta_V \approx -1$

Agrawal, Obied ’18

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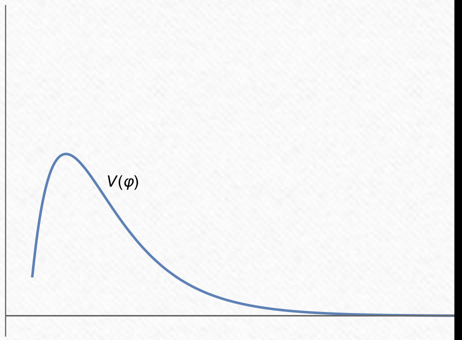

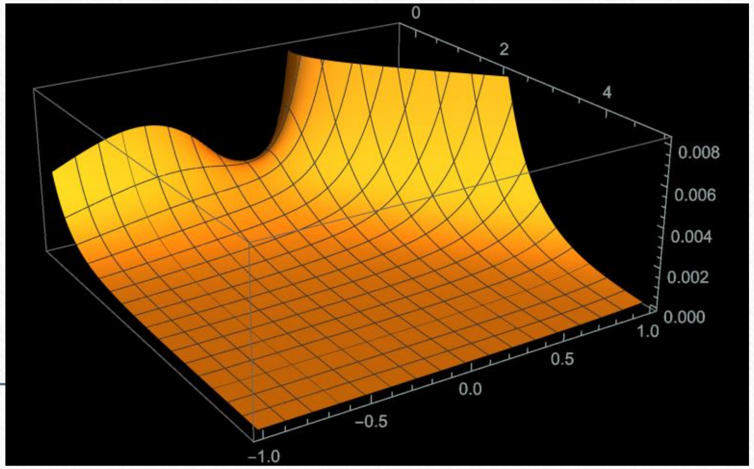

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Observational constraints: (model dependent!) DES '24

Λ CDM: $w = -1$ ✓ observations

w CDM: $w \approx -0.94$

$w_0 w_a$ CDM: $w_0 \approx -0.77$

$w = w_0 + w_a(1 - \frac{a}{a_0})$ $w_a \approx -0.83 < 0$

w closer to -1 in the near past

consistent with rolling down from de Sitter!

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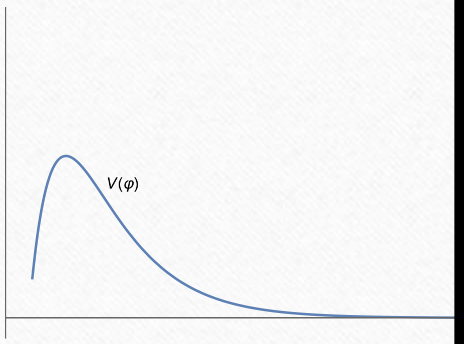

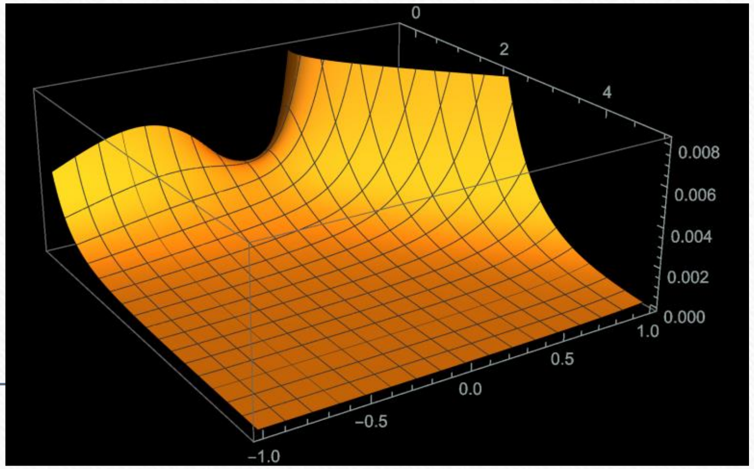

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De Sitter maximum is possible, **quintessence** is possible,
observational constraints are **very model dependent**

(more **quintessence models** later)

String theory realisation?

From string theory, we **easily** get $\int \mathrm{d}^4x \sqrt{|g_4|} \left(\frac{M_p^2}{2} \mathcal{R}_4 - \frac{1}{2} g_{ij} \partial_\mu \varphi^i \partial^\mu \varphi^j - V \right)$

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Challenge is $V > 0$ with the right shape for V: difficult!

(Trustable) de Sitter critical point

Appropriate (not large) slope $\frac{|V'|}{V}$ for quintessence

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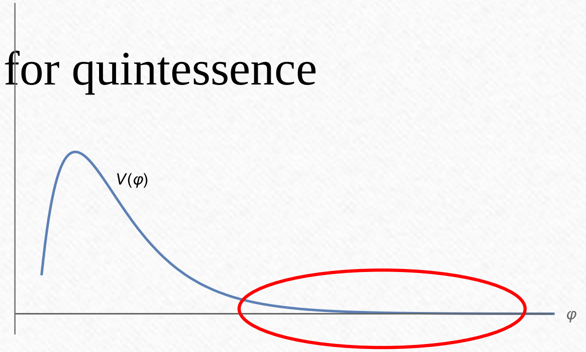

Conflict with ``typical'' behaviour in **asymptotic** of field space:

Strong de Sitter (swampland) conjecture: $\varphi \to \infty, \quad \frac{|\nabla V|}{V} \geq \sqrt{2}$

For $V(\varphi) \approx V_0 \, e^{-\lambda \varphi} : \quad \lambda \geq \sqrt{2}$

Bedroya, Vafa '19, Rudelius '21

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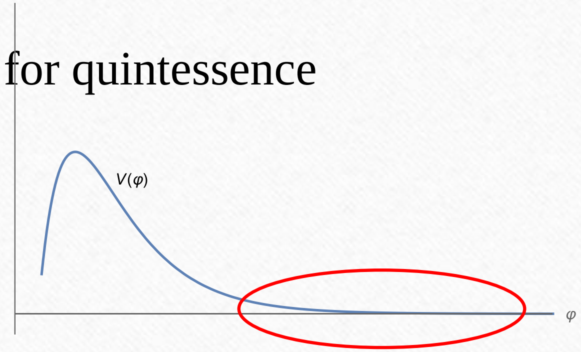

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⟶ No de Sitter in asymptotics ⟶ de Sitter solutions in **classical** string regime?

Concrete checks and arguments: Wrase et al '18, Junghans '18, Grimm et al '19, Andriot '19,'20, Cicoli et al '21

⟶ High slopes, problematic for quintessence: $\lambda < 0.5 - 1$ Agrawal et al, Akrami et al, Raveri et al, '18 Schöneberg et al '23

⟶ Cosmology away from asymptotics, i.e. towards bulk of field space? *See e.g. talk by Cicoli, Righi, Wiesner*

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Here: stick to large fields, if not **asymptotics** (control on corrections, naturally small Λ)

→ Possible way out for de Sitter solution
→ Possible way out for quintessence
(from string theory)

circumventing or contradicting previous stringy claims / results

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- **De Sitter**: 10d supergravity solution, with a scaling freedom (parameter $\gamma > 1$)

$$r_4, r_5 \to \gamma\ r_4, r_5, \quad r_1, r_2 \to \gamma^{\frac{1}{2}}\ r_1, r_2, \quad r_3, r_6, g_s \text{ invariant}$$

→ arbitrary large radii and volume

→ Contradict claim on no asymptotic de Sitter? Parametric control on corrections?

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- **Quintessence**: include spatial curvature (open universe $k=-1, \Omega_k \neq 0$)
  explore consequences, w.r.t. slope, acceleration, observational constraints

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expı s, w.r.t. ! Spies in the room ! onstraints

I. De Sitter solutions

Interested in **classical** string backgrounds that have a 4d de Sitter spacetime

In practice, solution of 10d supergravity + make sure, or check afterwards, that classical regime ✓

Long history: **no known classical de Sitter** solution up-to-date Andriot '19

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Compactification ansatz: $\mathrm{dS}_4 \times \mathcal{M}_6$

$\mathcal{M}_6 = G/\Gamma$ 6d compact group manifold, G 6d Lie group, Γ discrete subgroup/lattice (compactness)

E.g. twisted torus, nilmanifold, solvmanifold

Easy to handle + can have $\mathcal{R}_6 < 0$

H, F_q fluxes

O_p orientifolds, D_p -branes

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Classification of possible solutions Andriot, Horer, Marconnet '22

4d theory (kinetic terms, V) for each class after consistent trunc. Andriot, Marconnet, Rajaguru, Wrase '22

Solution examples: s^+_{6666} IIA, O_6, F_0 Wrase, Koerber, Lüst, Danielsson, Van Riet, Shiu, et al '08-'11

here → s^+_{55} IIB, O_5, F_1 Andriot, Marconnet, Wrase '20, '21

Compactification ansatz: more details:

$H, \, F_1, \, F_3$

Source Set I	Internal Dimension a					
	1	2	3	4	5	6
$I = 1$ (D_5 and O_5)	$\times$	$\times$				
$I = 2$ (D_5 and O_5)			$\times$	$\times$		
$I = 3$ (D_5 only)					$\times$	$\times$

Source contributions: T_{10}^{I}
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6d geometry $\mathcal{M}_6 = G/\Gamma$

$\mathrm{d}s_6^2 = g_{ab}\, e^a\, e^b$, one-forms: $e^a = e^a{}_m\, \mathrm{d}y^m$, basis used for all forms!

$\mathrm{d}e^a = -\frac{1}{2} f^a{}_{bc}\, e^b \wedge e^c$ (not closed, away from cycles…)

$f^a{}_{bc}$ $\sim$spin connection for G

+ structure constants of underlying algebra $\mathfrak{g}$

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Examples:

semi-simple: $[E_2, E_3] = f^1{}_{23}\, E_1$, $[E_1, E_3] = f^2{}_{13}\, E_2$, $[E_1, E_2] = f^3{}_{12}\, E_3 \ \rightarrow\ f^1{}_{23}, f^2{}_{13}, f^3{}_{12}$

solvable: $[E_2, E_3] = f^1{}_{23}\, E_1$, $[E_1, E_3] = f^2{}_{13}\, E_2$, $[E_1, E_2] = 0 \ \rightarrow\ f^1{}_{23}, f^2{}_{13}$

nilpotent: $[E_2, E_3] = f^1{}_{23}\, E_1$, $[E_1, E_3] = 0$, $[E_1, E_2] = 0 \ \rightarrow\ f^1{}_{23}$

Here: two copies of 3d solvable algebras: $\mathfrak{g}^0_{3.5} \oplus \mathfrak{g}^0_{3.5}$: $f^2{}_{35}$, $f^3{}_{25}$, $f^1{}_{46}$, $f^6{}_{14}$ $\longrightarrow$ 6d solvmanifold

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Convention: $\det g_{ab} = 1$ $\longrightarrow$ lengths or radii, $r_1, ..., r_6$, inside the e^a $(y^m \in [0, 2\pi])$

$\longrightarrow$ $f^1{}_{64} \sim \frac{r_1}{r_6 r_4} N_1$, $f^6{}_{14} \sim \frac{r_6}{r_1 r_4} N_6$, N_1, N_6 real numbers, subject to quantization conditions

$\leftrightarrow$ lattice, compactness

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$$e^6 = r_6 \left(\cos(\sqrt{|N_1 N_6|} y^4)\, \mathrm{d}y^6 - \left|\frac{N_6}{N_1}\right|^{\frac{1}{2}} \sin(\sqrt{|N_1 N_6|} y^4)\, \mathrm{d}y^1 \right)$$

$$e^1 = r_1 \left(\left|\frac{N_1}{N_6}\right|^{\frac{1}{2}} \sin(\sqrt{|N_1 N_6|} y^4)\, \mathrm{d}y^6 + \cos(\sqrt{|N_1 N_6|} y^4)\, \mathrm{d}y^1 \right)$$

$$e^4 = r_4\, \mathrm{d}y^4 \ .$$

Here: $\sqrt{|N_1 N_6|} = 1$

Ex.: $N_1 = -N_6 = 1$

But also: $N_1 \to \gamma^{-1} N_1$
$N_6 \to \gamma\, N_6$

Andriot, Goi, Minasian, Petrini, ’10
Grana, Minasian, Triendl, Van Riet, ’13

2 known supergravity solutions: Andriot, Marconnet, Wrase ’20 Andriot, Ruehle ’24

Solution 29: $\mathcal{R}_4^S = 0.020309$,

$$g_s T_{10}^1 = 10\ ,\ g_s T_{10}^2 = -0.079765\ ,\ g_s T_{10}^3 = -1.064125\ ,$$

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2 known supergravity solutions: Andriot, Marconnet, Wrase ’20 Andriot, Ruehle ’24

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5 requirements: $g_s < 1$, $r_a > l_s$ and

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Solution 29:

$$N_{s1} = 16\ ,\ N_{s2} = -67\ ,\ N_{s3} = -68\ ,\ N_{1\,5} = -46\ ,\ N_{\omega_1} = 1\ ,\ N_{\omega_2} = -18\ ,$$

$$N_1 = 0.020207\ ,\ N_2 = -0.002592\ ,\ N_3 = -1/N_2\ ,\ N_6 = -1/N_1\ , (...)\ ,$$

$$g_s = 0.532758\ ,\ r_1 = 4.704542\ l_s\ ,\ r_2 = 112.925701\ l_s\ ,\ r_4 = 14.968801\ l_s\ ,\ r_5 = 172.058417\ l_s\ ,$$

$$r_3 = 0.067605\ l_s\ ,\ r_6 = 0.077310\ l_s\ .$$

$g_s < 1\ ,\ r_1, r_2, r_4, r_5 > l_s\ ,\ r_3, r_6 < l_s$ $\longrightarrow$ **Classical?**

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Winding modes? But not exaclty radii, rather characteristic length in non-closed form so not volume of a one-cycle…

$$e^6 = r_6 \left(\cos(\sqrt{|N_1 N_6|} y^4)\, \mathrm{d}y^6 - \left|\frac{N_6}{N_1}\right|^{\frac{1}{2}} \sin(\sqrt{|N_1 N_6|} y^4)\, \mathrm{d}y^1 \right) \qquad \mathrm{d}e^6 \neq 0$$

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$\longrightarrow$ more investigation

Can’t we find **another solution** with better values?

Tried several numerical methods and tools for this, we did not manage.

But also clear **numerical difficulties** with this problem

$\longrightarrow$ **Theoretical obstruction** against classical de Sitter (not found here) or **numerical difficulty**?

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One **simple version**:

$r_{4,5} \to \gamma\, r_{4,5}$, $r_{1,2} \to \gamma^{\frac{1}{2}}\, r_{1,2}$, r_3, r_6, g_s invariant

$N_{\omega_{1,2}} \to \gamma^{\frac{1}{2}}\, N_{\omega_{1,2}}$,

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$4\ r_a \nearrow$ (6d volume $\nearrow$)

$N_{\rm flux} \nearrow$ (γ discretized)

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Effect on supergravity variables:

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$\longrightarrow$ $T^I_{10} \ \to\ \frac{1}{\gamma^2}\, T^I_{10}$, $\quad F_{1\,5}, F_{3\ldots}, H_{\ldots}, f^a{}_{bc} \ \to\ \frac{1}{\gamma}\, F_{1\,5}, F_{3\ldots}, H_{\ldots}, f^a{}_{bc}$

$\longrightarrow$ $\text{Eq. of motion} \ \to\ \frac{1}{\gamma^2}\, \text{Eq. of motion}$, $\text{Solution} \ \to\ \text{Solution}'$, $\mathcal{R}_4^S \ \to\ \frac{1}{\gamma^2}\, \mathcal{R}_4^S$,

(analogous to DGKT)

γ **scaling**:

- **Asymptotic de Sitter** solution (from 10d)…
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- Previous analytic arguments against asymptotic de Sitter: Junghans '18, Grimm et al '19, Andriot '19, Cicoli et al '21 ⟶ need for internal hierarchy (the case here) ↓ Dark Dimension?

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II. Quintessence

Consider $\int \mathrm{d}^4x \sqrt{|g_4|} \left(\frac{M_p^2}{2} \mathcal{R}_4 - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - V \right) \ , \qquad V = V_0 \, e^{-\lambda \, \varphi}$

Take FLRW metric with arbitrary space curvature, $k = 0, \pm 1$

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Observational constraints: $\lambda < 0.5 - 1$

If only ask **asymptotic acceleration**: $\ddot{a} > 0$ ⟶ theoretical bound: $\lambda < \sqrt{2}$ Halliwell '86, Copeland et al ’97 Shiu, Tonioni, Tran ’23

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(circumvent: non-canonical kin. terms: Brinkmann, Cicoli, Dibitetto, Pedro ’22)

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Explain/extend: 3 equations of motion $\longrightarrow$ rewritten as a **dynamical system**

$\longrightarrow$ study **fixed points,** relevant to **asymptotics!**

Find fixed point P_2 (attractor):

Allows for acceleration: $\ddot{a} > 0 \Leftrightarrow \lambda < \sqrt{2}$

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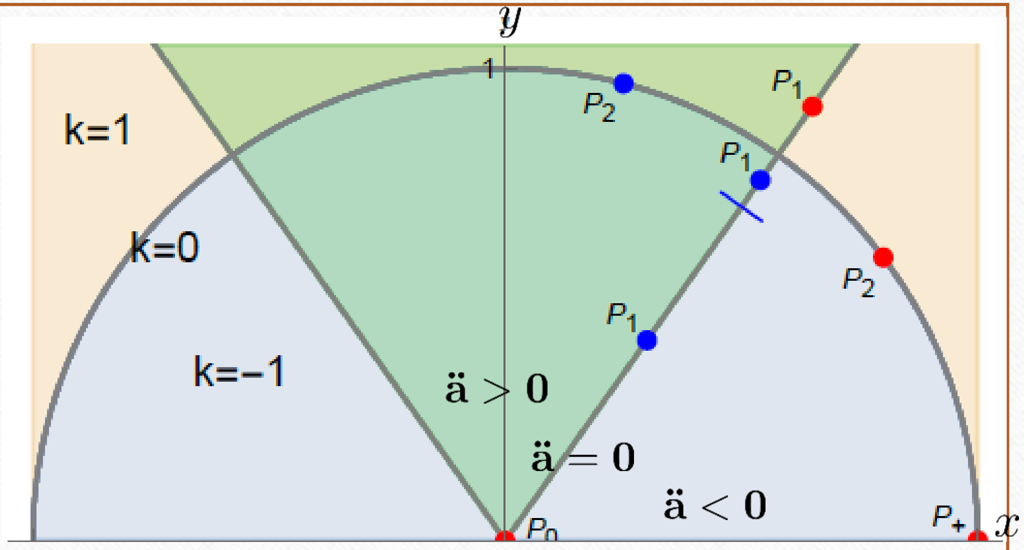

Phase space $(x, y) = \left(\frac{\dot{\varphi}}{H\sqrt{6}}, \frac{\sqrt{V}}{H\sqrt{3}} \right)$

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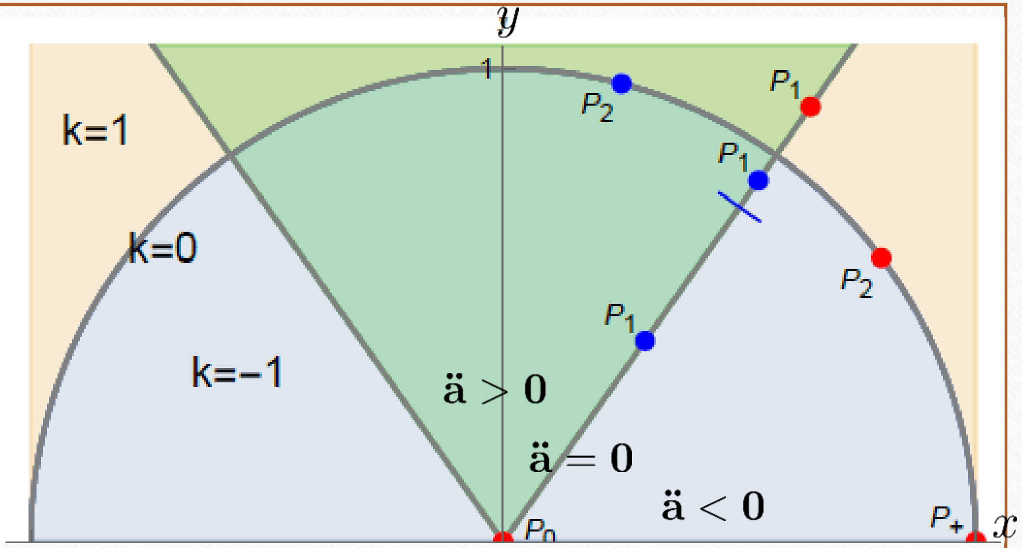
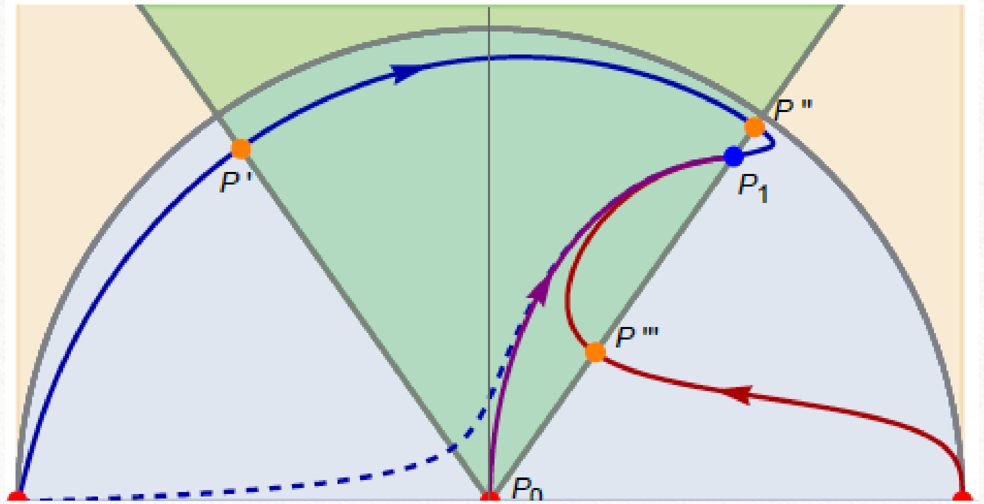


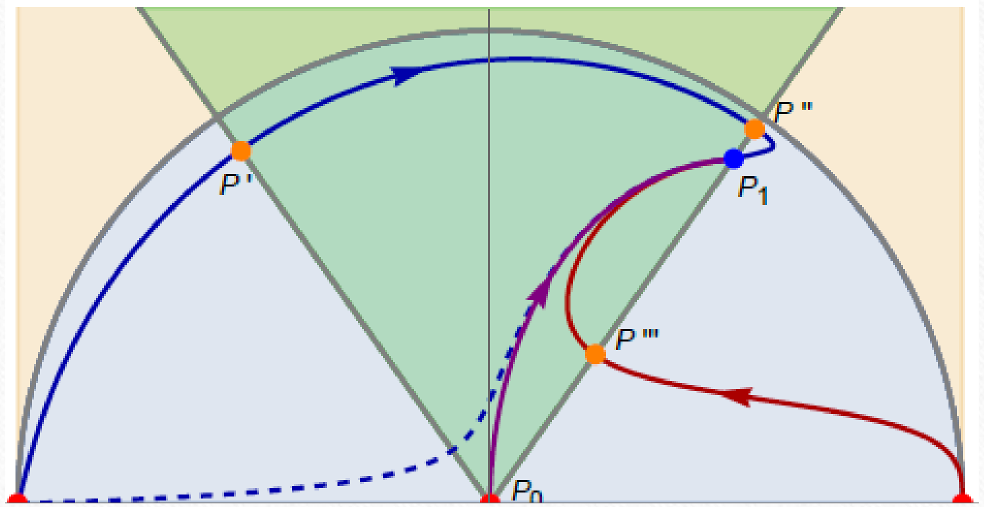
Cosmological solutions asymptoting to
Acceleration: eternal, semi-eternal, transient



With open universe ($k = -1$), and string model $V = V_0\, e^{-\lambda\,\varphi},\ \lambda > \sqrt{2}$

⟶ **asymptotic acceleration** (and **transient**)

How **realistic**?



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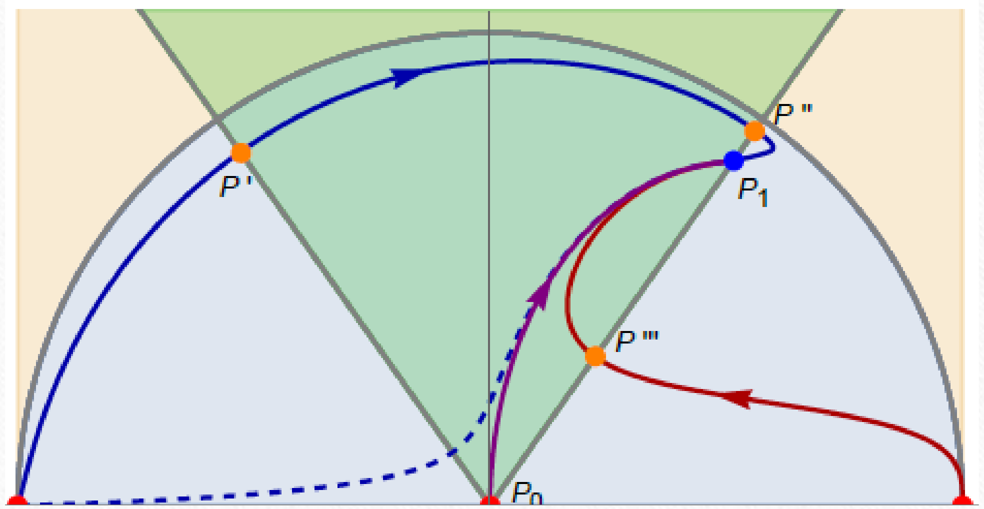

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(not the right model…!)

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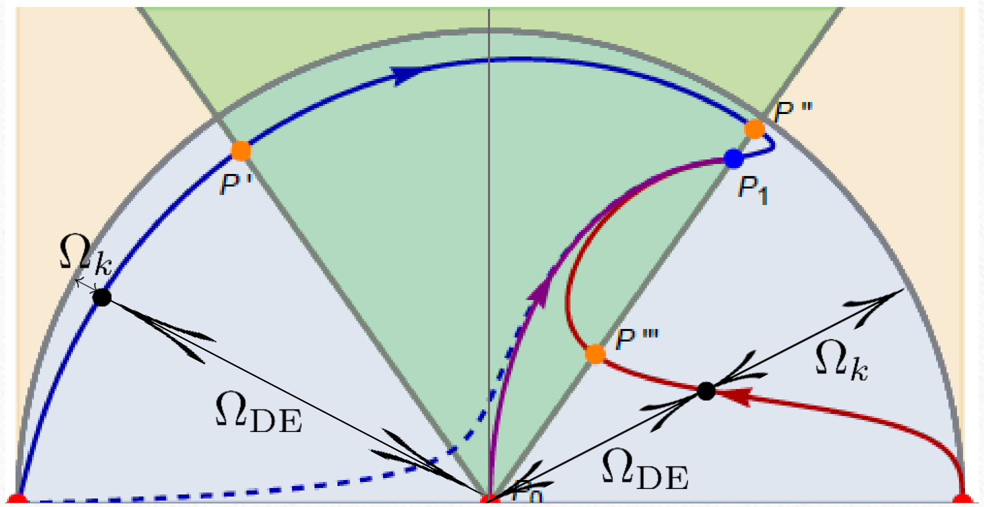

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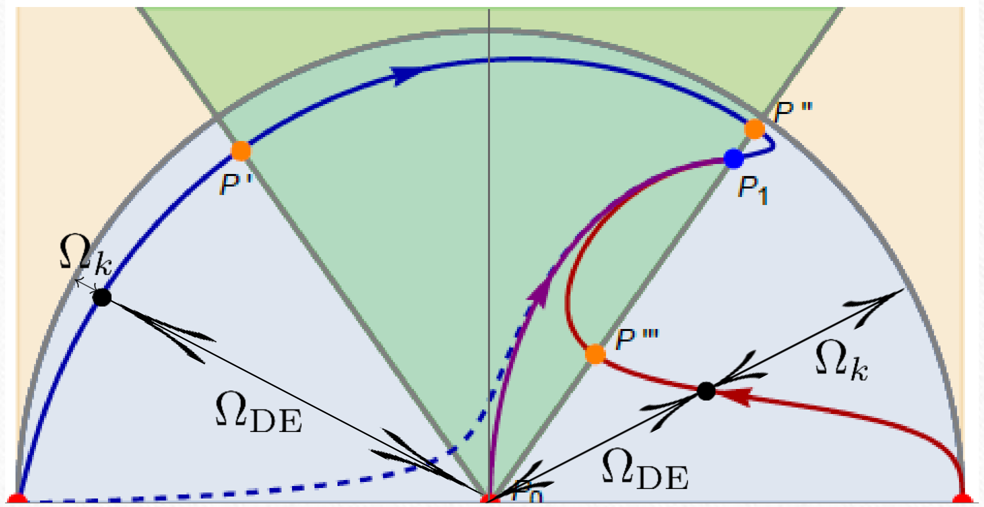

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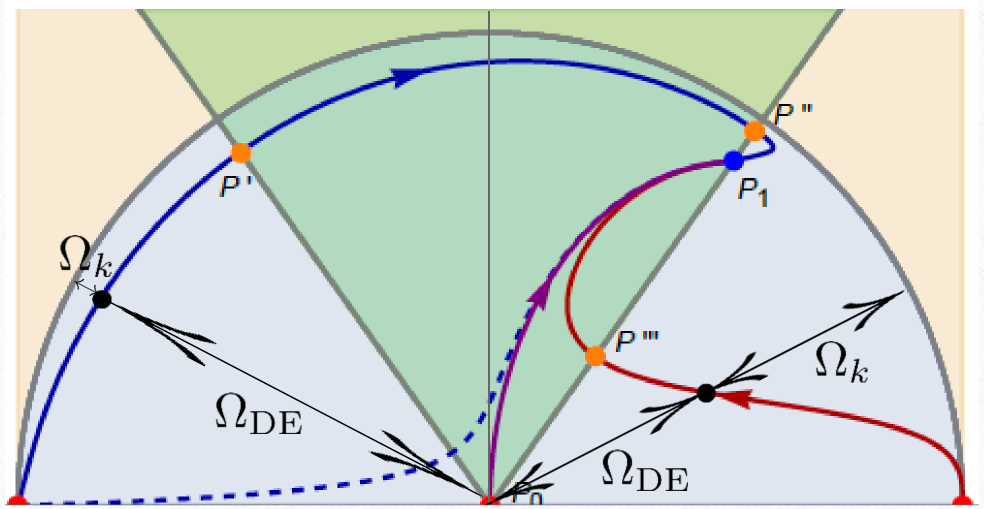

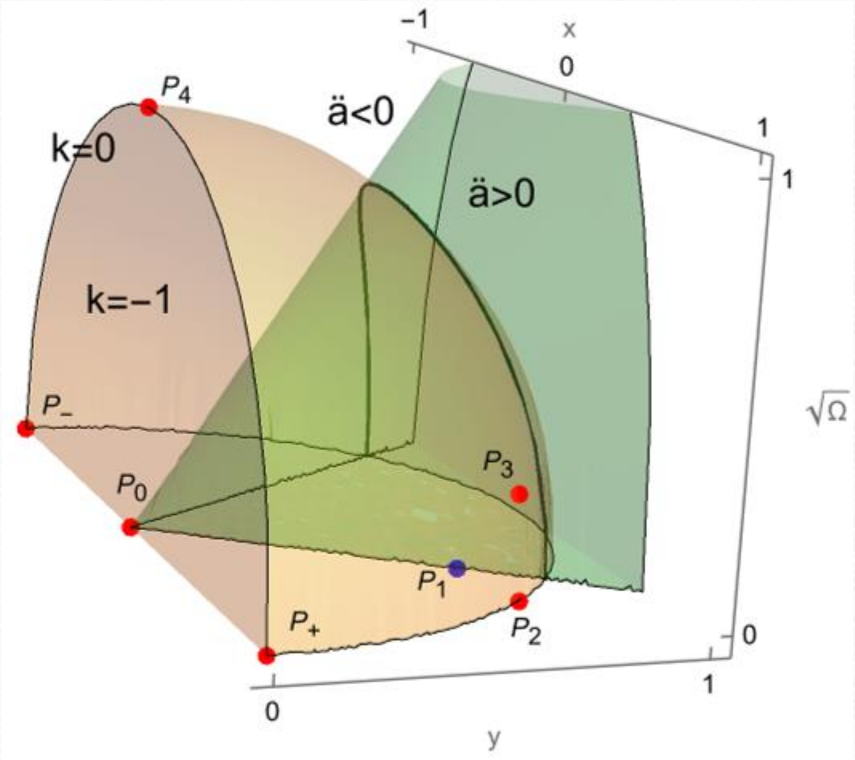

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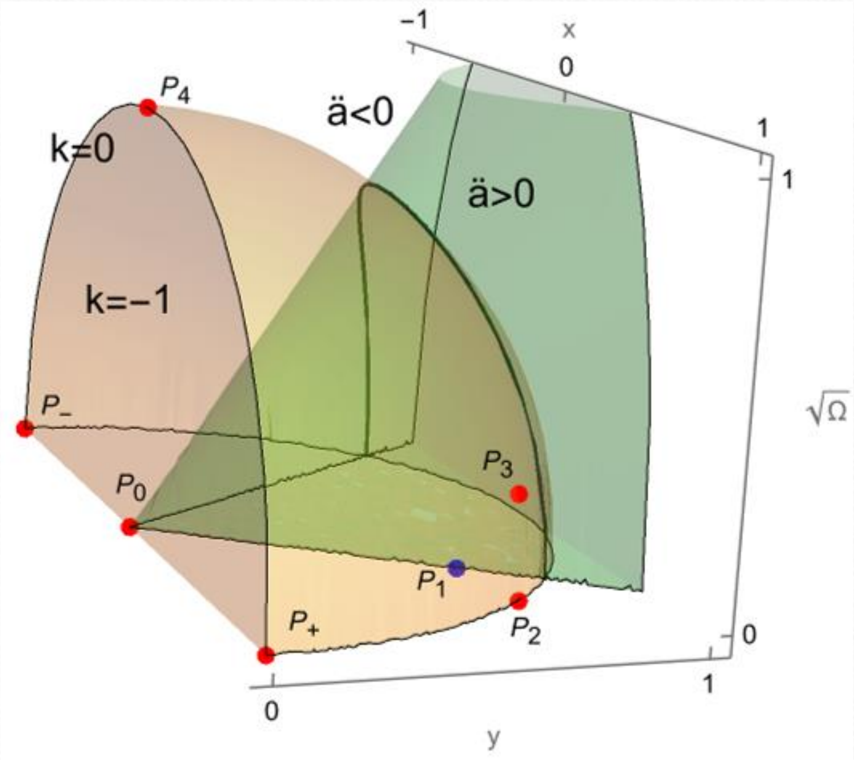
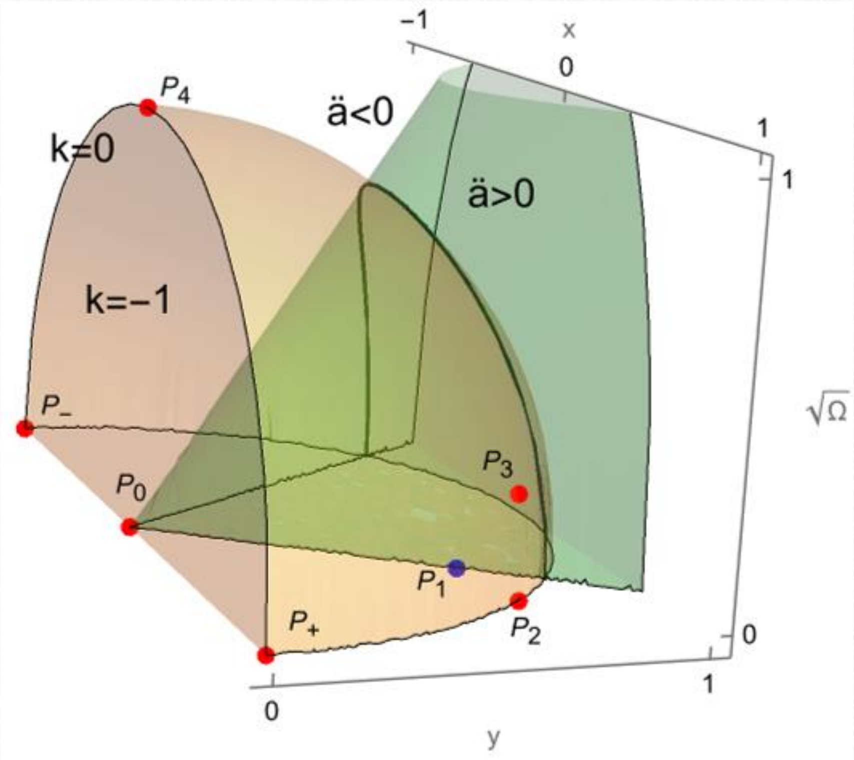
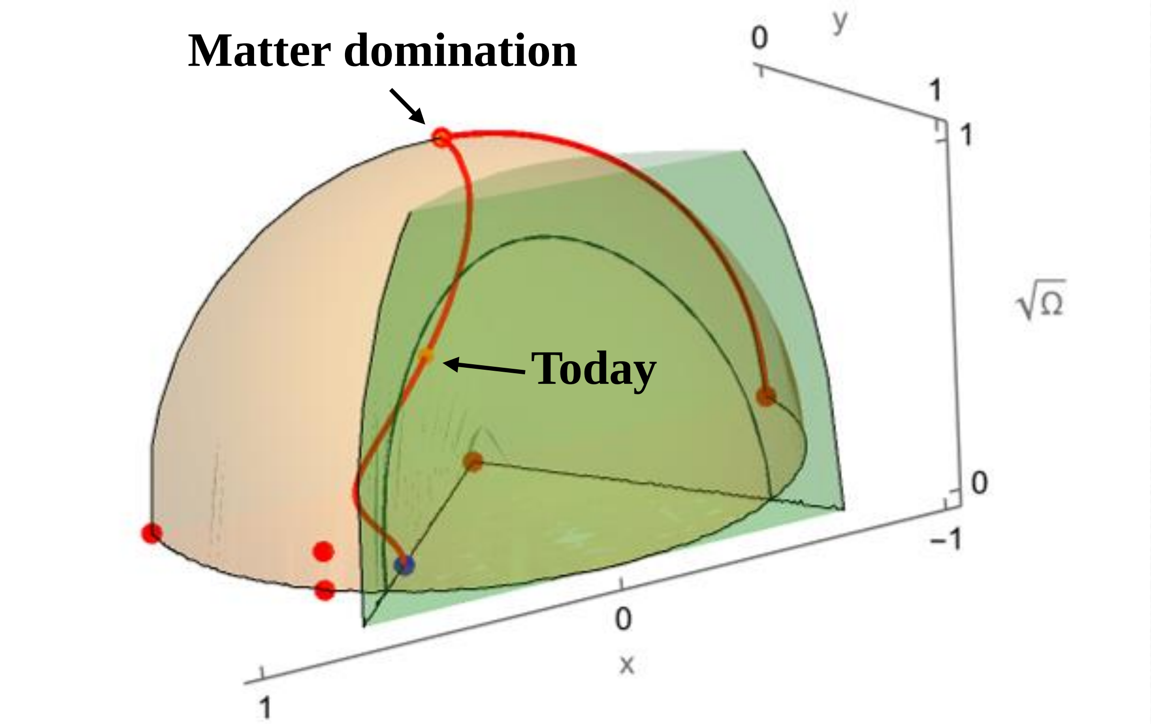


k=0
k=−1
ä<0
ä>0
P_4
P_-
P_0
P_3
P_1
P_2
P_+
x
−1
0
1
y
0
1
$\sqrt{\Omega}$
0
1



Cosmological solutions in phase space: example: $\lambda \approx \sqrt{3}$

``Today'': $\Omega_{\rm DE} \approx 61\%,\ \Omega_m \approx 34\%,\ \Omega_k \approx 5\%$



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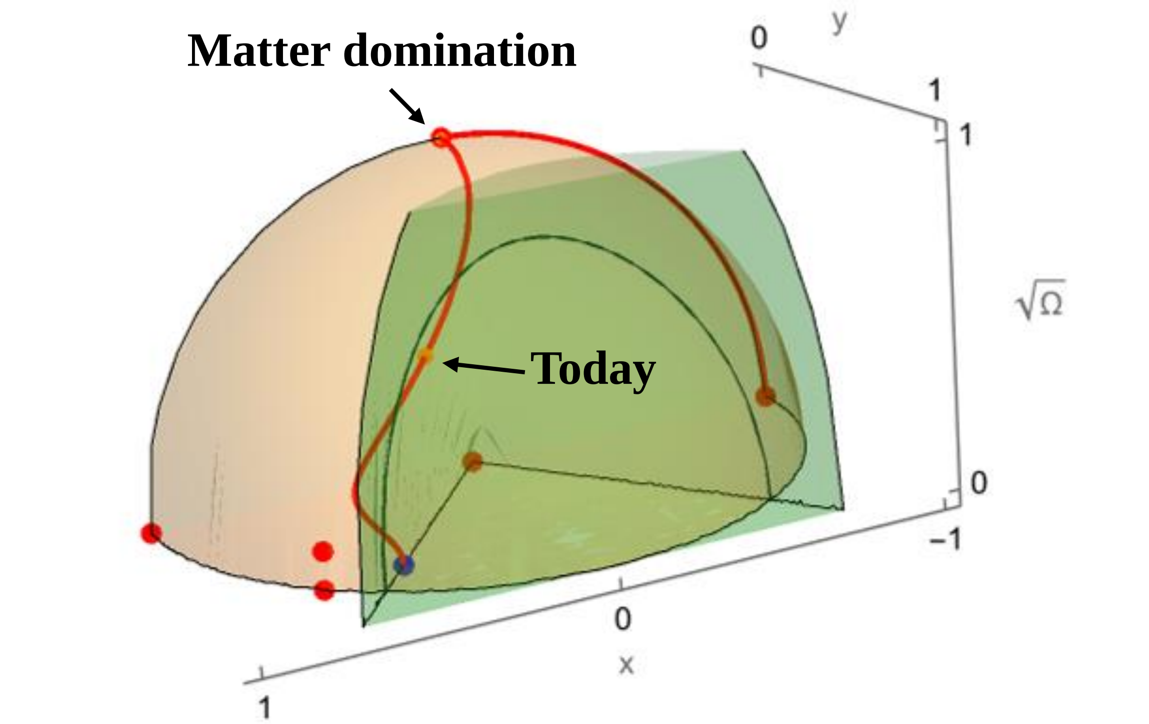

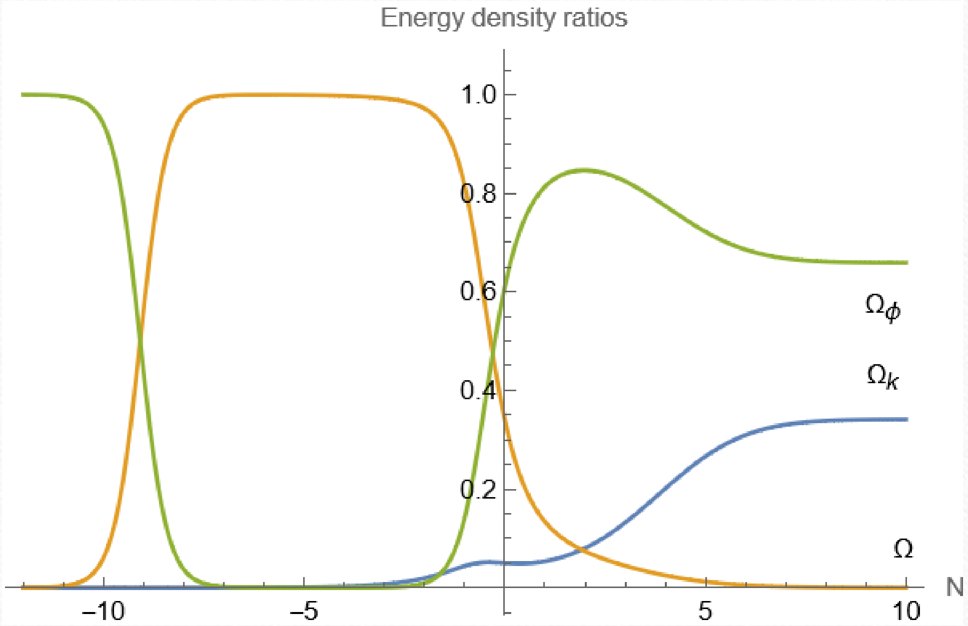

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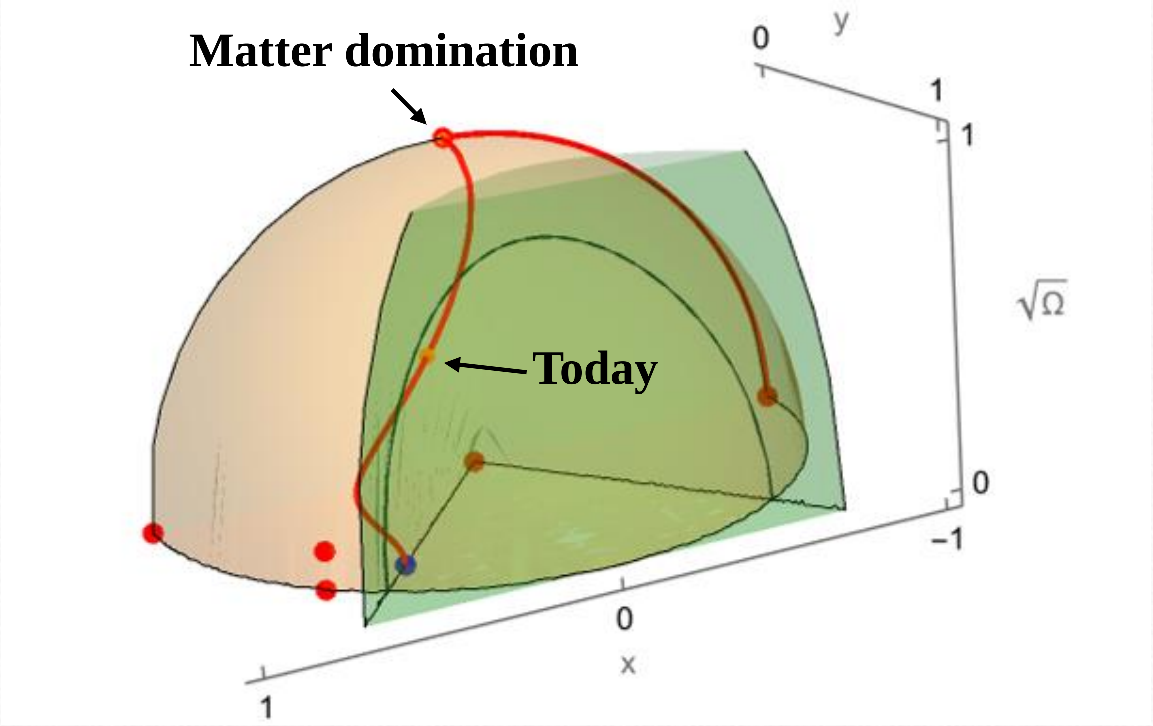

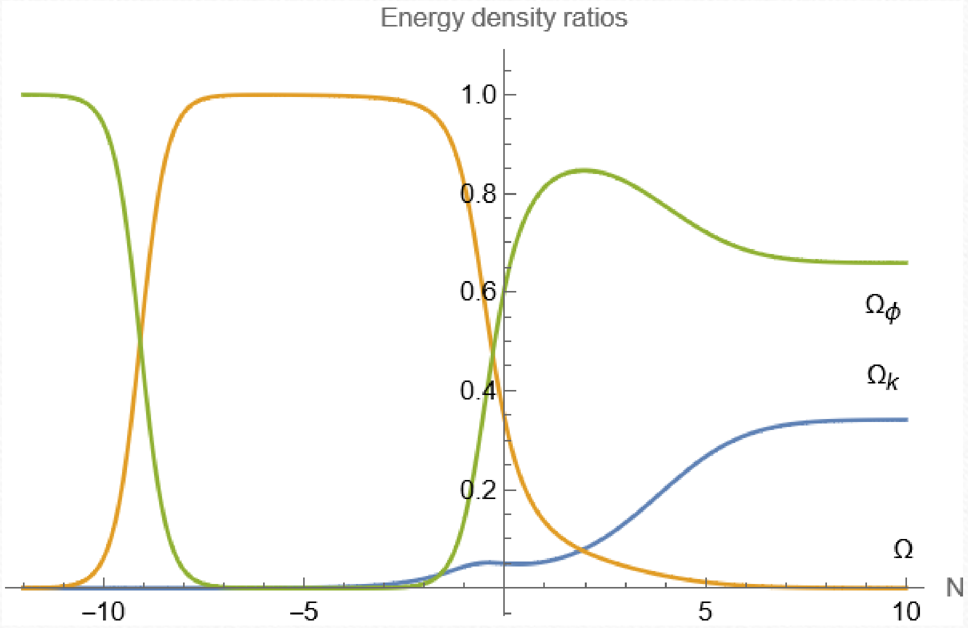

→ Transient acceleration phase

→ **Realistic** cosmological solution? Impact of having spatial curvature?

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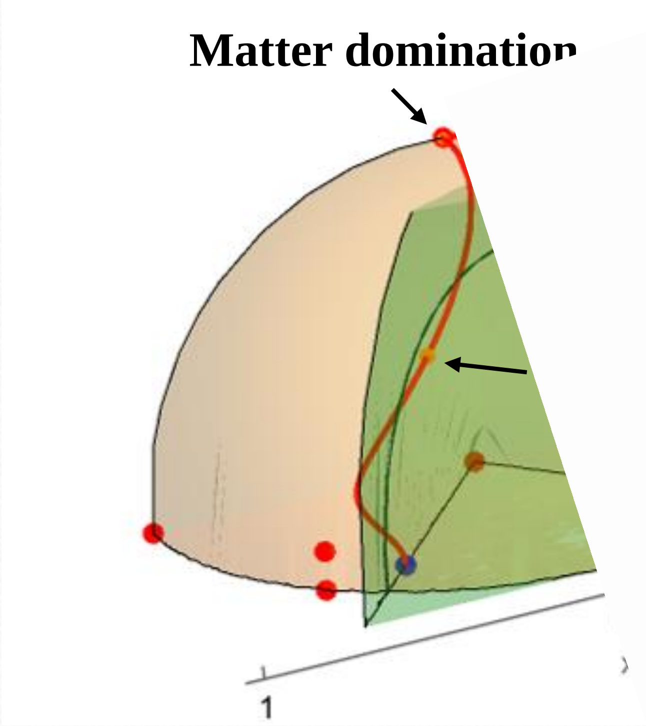

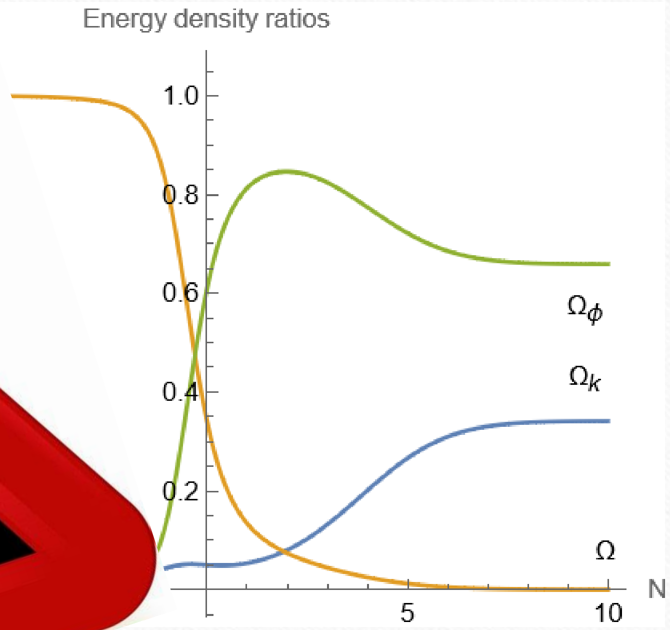

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Summary

- Reproduce **dark energy** as **de Sitter** solution or **quintessence** for large field/**asymptotics**

- Do this from **string theory** ⟶ obstruction by strong dS conjecture: $\lambda \geq \sqrt{2}$

  ⟶ no dS solution

  ⟶ quintessence: no asymptotic acc. + obs. constraints

  ⟶ Here: ways out

- Reproduce **dark energy** as **de Sitter** solution or **quintessence** for large field/**asymptotics**

- Do this from **string theory** ⟶ obstruction by strong dS conjecture: $\lambda \geq \sqrt{2}$

  ⟶ no dS solution

  ⟶ quintessence: no asymptotic acc. + obs. constraints

  ⟶ Here: ways out

- **De Sitter** solution of 10d IIB supergravity on 6d solvmanifold

  $g_s < 1\ ,\ r_1, r_2, r_4, r_5 > l_s\ ,\ r_3, r_6 < l_s$ ⟶ **ambiguous classicality**

  scaling freedom in ansatz: parametrically large 4 radii, volume ⟶ **asymptotic dS ?**

  ⟶ **parametric control on classicality/corrections?**

- **Quintessence**: open universe ($k = -1$) ⟶ **asymptotic acceleration** with $\lambda > \sqrt{2}$

  ⟶ include **matter** to have realistic solutions with $\lambda > \sqrt{2}$