New Insights on Black Hole Evaporation and the Dark Matter Fraction Composed of Primordial Black Holes

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OUTLINE

1.-Hidden agenda

.primordial black holes as dark matter

.the dark dimension: a quick glance

2.- Hawking's semiclassical approximation

.Schwarzschild black holes

.Near extremal black holes

3.- Quantum effects ☛ Memory burdem

4.- Summary:

Limits on all-dark-matters interpretation of 5D primordial black holes

Primordial black holes as dark matter

Speculation BH could be formed from collapse of large amplitude fluctuations in early universe

Zeldovich 1967; Hawking 1971

What's M_{BH} ?

- Scaling of cosmological energy density with time ☛ $\rho \sim M_p^2/t^2$
- Required density for M_{BH} to collapse within its Schwarzschild radius ☛ $\rho \sim M_p/M_{BH}^2$

$$\boxed{M_{BH} \sim tM_p^2 \sim 10^{15}\,(t/\;10^{-23}\;s)\;g}$$

PBHs would initially have around the cosmological horizon mass

- $M_{BH} \sim M_p \sim 10^{-5}$ g ☛ formed @ Planck time 10^{-43} s

- $M_{BH} \sim M_\odot \sim 10^{33}$ g ☛ formed @ QCD epoch 10^{-5} s

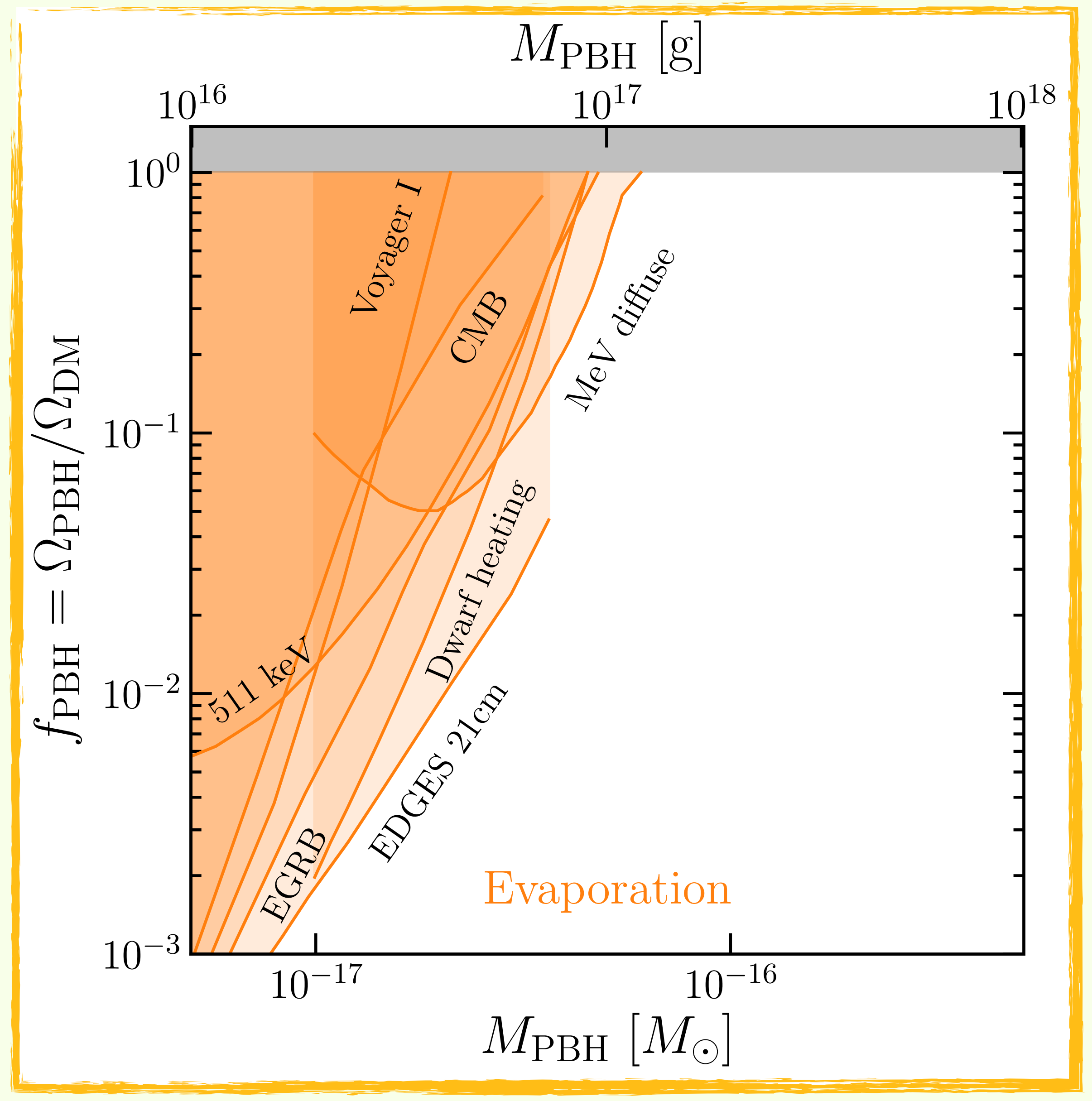
- $M_{BH} \sim 10^5\, M_\odot \sim 10^{38}$ g ☛ formed @ t ~ 1s

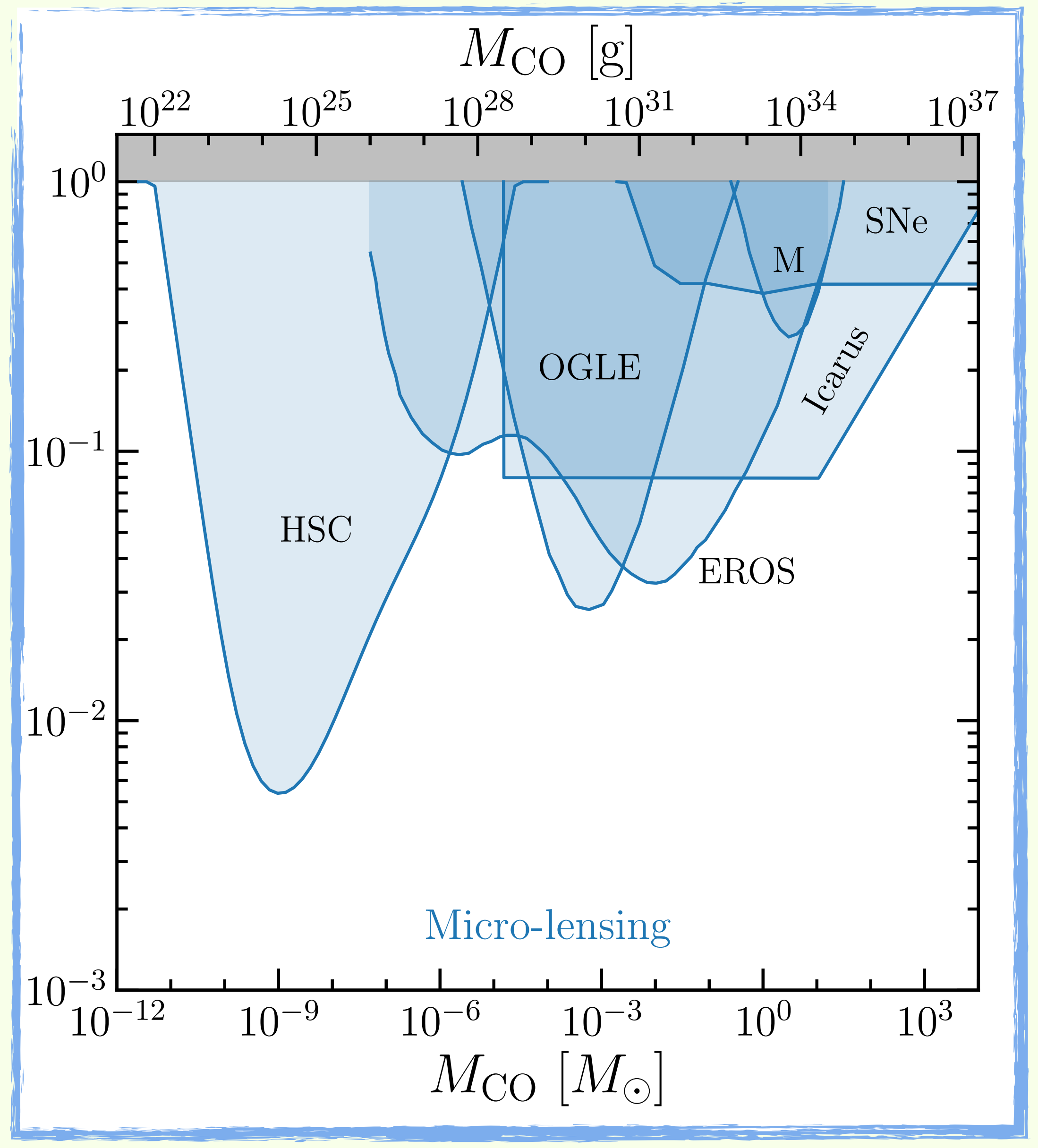
PBHs could span an enormous mass range

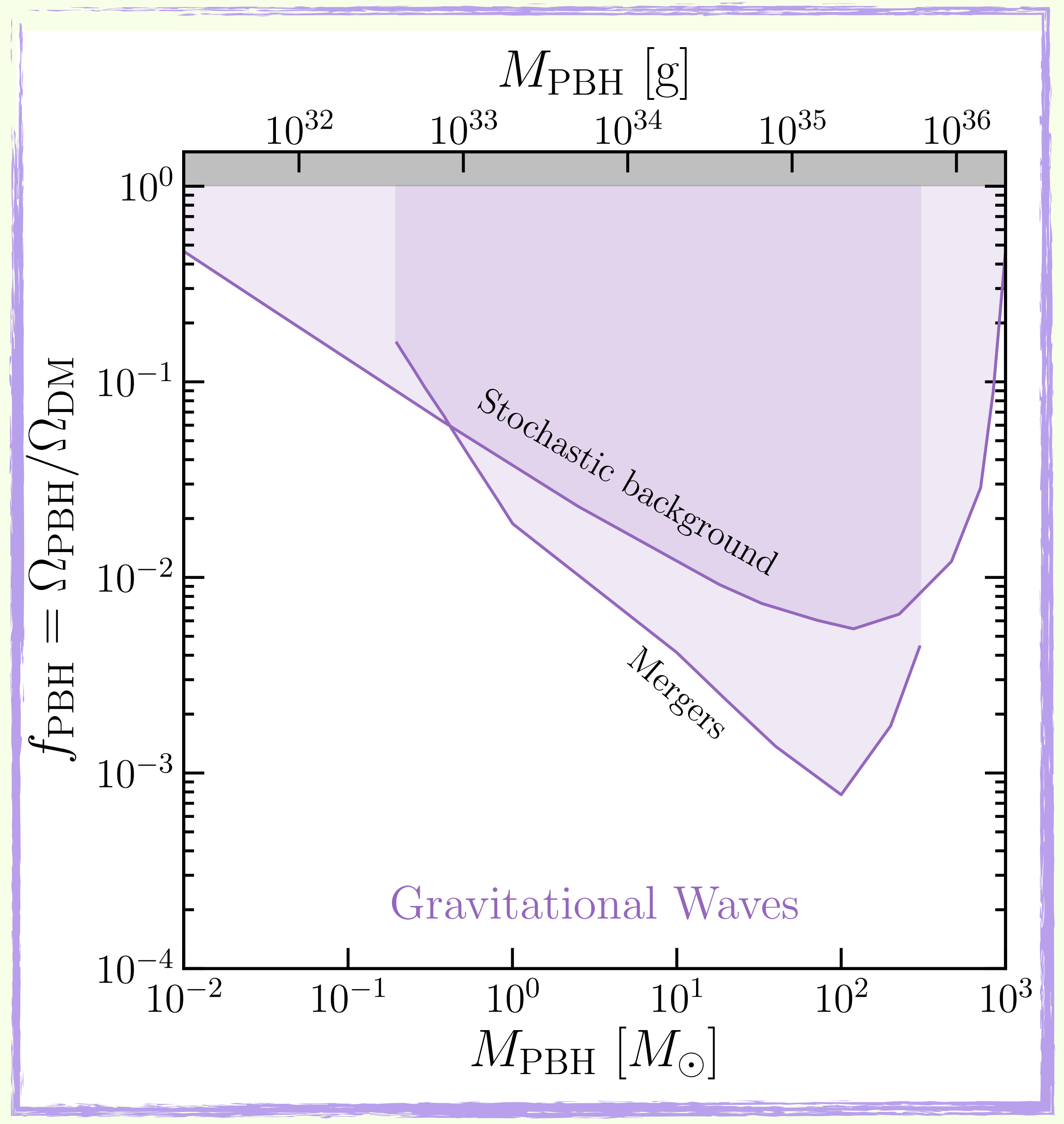
Mass spectrum is yet to be shaped

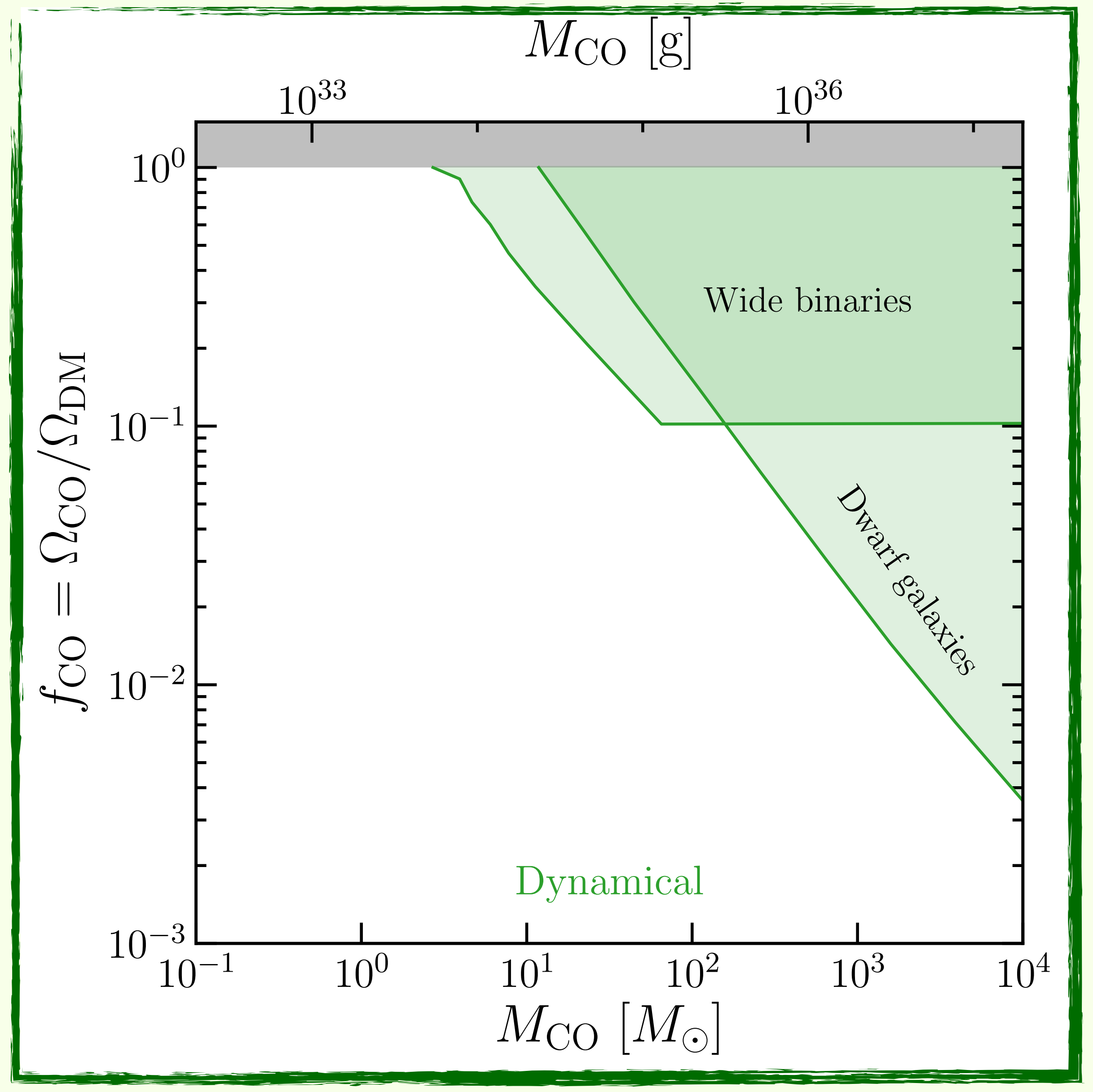
Behave like typical CDM particles on cosmological scales

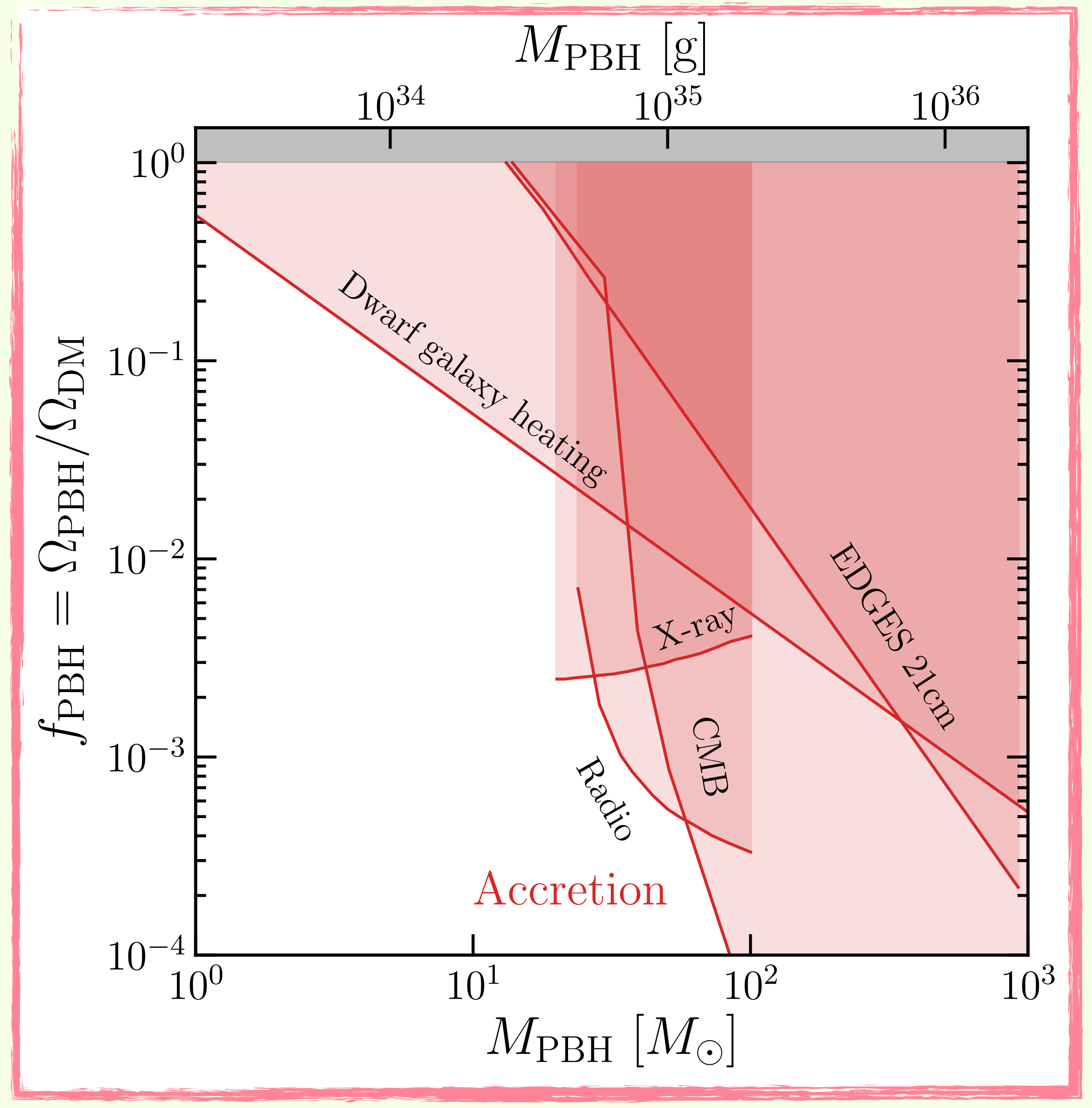
A PBH all-dark-matter interpretation is severely constrained by observations











Green, Kavanagh 2020

Swampland distance conjeture

- Infinite distance limits $\phi \to \infty$ in the field space of massless scalars are accompanied by an infinite tower of exponentially light states $m \sim e^{-\alpha\phi}$ ☛ $\alpha \sim \mathcal{O}(1)$

  distance and masses are measured in Planck units

Ooguri, Vafa 2006

- Kaluza-Klein tower ☛ decompactification of extra dimensions

$$\phi = \ln R \;☛\; m \sim 1/R$$

- Smallness of some physical parameters might signal that we live in asymptotic corner of field space

- Such parameters can be ☛ scales of dark energy and neutrino masses

The Dark Dimension: a quick glance

1.- AdS distance conjecture $\phi = -\ln|\Lambda|$

Lüst, Palti, Vafa 2019

2.- Extension to dS:

- could help elucidate radiative stability of cosmological hierarchy

$$\Lambda \sim 10^{-120}\, M_p^4$$

because it connects size of compact space $R_\perp$ to dark energy scale Λ

$$R_\perp \sim \lambda\ \Lambda^{-\alpha}$$

- Null results on deviations from Newton's law ☛ $R < 30\ \mu m \Rightarrow \alpha = 1/4$

- Astrophysical constraints ☛ dark dimension

$R_\perp \sim 1$ to $10\ \mu m$ $\qquad 10^{-3} \lesssim \lambda \lesssim 10^{-2}$

Montero, Vafa, Valenzuela 2022

- $\alpha = 1/4$ ☛ consistent with string computations

LAA, Antoniadis, Lüst, Lüst 2023

1.- Species scale where gravity becomes strong

$$M_* \sim m^{1/3} M_p^{2/3} \sim 10^9 \text{ GeV}$$

Dvali 2007

2.- Interesting dark matter candidates

- 5D PBHs

LAA, Antoniadis, Lüst, 2022

- KK graviton tower

Gonzalo, Montero, Obied, Vafa 2022

(explicit realization of dynamical dark matter)

Dienes, Thomas 2012

Stunning coincidence

size of the dark dimension ~ wavelength of visible light

- Schwarzschild radius of 5D black holes ☛ well below wavelength of light

- For point-like lenses:

  this is precisely critical length where geometric optics breaks down

  and effects of wave optics suppress magnification,

  obstructing sensitivity to 5D PBH microlensing signals!

Assumptions

1.- Gravity propagates in bulk spacetime while SM fields lie on a brane

2.- Rely on probe brane approximation

- Ensures only effect of brane field is to bind black hole to brane

- Adequate approximation provided $M_{BH} \gg$ brane tension

presumably of order of but smaller than M_*

3.- Black hole can be treated as flat d-dimensional object

- Assumption valid for ☛ $r_s \sim (M_{BH} / M_*)^{1/(d-3)} M_*^{-1} \ll R_\perp$

Tangherlini 1963; Myers, Perry 1986

Schwarzschild black holes

$$ds^2 = -f(r)\,dt^2 + f^{-1}(r)\,dr^2 + d\Omega_{d-2}^2$$

- $f(r) = 1 - r_s/r$

- $d\Omega_{d-2}$ ☛ interval of spherical solid angle in dimension d-2

Hawking's semiclassical approximation

BH emits thermal radiation as it were a blackbody

- Temperature $\boxed{T_H \sim 1/r_s \sim M_* (M_{BH}/M_*)^{-1/(d-3)}}$
- Entropy $\boxed{S_{BH} \sim M_{BH}\, r_s \sim (M_{BH}/M_*)^{(d-2)/(d-3)}}$
- Particle emission rate $\boxed{\Gamma_H \sim T_H \sim M_* (M_{BH}/M_*)^{-1/(d-3)}}$
- Hawking's lifetime $\boxed{\tau_H \sim S_{BH}\, r_s \sim (M_{BH}/M_*)^{(d-1)/(d-3)}}$

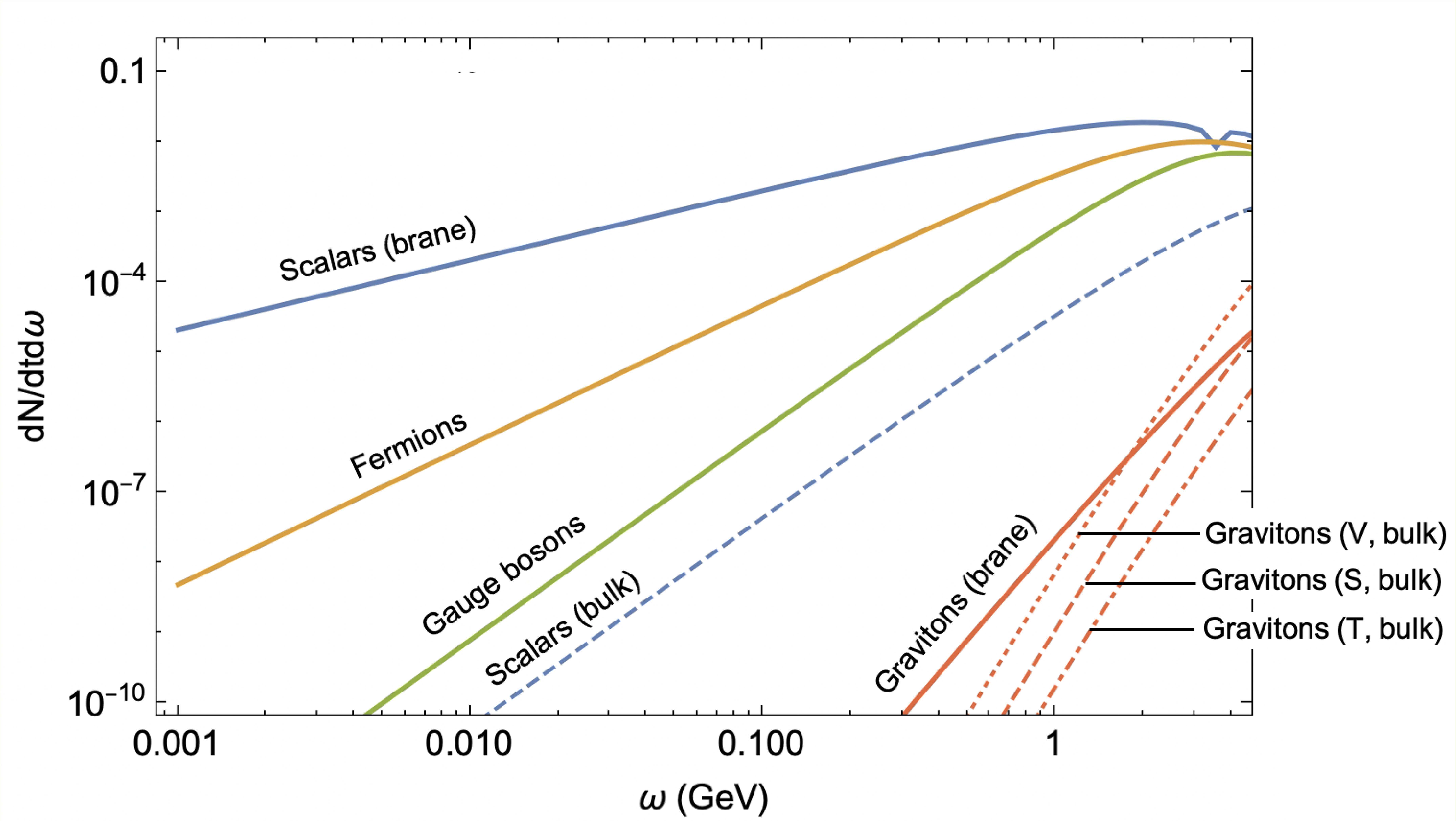
Black holes radiate mainly on the brane

KK modes are excitations in full transverse space and so their overlap with small (higher dimensional) black holes is suppressed by geometric factor $(r_s/R_\perp)^{d-4}$ relative to brane fields

This geometric suppression precisely compensates for the enormous number of modes and total KK contribution is only of same order as that from single brane field

Emparan, Horowitz, Myers 2000

Recent numerical analysis



Ireland, Profumo, Scharnhorst 2023

- Emission rate per degree of particle freedom i of particles of spin s with initial total energy between (Q, Q + dQ)

$$\frac{d\dot{N}_i}{dQ} = \frac{\sigma_s}{8\,\pi^2}\, Q^2 \left[\exp\left(\frac{Q}{T_{\rm H}}\right) - (-1)^{2s}\right]^{-1}$$

- Average total emission rate for particle species i is then

$$\Gamma_i \equiv \dot{N}_i = f\,\frac{\Gamma_s}{32\,\pi^3}\,\frac{(d-1)^{(d-1)/(d-3)}\ (d-3)}{2^{2/(d-3)}}\,\Gamma(3)\,\zeta(3)\,T_{\rm H}$$

- greybody factor taken as dimensionless contant $\Gamma_s = \sigma_s / A_4$

normalized to the horizon surface area $A_4 = 4\pi \left(\frac{d-1}{2}\right)^{2/(d-3)} \frac{d-1}{d-3}\, r_s^2$

seen by the SM fields $\Gamma_{s=0} = 1$, $\Gamma_{s=1/2} \approx 2/3$, $\Gamma_{s=1} \approx 1/4$

- $f = 1$ ($f = 3/4$) for bosons (fermions)

LAA, Goldberg 2002

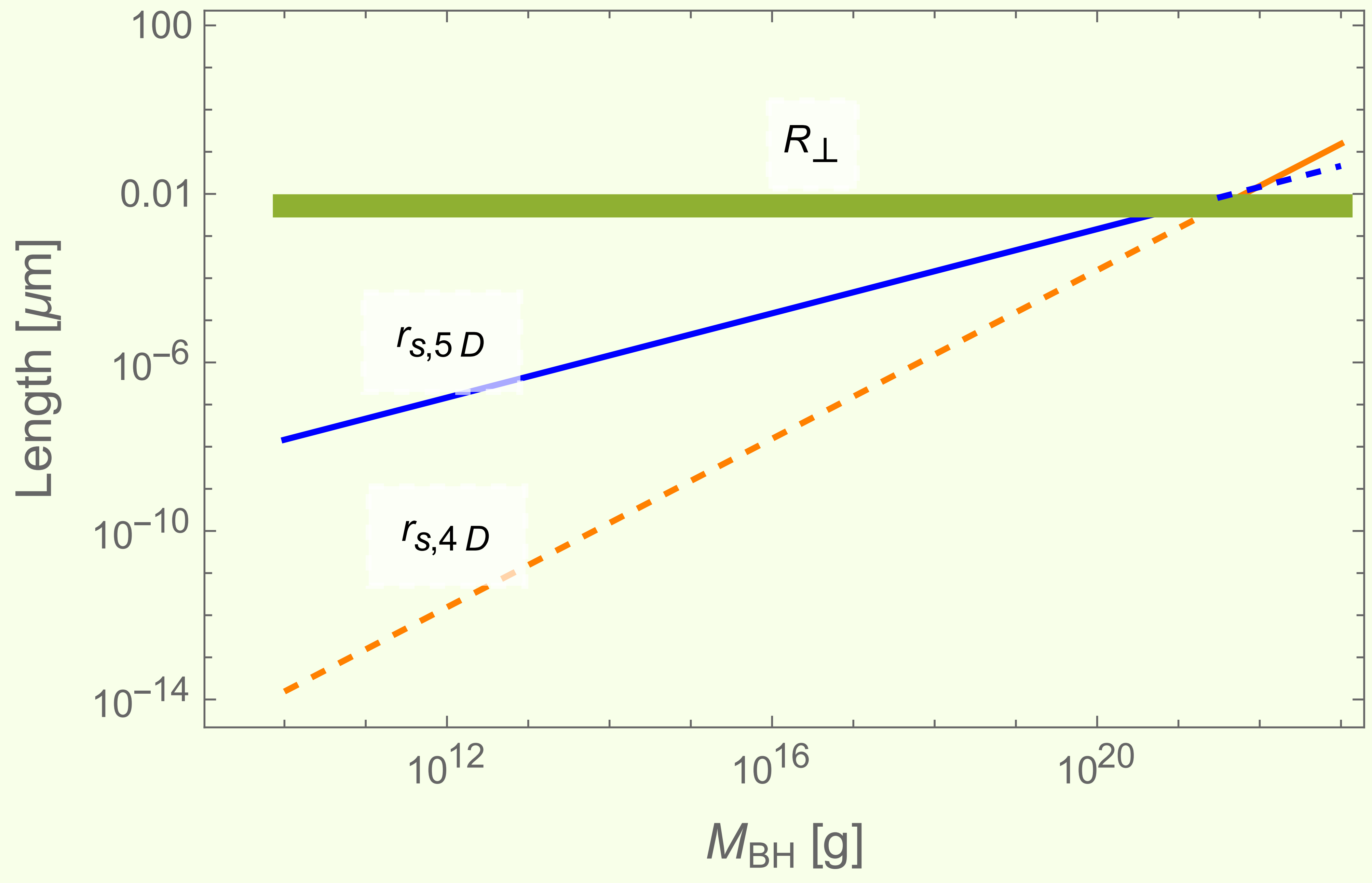
4D versus 5D

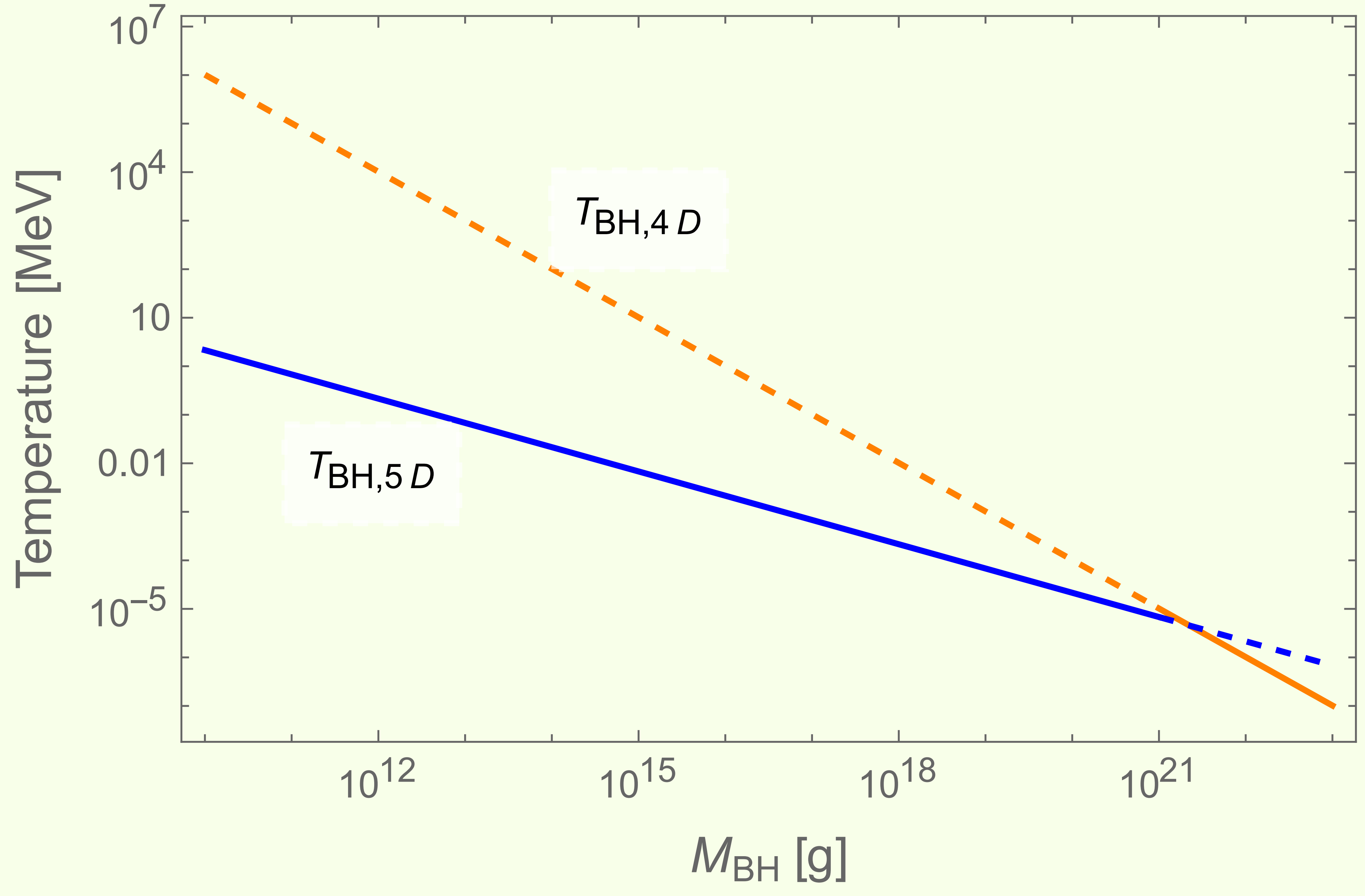
$T_H \sim (M_{BH}/10^{16}\ g)^{-1}$ MeV

$T_H \sim (M_{BH}/10^{12}\ g)^{-1/2}$ MeV

$\boxed{\tau_H \sim 13.8\ (M_{BH}/10^{14.7}\ g)^{3}\ \text{Gyr}}$

$\boxed{\tau_H \sim 13.8\ (M_{BH}/10^{12}\ g)^{2}\ \text{Gyr}}$





LAA, Antoniadis, Lüst 2022

PBH all-dark-matter interpretation

- Difference in number density

$$n_{PBH}^{4D}(T_H) < n_{PBH}^{5D}(T_H)$$

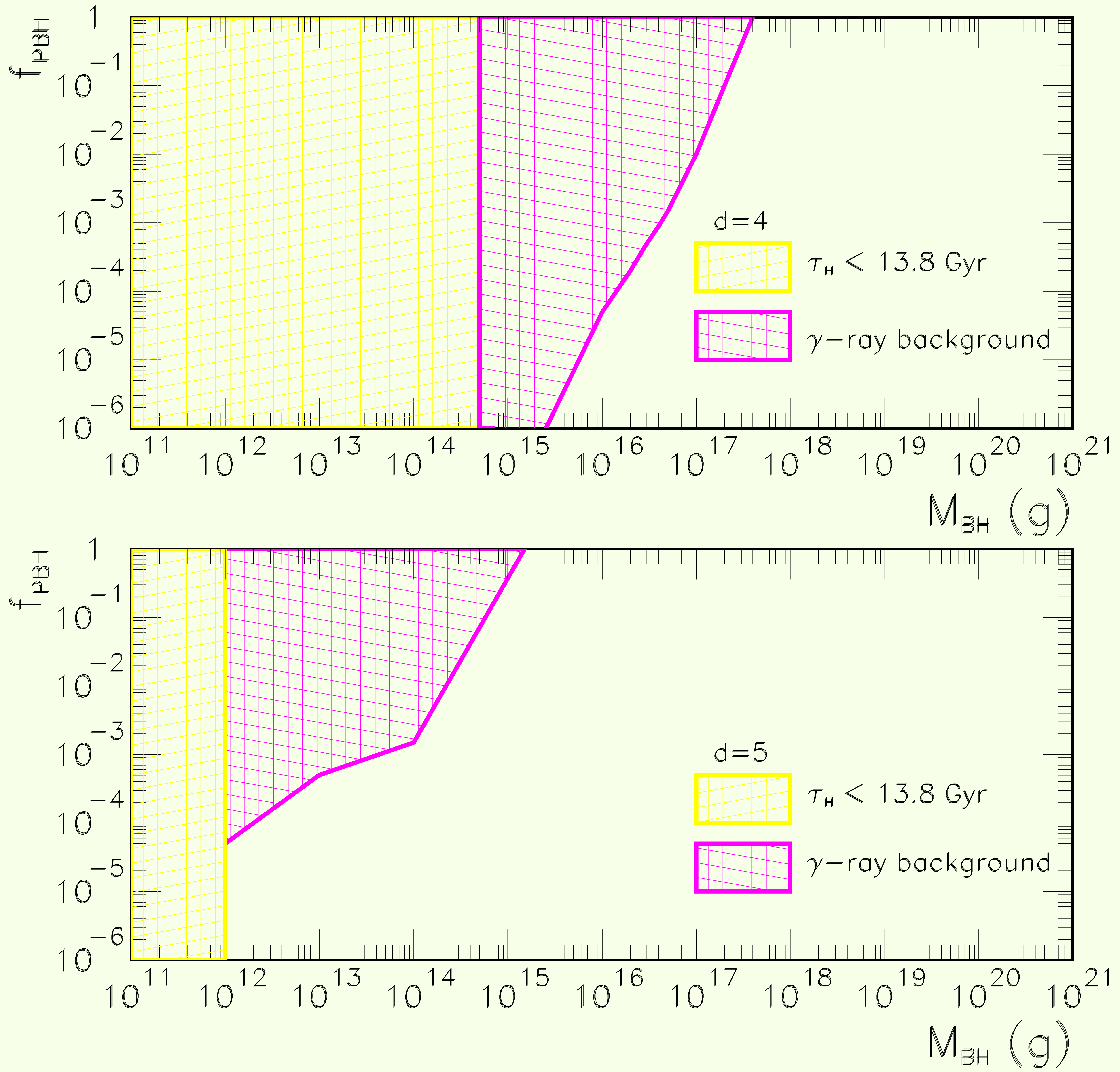
- Compensated by black hole mass

$$M_{BH}^{4D}(T_H) > M_{BH}^{5D}(T_H)$$

- Same PBH density

$$\rho_{PBH}^{4D}(T_H) \sim \rho_{PBH}^{5D}(T_H)$$

Limits on f_{PBH}



$$10^{15} < M_{BH}/g < 10^{21}$$

LAA, Antoniadis, Lüst to appear

Reissner-Nordström black holes

$$ds^2 = u(r)\,dt^2 - u^{-1}(r)\,dr^2 - d\Omega^2_{d-2}$$

unit (d−2) sphere

$$u(r) = 1 - 2M_{BH}/\left(M_*^{d-2}\, r^{d-3}\right) + Q^2/\left(M_*^{d-2}\, r^{2d-6}\right)$$

$$c = \left(M^2_{BH} - Q^2 M_*^{d-2}\right)^{1/2}$$

$M^2_{BH} - Q^2 M_*^{d-2} > 0$ ☛ two horizons

$M^2_{BH} - Q^2 M_*^{d-2} = 0$ ☛ extremal black hole

$M^2_{BH} - Q^2 M_*^{d-2} < 0$ ☛ naked singularity

Chamblin, Emparan, Johnson, Myers 1999

Near-extremal black holes

- For $Q\, M_*^{(d-2)/2} / M_{BH} \ll 1$ ☛ $c \sim M_{BH}$

  leading to non-extremal relation between $c \sim M_*\, S_{BH}^{(d-3)/(d-2)}$

- For near-extremal case ☛ $M_{BH} \sim Q\, M_*^{(d-2)/2}$

  expanding square root it follows that leading term cancels and sub-leading term gives

$$\boxed{c \sim M_*\, \beta^{1/2}\, S_{BH}^{(d/2-2)/(d-2)}}$$

β ☛ order-one parameter controling difference between M_{BH} and Q

Cribiori, Dierigl, Gnecchi, Lüst, Scalisi 2022 ; Basile, Cribiori, Lüst, Montella 2024

- Near-extremal temperature

$$T_{ne} = T_H \, (\beta / S_{BH})^{1/2}$$

- Particle emission rate

$$\Gamma_{ne} \sim T_{ne} = (\beta / S_{BH})^{1/2} \, \Gamma_H$$

Evaporation rate of near-extremal black holes

would be suppressed by a factor of $(\beta / S_{BH})^{1/2}$

with respect to Schwarzschild black holes of same mass

LAA, Antoniadis, Lüst 2024

- For 10^5 g ☛ $T_H \sim 4$ GeV **BUT** $T_{ne} \sim 10^{-5}\ \beta^{1/2}$ eV

- For 10^5 g ☛ $T_H \sim 4 \times 10^{-5}$ yr

- For near-extremal black hole of the same mass ☛ $T_{ne} \sim 20 / \beta^{1/2}$ Gyr

- Order of magnitude estimate:

  If there were 5D primordial neBH in nature

  a PBH all-DM interpretation would be possible in mass range

$$10^5 < M_{BH} / g < 10^{21}$$

- $\hat{C} = c / M_{BH} = (\beta / S)^{1/2}$

☟

quantifies near-extremality is very small because of large entropy

LAA, Antoniadis, Lüst 2024

Self-similarity

1.- Semiclassical approximation relies on assumption of self-similarity: black hole gradually shrinks in size while maintaining the standard semi-classical relations between its parameters: r_s, M_{BH}, T_H

2.- Would semiclassical approximation hold throughout entirety of the black hole lifetime?

3.- Assuming self-similarity ☛ $S_{BH} \sim S_{emission}$ at τ_{Page}

$$M_{BH} \to M_{BH}/2 \qquad r_s \to r_s/2 \qquad S_{BH} \to S_{BH}/4$$

for d=4

Page 2013

Quantum decay rate: effect of memory burden

1.- Self-similarity implies that at each stage radiation is thermal and information is maintained internally

2.- However ☛ remaining black hole has only 1/4 of its initial entropy and so much less information storage capacity

3.- Analysis of prototype models shows ☛ after $\tau_{\rm Page}$ system gets effectively stabilized and gains much longer lifetime formula

4.- Assume evaporation is slowed down by further n powers of entropy S_{BH}

$$\tau_H^{\{n\}} \sim S_{BH}^{1+n}\, r_s$$

Dvali 2018; Dvali, Eisemann, Michel, Zell 2020; Alexandre, Dvali, Koutsangelas 2024

- Hawking decay rate of Schwarzschild black holes

$$\Gamma_H^{\{0\}} \sim T_H \sim M_* S_{BH}^{-1/(d-2)} \sim M_* (M_{BH}/M_*)^{-1/(d-3)}$$

- Compared to Hawking decay rate ☛ quantum decay rate has additional $1/S_{BH}^n$ suppresion

$$\Gamma_H^{\{n\}} \sim T_H / S_{BH}^n \sim M_* S_{BH}^{(-1 - nd + 2n)/(d-3)} \sim M_* (M_{BH} / M_*)^{(-1 - nd + 2n)/(d-3)}$$

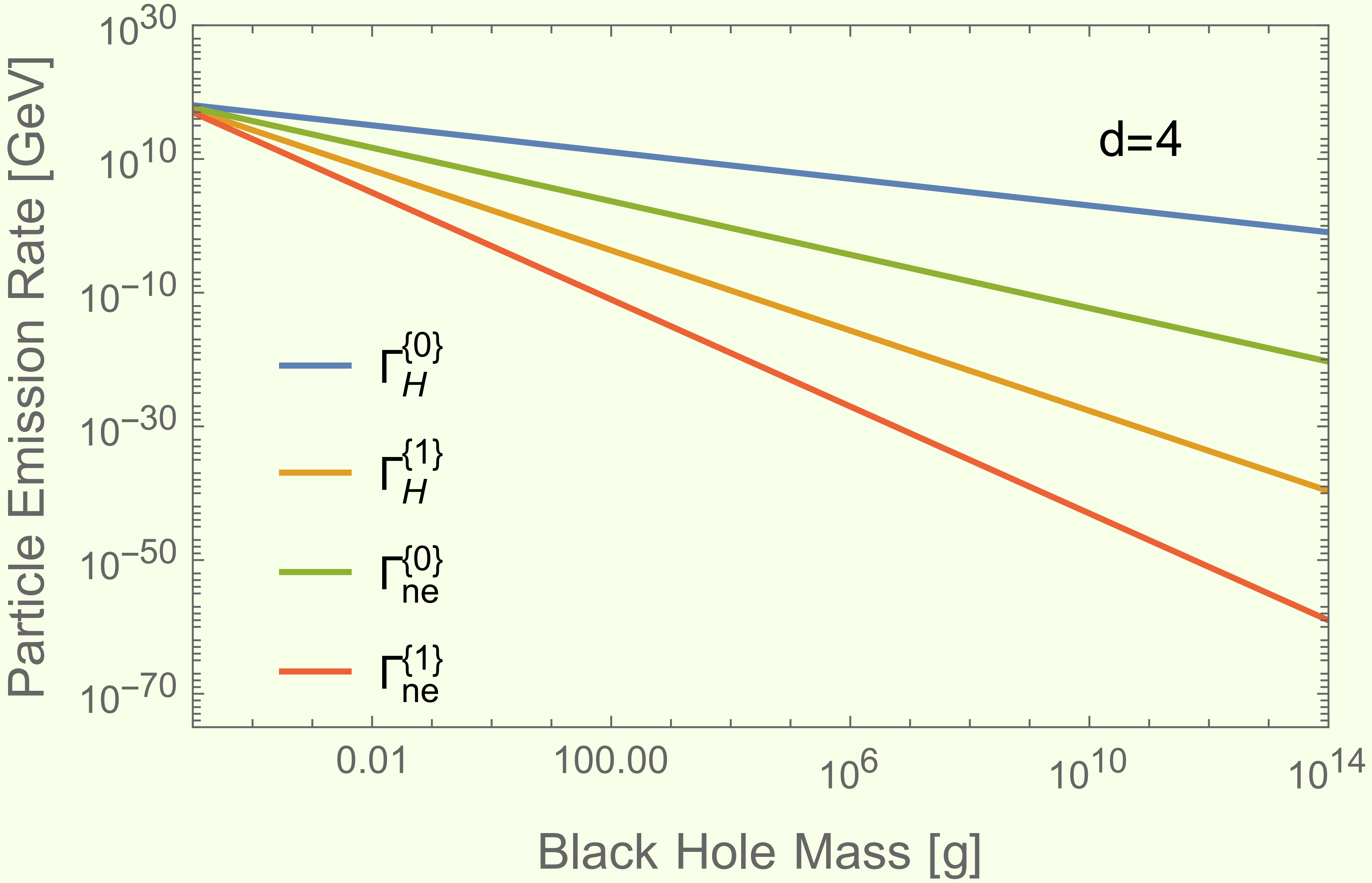
- Compared to evaporation of Schwarzschild black holes ☛ near-extremal black holes have additional $(\beta / S_{BH})^{1/2}$ suppression

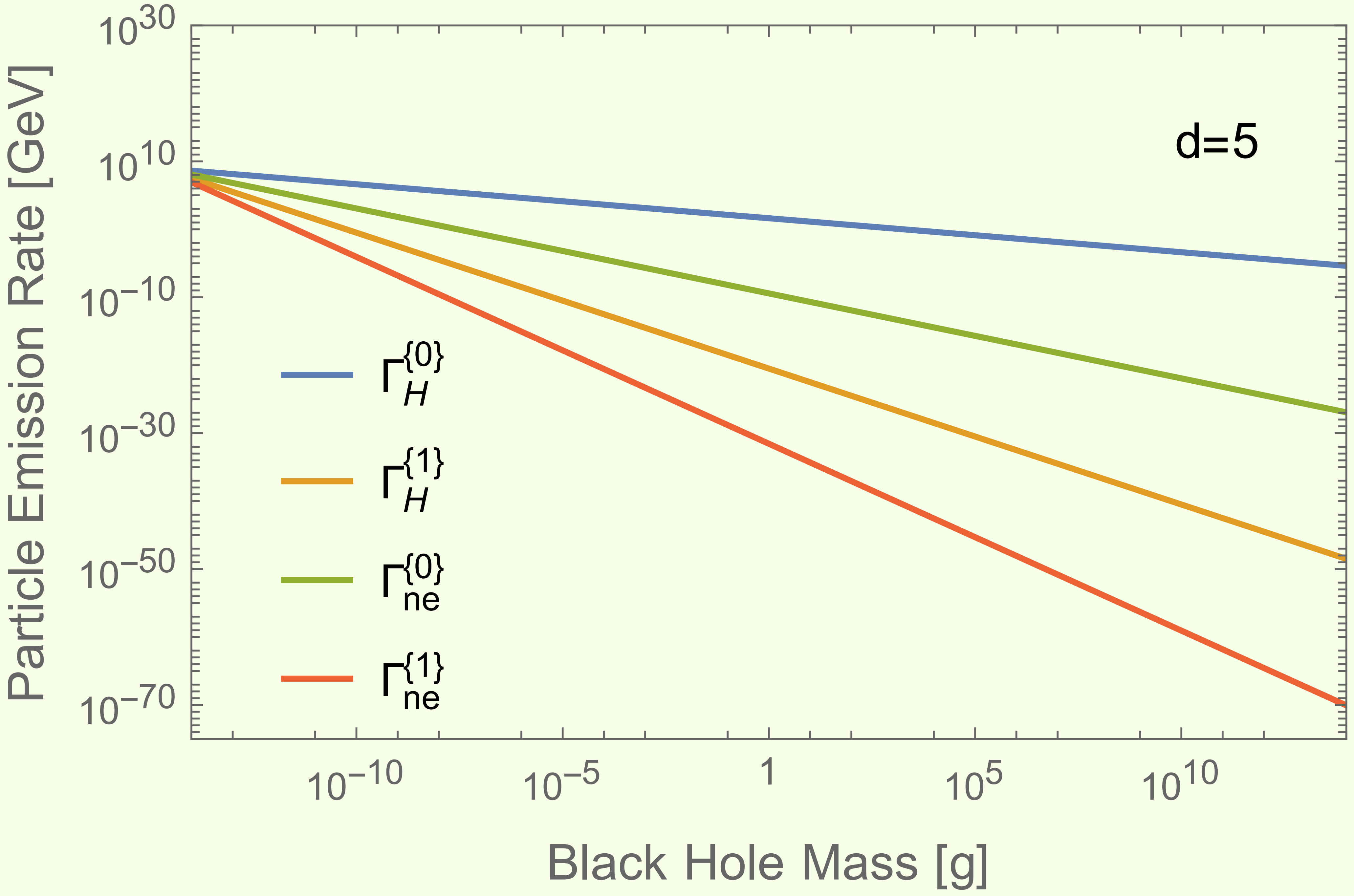
$$\Gamma_{ne}^{\{0\}} \sim \beta^{1/2} T_H / S_{BH}^{1/2} \sim \beta^{1/2} M_* S_{BH}^{-d/(2d-4)} \sim \beta^{1/2} M_* (M_{BH} / M_*)^{-d/(2d-6)}$$

- Quantum decay rate of near-extremal black holes

$$\Gamma_{ne}^{\{n\}} \sim \beta^{1/2} T_H / S_{BH}^{n+1/2} \sim \beta^{1/2} M_* S_{BH}^{(-d/2 - nd + 2n)(d-2)} \sim \beta^{1/2} M_* (M_{BH}/M_*)^{(-d/2 - nd + 2n)(d-3)}$$

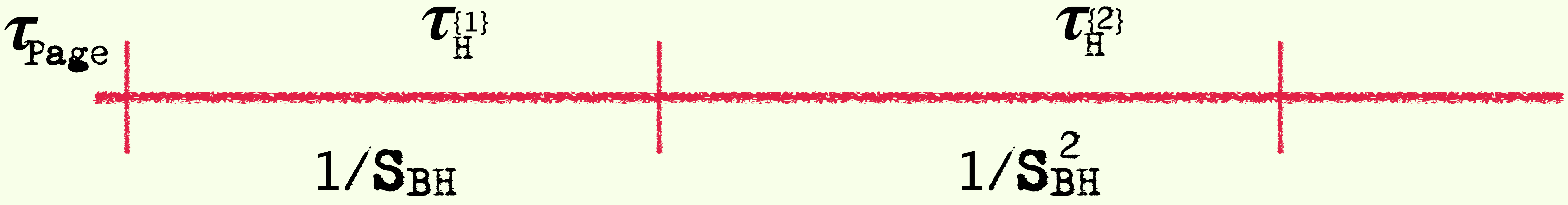
4D versus 5D





LAA, Antoniadis, Lüst to appear

How would memory burden impact the mass range of PBH-DM interpretation?



Assuming only two phases in evaporation process

$\tau_{\mathrm{Page}} \sim 180\ (M_{BH}\ /\ 10^4\ \mathrm{g})^2\ \mathrm{s}$ $\qquad$ $\tau_H^{\{1\}} \sim 4 \times 10^{17}\ (M_{BH}\ /\ 5\ \mathrm{g})^2\ \mathrm{s}$

LAA, Antoniadis, Lüst to appear

~~Conclusions~~

1.- 5D black holes are bigger, colder, and longer-lived than usual 4D black holes of same mass

2.- Adopting Hawking's semiclassical approximation ☛ PBH all dark matter interpretation would be possible if

- Schwarzschild black holes $10^{15} < M_{BH}/g < 10^{21}$
- Near extremal black holes $10^{5} < M_{BH}/g < 10^{21}$

3. Memory Burden ☛ stay tuned


That's all Folks!

Extra slides

4D versus 5D

$$\tau_H \sim 14 \left(\frac{M_{\rm BH}}{10^{14.5}\ \rm g}\right)^3 \left(\frac{4}{\sum_i c_i(T_H)\ \tilde{f}\ \Gamma_s}\right) \quad \rm Gyr$$

$$\tau_H \sim 14 \left(\frac{M_{\rm BH}}{10^{12}\ \rm g}\right)^2 \left(\frac{4}{\sum_i c_i(T_H)\ \tilde{f}\ \Gamma_s}\right) \quad \rm Gyr$$

- $c_i\ (T_H)$ ☛ counts number of internal degrees of freedom of particle species i of mass m_i satisfying $m_i \ll T_H$
- $\tilde{f} = 1$ ($\tilde{f} = 7/8$) for bosons (fermions)

LAA, Antoniadis, Lüst 2022